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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

THE ELIMINATION OF THERMOMETRIC LAG AND ACCIDENTAL LOSS OF HEAT IN CALORIMETRY.

By Theodore William Richards, Lawrence J. Henderson, and George S. Forbes.

1. *TWO NEW METHODS OF AVOIDING ERROR IN CALORIMETRY.* — By Theodore William Richards.
2. *THE CONSTANCY OF RESULTS OBTAINED BY ONE OF THE NEW METHODS.* — By Theodore W. Richards and George S. Forbes.
3. *THE COMPARISON OF THE TWO METHODS, AND THE EXACT ESTIMATION OF ADIABATIC RISE OF TEMPERATURE.* — By Theodore W. Richards and Lawrence J. Henderson.

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Presented March 8, 1905. Received March 8, 1905.

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1. TWO NEW METHODS OF AVOIDING ERROR IN CALORIMETRY.

By Theodore W. Richards.

It is very well known that the error most difficult to overcome in calorimetric work is the accidental exchange of heat with the environment by radiation, convection, and conduction. Very few calorimetric results are wholly free from error due to this cause, even comparative methods, like that of Pfaundler,* being dependent upon the assumption that the loss of heat in two similar sets of apparatus are equal, one to the other. Thermo-chemical results, especially those involving deliberate reactions, are notoriously subject to uncertainty on account of the magnitude of their corrections for cooling. The effort to annihilate such error as this was the guiding thought in a preceding investigation of

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* Sitzungsber. d. Akad. Wiss. Wien, **62**-2, 379 (1870).

Richards and Lamb,* and the present paper is written to elaborate the argument there presented, as well as to suggest new methods of overcoming the difficulty.

An analysis of the sources of uncertainty due to this cooling effect in a reaction evolving heat easily reveals two chief factors: first, the deficiency of the actual maximum temperature in the calorimeter due to loss of heat, and, secondly, the inability of the thermometer to follow with adequate speed the actual temperature. The first of these causes of error receives a more or less satisfactory elimination by the methods of Regnault and Rumford; the second seems never to have been adequately treated.

First, the former question may be briefly discussed. The method of Regnault as formulated by Pfaundler depends upon Newton's law of cooling, which is known to hold with reasonable accuracy for small temperature intervals.† It must be noted, however, that this application of Newton's law is not wholly free from assumption; for the law has been tested only during long-continued cooling, not when the temperature is subject to rapid change. In this latter case it seemed quite possible that the convection currents, to which the cooling is partly due, are not at once started in full activity; and hence that the rate of cooling after a sudden change of temperature is not identical with that during a slow change.

Rumford's method of correction is subject to a somewhat similar possible uncertainty of unknown magnitude, and this method (of starting a reaction as much below the temperature of the room as the reaction finishes above the temperature of the room) is moreover only applicable to reactions which progress evenly throughout their course. The consideration of these circumstances led to a wish to devise a method free from the objections to which these corrections for cooling are open; and this wish was intensified by the consideration of the second of the causes of error already alluded to,— namely, the lag of the thermometer.

Although this latter cause of error is not usually considered in the treatment of calorimetric work, it necessarily enters to a greater or less extent into all results in which the temperature is read on a moving thread.

The rate of cooling of the mercury in the thermometer, and therefore the lag of its reading, varies very greatly with the construction, form, and size of the thermometer, and also with the specific heat, conductivity, and agitation of the surrounding liquid. Therefore it must be determined for each thermometer under the exact conditions employed in each case.

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* These Proceedings, **40**, 657 (1905). † Pogg. Ann., **129**, 113 (1866).

Nevertheless, it may be of interest to record two typical tests of a Beckmann thermometer, such as is usually furnished by Fuess or Goetze, because this type of thermometer is now so much used for calorimetric work.

The lag was determined very simply by plunging the slightly warmed thermometer into a large flask full of water at the temperature of the room, which was itself maintained at a constant point by a thermostat attachment to the steam-heating coil. The change of reading of the thermometer with time was then recorded. During the time of an experiment the bath did not vary in temperature by an appreciable amount. Two series of experiments were made at different times,— one in which the thermometer was clamped and the liquid was not stirred, and the other in which the liquid was stirred by means of the thermometer itself. The lag was of course much greater in the former case than in the latter. Below are given the data and corresponding curves of two experiments typical respectively of these two series, and representing a fair average of their somewhat irregular results.

Lag of Beckmann Thermometer under Different Conditions.

Time Seconds.	Curve A.		Curve B.	
	Reading when not Stirred.	Lag.	Reading when Stirred by Thermometer.	Lag.
0	6.00	. . .	6.00	. . .
15	. . .	. . .	3.450	0.053°
30	. . .	. . .	3.428	0.031°
45	. . .	. . .	3.410	0.013°
60	3.36	0.049°	3.401	0.004°
75	3.345	0.034°	3.3985	0.0015°
90	3.335	0.024°	3.397	0.000°
120	3.324	0.013°	3.397	0.000°
180	3.314	0.003°	3.397	0.000°
240	3.312	0.001°	. . .	. . .
300	3 311	0.000°	. . .	. . .
360	3.311	0.000°	. . .	. . .

Curves *A* and *B* given in the above table are plotted in the diagram, which explains itself.

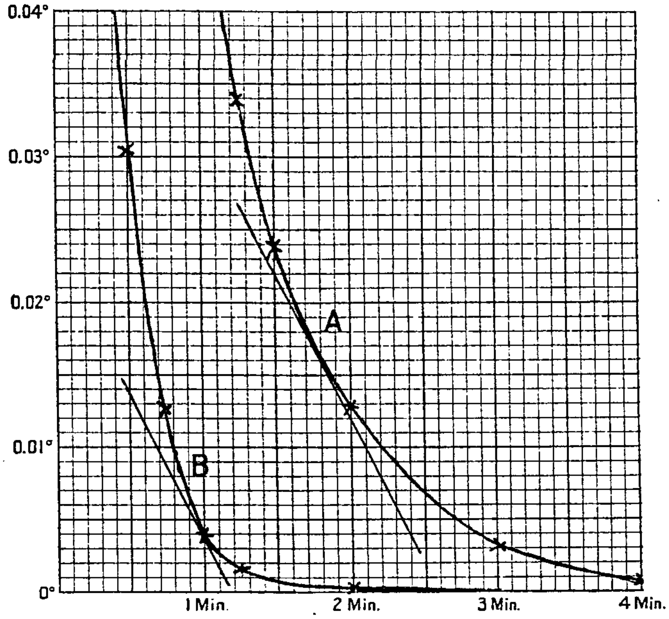

FIGURE 1.

The application of these results to a calorimetric experiment is very simple. Obviously, the thermometer will always lag as much behind the temperature of a given bath of changing temperature as it was behind the bath at the time when its own temperature was changing at the same rate as that of the bath. Because the tangent of a curve at any point indicates the direction of the curve at that point, the lag is simply found by determining the point at which a line drawn at the required angle just touches the curve. For example, if the bath were cooling 0.020° per minute, the ordinate of the point of contact of the curve with a tangent drawn at the corresponding angle (0.02° per minute) in the diagram will give at once the lag. Thus in the case of the present thermometer, the lag would be 0.017° (Curve A) if the liquid were not stirred, and 0.004° (curve B) if it were stirred, as indicated by the tangents drawn on the diagram. Even in the most favorable case, the correction would probably amount to over 0.001°, and should therefore always receive consideration.

As has been said, each experimenter must of course determine this value for his own exact conditions; therefore it is impossible here to apply a suitable correction to previous investigations, although all must have been more or less affected by this cause of error. The Rumford method of eliminating the error from radiation is little better than the Regnault method in this respect; for the rising thermometer just before the beginning of the reaction will always give too low a reading, while the conditions at the close of the reaction are highly uncertain.

On account of this phenomenon of thermometric lag, and because of the uncertain application of Newton's law of cooling to systems suffering sudden changes of temperature, it seemed highly desirable to devise a method of wholly eliminating both irregularities. It appeared probable that this object might be accomplished by artificially changing the temperature of the environment of a reacting system at the same rate as the system itself changed in temperature.* Thus a given reaction might be made really adiabatic, neither losing nor gaining heat from its equally hot surroundings at any time during the reaction.

Obviously there are several ways in which the outside water jacket in a calorimeter might be heated in order to accomplish this purpose. The simple device of pouring in hot water might be employed, or the water might be warmed by an electrically heated resistance coil, or the jacket itself might be made the scene of a chemical reaction of the same speed and thermal intensity as that within the calorimeter itself.

Of these and other methods which suggested themselves the last named seemed the most convenient and suitable for a chemical laboratory. It has the special advantages that before the beginning of operations all the apparatus and material employed may be at the temperature of the room; that the maximum temperature attained may be easily calculated with great nicety; that no point in the system can ever exceed this maximum temperature, if the reaction is suitably chosen; and that the speed of the reaction may be simply regulated by a stop-cock admitting one of the reacting substances.

The following sections of this paper show how this suggestion has already proved serviceable in two important investigations in this laboratory, and serve to give an approximate idea of the conditions necessary for its satisfactory fulfilment.

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* After the present paper had gone to press it was found that S. W. Holman had suggested this, although without in any way testing the suggestion in practice. These Proceedings, **31**, 252 (1895).

2. THE CONSTANCY OF RESULTS OBTAINED BY ONE OF THE NEW METHODS.

By Theodore W. Richards and George S. Forbes.

The experiments whose description follows in this section show how, in cases of comparative calorimetry, measurements can be conducted by heating the environment with constancy greater than that attainable with the same apparatus by the use of a cooling correction. It will also be shown that furthur study and improvement of the method may render possible the direct determination of the true adiabatic rise of temperature in any calorimetric process.

In the course of a certain investigation, to be described later in detail, it became desirable to compare the heat capacities of mercury and of liquid amalgams of zinc and cadmium. It is not necessary to describe the apparatus in great detail here. A sheet-iron calorimeter containing eleven hundred grams of toluol was supported in the center of a tinned iron vessel, which was immersed in a large bath of water contained in a galvanized-iron barrel. An annular cover protected the air space around the calorimeter from disturbance. Twenty-three hundred grams of mercury were delivered from a warm reservoir of very constant temperature and distributed into a series of shallow pans; all of these were insulated from the walls of the calorimeter by the toluol, which was kept in rapid circulation by a complicated and efficient stirrer. The warming effect due to the agitation of the liquid was considerable, but very constant. The maximum temperature was always attained within four minutes, and was read by a Beckmann thermometer graduated to hundredths; an electric buzzer prevented errors due to the friction of the thread. The thermometer was not calibrated, but the same part of its scale was used in every experiment. After the apparatus was in running order, and the details of its manipulation mastered, a series of determinations of the heat capacity of mercury, in certain arbitrary units, was made. The indications of the thermometer were carefully noted, and interpreted in the usual way, applying the usual Regnault-Pfaundler method for correcting the error from radiation.*

Four successive determinations of the heat capacity of mercury in these arbitrary units were 608, 613, 614, 607, average 611 ± 1.

The probability is that this mean value represents approximately the mean of an indefinite number of results by this process, but nevertheless the considerable deviation between the extreme values is greater than we can easily ascribe to uncertainty in the reading of a thermometer, or inaccuracy of weighing. Hence it appeared that the cooling correction, which here amounted to about 5 per cent of the rise of temperature, was probably at fault.

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* Pfaundler, Pogg. Ann., **129**, 113 (1866).

A series of determinations was next made with the same apparatus, increasing the temperature of the surrounding jacket at the same rate as that of the calorimeter. To accomplish this, a suitable weight of sodic hydroxide was dissolved in the jacket, and just at the instant when the mercury was delivered from its reservoir an equivalent mass of sulphuric acid was poured into the caustic solution from four different outlets, so placed as to make the initial distribution of the acid as uniform as possible. A powerful stirrer completed the mixing within a short time, as was proved by the indications of a thermometer hung in the reacting mixture. The total rise of the bath under these circumstances was almost exactly the same as that observed in the calorimeter, and ran nearly parallel to it; hence radiation from the sides and bottom of the calorimeter must have been minimized if the influence of the adjacent body of air is negligible. The partial exposure of the top of the liquid still left a slight cooling effect, which had to be corrected arithmetically; but this could easily have been avoided in more elaborate experiments.

Three determinations made in this way gave the results 605, 606, 608, average 606 ± 0.06, thus possessing a smaller probable error than any three determinations chosen from among the previous series. Such irregularity as exists was probably due to the fact that the bath was not exactly regulated to the proper rate of temperature increase. The result is also distinctly lower than the preceding series, showing that the temperature interval had previously been estimated as too large. The difference of 0.8 per cent in the averages of the two series is a very important one. Possibly the value 606 may have been slightly too low, owing to the large air-space between the calorimeter and jacket, but it could hardly have been as much too low as 0.8 per cent.

In order to obtain further light upon the matter, another series of determinations was made, in which the environment was warmed in such a way as certainly to yield too high a result. The acid and alkali in the jacket of the calorimeter were mixed exactly two minutes before the hot mercury was run into the calorimeter itself, and the final temperature of the environment was made high enough to counterbalance the slight cooling effect on top. Therefore during the first two minutes there was undoubtedly an excessive warming effect. The results thus found were

610, 613, 611, 612; average 611.5 ± 00.4, or almost exactly identical with value 611 of the first series calculated by Regnault's method. Because, however, the results of this last series must certainly have been too high, the results calculated by Regnault's method must also be too high; and it is clear that one cause, if not the only cause, of this error must be the lag of the thermometer.

It is to be noted that in the last series, where irregular radiation-effects took place only during two minutes, the results are more concordant with one another than in the first series, where the total period of irregular radiation was fifteen minutes. This fact again indicates that the calculated cooling correction was at fault.

On studying these figures, it is clear that where only comparative results are required, the method of heating the environment will give satisfactory results so long as the conditions in every experiment are identical. Even when the environment is warmed two minutes too soon, the results are constant; but of course if this procedure is adopted once in a series, it must be always adopted.

Therefore, even if the method of warming the environment were incapable of yielding an absolute value, it would nevertheless be a distinct addition to comparative calorimetry. In the following section it will be shown that under proper precautions the method will probably yield not only a constant value, but a result very close to the true value.

3. THE COMPARISON OF THE TWO METHODS, AND THE EXACT ESTIMATION OF ADIABATIC RISE OF TEMPERATURE.

BY THEODORE W. RICHARDS AND L. J. HENDERSON.

The foregoing sections of this paper show indubitably that whenever there is any considerable rate of cooling in a calorimetric experiment, the lag of the thermometer will cause the result to be higher than the truth.

It becomes now a matter of great interest to determine, if possible, the method of procedure which is capable of yielding the true value for the temperature increment in a strictly adiabatic reaction. The question assumed especial importance because of an extended series of measurements of heats of combustion now in progress here. In order to procure further light, it was decided to execute several series of experiments in which the cooling correction was calculated in the usual way, and to correct these results further for the lag of the thermometer. Subsequently, the outcome of these experiments was to be compared with that of a series in which the temperature of the environment was changed simultaneously with that of the calorimeter itself. The accomplishment of this program and the comparison of the results is recorded below.

The reaction chosen to be measured was that of sodium hydroxide on sulphuric acid. This reaction has the advantage of easy repetition, as well as the advantage of a less considerable change of heat evolution with temperature than is the case in many reactions. In order to exclude all error from this cause, the maximum temperature was about the same in all cases. A slight excess of the alkali was used to avoid error from decomposition of carbonate.

In a preliminary series of experiments the divided beaker calorimeter used by Richards and Lamb * was at first employed; but because with this apparatus the speed of the reaction is not easily regulated, another form depending upon the same principle was employed. This form is shown in section in Figure 2.

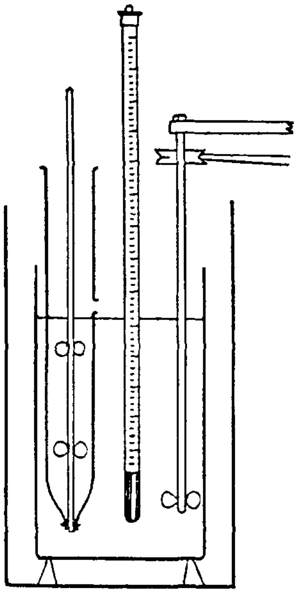

FIGURE 2.

The sodic hydroxide was contained in a large test-tube provided with two holes, one on the side above the liquid level, and the other in the middle of the bottom. The latter was closed until the moment of the reaction by a rubber stopper upon a rod running down through the axis of the tube. Before this moment arrived the test-tube was immersed in the calorimeter vessel, full of dilute acid, to the level of the alkali within. When it was desired to start the reaction, the stopper was displaced and the test-tube still further immersed, so that both holes were open under the liquid level. Screw-propeller vanes on the stopper-rod made it possible to circulate the liquid through these two holes, so that the mixing could be accomplished as quickly or as slowly as was desired. Another screw-stirrer in the calorimeter vessel itself assisted the mixing. The heat developed by the friction of the stirring was found to be barely perceptible; it was moreover identical in each case, and may therefore be wholly neglected.

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* These Proceedings, **40**, 675 (1905).

With this apparatus experiments were first made according to each of the usual methods of calorimetric work. Two distinct procedures have been used by various experimenters; in one case the experiment is started at about the temperature of the room, and a large final cooling correction is added, while in the other case the experiment is started below the temperature of the room, and a small initial correction for warming is subtracted.

In previous experiments in this laboratory it has been found that the latter of these methods gives the more constant results; and the same conclusion must have been reached by Stohmann and by Atwater, since they have usually used it.

Because of the lesser accuracy of the former of these methods, more experiments were made by it than by the latter, in order that the averages might have about equal weight.

Below are given the results of successive experiments by these two methods. The first column of the table designates the method; the second, the number of the experiments; the third, the rise in temperature as calculated by the Regnault-Pfaundler method, but not corrected for thermometric lag; the fourth, the correction for thermometric lag, as found in the way already described; and the fifth, the final value with all corrections.

PRELIMINARY EXPERIMENTS.

Method.	No. of Experiment.	Rise of Temperature as usually Calculated.	Correction for Thermometric Lag.	Final Value.	Average Rise of Temperature.
I.	12	3.382°	–0.007°	3.375°	
	13	3.385°	–0.007°	3.378°	3.3747°
	14	3.378°	–0.007°	3.371°	
II.	15	3.385°	–0.010°	3.375°	
	16	3.388°	–0.009°	3.379°	
	17	3.386°	–0.007°	3.379°	3.3777°
I.	18	3.385°	–0.007°	3.378°	
	19	3.385°	–0.008°	3.377°	3.3793°
	20	3.391°	–0.008°	3.383°	

It will be noticed that the correction for thermometric lag averages about 0.008°, or about 0.23 per cent of the total rise in temperature. This thermometric lag was not precisely similar in all the experiments, because the rate of cooling was not always exactly the same in each case.

It is further noteworthy that the two methods give essentially the same result, the average of the six determinations with the first method being 3.377° ± 0.001°, while that of the three determinations with the second method is 3.378° ± 0.001°. Thus each confirms the other.

Moreover, attention should be called to the fact that twice as many determinations were needed to secure a given degree of probability by the first method as by the second, although indeed the series are too brief to make the application of the method of least squares satisfactory.

After this comparison of the various methods of calculation formerly used, and the correction of the results for the newly applied correction for thermometric lag, it became a matter of great interest to determine whether or not the final result thus obtained is identical with that yielded in the same reaction by the new method of experimentally eliminating the correction for cooling. The results of this new comparison, even more carefully made than before, are given below.

The solutions previously used having been exhausted, new, less concentrated ones were prepared. The apparatus also was somewhat altered. A large platinum calorimeter, capable of holding over a litre, was employed to contain the reacting acid, and this was fitted into a deep copper cylinder, so as to leave an annular space of air between them only a few millimeters in thickness. It is obvious that this air jacket should be as thin as possible in order to reduce its heat capacity. The calorimeter was covered by a cardboard diaphragm, pierced with suitable holes for the thermometer, stirrer, and alkali vessel. The last was in this case not immersed in the calorimeter as before, but raised above it, with the delivery jet just touching the acid into which the alkali was to be discharged,— a change adopted in order to make the delivery as rapid as possible. The temperatures of both acid and alkali were read with great care; and because the volume and heat capacity of the latter were small, it was easy to apply a small correction for its deviation in initial temperature, that of the acid being taken as the true initial temperature. The small correction necessary is given in each case in the tables below.

The copper cylinder was highly polished inside, and was sunk deeply in a bath of water whose temperature could either be maintained at a constant point or varied at will. A second cardboard cover protected the alkali tube from accidental changes of temperature. It would of course have been better to have a hollow cover also with provision for changing its temperature; this further improvement has since been successfully tried.

The apparatus is outlined in Figure 3, a glance at which, in connection with the foregoing description, will suffice to explain it.

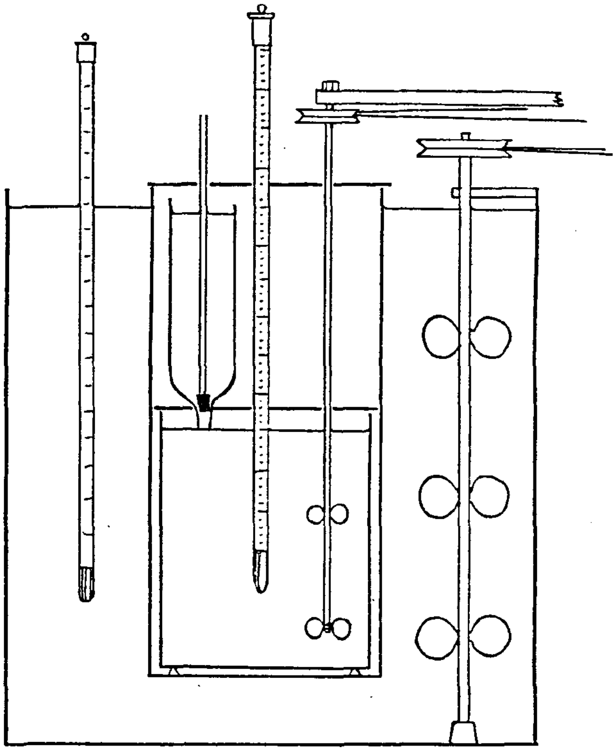

Figure 3.

With it, two series of determinations were made,— one, in which the temperature of the outside bath was kept constant at a temperature somewhere between the initial and final temperatures; and a second, in which the temperature of the bath was made to run parallel with the temperature of the quantitative mixture.

The first of these series is recorded in the table on the opposite page, the corrected rise of temperature being determined by the Regnault-Pfaundler formula,

$$C = n v + \frac{v' - v}{t' - \theta} + \left(\frac{\theta_n + \theta_0}{2} + \sum_1^{n-1} \theta - N\theta\right).$$

According to this formula, the reaction is divided into three periods or stages; namely, an initial period before the reaction begins, a reaction period, and a final period after the reaction is over.

In this case, $n = 5$, the number of time units (minutes) in the reaction period.

v = rate of fall of temperature in the initial period.

v' = rate of fall of temperature in calorimeter during the final period.

θ_0 = temperature at beginning of temperature rise.

$\theta_1, \theta_2 \ldots$ = temperatures during successive minutes.

θ and θ' = the mean temperatures of the calorimeter during the initial and final periods respectively (easily found from θ_0 and θ_5 with the help of v and v'). All the data necessary are given in the table, including the small correction due to a deviation in the initial temperature of the alkali.

Rise of Temperature during Reaction, Corrected in Usual Manner.

	(21)	(22)	(23)	(24)	(25)
θ_0	1.950°	1.622°	1.542°	1.920°	1.968°
θ_5	4.718	4.413	4.417	4.739	4.786
$\theta_5 - \theta_0$	2.768	2.791	2.875	2.819	2.818
v	0.0036	−0.0002	−0.0068	−0.0042	−0.0140
v'	0.0184	0.0168	0.0086	0.0132	0.0126
Quantity in parenthesis of formula	12.6	12.7	13.0	13.0	12.6
$v' - v$	0.0148	0.0170	0.0154	0.0174	0.0266
$\theta' - \theta$	2.716	2.750	2.870	2.796	2.822
Total correction for cooling	0.087	0.077	0.036	0.061	0.049
Correction for Alkali	+0.017	−0.005	−0.032	−0.013	0.000
Corrected rise . .	2.872°	2.863°	2.879°	2.867°	2.867°

According to the preceding evidence, the rise of temperature recorded in the last line of this table must be somewhat too high, because of the lag of the thermometer both before and after the experiment. In order to determine this amount, the lag of the thermometer was carefully studied, as on pages 5 and 6, both with a large fall of temperature and with a small fall in temperature, under precisely the same conditions of stirring as those maintained in the foregoing experiments. In this way, consistent results were obtained, the averages of which are recorded in the curve (Figure 4).

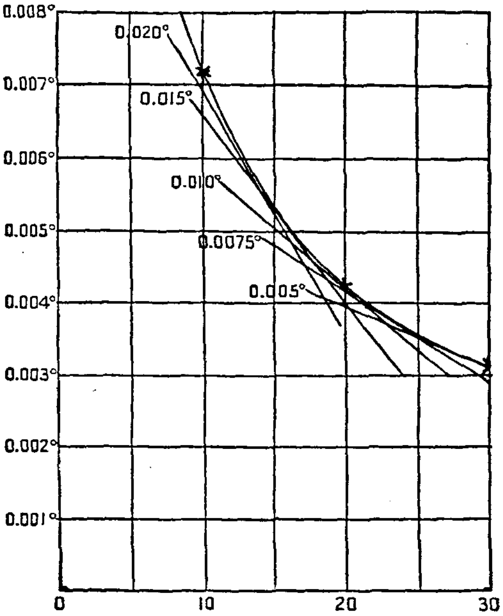

Figure 4.

Lag of Thermometer Employed in Foregoing Experiments.

The time after the immersion of the slightly warmed thermometer is recorded in the direction of abscissae in seconds; the lag of the thermometer in the direction of ordinates in 0.001° intervals. The curve represents the average of many figures. Upon it are plotted the tangents corresponding to several rates of cooling, expressed in fractions of a degree per minute. From these tangents comes the table below.

Rate of Cooling per Minute.	Lag of Thermometer.	Rate of Cooling per Minute.	Lag of Thermometer.
0.030	0.0076°	0.010	0.0042°
0.020	0.0058°	0.0075	0.0038°
0.015	0.0051°	0.0050	0.0031°

As before, the lag of the thermometer was found, both for the initial and the final readings, by drawing a tangent to the curve C at the angle determined respectively by v and v', the rates of cooling. The results of this graphic computation are recorded in the table below.

CORRECTION OF ABOVE RESULTS FOR THERMOMETRIC LAG.

Rise of T° (Regnault-Pfaundler) in C°.	Lag of Thermometer in C°.			Corrected Rise of Temperature in C°.
	Initial.	Final.	Total.	
2.872	−0.002	+0.006	0.004	2.868
2.863	±0.000	+0.006	0.006	2.857
2.879	+0.004	+0.004	0.008	2.871
2.867	+0.003	+0.005	0.008	2.859
2.867	+0.001	+0.005	0.006	2.861
2.870	. . .	. . .	0.0064	2.863°

Thus the true average result is lower than that given by the Regnault-Pfaundler formula by 0.0064°, or by 0.22 per cent.

A satisfactory value having been thus found by one method, it became a matter of great interest to execute the second series by the method of warming the environment, in order to determine whether or not this would yield the same result. For this purpose the dilute acid placed in the outer jacket of the calorimeter was strongly agitated by a powerful stirrer, and at the moment of the experiment a beakerful of alkali of suitable amount was very rapidly poured into it. The temperatures inside and out were so arranged that the final temperatures were read on almost if not quite stationary thermometers, the lag of which was negligible. To accomplish this the final outside temperature had to be slightly above the final inside temperature, because of the partially unprotected top of the calorimeter. The initial and final temperatures were about the same as before, and every other condition of the experiment remained unchanged. The expression of the data is far simpler than before, it being necessary to give only the initial and final temperatures and the correction for the temperature deviation of the alkali delivered into the inner vessel; and even the latter might have been omitted if somewhat more time had been allowed it in order that it might, before the experiment, assume the same initial temperature as the acid. The final temperature was designated θ_3, since it was attained in three minutes.

RISE OF TEMPERATURE AS DETERMINED BY NEW EXPERIMENTAL METHOD.

	(26)	(27)	(28)	(29)	(30)
θ_0	1.658	1.665	1.851	1.503	1.798
θ_3	4.520	4.540	4.699	4.333	4.668
Correction for Alkali	0.000	–0.013	+0.012	+0.022	–0.007
$\theta_3 - \theta_0$	2.862°	2.862°	2.860°	2.852°	2.863°

The average rise of temperature as indicated by these experiments is therefore 2.860 ± 0.001, while that obtained in the previous series after the new correction for thermometric lag had been applied was 2.863 ± 0.002. Thus the two series, when all corrections were applied, gave results not differing by an amount greater than the sum of their probable errors, and the average by the new method had only half as large a probable error as that obtained with the old method.

While it is of course true that this agreement might have been accidental, such an interpretation to us seems improbable.

That these two methods, each of which is apparently satisfactory, should unite on the same value, gives that value a strong presumptive right to be considered as the true value. If this is so, nearly all calorimetric results which have ever been published, where any considerable rate of cooling or warming is involved, have been incorrectly calculated. In cases where the rate of cooling or warming is very slight, as in the admirable work of E. Fischer and Wrede,* the error is probably so small as to be negligible.

It is a pleasure to acknowledge our indebtedness to the Rumford Fund of the American Academy of Arts and Sciences for the apparatus used in this investigation.

SUMMARY.

1. A modification of one of the devices used by Richards and Lamb, permitting more careful regulation of the speed of reaction, is described.

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* Fischer and Wrede, Sitzungsber. könig. preuss. Akad. **20**, 687 (1904).

2. It is shown that the lag of the thermometer behind the temperature of even a slightly cooling or warming environment causes an appreciable error in estimating the temperature of the environment.

3. It is further shown that this lag may be easily determined and a suitable correction applied.

4. A new method for entirely obviating this and all other corrections for cooling in calorimetric work is shown to consist in systematically altering the temperature of the environment at the same rate and to the same degree as that of the calorimeter proper.

5. This method is shown in several series of experiments to give a more constant result than the old method of calculation.

6. It is shown, moreover, to give essentially the same value for an adiabatic rise of temperature as that afforded by the old method when the latter is further corrected for the lag of the thermometer.

7. Therefore it appears probable that the true result of a calorimetric experiment can be attained only with the help of one or the other of these new methods, or some such method as that suggested by Richards and Lamb.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

MEASUREMENT OF VARIOUS THERMAL AND ELECTRICAL EFFECTS, ESPECIALLY THE THOMSON EFFECT, IN SOFT IRON.

BY EDWIN H. HALL, MESSRS. CHURCHILL, CAMPBELL, AND SERVISS.

INVESTIGATIONS ON LIGHT AND HEAT MADE OR PUBLISHED, WHOLLY OR IN PART, WITH APPROPRIATIONS FROM THE RUMFORD FUND.

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MEASUREMENT OF VARIOUS THERMAL AND ELECTRICAL EFFECTS, ESPECIALLY THE THOMSON EFFECT, IN SOFT IRON.

By Edwin H. Hall, Messrs. Churchill, Campbell, and Serviss.*

Presented December 14, 1904. Received March 9, 1905.

Part I: General Account of Methods, Apparatus, and Results.

Let the end E of a bar of metal be maintained at some fixed temperature, T, and let the end E' of the same bar be maintained at a different fixed temperature, T'. If an electric current of given strength is then driven through the bar, the amount of heat generated by this current will depend on the direction of the current, being either greater or less when the current flows from E to E' than when it flows in the opposite direction. This directional relation between an electric current and the heat generated by it is the simplest and most direct aspect of the "Thomson effect."

Let us suppose that we have initially, before the electric current is put on, a uniform ascending gradient of temperature from the end E to the end E', indicated by the straight line S in Figure 1. Let J in the same figure represent the line of temperature gradient which would be established in the bar by a permanent electric current of strength i, if there were no Thomson effect. Let the line A represent the temperature gradient which would actually be maintained in the bar by the permanent current

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* Most of the preliminary work of this investigation, making the main thermo-electric couples, testing them, varnishing the iron bars and fixing the thermo-electric couples upon them, assembling the apparatus as a whole and getting it into good working order, was done by Mr. Churchill, who gave much time to the undertaking during the academic years 1902–3 and 1903–4. Unfortunately Mr. Churchill, leaving Cambridge early in September, 1904, was unable to take much part in the main observations, which were made during the fall of that year and the early part of 1905, largely by Mr. L. L. Campbell and Mr. S. B. Serviss of the Harvard Graduate School.

i flowing from E to E'; and let B represent the gradient which would be maintained by an equal current flowing from E' to E. The general disposition of the lines A and B with respect to J in this figure corresponds to the phenomena observed in iron, more heat being generated by a given current in this metal when the flow is from cold to hot than when it is from hot to cold.

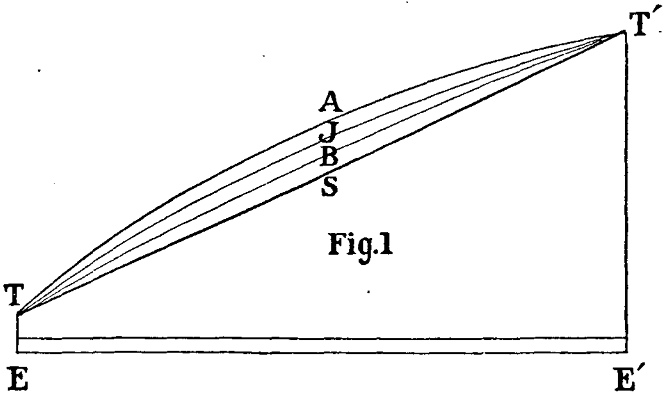

Fig. 1

If we have two similar cylindrical bars of iron, α and β, and send the same current from cold to hot in α and from hot to cold in β, we may have simultaneously gradient A of Figure 1 in bar α and gradient B in bar β. We shall in the discussion that follows assume this condition of things.

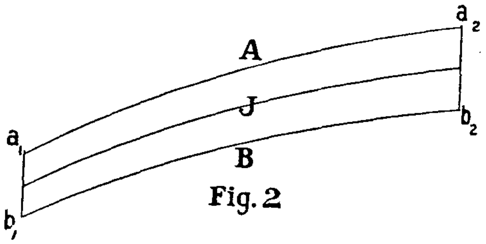

Fig. 2

We shall assume also, for the present, that the bars are so protected that no heat escapes from them laterally.

In Figure 2 we have parts of the curves A and B, the part $a_1\ a_2$ belonging to any part of the bar α and the part $b_1\ b_2$ belonging to the corresponding part of the bar β. Let the length taken on each bar be L cm., and let the area of cross-section of each bar be S sq. cm.

We shall assume that the specific electric resistance of the iron is a linear function of the temperature, and accordingly shall write

$K(1 + \gamma t_a)\ L \div S$ as the resistance of the L cm. of α
and $K(1 + \gamma t_b) L \div S$ as the resistance of the L cm. of β,

K being the specific resistance at 0° C., γ the temperature coefficient of resistance, t_a the mean temperature of the L cm. of α, above 0° C., and t_b the corresponding mean for β.

Let the specific thermal conductivity of the iron be k_{a_1} at the temperature t_{a_1}, k_{a_2} at the temperature t_{a_2}, k_{b_1} at t_{b_1} and k_{b_2} at t_{b_2}.

Let q_a be the Thomson effect heat in $a_1\,a_2$; that is, the amount of heat generated per second in $a_1\,a_2$ in excess of the amount accounted for by Joule's law. Let q_b be the Thomson effect heat in $b_1\,b_2$; that is, the amount by which the heat generated per second in $b_1\,b_2$ falls short of the amount given by Joule's law.

Let the temperature gradient at any point in either bar be called g, with the subscript approximate to the point. The current is to be expressed in amperes, the resistance in ohms, and the heat in calories.

The heat carried out by conduction at a_1 must exceed the heat brought in by conduction at a_2 by the amount generated between a_1 and a_2. Hence

$$S\,(k_{a_1}g_{a_1} - k_{a_2}g_{a_2}) = q_a + 0.2387\, i^2 K (1 + \gamma t_a)\, L \div S,$$

or

$$(1) \qquad q_a = S\,(k_{a_1}g_{a_1} - k_{a_2}g_{a_2}) - 0.2387\, i^2 K (1 + \gamma t_a)\, L \div S.$$

For the other bar we have

$$(2) \qquad q_b = -\, S\,(k_{b_1}g_{b_1} - k_{b_2}g_{b_2}) + 0.2387\, i^2 K (1 + \gamma t_b)\, L \div S.$$

From (1) and (2) we get

$$(3) \qquad q = \frac{q_a + q_b}{2} = \tfrac{1}{2}\, S\,[(k_{a_1}g_{a_1} - k_{b_1}g_{b_1}) + (k_{b_2}g_{b_2} - k_{a_2}g_{a_2})] \\ - 0.1194\, i^2 K \gamma\, (t_a - t_b)\, L \div S.$$

As the difference of temperature between any part of α and the corresponding part of β is likely to be small, always much less than one degree in the experiments to be described, and as the temperature coefficient of k is small, we may provisionally write in place of (3)

$$(4) \quad q = \tfrac{1}{2} S[k_1 (g_{a_1} - g_{b_1}) + k_2 (g_{b_2} - g_{a_2})] - 0.1194 i^2 K\gamma\, (t_a - t_b)\, L \div S,$$

where k_1 is a mean between k_{a_1} and k_{b_1}, while k_2 is a mean between k_{a_2} and k_{b_2}. The magnitude of the error made in the substitution of (4) for (3) will be considered later. It is not very important.

Of the quantities in the second member of (4), S, L, K, i, and γ are easily measured; k_1 and k_2 can, from previous study of the thermal conductivity of the iron used, be found as soon as the corresponding temperatures are known; the differences of gradient and the differences of temperature at corresponding parts of the two bars are the difficult things. Let us now look at the apparatus by means of which an attempt has been made to determine these latter quantities.

Figure 3 shows a horizontal section through the main part of this apparatus, the scale of the figure being 2 to 5. Each of the bars α and β is 20 cm. long in its narrow cylindrical part, and this part of each bar was ground by Brown and Sharpe to a diameter of 1 cm., with such accuracy that the deviations are difficult to detect and need not be further considered. One enlarged end of each bar is submerged in a mixture of ice and water, which is agitated by a plunging stirrer, *P*, which makes about forty-five strokes a minute, entering and leaving the water at each stroke and causing about 1.5 cm. fluctuation of the surface level. The other enlarged end of each bar is immersed in boiling water. Each bar is of course electrically insulated from the metal boxes which contain the water. The arrows indicate the general direction of the electric current in what will be called condition *B*, the current running from cold to hot in the rod which was farthest from the observer, the *back* rod. Half of the time the current ran from cold to hot in the *front* rod, and it was then said to be in condition *F*. *G* and *G* are two of twenty iron bars, each about 6 mm. in diameter, which form a guard-ring around the bars α and β. See also Figure 4. The space between α, β, and the bars *G* is filled with cotton, rather closely packed. Outside the guard-ring is a wrapping of cotton batting about 5 cm. thick. In spite of the guard-ring and the cotton there is a considerable lateral flow of heat, which must be considered later.

The long cylindrical parts of α and β are covered with an insulating coating of asphaltum varnish well baked on. The points *K* and *K'* on α (Figure 3) are the two junctions of a thermo-électric couple consisting of a piece of well-annealed german-silver wire (18 per cent nickel), 0.02 cm. in diameter and 13 cm. long, neatly joined, by means of a small amount of hard solder, to copper wires, also 0.02 cm. in diameter. Both the german-silver wire and the copper wire are wound in a groove, for the most part a spiral groove, made in a second coating of asphaltum varnish while it was yet comparatively soft. The junction *K* was to be placed 1.5 cm. distant from the neighboring shoulder of α, and the junction *K'* was to be 2.5 cm. distant from the same shoulder. Accordingly, the groove, already mentioned, into which the wire was to be wound, was made very carefully, by the use of a special tool and a gear-cutting lathe, and was of such shape that, if the layer of varnish could have been removed from the rod and laid out flat, the groove would have looked like Figure 5, the scale of which is 4 to 5. The two dots shown in this figure mark the positions of the junctions K and K'. It will be seen that each of these points lies in a part of the groove which is not of the spiral, a part which is in a plane at right angles with the axis of the bar.

These two plane half circles in which the junctions rest are very nearly 1 cm. apart, perhaps as much as 1.003 cm. In three trial cases the distances between such grooves, after baking, were 1.0021 cm., 1.0018 cm., 1.0032 cm., according to the measurements made by Mr. Churchill by means of a dividing engine. The small lumps of solder at the junctions are probably not quite symmetrical, though they were filed down as much as was considered safe, and therefore the junctions probably do not lie exactly in or over the middle of the grooves. But the conditions of the experiment are so varied, by reversing the current and by reversing the temperature gradient, that the result must be very nearly the same as if each junction were placed accurately in the groove made for it.

The couple LL' has its left junction 9.5 cm. distant from the left shoulder of α, and its right junction 9.5 cm. from the right shoulder. The couple MM' has M distant 2.5 cm. and M' distant 1.5 cm. from the right-hand shoulder of α. The couples NN', OO', and PP' have corresponding positions on β.

Figure 4 shows a vertical cross-section through the bars near their middle. The small copper wires, w and w, leading up from α and β, pass through glass tubes, t and t, set in the board B, and above are soldered to somewhat larger copper wires, which are gripped near the end by the binding posts p and p, but extend beyond these posts and out of the main apparatus to make connections with the sensitive galvanometer used to measure the thermo-electric currents. Before the copper wires were soldered together they were hooked to each other, and during the soldering the hooks were kept taut, so that after the soldering the two copper surfaces were almost, if not quite, in direct contact. An actual interruption of the copper connection by as much as half a millimeter of solder at this point would probably make serious trouble. The board B was baked after receiving a coating of asphaltum varnish.

Z' is a thermo-electric junction of copper and german-silver laid on the outer side of the bar G_3, with a layer of thin sheet rubber between. Wires leading from this junction pass around the whole guard-ring in a vertical plane, and the german-silver wire joins another copper wire in a second junction, which can be placed in ice-water or in steam. The object of this arrangement is to give an indication of the temperature at the point where the inner junction lies against the guard-ring bar G_3. For a like purpose the junction Z is placed on the bar G_8, 90° around the guard-ring from G_3.

The line W W indicates the boundary of the cotton wrapping of the rods.

E, F, G and H, I, J, in Figure 3, are copper-german-silver junctions, which have, with respect to certain points on α and β, a function like that which the junctions Z and Z' shown in Figure 4 have with respect to points on the guard-ring bars. Thus E, with its corresponding external junction placed in ice-water or in steam, indicates the temperature of a point midway between the junctions K and K'. H does the same for the junctions N and N', etc.

Figure 6 shows a vertical longitudinal section of the apparatus on the scale 1 to 6. C and C are the conductors carrying the main current into and out of the apparatus. This current, about 25 amperes, is measured by an ammeter not here shown.

Figure 7 shows the arrangements on the top of the board B. It is to be noted that each couple on α is joined in opposition to the corresponding couple on β. For example, the wires coming up from the junctions K and N (Figure 3) are soldered to one and the same wire, which leads away from B and may run to the galvanometer, though it is usually disconnected at its outer end. The wires from junctions K' and N', respectively, on the other hand, connect above B with two separate wires, which lead off toward the galvanometer. The three wires thus running from the couples $K K'$ and $N N'$ make it possible to put into the galvanometer circuit either of these couples alone or both in opposition to each other. The other pairs of couples are similarly connected, L L′ with O O', and $M M'$ with P P'.

It is unnecessary to depict here the other apparatus used, concerning which some details will be given later.

The general order of procedure on October 22, 1904, a fair sample day, was substantially as follows:

	Time.	
A.M.,	8:05	Water ice-cold in left box, boiling in the other.
	9:00	Main current put on in condition F.
	10:35	Measurement of sensitiveness of galvanometer.
		“ “ disturbing thermo-electric effects.
		“ “ main current.

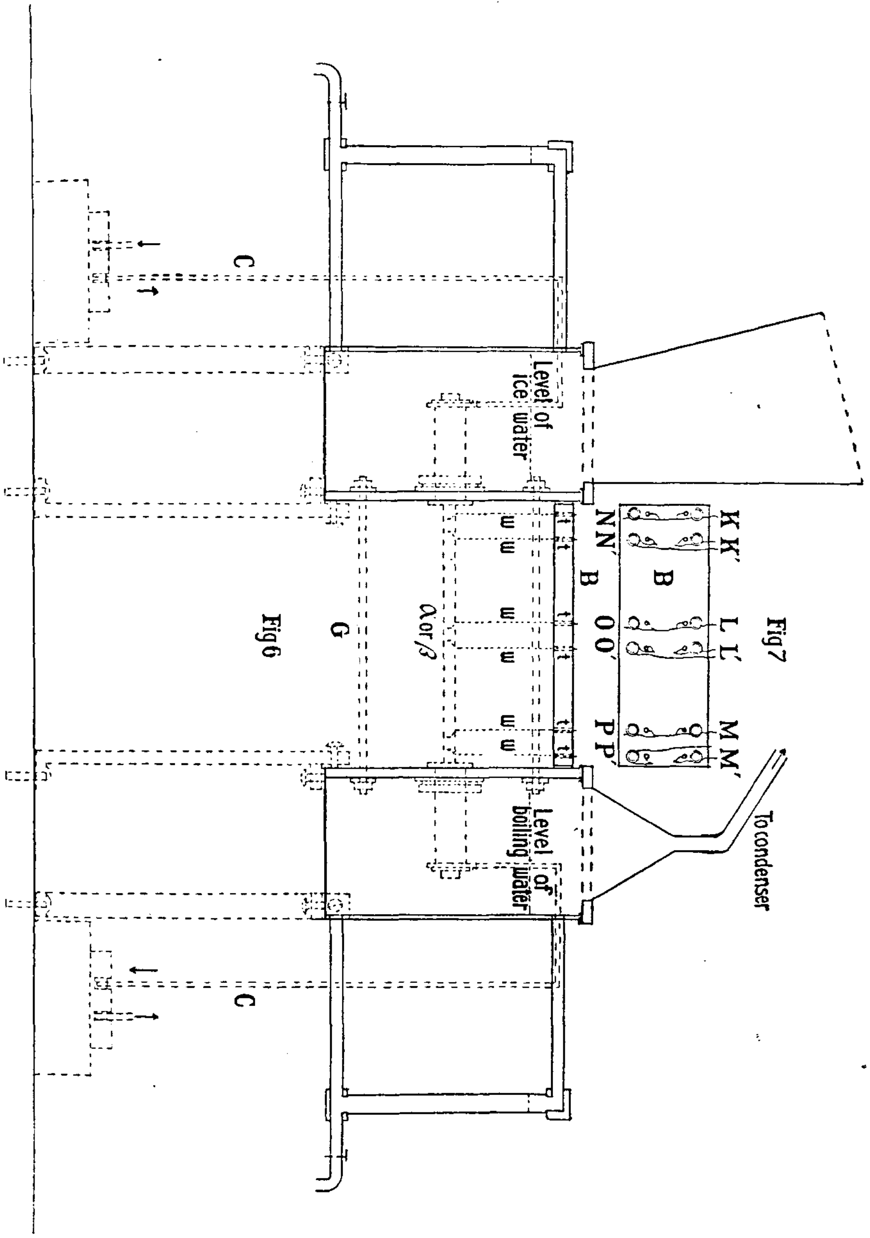
Fig 7
K K'
L L'
M M'
B
N N'
B
O O'
P P'
To condenser
Level of ice water
Level of boiling water
C
C
G
α or β
t
m
Fig 6

Time.
10:52 Observations with couples MM' and PP' opposed.
“ “ “ LL' “ OO' “
“ “ “ KK' “ NN' “
“ “ “ LL' “ OO' “
11:22 “ “ “ MM' “ PP' “
Measurement of main current.
“ “ disturbing effects.
11:35 “ “ sensitiveness.
“ “ main current.
11:45 Observations with couples MM' and PP' opposed.
“ “ “ LL' “ OO' “
“ “ “ KK' “ NN' “
“ “ “ LL' “ OO' “
P.M., 12:18 “ “ “ MM' “ PP' “
Measurement of main current.
“ “ disturbing effects.
12:35 “ “ sensitiveness.
“ “ main current.
12:54 Observations with couples MM' and PP' opposed.
“ “ “ LL' “ OO' “
“ “ “ KK' “ NN' “
“ “ “ LL' “ OO' “
1:30 “ “ “ MM' “ PP' “
Measurement of main current.
1:36 *Reversal* “ “ “ to condition *B*.
Measurement of disturbing effects.
1:45 “ “ sensitiveness.

Wait for effects of reversal to occur.

2:26 Measurement of sensitiveness.
“ “ disturbing effects.
“ “ main current.
2:42 Observations with couples MM' and PP' opposed.
“ “ “ LL' “ OO' “

And so on till the observations made with main current in condition *B* correspond in number and variety to
5:50 those made with current in condition *F*.

Throughout the day various incidental observations, of the barometer, the temperature of the galvanometer, etc., were occasionally made.

It will be observed that such a day's work includes six complete cycles, or runs, through all the pairs of corresponding couples from MP to KN and back, three runs with the main current in condition F, and three with this current in condition B. If we let

A = strength of main current in amperes,
t_w = temperature of boiling water at mid depth,
E_c = net e. m. f., in volts, of the opposed cold couples,
E_m = " " middle " ,
E_h = " " hot " ,

the sign + indicating that the couple on the warmer bar, the bar in which the current runs from cold to hot, prevails, and *vice versa*, we can express the results of the observations of October 22 as follows:

	A	E_c	E_m	E_h	t_w
Condition F	25.06	-257×10^{-9}	$+585 \times 10^{9}$	-120×10^{-9}	100°
Condition B	25.02	$+1450 \times 10^{-9}$	-314×10^{9}	-1185×10^{-9}	100°
Means . .	25.04	$+597 \times 10^{-9}$	$+136 \times 10^{9}$	-653×10^{-9}	100°

It is evident that observations made without reversal of the main current would be useless. This fact is probably due in part to the slight misplacements of the junctions which have already been suggested and in part to certain accidental causes which may vary somewhat with the conditions of the experiment.

To find an approximate value for the difference of temperature gradient which yields any one of the E's given above, we have only to divide this E by the "thermo-electric height" of copper and german-silver at the mean temperature of the two opposed couples which have this net E. Thus we get from the observations of October 22, if Δ stands for difference of temperature gradient in degrees per centimeter, the + sign indicating that the gradient is steeper in the warmer bar, and *vice versa*,

$$\begin{array}{ccc} \Delta_c & \Delta_m & \Delta_h \\ +0.0352 & +0.0072 & -0.0316 \end{array}$$

It so happens that on this day the value found for Δ_h was numerically less than the value found for Δ_c; but the opposite condition was the common one, as the following table, which gives the results of all the fifteen days of successful observation of the main phenomena, will show:

Date.	Hot End.	A	Δ_c	Δ_m	Δ_h	t_w
1904. Aug. 31	K–N	24.98	0.0344	0.0070	– 0.0348	100.6°
Sept. 7	M–P	25.02	0.0346	0.0052	– 0.0341	100.4°
" 8	"	25.18	0.0350	0.0047	– 0.0355	100.3°
" 9	"	25.06	0.0367	0.0080	– 0.0347	100.5°
" 21	"	25.24	0.0320	0.0085	– 0.0385	100.5°
" 22	"	25.14	0.0346	0.0071	– 0.0349	100.8°
" 26	K–N	25.06	0.0346	0.0072	– 0.0390	100.1°
" 28	"	25.20	0.0337	0.0064	– 0.0386	100.6°
" 29	"	25.30	0.0380	0.0110	– 0.0335	100.2°
" 30	"	25.22	0.0315	0.0042	– 0.0397	99.7°
Oct. 12	"	25.44	0.0345	0.0091	– 0.0351	100.6°
" 22	M–P	25.04	0.0352	0.0072	– 0.0316	100.0°
" 25	"	25.12	0.0326	0.0047	– 0.0344	100.5°
" 27	"	25.12	0.0328	0.0079	– 0.0357	100.3°
" 28	"	25.10	0.0357	0.0067	– 0.0368	100.7°
Means		25.15	0.0344	0.0070	– 0.0359	100.4°

There is a certain inaccuracy in the method, already described, by which the Δ's are obtained from the E's. Each E has been treated as if each of the two opposing couples producing it had the same mean temperature. But observations to be described later, made by means of the junctions E, F, etc., shown in Figure 3, indicate that the temperature of the warmer bar exceeds that of the cooler by 0.055° at the mid section of the cool couples, by 0.265° at the mid section of the middle couples, and by 0.145° at the mid section of the hot couples. These differences would, because of the variation of "thermo-electric height" of copper and german-silver with temperature, give appreciable values of E if there were no differences of gradient in the opposing couples. Applying to the mean Δ's given above the corrections made necessary by this consideration, we get, as the improved Δ's,

Δ_c	Δ_m	Δ_h
+ 0.0337	+ 0.0040	— 0.0374

Dividing each of these quantities by 1.002, which according to a previous statement may be taken as the average distance between the two junctions of a single couple, we get

Δ_c	Δ_m	Δ_h
0.03363	0.00400	— 0.03733

The mean temperatures found for the mid points of the couples KK' NN', etc., by use of the junctions E, H, etc., are

t_c	t_m	t_h
13.0°	51.1°	90.3°

According to a previous investigation,* the value of the thermal conductivity of the iron here studied is 0.1528 at 28° C., with a mean temperature coefficient — 0.0003 for the range from 28° to 58°. A very recent investigation (see p. 52), made with the same apparatus that has been described in this paper, gives for the mean temperature coefficient of thermal conductivity from 13° C. to 87° C. the value — 0.0007. Using this later value and taking k as 0.1528 at 28°, we get

k_c	k_m	k_h
0.1543	0.1493	0.1461

From the difference of temperature of the two bars, as already given, at the sections c, m, and h respectively, we can make the somewhat uncertain estimate that the difference $(t_a - t_b)$, occurring in equation (4), is, between the sections c and h, 0.21°.

Recent measurements, by Mr. Campbell, of the electrical conductivity of the iron here used give for K, in equation (4), the value 11365×10^{-9} ohms and show that γ is approximately 0.005.

Putting, in equation (4),

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* Physical Review, May–June, 1900, p. 305.

$$\begin{aligned}
S &= 0.785,\\
L &= 16,\\
k_1 &= k_c \text{ above} = 0.1543,\\
k_2 &= k_h \quad \text{"} \quad = 0.1461,\\
g_{a_1} - g_{b_1} &= \Delta_c \text{ last given} = 0.03363,\\
g_{b_2} - g_{a_2} &= -\Delta_h \quad \text{"} \quad \text{"} \quad = 0.03733,\\
i &= 25.15,\\
K &= 11365 \times 10^{-9},\\
\gamma &= 0.005,\\
t_a - t_b &= 0.21,
\end{aligned}$$

we get, as the first approximation to the value of the Thomson effect heat,

$$q = 0.002037 + 0.002142 - 0.000018. \tag{5}$$

In this evaluation of q we have taken full account of the temperature gradients; but we have, in using a mean k_1 for the cold couples and a mean k_2 for the hot couples, ignored the fact that the thermal conductivity is slightly less at any point in the warmer bar than at the corresponding point in the cooler bar. In fact, if there were no difference of gradient at the cold couples, the difference of temperature, about 0.055°, between the bars at the section c would, with the temperature gradient 4.6, which is, approximately, the general gradient along each bar, send about 0.000020 calorie per second more along the cooler bar than along the other, and accordingly the first term in the second member of equation (5) should be diminished by half this amount. At the h section the excess of temperature of the warmer bar is apparently about 0.145 °, and accordingly, if there were no difference of gradient in the two bars at this section, the cooler would here transmit about 0.000056 calorie per second more than the warmer. Therefore the second term of the second member of equation (5) should be increased by half this amount The result is,

$$q = 0.002027 + 0.002170 - 0.000018 = 0.004179. \tag{6}$$

This would be the final value of q, if there were no correction for loss of heat by lateral flow from the bars α and β. Unfortunately such a correction is necessary and is large. It is estimated by the following method: When both ends of both bars have been for several hours in boiling water, the couples, KK', NN', MM', and PP' indicate a permanent temperature gradient of about 0.1° per centimeter from the ends toward the middle of the bars. Under the same conditions, observations made with the junctions which are attached to the guard-ring bars (see Figure 4) indicate that the mean temperature at the outer surface of the guard-ring, for the 16 cm. corresponding to the distance between sections KN and MP of the main bars, is approximately 4.5° below the temperature of the boiling water in which the ends of α and β are immersed. Taking the mean temperature of the main bars in this case for the stretch between section KN and section MP as 0.3° below the temperature of the water, which in view of all the observations seems a reasonable estimate, we get 4.2° as the mean difference of temperature maintaining lateral flow of heat, a flow sufficient to maintain a gradient of 0.106° per centimeter along each bar toward the middle at both of the end sections. The amount of heat thus transmitted per second is about 0.0242 calorie from each bar.

The lateral loss from each bar when one end is hot and the other cold is probably considerably less than half this amount, as the difference of temperature between corresponding points on the main bars and the guard-ring bars is in this case apparently less than 2°; but even this diminished flow is greater than the whole Thomson effect heat, and would be very serious indeed if it were not very nearly the same from both bars. The warmer bar being, according to the estimate already made for $(t_a—t_b)$, about 0.21 warmer than the other, the excess of lateral flow from the warmer bar may be estimated as 0.0242 × (0.21 ÷ 4.2) calorie per second. This is 0.001209 calorie per second, and the value found for q in equation (6) should be increased by one half this amount, which will give as our final value

$$q = 0.00478.^{*} \tag{7}$$

To find σ, the mean amount of heat produced per second by sending 10 amperes, 1 c. g. s. absolute unit of current, through a temperature interval of 1° downward in the region between 13.0° and 90.3°, we have

$$\sigma = - q \div 2.515 \times (90.3 - 13.0) = - 0.0000246.$$

If we assume that the heat generated or absorbed per second by a given current flowing through an interval of 1° is proportional to the absolute temperature of the metal at the point considered, — that is, if we assume that the line representing iron on the ordinary thermo-electric diagram should be a straight line within the range of temperature here considered, we get as the Thomson effect coefficient,

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* The value of q would be 0.00483, if −0.0003 had been taken as the temperature coefficient of thermal conductivity. See p. 33.

$$\nu = \sigma \div (273 + \tfrac{1}{2}\,(90.3 + 13.0)) = -\,757 \times 10^{-10}.$$

Attempts to determine the Thomson effect directly in precise units have, apparently, been very few. Le Roux in 1867 * gave the result of a rough absolute measurement of this effect in a certain alloy of bismuth and antimony, called "bismuth of Becquerel." Battelli published in 1886 † the results of measurements made by himself with various metals, one of which was iron. Mr. R. O. King published in 1898 ‡ the method and results of a study in absolute measure of the Thomson effect in copper, his work having been done at the Jefferson Physical Laboratory of Harvard University, though the general plan of his investigation came from Professor Callendar at Montreal.

Battelli, whose method seems not capable of very great accuracy, though it was probably a great improvement over that of Le Roux, got data which yield the following values of ν for iron:

$-\,283 \times 10^{-10}$	at	53° C.
$-\,319 \times 10^{-10}$	"	108° C.
$-\,331 \times 10^{-10}$	"	243° C.
$-\,369 \times 10^{-10}$	"	308° C.

He concludes from his own experiments: "1. That in all the substances studied, cadmium, iron, antimony, bismuth, *pakfong* [german-silver?], an alloy of 10 Bi + 1Sb by weight, the Thomson effect is proportional to the intensity of the current. 2. That in all these substances, with the exception of iron, the Thomson effect [the σ, not the ν, of this paper] is proportional to the absolute temperature."

The value of ν obtained from Battelli's data for the temperature 53°, which is very near the mean temperature, 51.7°, of the range used in the experiments described in this paper, is about 0.37 of the value here found for ν. Battelli gives no description whatever of the quality of the iron which he studied. It was "*ferro*." His apparent discovery that the ν for iron increases with rise of temperature, which requires that the line representing iron in the ordinary thermo-electric diagram shall be a curve growing steeper at high temperatures, has probably not been generally accepted; though the fact that he does not find a change of slope for the lines of the other substances examined by him may well make us hesitate to reject, as based on experimental errors, his conclusion concerning iron.

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* Ann. de Chimie et de Physique, 4^me^ série, t. X, p. 201.

† Atta della Reale Accademia delle Scienze di Torino, **22**, 48, 539.

‡ These Proceedings, **33**, 19, June, 1898.

The data of this paper enable us to make a separate calculation of the Thomson effect for the stretch from section *c* to section *m* and for the stretch from *m* to *h*; and this calculation is perhaps worth making, though the results will naturally be less reliable than the one result for the whole stretch from *c* to *h*.

Putting, in equation (4),

$$\begin{aligned} S &= 0.785 \\ L &= 8.0 \\ k_1 &= k_c = 0.1543 \\ k_2 &= k_m = 0.1493 \\ g_{a_1} - g_{b_1} &= \Delta_c = 0.3363 \\ g_{b_2} - g_{a_2} &= -\Delta_m = -0.00400 \\ i &= 25.15 \\ K &= 11365 \times 10^{-9} \\ \gamma &= 0.005 \\ t_a - t_b &= 0.21, \end{aligned}$$

we get, as the first approximation to the Thomson effect heat in the stretch from section *c* to section *m*,

(5′) $$q' = 0.002037 - 0.000235 - 0.000010.$$

Correcting for the effect of the higher temperature of the warmer bar in diminishing its thermal conductivity, we get

(6′) $$q' = 0.002027 - 0.000186 - 0.000010 = 0.001831;$$

with allowance for lateral leakage we get

(7′) $$q' = 0.002090.$$

To find q'', the Thomson effect heat for the stretch *m* to *h*, we have

(7″) $$q'' = q - q' = 0.00478 - 0.00209 = 0.00269.$$

The value of the Thomson effect coefficient for the stretch *c* to *m* would, according to the data pertaining to this stretch alone, be

$$\nu' = -q' \div (2.515 \times 38.1 \times 305) = -715 \times 10^{-10}.$$

The value of this coefficient found from the data belonging to the stretch *m* to *h* alone is

$$\nu'' = -q'' \div (25.15 + 39.2 \times 344) = -793 \times 10^{-10}.$$

The difference between ν' and ν'', about 10 per cent, is too large to be readily accounted for by experimental errors. Taken with the evidence found by Battelli, it may perhaps be regarded as raising a presumption in favor of the opinion that ν does increase, numerically, with rise of temperature in iron.* On the other hand, the common practice of drawing a straight line for iron, at low temperatures, on the thermo-electric diagram is not based on pure imagination or on mere theory. For example, experiments made on a thermo-electric couple, consisting of the iron here studied combined with copper, indicated † that the "thermo-electric height" of this couple diminished at a nearly uniform rate with rise of temperature, which may seem at first sight to require either lines very nearly straight for both iron and copper on the thermo-electric diagram, or lines having very nearly equal changes of curvature with the region of temperature used, 26.6° to 71.1°. Inspection of the ordinary diagram will show, however, that in this range of temperature a considerable relative change in the inclination of either line would make a much smaller relative change in the thermo-electric height, which is represented by the vertical distance between the two lines. Such change as there was in the rate of change of this height with rise of temperature was in the right direction to be accounted for by an increase in the steepness of the slope of either the iron line or the copper line. The values found for the "thermo-electric height" of the copper-iron couple were:

at 26.6° C.	1028×10^{-8} volt
" 41.3°	980×10^{-8} volt
" 54.5°	938×10^{-8} volt
" 71.1°	870×10^{-8} volt

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* Tait, Trans. Roy. Soc. Edin., **27** (1872–76), 137, found that at high temperatures the iron line has at least two curvatures, each sufficient to change the sign of its inclination. Tait made no absolute determinations of the Thomson effect, but based the inclination of the lines of his diagram on the assumption (supported by the experiments of Le Roux) that this effect is null in lead. Battelli (Atti della R. Accademia dei Lincei, serie 4, vol. 3, 1887, p. 212) found data which give 4.3×10^{-10} as the value of ν in lead.

† These Proceedings, **36**, 130, August, 1900.

To establish a uniform gradient the intermediate values should have been

at 41.3°	975×10^{-8}
" 54.5°	929×10^{-8}

With these data, and with the value of ν found in this paper for iron, it is possible to go through the form, at least, of calculating the value of ν for copper. The process is as follows:

E. m. f. of copper-iron couple with junctions at 26.6° C. and 71.1° C., respectively, is

$$E = \frac{1028 + 870}{2} \times 10^{-8} \times (71.1 - 26.6) \text{ volt} = 422 \times 10^{-6} \text{ volt.}$$

Heat, calories, which would be absorbed by this net e. m. f. in one second by 10 amperes is

$$q_n = 422 \times 10^{-6} \times 10 \times 0.2387 = 0.00101.$$

Heat, calories, which would be absorbed at the hot junction in one second by 10 amperes is

$$q_h = 870 \times 10^{-8} \times (273 + 71.1) \times 10 \times 0.2387 = 0.00715.$$

Heat which would be given out at the cold junction by the same current in the same time is

$$q_c = 1028 \times 10^{-8} \times (273 + 26.6) \times 10 \times 0.2387 = 0.00735.$$

Hence the heat which would be absorbed in the Thomson effect in both metals by the same current in the same time is

$$q_T = 0.00101 - (0.00715 - 0.00735) = 0.00121.$$

Letting ν_c and ν_i stand for the Thomson coefficients in copper and iron respectively, we have

$$\nu_c - \nu_i = q_T \div (71.1 - 26.6)\left(273 + \frac{26.6 + 71.1}{2}\right) = 844 \times 10^{-10}.$$

Taking $\nu_i = -757 \times 10^{-10}$, the mean value found in this paper, we get

$$\nu_c = 844 \times 10^{-10} - 757 \times 10^{-10} = 87 \times 10^{-10}.$$

King, in the paper already referred to, gave evidence from which he concluded that the Thomson effect σ for copper diminishes with rise of temperature. The ν would of course diminish still more rapidly. He worked out values of σ for several temperatures, with various assumptions as to the thermal conductivity of his copper and the radiation from its surface. The smallest value of ν which comes from any of his estimates of σ is 103×10^{-10} at 208°; the largest value, 174×10^{-10} at 122°.

The fact that King's smallest value of ν is considerably larger than the value of ν_c deduced from the ν_i found in this paper for iron may indicate that the ν_i as here found is somewhat too large. It is to be observed that a reduction of a little more than two per cent in the numerical value used for ν_i would bring the value of ν_c up to the minimum value found from King's data. It is quite possible that the value 757×10^{-10} found for ν_i is several per cent in error.

We are at present making preparations for a study of the properties of our iron between 100°C. and 200 ° C.

It may be advisable to make a direct determination of the Thomson effect in copper and various other metals with apparatus like that described in this paper, but the difficulty of a direct study of copper would be very considerable.

The properties of the very soft iron studied in this paper, so far as they are quantitatively known, are summarized below:

Composition: iron, 99.93 %; carbon, 0.059%.
Density: about 7.785 at 0° C.
Thermal conductivity, k: 0.1528 at 28.2° C.,
with a temperature coefficient 0.0003 (?) from 28° to 58°,
" " " " 0.0007 (?) " 13° " 87°,
the latter value being the more reliable.
Electric resistance, absolute: 11365 at 0° C.,
with a mean temperature coefficient 0.00519 from 0° to 100°.

"Thermo-electric height" with copper:

$$\text{approximately} \begin{cases} 1028 \times 10^{-8} \text{ volt at } 26.6^\circ \text{ C.,} \\ 980 \text{ " " " } 41.3^\circ \text{ ,} \\ 936 \text{ " " " } 54.5^\circ \text{ ,} \\ 870 \text{ " " " } 71.1^\circ \text{ ,} \end{cases}$$

Thomson effect coefficient, ν:

$$\text{approximately} \begin{cases} -757 \times 10^{-10} \text{ mean, from } 13^\circ \text{ to } 90^\circ, \\ -715 \text{ " " " " " } 51^\circ. \\ -793 \text{ " " " } 51^\circ \text{ " } 90^\circ. \end{cases}$$

PART II: DETAILS.

Preparation of the Main Bars.

The two main soft-iron bars, α and β in Figure 3, were cut from the same piece of metal, and each was marked at one end. The two marked ends came from the same end of the original bar, and were placed at the same end of the apparatus for the main experiment.

For an insulating layer on the narrow part of these bars, paper baked on with asphaltum varnish was at first tried; but it did not adhere well, and the varnish alone was finally used, baked on in an electric oven.

An attempt was made to *cut* a groove for the fine wires of the thermo-electric couples in a second layer of asphaltum, baked on at 100° C. over a first layer baked on at 200° C.; but this failed, the varnish chipping off under the action of the cutting tool.

These experiments, and various others which need not be described, were made with trial bars of iron, not with the bars α and β, as the complete removal of the varnish from a rod in case of failure is a troublesome operation. The method finally adopted and used with bars α and β was substantially as follows: A thin layer of asphaltum varnish was laid on and, before baking, was allowed to harden at the ordinary temperature of the laboratory for three hours. This partial slow drying was to prevent the varnish from running and distributing itself unevenly during the baking, which lasted four or five hours at 200° C. Then a second thin coating of varnish was put on and allowed to dry at the temperature of the laboratory for two days; then a third thin layer, which also was left to dry during two days. In the unbaked double layer thus provided a groove (see Figure 5) was made by means of a small, sharp-edged, brass disk, held in such a way as to allow revolution with little or no lateral motion and applied with moderate pressure to the rod, which was carried beneath the disk in a screw-like motion by the action of a gear-cutting lathe. After the grooves were thus made the bars were baked about four hours at 200° C. The varnish ran a little into the grooves during this heating, but not enough to cause serious difficulties.

The construction of the six thermo-electric couples which are placed on bars α and β has been described in connection with Figure 5. As a cement to be used temporarily during the placing of the couples in the grooves prepared to receive them, a mixture of beeswax and honey in equal parts was found satisfactory. After this placing, the windings of wire were covered with asphaltum, which was allowed to dry over night, after which the distance between the junctions of each couple was tested by means of calipers; and one or two junctions were moved a trifle in the still yielding varnish in order to make this distance, as nearly as might be, 1 cm. The windings were then recoated with asphaltum and baked at 100° for some hours, then cooled and re-examined, then coated once more at the points where the copper wires leave the rods, then finally baked for six hours at 200° C.

No grooves were made to receive the supplementary thermo-electric junctions (see F and I, Figure 4), which were put on to give the temperatures of the midway points of the various couples now fixed, according to the method just given, on the bars; for it was not necessary to place these supplementary junctions with any great exactness. They were at first put on with a thick layer of asphaltum, which was allowed to dry for some ten days before baking; but when baking was attempted the heat so softened this coating that it was necessary to start anew. Success in fastening these junctions was finally attained by keeping the new coating of varnish in which they were imbedded at 50° for several hours before the final baking. The worst placed of these junctions at the last was the middle one on bar a, which was about half a millimeter too far toward the KK' end of the bar. Such a displacement must have produced certain temporary dissymmetries in the observations, but, with the various reversals which were practised, cannot have had any serious effect on the final result.

Calibration of Thermo-electric Couples.

With the same kinds of wire, and the same methods of annealing and soldering, Mr. Churchill made, for calibration use, four more thermo-electric couples, *A*, *B*, *C*, and *D*. The german-silver wire in each of these was about 1.6 m. long. Near each end it ran through a glass tube about 40 cm. long, to keep it from accidental contact with the copper wire, which was lashed to the outside of the tube. For a calibration test each junction, with a considerable part of the glass tube above it, was placed in a mixture of ice and water or in water near the temperature of the room, or in the vapor of some boiling liquid, as the bulb and stem of a thermometer under test would be placed. The temperatures, except those of melting ice and boiling water, were determined by means of one or the other of a pair of Baudin thermometers, which Mr. Churchill had carefully compared with a Tonnelot corrected to the hydrogen scale by the Bureau International des Poids et Mesures.

Several temperatures between 0° and 100° were used, the room temperature, about 18°, and the boiling-points of ether, bisulphide of carbon, chloroform, ethylacetate, and benzene, respectively. Using all four of the couples *A*, *B*, *C*, and *D* in each test, Mr. Churchill found substantially the following mean values of the "thermo-electric height" for the temperatures given alongside :

Temp.	Volt.
10° C.	1686×10^{-8}
20°	1734×10^{-8}
30°	1782×10^{-8}
40°	1835×10^{-8}
50°	1878×10^{-8}
60°	1925×10^{-8}
70°	1972×10^{-8}
80°	2022×10^{-8}
90°	2073×10^{-8}

These numbers indicate a very nearly constant rate of increase of the sensitiveness of a copper-german-silver couple with rise of mean temperature on the hydrogen scale.

As Mr. Churchill left in Cambridge no detailed account of this calibration work, it seemed advisable to repeat some of it as a check. Accordingly on November 5, 1904, couples *A*, *B*, *C*, and *D* were tested again, each with one junction in a mixture of ice and water, and the other junction in steam, each junction being subjected to both conditions in the course of the trial. The result was somewhat alarming, as it indicated for the sensitiveness of the couples a value about 6 per cent less than Mr. Churchill had found; and further study of the matter seemed necessary.

In the experiment of November 5, as in those of Mr. Churchill, the junctions had been exposed naked to the water and to the steam, a test made with one junction in water and one in alcohol, the two being at or very near the same temperature, having appeared to show that no ill effects need be feared from the chemical action or the electrical conductivity of the water. After November 5, however, this practice of direct exposure of the junctions was abandoned, and in all subsequent calibration tests the glass tubes carrying the junctions were placed in glass tubes, about 38 cm. long and 0.8 cm. in diameter of bore, containing paraffine oil ; and these outer tubes were exposed to the water and the vapors used. The thermometers also were placed in similar oil tubes, lest the direct action on them of the water and the vapors might affect them more rapidly than the junctions would be affected through the oil.

With this arrangement another test of couples *A*, *B*, *C*, and *D* was made November 18, with ice and water and steam, as on November 5. This gave a result about 1.5 per cent less than the one found by Mr. Churchill for the same range of temperature.

On December 2, a test of the same couples was made with a range of temperature from 0° to 56.9° (vapor of boiling acetone); and the result was a value about 2 per cent less than Mr. Churchill had found for the same range.

About the first of December, 1904, four new calibration couples were made from the same kinds of wire that had been used before. In the present case the german-silver wire was annealed, in pieces about 1.2 m. long, by sending through each piece a current of about three amperes (requiring about thirty-five volts) until the wire glowed brightly in a darkened room. Each piece stretched a good deal during the heating and was allowed to hang as a free catenary in the air; but on cooling it returned, apparently, to its original length. Soft solder was used in making these new couples, which will be called A', B', C', and D', respectively. These couples were mounted and were tested in oil, just as the couples *A*, *B*, *C*, and *D* had been mounted and tested at the last. On December 5, experiments with A', B', C' and D' gave a result about 1.2 per cent greater for the range from 0° to 56.7° than Mr. Churchill had found for the same range. On December 6, with a range from 0° to 100°, experiments with the same junctions gave a result about 1.7 per cent greater than Mr. Churchill's for the same range.

Taken all together, the tests now described, made with couples *A*, *B*, *C*, *D*, A', B', C', and D', in oil, confirmed the values found by Mr. Churchill; and accordingly his values were used in reducing the observations made with the couples which he had made and had attached to the main iron bars.

Perhaps more attention has been given to this matter than the immediate object in view, the determination of the Thomson effect, required. For, as the couples were used, an error of, for example, 1 per cent in the reduction factor applied to their indications would make, to be sure, an error of one per cent in the estimation of the amount of heat generated, but it would make also an error of one per cent in the same direction in the estimation of the interval of temperature between the junctions on the bars. These two errors would balance each other, except in this particular, that they would leave an error of about one sixth of one per cent in the estimation of the mean temperature of the bars in which the Thomson effect occurs.

Further and more exacting uses of these couples, or others similar, are in view, however; and the study of their behavior will be continued. See p. 54.

Resistances.

The resistances used in this investigation were, for the most part, in the form of detached coils of manganine, made and tested by the methods which Professor P. O. Peirce has established at the Jefferson Laboratory. The greatest proportional error found in any of these coils when they were tested at the end of the work, in December, 1904, was about one third of one per cent in a certain 10 ohm coil, which proved not to be made of the best wire. Error from the use of this coil was probably not more than one seventh of one per cent in any important place, and this error has been disregarded in the calculations.

The resistances of the thermo-electric couples and of the connecting wires of the various circuits were usually measured by means of a well-tested Wheatstone bridge, in box form.

Current Measurers.

On and after August 31, 1904, the instrument used for the measurement of thermo-electric currents was a three-cased *Panzer-galvanometer* of the du Bois and Rubens design. This was entirely satisfactory in regard to sensitiveness and freedom from external magnetic disturbances; but on the other hand its sensitiveness, when great, was subject to large and quick changes, sometimes amounting to several per cent in a single hour, and accordingly it was necessary to make frequent measurements of its reduction factor. For this purpose a potentiometer method was used, with a cadmium cell as a standard of electromotive force and with a number of well-tested resistances. Compared, about the middle of November, 1904, with a number of other cells in the care of Professor B. O. Peirce, the cell here used appeared to have an electromotive force of 1.0178 volt. Tested more carefully in March, 1905, under the direct supervision of Professor Peirce, it showed, near 16° C., an electromotive of 1.0184 volt.

The galvanometer coils were marked by the manufacturer as having 5.5 ohms and 5.3 ohms resistance, respectively, at 20° C., and this certificate was found to be correct.

The heavier of the two suspension systems was used, and no attempt was made to attain the highest sensitiveness. With a scale distance not far from 2.5 meters this galvanometer usually gave about 1 cm. deflection for a current of 10^{-8} amperes, without reversal.

The current, about 25 amperes, which was sent through the iron bars, was measured by means of a shunted ammeter, which, tested * in January, 1905, appeared to have, in the part of the scale used in the experiments of this paper, an error less than one part in one thousand. Tested two or three months later at the Jefferson Laboratory, on the basis of a Crompton standard resistance and a cadmium cell, it read about one half of one per cent low in the same part of the scale. It is assumed in this paper that the instrument gave correct readings as it was used in 1904.

Disturbing Currents.

Much care was taken to avoid accidental thermo-electric currents. For example, unnecessarily thick layers of solder at joints between pieces of copper were studiously avoided. It was not practicable, however, to get rid of mercury contacts, and slight differences of temperature near such contacts sometimes gave currents sufficiently great to require especial attention. The mere operation of a commutator having pieces of copper dipping into little mercury wells, the reversals occurring rather oftener than once a minute, appeared to produce a cumulative disturbing effect, the slow increase going on for hours. The various reversals of other connections which were regularly practised had a tendency to eliminate the effect of such disturbances from the final result; but in some cases it was necessary to measure the illegitimate current regularly and make allowance for it.

Deflections of the galvanometer were sometimes noticed which appeared to be due to slight currents entering the galvanometer circuit from some external non-metallic path. For example, when connections were made for getting the reduction factor of the galvanometer, if the circuit of this instrument remained connected at one point with the thermo-electric couple wound on the *cold* end of either iron bar, α or β, and if at the same time the commutator leading the main current into the iron bars was closed, the galvanometer showed a considerable illegitimate current, the direction and magnitude of which were not changed by reversing the commutator of the twenty-five ampere current, though the disturbance disappeared when this commutator was opened. The storage battery furnishing the main current was in the basement, and the galvanometer rested on a brick pier which rose from this basement. The disturbance was apparently not due to the storage battery, as its direction was independent of the direction of the current entering the Thomson effect bars from this battery. It probably was due to battery action between water and metals at the cold end of the apparatus, causing a slight current to creep along the varnished surface of the board B (Figure 6), this surface being slightly dampened by precipitation of water from the air, thence along a metal path through the galvanometer, thence by way of the pier surface, or some other damp non-metallic path, to the basement, thence by way of the main current leads to the Thomson apparatus again. When connection was made with the wires leading from the *hot* end of the board *B*, no disturbing effect due to this connection could be made out with certainty. There was, of course, no need of having any metallic connection between the Thomson effect apparatus and the galvanometer circuit during the measurement of the reduction factor; but for convenience the one-point connection described above was frequently left during the early part of the work, before the possibility of trouble from it was thoroughly realized. Fortunately this connection was made, as a rule, with the hot end of the Thomson effect apparatus, so that when the danger was discovered it was necessary to reject on this score the work of one or two days only.

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* By the kindness of Professor Laws at the Massachusetts Institute of Technology.

Moisture at the ends of the board *B* was, indeed, a frequent and serious cause of trouble, especially at first. Sometimes it came from leakage past the rubber gasket which separated each vessel from its cover or top. To avoid this leakage the level of the water in each vessel was, after a time, kept below the level of the gasket, and care was taken not to let the boiling become so violent as to clog the condenser return pipe and raise the pressure of steam in the boiler. At the cold end, especially in warm weather, profuse precipitation of water from the air was capable of drenching the cotton lying on the board. To prevent this, the top of the cold vessel was swathed in cotton to a depth of about 2 cm., and outside this cotton was placed a covering of thin sheet rubber.

Much trouble would have been saved by placing the board *B* considerably higher, and by having the end posts upon it placed nearer the middle; but it is unlikely that any serious error due to the presence of water on this board survived the daily reversals of current and the long course of observation to which the apparatus was subjected.

Was a Stable Condition Reached?

The question how long the Thomson effect apparatus would require in order to reach a practically stable condition of temperature, after the heating at one end and the cooling at the other had begun, was one which presented itself and was considered long before the apparatus was set up. With the assumption of no lateral flow of heat from the bars, calculation, by a well-known use of Fourier's series, showed that two hours would be ample time. But such a calculation could not be decisive for the actual conditions of the experiment, where lateral flow did exist to a considerable extent, and where the thermal condition of the board B (Figure 6) might affect in some measure the thermo-electric currents passing through the wires lying upon it. The usual practise in a day's work was to have the thermal conditions at the ends of the apparatus established at least two hours, and the electric current in the bars at least one hour, before observations on the Thomson effect were begun; but it was not taken for granted that a stable condition was reached by this preliminary delay. On the contrary, of the six cycles, or runs, made, as a rule, each day (see p. 31), the first cycle and the first cycle after reversal of the current through the bars, which latter cycle usually began about one hour after the reversal, were at first regarded with much suspicion. Accordingly, the mean result obtained by a rather summary calculation from these suspected cycles in fourteen * days of work was compared with the mean result obtained, in a similar way, from all the cycles of these fourteen days. The difference of the two means was found to be less than one third of one per cent of either mean, and therefore all cycles were treated as entitled to equal confidence.

Nevertheless, observations made with the express purpose of getting data for an estimation of the lateral flow of heat from the apparatus showed that perceptible progressive changes of gradient were going on in the bars after several hours of fixed terminal condition. These changes were probably not of a character to effect perceptibly the differential of gradients, with which we are mainly concerned.

The Lateral Loss of Heat.

This loss has already been referred to, and the general method by which it was estimated has been given on pp. 34 and 35; but some details of the methods used in getting the data needed in making this estimate may be desirable.

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* There were fifteen such days of work; but the data of the first day were not sufficiently full to be used here.

On November 23, with boiling water in both boxes of the Thomson effect apparatus from 12.45 o' clock on, observations made first with the couples *K* and *M* (see Figure 3) joined in series, then with the junctions *N* and *P* joined in series, then with *K* and *M* once more, and so on alternately, indicated gradients of temperature proportional, approximately, to the numbers given below:

Time	K-M		N-P	
3.11	165			
3.24			169	
3.33	151			
4.10	134			
4.21			129	
4.29	125			
4.58	121			
5.08			112	
5.15	114	116		110
5.23			109	
5.31	114			

It is evident that an approximately stable condition had been reached during the last hour of the observations; and the last three values found from *K–M*, with the last two found from *N–P*, were taken as indicative of the temperature gradient in this stable condition. The average value of this gradient at the four ends, as estimated from these observations, was 0.104° per centimeter. A corresponding set of observations made December 8 gave 0.108° per centimeter as the mean gradient at the four ends.

As the gradient at the middle of the bars must have been practically zero, we cannot err greatly in taking the mean gradient from the ends to the middle of each 20 cm. cylindrical part as 0.06° per centimeter, and the temperature at the middle as 0.6° lower than that at the ends of this part. This estimate is in fair agreement with the result of a rough direct measurement of this difference made October 29. The mean temperature of that part of bar α which lies between the mid point of KK' and the mid point of MM' must have been not far from 0.3° lower than the temperature of the water at the ends; and a like estimate holds for bar β.

On December 8 an attempt was made to find the temperatures at various points on the guard-ring bars, by means of thermo-electric couples *X*, *Y*, *Z*, *Z′*, *Y′* and *X′*. One junction of each of these couples was attached, with a thin layer of sheet rubber between, to one of the guard-ring bars. Thus, one junction in the case of couple *Z* (see Figure 4) was at the middle of bar 3; *Y* was on the same bar, 4.25 cm. from the middle toward the left end; *X* on the same bar 4.25 cm. beyond *Y*. *Z′* was on bar 3 at the middle; *Y* was on the same bar 4.25 cm. from the middle toward the right end; *X′* on the same bar 4.25 cm. beyond *Y′*.* The placing of these junctions was not very exact. The outer junction of each couple was placed in oil in a glass tube about 10 cm. long and perhaps 0.4 cm. in diameter of bore. These tubes were placed in a mixture of ice and water. The couples *X*, *Y*, *Z*, etc., had not been directly calibrated; but similar junctions, *A′*, *B′*, *C′*, and *D′*, made at the same time from the same wire treated in the same way, had been tested for the purpose of getting the reduction factors to be used with *X*, *Y*, *Z*, etc. The use of these factors, applied to the observations made December 8, gave as the temperature of the middle points of the guard-ring bars, 95.1° C.; as the temperature of points 4.25 cm. nearer the ends, 96.0°; as the temperature of points 4.25 cm. nearer still to the ends, 98.3°. It was evident, however, that no great confidence could be placed in the exactness of these numbers; for, with nearly a hundred degrees difference of temperature between the junctions of a couple, an error of one per cent in the reduction factor would make an error of nearly one degree in the estimation of the temperature of the hot junction. Accordingly, on December 22 a new set of observations was made with couples *X*, *Y*, *Z*, etc., the oil tubes containing the outer junctions being in this case placed in steam over boiling water. The temperature of this steam, as found from the barometric pressure, was 100.3° C. The temperature found for the *X* point of the guard-ring bar 8 was 97.8°; for the *Y* point of the same bar, 95.4°; for the *Z* point, 94.2°. For the *X′* point of the bar 3 the temperature found was 96.5°; for the *Y′* point, 94.7°; for the *Z′* point, 93.5°. Thus, the mean temperature indicated for bar 3 was about 0.9° lower than the mean temperature indicated for bar 8. This is hardly surprising, as these two bars lie a quarter circle apart in the guard-ring, and the region of the lower one, No. 8, is somewhat more shielded than the region of the upper bar. Moreover, the ends of the uppermost guard-ring bars were not all the time fully covered by the water in the end vessels. In fact, these two bars were chosen to carry the junctions X, Y, Z, etc., for the reason that they seemed fairly representative, one of conditions in the upper half of the guard-ring, the other of conditions in the lower half.

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* The intention was that junction *X* should be just opposite, just as far from the adjacent end vessel as the mid points of the couples *K K′* and *N N′*, and that *X′* should have a like relation to *M M′* and *P P′*. All the junctions on α and β were entirely hidden when *X*, *Y*, *Z*, etc., were placed, and the mid points of *K K′* and *M M′* were supposed to be 17 cm. apart instead of 16 cm.

The observations of December 22 indicate that the mean temperature of the guard-ring bars, for the 17 cm. length extending from the vertical plane of the point X to the plane of the point X', was at the time of these observations about 5° below the temperature of the boiling water in which the ends of the bars were immersed; but a close examination of the observations shows that the apparatus as a whole had not yet reached a perfectly stable condition of temperature. The thermoelectric observations began when the water at the ends of the bars had been boiling not much more than two hours and ended about two hours later; and the earlier observations of the series showed a slightly lower temperature for the bars than the later ones. It is not unlikely that, if the experiment had continued some hours longer, the mean difference of temperature between the boiling water and that part of the guard-ring which lies between the planes of X and X' would have fallen as low as 4.5°; and this is taken as the final value. See p. 35.

On December 9, 1904, with all the usual conditions of an experiment on the Thomson effect, observations were made for the purpose of determining, approximately, the temperatures of the mid points of the couples KK', NN', etc., on the bars α and β. The outer junctions belonging to E, F, G, etc., were placed in oil tubes, in ice and water, and after the terminal thermal conditions of the main apparatus had been established, and the main electric current had been running nearly eight hours, observations on the current from the couples J and G., joined in series with each other, were made. They were followed by observations on couples I and F, joined in series; and so on. The general course of these observations was very satisfactory, and it indicated the following values:

Mean temperatures of the points J and $G = 90.2°$
“ “ “ I “ $F = 51.0°$
“ “ “ H “ $E = 13.0°$

On the same day, and after the observations just described, similar observations were made with the couples X, Y, Z, etc., taken singly. The following values were indicated:

Temperatures of the point $X' = 88.7^\circ$
" " $Y' = 67.8^\circ$
" " $Z' = 49.0^\circ$
" " $Z = 49.4^\circ$
" " $Y = 33.2^\circ$
" " $X = 12.0^\circ$

As the junctions X, Y, etc., were not very accurately placed on their respective bars, it would have been desirable to make a corresponding set of observations on these junctions with the X end hot and the X' end cold, if there had been any need of using these last guard-ring temperatures in the calculation of the Thomson effect. That there was no such need is shown by the considerations given on p. 35.

TEMPERATURE COEFFICIENT OF THERMAL CONDUCTIVITY.

The measurements designed especially for the determination of this coefficient (see p. 33) were made in February, 1905, after the observations on the Thomson effect were ended. These especial measurements involved few, if any, considerations that have not been set forth in the preceding pages.

With no electric current in α and β, but with a permanent gradient of temperature established therein, let

g_h = gradient of temperature at the hot end,
g_c = " " " " " cold " ,
t_h = temperature of mid point of hot end couple,
t_c = " " " " " cold " " ,
k_h = thermal conductivity at temperature t_h,
k_c = " " " " t_c,
f = temperature coefficient of thermal conductivity.

Then, if there were no lateral flow of heat from α and β, we should have, neglecting the small effect of expansion of the iron,

$$k_h \times g_h = k_c \times g_c, \quad \text{or} \quad \frac{k_h}{k_c} = \frac{g_c}{g_h}, \quad \text{and so}$$

$$(8) \quad f = \frac{k_h - k_c}{k_c} \div (t_h - t_c) = \left(\frac{k_h}{k_c} - 1\right) \div (t_h - t_c) = \left(\frac{g_c}{g_h} - 1\right) \div (t_h - t_c).$$

Observations on the ratio $g_c \div g_h$ were made on four days, with the following results:

	$g_c \div g_h$	t_c	t_h	
Feb. 1	0.9312	13.0°	[87.3°]	Right end hot
" 4	0.9320		[87.5°]	" " "
" 7	0.9478	12.0°	87.3°	" " cold
" 9	0.9460	12.7°	[87.2°]	" " "
Means	0.9392	12.6°	87.3°	

The marked difference between the first two and the last two values found for $g_c \div g_h$ is due to the reversal of the direction of flow, there being, no doubt, small differences in the placing of the thermo-electric couples at the ends of the bars. See p. 27. The reversal should eliminate error due to this source.

The value 87.3°, found for t_h on February 7, was obtained from observations made with thermo-electric couple H, the outer junction of this couple being placed, on this day, in oil heated to a temperature not far from that of the inner junction, so that uncertainty of calibration of the couple had little effect on the result. The other values of t_h, which are given in brackets, were estimated from this value, with use of the barometer observations of the various days as an indication of the various temperatures of the boiling water.

Estimation of the loss of heat by lateral flow required observations on the difference of temperature between points on the main bars, with no electric current on, and corresponding points on the guard-ring bars. Such observations were obstructed, and in some measure restricted, by the breaking of one of the thermo-electric wires and by the inadequate length of several others, the full scope of their possible use not having been foreseen when the couples were made. The results were as follows:

	On α and β.			On guard-ring.		
	t_c	t_m	t_h	t_c	t_m	t_h
Feb. 1	13.0° [*E–H.*]					
" 4		49.3° [*I*]			48.6° [*Z*] 47.8° [*Z′*]	87.1° [*X′*]
" 7	12.0° [*G*]	48.7° [*I*]	87.3° [*H*]	11.3° [*X′*]	47.4° [*Z′*] 48.0° [*Z*]	86.7° [*X*]
" 9	12.7° [*G*]			12.1° [*X′*]		
	12.6°	49.0°	87.3°	11.7°	48.0°	86.9°

Taking account now of the fact that the junctions X and X' were not placed just opposite H and G (see p. 50), and estimating for points which are opposite H and G, we get

On α and β.			On guard-ring.		
t_c	t_m	t_h	t_c	t_m	t_h
12.6°	49.0°	87.3°	14.0°	48.0°	84.6°

Accordingly we get 0.8° as an approximate value for the mean excess of temperature of the bars α and β, for the significant 16 cm. between mid points of hot and cold couples, above the temperature of the corresponding part of the guard-ring bars. Now from p. 35 it appears that the lateral flow from the same 16 cm. of each bar, when it is 4.2° warmer than the guard-ring, is 0.0242 calorie per second. With a difference of 0.8° we may take the lateral flow from each bar as 0.0046 calorie per second, about 0.9 per cent of the amount which is conducted along the hot end of the bar by a gradient of 4.6° per centimeter.

Making correction accordingly, and taking account also of the slight effect produced by the expansion of the main bars, we get from the value 0.9392, given above for $g_c \div g_h$, the ratio $k_h \div k_c = 0.947$; and hence

$$f = \left(\frac{k_h}{k_c} - 1\right) \div (t_h - t_c) = (0.947 - 1) \div (87.3 - 12.6) = -0.0007.$$

This study of the value of f raised again the question of the accuracy of calibration of the thermo-electric couples; and a good deal of additional attention was given to this matter in February, as the following summary will show:

	Thermo-electric Height (mean) for A', B', C', D.	Mean t.	Range of t.
Feb. 14	2056×10^{-8}	89.3°	78.5° to 100.2°
" 16 and 17	1697×10^{-8}	17.3°	0° to 34.6°
	A, B, C, D.		
" 20 and 21	2054×10^{-8}	89.5°	78.6° 100.4°
" 23	1709×10^{-8}	17.4°	0° 34.8°

Combination gives

2055×10^{-8} at 89.4°,
1703×10^{-8} at 17.4°.

Mr. Churchill found values from which we get

2070×10^{-8} at 89.4°,
1722×10^{-8} at 17.4°.

Ratios of Churchill values to February values

$$2070 \div 2055 = 1.0073 \text{ at } 89.4°,$$
$$1722 \div 1703 = 1.0111 \text{ at } 17.3°.$$

Change from one set of calibration values to the other would make but little difference in the value found for the Thomson effect coefficient and no large difference in the value found for f. Accordingly, the thermo-electric heights found by Mr. Churchill have been retained in all the important calculations of this paper.

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THE SCALAR FUNCTIONS OF HYPERCOMPLEX NUMBERS.

By Henry Taber.

THE SCALAR FUNCTIONS OF HYPERCOMPLEX NUMBERS.

BY HENRY TABER.

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IN a recent number of the *Transactions of the American Mathematical Society* [*] I have extended the quaternion scalar function to hypercomplex number systems in general, establishing by the aid of this function certain important theorems in the theory of hypercomplex numbers. [†] I find that there are two generalizations of the quaternion scalar function. Thus, denoting (as, throughout this paper) the constants of multiplication of any given hypercomplex number system $e_1, e_2, \ldots e_n$, by γ_{ijk}, for $i, j, k = 1, 2, \ldots n$, — when we have

$$e_i e_j = \sum_{1}^{n}{}_{i} \gamma_{ijk} e_k \qquad (i,j = 1, 2, \ldots n),$$

and, by

$$A = \sum_{1}^{n}{}_{i} a_i e_i,$$

any number of the system, I employ SA and $\bar{S}A$ to denote those functions of the coefficients a and of the constants of multiplication defined as follows:

$$SA \equiv \frac{1}{n} \sum_{1}^{n}{}_{i} \sum_{1}^{n}{}_{j} a_i \gamma_{ijj}, \qquad \bar{S}A \equiv \frac{1}{n} \sum_{1}^{n}{}_{i} \sum_{1}^{n}{}_{j} a_i \gamma_{jij}.$$

When $n = 4$, and e_1, e_2, e_3, e_4 are the units of quaternions, these functions both coincide with the quaternion scalar function as customarily defined. Thus, if $e_4 = 1$, and e_1, e_2, e_3 are three mutually normal unit vectors, we have

$$\gamma_{ijj} = 0 = \gamma_{jij}, \quad \gamma_{4jj} = 1 = \gamma_{j4j} \quad (i = 1, 2, 3\,;\; j = 1, 2, 3, 4)\,;$$

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[*] Vol. 5 (Oct., 1904), p. 514.

[†] In the Proc. Lond. Math. Soc. for Dec., 1890, vol. 22, p. 67, I had previously extended the scalar function of quaternions to matrices in general, employing this generalization to prove certain theorems of Sylvester's.

and, therefore,

$$\bar{S}\,(a_1 e_1 + a_2 e_2 + a_3 e_3 + a_4 e_4) = a_4 = S\,(a_1 e_1 + a_2 e_2 + a_3 e_3 + a_4 e_4)$$

by the above definition. For hypercomplex number systems in general,

$$\sum_{1}^{n}{}_{i} \sum_{1}^{n}{}_{j}\, a_i\,\gamma_{jij} \neq \sum_{1}^{n}{}_{i} \sum_{1}^{n}{}_{j}\, a_i\,\gamma_{ijj},$$

that is, $\bar{S} A \neq S A$ for every number A of the system.[*] The chief properties of the two scalar functions, $S A$ and $\bar{S} A$, are enumerated in *Theorems I* and *II* below; and in *Theorem III*, I give certain properties of these functions relating to nilpotent numbers.

Throughout this paper I shall denote by R the domain of rationality of any arbitrary aggregate of scalars including the constants γ_{ijk} of multiplication of the number system $e_1, e_2, \ldots e_n$, and by $\mathfrak{B}\,(R, e_i)$ the hypercomplex domain of rationality constituted by the totality of numbers

$$A = \sum_{1}^{n}{}_{i}\, a_i\, e_i$$

of the system for which the a's are rational in R.[†] Any such number A will be termed *rational* in this hypercomplex domain. Further, I shall denote the units of the system reciprocal to $e_1, e_2, \ldots e_n$ by $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$, — when, if $\bar{\gamma}_{ijk}$ $(i, j, k = 1, 2, \ldots n)$ are the constants of multiplication of the latter system, that is, if

$$\bar{e}_i\,\bar{e}_j = \sum_{1}^{n}{}_{i}\, \bar{\gamma}_{ijk}\,\bar{e}_k \qquad (i, j, = 1, 2, \ldots n),$$

we have

$$\bar{\gamma}_{ijk} = \gamma_{jik} \qquad (i, j, k, = 1, 2, \ldots n);\text{[‡]}$$

and I shall write

$$\bar{A} = \sum_{1}^{n}{}_{i}\, a_i\,\bar{e}_i$$

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[*] Thus, let $n = 3$, and let e_3 be a modulus of the system,

$$e_1^{\,2} = e_1\,e_2 = 0, \qquad e_2\,e_1 = e_1, \quad e_2^{\,2} = e_2.$$

We then have

$$S A = a_3 + \tfrac{2}{3}\,a_2;$$

whereas,

$$\bar{S} A = a_3 + \tfrac{1}{3}\,a_2.$$

[†] See Trans. Am. Math. Soc., **5**, 513.

[‡] Encycl. d. Math. Wissensch., **1**, 163.

to denote the number obtained from the number

$$A = \sum_{1}^{n}{}_{i}\, a_i e_i$$

by replacing, severally, the units of the original system $e_1, e_2, \ldots e_n$ by the units $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$, respectively, of the reciprocal system.

Theorem I. Let R *denote the domain of rationality of any arbitrary aggregate of scalars including the constants* γ_{ijk} *of multiplication of any given hypercomplex number system* $e_1, e_2, \ldots e_n$. *Let*

$$A = \sum_{1}^{n}{}_{i}\, a_i e_i$$

be any number of the system; and let

$$S A \equiv \frac{1}{n} \sum_{1}^{n}{}_{i} \sum_{1}^{n}{}_{j}\, a_i \gamma_{ijj}, \qquad \bar{S} A = \frac{1}{n} \sum_{1}^{n}{}_{i} \sum_{1}^{n}{}_{j}\, a_i \gamma_{jij}.$$

Then both $S A$ *and* $\bar{S} A$ *are invariant to any linear transformation of the units of the system; and, if* ρ *is any scalar and* B *any second number of the system,*

$$\begin{aligned} S \rho A &= \rho S A, & \bar{S} \rho A &= \rho \bar{S} A, \\ S(A + B) &= S A + S B, & \bar{S}(A + B) &= \bar{S} A + \bar{S} B, \\ S A B &= S B A, & \bar{S} A B &= \bar{S} B A. \end{aligned}$$

If ε *is a modulus of the system,*

$$S \varepsilon = 1 = \bar{S} \varepsilon.$$

If A *is rational in the hypercomplex domain* $\mathfrak{R}\,(R, e_i)$, *then both* $S A$ *and* $\bar{S} A$ *are rational in* R; *and if, moreover,* A *is idempotent,*[*] *there are* $\begin{matrix} n\, S A > 0 \\ n\, \bar{S} A > 0 \end{matrix}$ *independent hypercomplex numbers, rational in* $\mathfrak{R}\,(R, e_i)$, *that are* $\begin{matrix} \textit{idemfaciend}^{\dagger} \\ \textit{idemfacient} \end{matrix}$ *with respect to* A, *in terms of which every number of the system* $\begin{matrix} \textit{idemfaciend} \\ \textit{idemfacient} \end{matrix}$ *to* A *can be linearly expressed, and there are*

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[*] If $A^2 = A \neq 0$, A is *idempotent*. Benjamin Peirce, Am. Journ. Maths., **4**, 104.

[†] If $A B = B$, B is *idemfaciend* with respect to A; if $B A = B$, B is *idemfacient* with respect to A. Peirce, loc. cit., p. 104.

$\begin{matrix} n\,(1 - S\,A) \geqq 0 \\ n\,(1 - \bar{S}\,A) \geqq 0 \end{matrix}$ *independent hypercomplex numbers, rational in* $\mathfrak{R}\,(R, e_i)$, *that are* $\begin{matrix} \textit{nilfaciend}\,^{*} \\ \textit{nilfacient} \end{matrix}$ *with respect to* A, *in terms of which any number of the system* $\begin{matrix} \textit{nilfaciend} \\ \textit{nilfacient} \end{matrix}$ *with respect to* A *can be linearly expressed. If* A *is nilpotent*,†

$$S\,A^p = 0, \qquad\qquad \bar{S}\,A^p = 0$$

for any positive integer p; *and, conversely, if either* $S\,A^p = 0$ *for every positive integer* p, *or* $\bar{S}\,A^p = 0$ *for every positive integer* p, A *is nilpotent. Finally, if the system contains* n *independent numbers* $A_1, A_2, \ldots A_n$ *for which*

$$S\,A_1 = S\,A_2 = \ldots = S\,A_n = 0,$$

or

$$\bar{S}\,A_1 = \bar{S}\,A_2 = \ldots = \bar{S}\,A_n = 0,$$

the system is nilpotent.‡

The proof of *Theorem I*, so far as it relates to $S\,A$, the first of the two scalar functions, I have given in the paper above referred to.§ This theorem may be demonstrated for the second scalar function, $\bar{S}\,A$, by the aid of the following theorems relating to the reciprocal systems $e_1, e_2, \ldots e_n$ and $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$.

Theorem (1). $\bar{S}\,A = S\,\bar{A}, \quad \bar{S}\,\bar{A} = S\,A.$

Theorem (2). If

$$e'_h = \tau_{h1}\,e_1 + \tau_{h2}\,e_2 + \ldots + \tau_{hn}\,e_n \quad (h = 1, 2, \ldots n),$$

and

$$\bar{e}'_h = \tau_{h1}\,\bar{e}_1 + \tau_{h2}\,\bar{e}_2 + \ldots + \tau_{hn}\,\bar{e}_n \quad (h = 1, 2, \ldots n),$$

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* If $A\,B = 0$, B is *nilfaciend* with respect to A; if $B\,A = 0$, B is *nilfacient* with respect to A. Peirce, loc. cit., p. 104.

† If $A^m = 0$, for some positive integer m, A is *nilpotent*. Peirce, loc. cit., p. 104.

‡ A system is *nilpotent* which contains no idempotent number. Peirce, loc. cit., p. 115.

§ Loc. cit., p. 514 et seq., and p. 531.

then the systems $e'_1, e'_2, \ldots e'_n$ and $\bar{e}'_1, \bar{e}'_2, \ldots \bar{e}'_n$ are reciprocal provided the determinant of the transformation is not zero.*

Theorem (3). If ρ is any scalar, and $\rho A = C$, then $\rho \bar{A} = \bar{C}$; and conversely. If $A + B = C$, then $\bar{A} + \bar{B} = \bar{C}$; and conversely. If $A B = C$, then $\bar{B} \bar{A} = \bar{C}$;† and conversely.

Theorem (4). If ε is a modulus of the system $e_1, e_2, \ldots e_n$, then $\bar{\varepsilon}$ is a modulus of the reciprocal system $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$; and conversely. If A is idempotent, so also is $\bar{A}$, and conversely; if A is nilpotent, so also is $\bar{A}$, and conversely. If B is either idemfaciend, or idemfacient, with respect to A, then $\bar{B}$ is either idemfacient, or idemfaciend, respectively, with respect to $\bar{A}$; and conversely. And, if B is either nilfaciend, or nilfacient, with respect to A, then $\bar{B}$ is either nilfacient, or nilfaciend, respectively, with respect to $\bar{A}$.‡

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* For let

$$e'_i e'_j = \sum_1^n{}_k \gamma'_{ijk} e'_k \qquad (i, j = 1, 2, \ldots n),$$

$$\bar{e}'_j \bar{e}'_i = \sum_1^n{}_k \bar{\gamma}'_{jik} \bar{e}'_k \qquad (i, j = 1, 2, \ldots n).$$

Then

$$\sum_1^n{}_h \sum_1^n{}_k \tau_{ih} \tau_{jk} \gamma_{hkl} = \sum_1^n{}_k \tau_{kl} \gamma'_{ijk} \qquad (i, j, l = 1, 2, \ldots n),$$

$$\sum_1^n{}_h \sum_1^n{}_k \tau_{jk} \tau_{ih} \bar{\gamma}_{khl} = \sum_1^n{}_k \tau_{kl} \bar{\gamma}'_{jik} \qquad (i, j, l = 1, 2, \ldots n);$$

therefore,

$$\sum_1^n{}_k \tau_{kl} (\gamma'_{ijk} - \bar{\gamma}'_{jik}) = \sum_1^n{}_h \sum_1^n{}_k \tau_{ih} \tau_{jk} (\gamma_{hkl} - \bar{\gamma}_{khl}) = 0 \qquad (i, j, l = 1, 2, \ldots n).$$

Whence follows

$$\bar{\gamma}'_{jik} = \gamma'_{ijk} \qquad (i, j, k = 1, 2, \ldots n),$$

since, otherwise, the determinant of the transformation is zero.

† If

$$A\, B = \sum_1^n{}_i a_i e_i \cdot \sum_1^n{}_i b_i e_i = \sum_1^n{}_i c_i e_i = C,$$

then

$$\sum_1^n{}_i \sum_1^n{}_j a_i b_j \bar{\gamma}_{jik} = \sum_1^n{}_i \sum_1^n{}_j a_i b_j \gamma_{ijk} = c_k \qquad (k = 1, 2, \ldots n);$$

and, therefore,

$$\bar{B}\, \bar{A} = \sum_1^n{}_i b_i \bar{e}_i \cdot \sum_1^n a_i \bar{e}_i = \sum_1^n{}_i c_i \bar{e}_i = \bar{C}.$$

‡ This theorem follows from the last clause of (3).

Theorem (5). If the system $e_1, e_2, \ldots e_n$ is nilpotent so also is the reciprocal system $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$; and conversely.*

Theorem (6). If A is rational in $\mathfrak{R}\,(R, e_i)$, then $\bar{A}$ is rational in $\mathfrak{R}\,(R, \bar{e}_i)$; and conversely.

Theorem (7). If $A_1, A_2, \ldots A_\mu$ are independent, so also are $\bar{A}_1, \bar{A}_2, \ldots \bar{A}_\mu$; and conversely.†

Assuming *Theorem I*, in so far as it relates to the first scalar function, SA, we are now in position to establish the theorem for $\bar{S}A$.

First, let

$$e'_h = \sum_1^n {}_j\, \tau_{hj}\, e_j \qquad (h = 1, 2, \ldots n),$$

$$\bar{e}'_h = \sum_1^n {}_j\, \tau_{hj}\, \bar{e}_j \qquad (h = 1, 2, \ldots n);$$

and let

$$e'_i\, e'_j = \sum_1^n {}_k\, \gamma'_{ijk}\, e'_k \qquad (i, j = 1, 2, \ldots n),$$

$$\bar{e}'_i\, \bar{e}'_j = \sum_1^n {}_k\, \bar{\gamma}'_{ijk}\, \bar{e}'_k \qquad (i, j = 1, 2, \ldots n).$$

Then, by (2),

$$\bar{\gamma}'_{ijk} = \gamma'_{jik} \qquad (i, j, k = 1, 2, \ldots n).$$

If

$$A' = \sum_1^n {}_i\, a'_i\, e'_i = \sum_1^n {}_i\, a_i\, e_i = A,$$

then

$$\sum_1^n {}_i\, a'_i\, \tau_{ij} = a_j \qquad (j = 1, 2, \ldots n);$$

and, therefore,

$$\bar{A}' = \sum_1^n {}_i\, a'_i\, \bar{e}'_i = \sum_1^n {}_i\, a_i\, \bar{e}_i = \bar{A}.$$

But, by *Theorem I*, $S\bar{A}$ is invariant to any linear transformation of the units of the system $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$; that is,

$$\sum_1^n {}_i \sum_1^n {}_j\, a'_i\, \bar{\gamma}'_{ijj} = \sum_1^n {}_i \sum_1^n {}_j\, a_i\, \bar{\gamma}_{ijj}.$$

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[*] For, if $e_1, e_2, \ldots e_n$ contains an idempotent number A, then $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$ contains an idempotent number $\bar{A}$ by (4).

[†] This follows from the first two clauses of (3).

Whence follows

$$\sum_1^n{}_i \sum_1^n{}_i \, a'_i \, \gamma'_{jij} = \sum_1^n{}_i \sum_1^n{}_j \, a_i \, \gamma_{fij};$$

that is, $\bar{S} A$ is invariant to any linear transformation of the units of the system $e_1, e_2, \ldots e_n$.

Let ρ be any scalar, and A and B any two numbers of the system $e_1, e_2, \ldots e_n$. Then, by (1), (3), and *Theorem I*,

$$\bar{S}(\rho A) = S(\rho \bar{A}) = \rho S \bar{A} = \rho \bar{S} A,$$
$$\bar{S}(A + B) = S(\bar{A} + \bar{B}) = S \bar{A} + S \bar{B} = \bar{S} A + \bar{S} B,^*$$
$$\bar{S}(A B) = S(\bar{B} \bar{A}) = S(\bar{A} \bar{B}) = \bar{S}(B A).^\dagger$$

If ε is a modulus of the system $e_1, e_2, \ldots e_n$, then by (4), $\bar{\varepsilon}$ is a modulus of the system $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$; and therefore, by (1) and *Theorem I*,

$$\bar{S} \varepsilon = S \bar{\varepsilon} = 1.$$

Let A be rational in $\mathfrak{R}$ (R, e_i). Then by (6), $\bar{A}$ is rational in $\mathfrak{R}$ $(R, \bar{e}_i)$; therefore by *Theorem I*, $S \bar{A}$, and thus, by (1), $\bar{S} A = S \bar{A}$ is rational in R. Let, moreover, A be idempotent. Then, by (4), $\bar{A}$ is idempotent. Therefore, by *Theorem I*, there are

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* The equations $\bar{S} \rho A = \rho \bar{S} A$ and $\bar{S}(A + B) = \bar{S} A + \bar{S} B$ are immediate consequences of the definition of $\bar{S} A$.

† Since $e_h \cdot e_i e_j = e_h e_i \cdot e_j$ we have

$$\sum_1^n{}_k \gamma_{ijk} \gamma_{hkl} = \sum_1^n{}_k \gamma_{hik} \gamma_{kjl} \qquad (i, j, h, l = 1, 2, \ldots n);$$

and, therefore,

$$\bar{S} A B = \frac{1}{n} \sum_1^n{}_i \sum_1^n{}_j \sum_1^n{}_k \sum_1^n{}_h \, a_i b_j \gamma_{ijk} \gamma_{hkh}$$
$$= \frac{1}{n} \sum_1^n{}_i \sum_1^n{}_j \sum_1^n{}_k \sum_1^n{}_h \, a_i b_j \gamma_{hik} \gamma_{kjh},$$
$$\bar{S} B A = \frac{1}{n} \sum_1^n{}_i \sum_1^n{}_j \sum_1^n{}_k \sum_1^n{}_h \, a_j b_i \gamma_{ijk} \gamma_{hkh}$$
$$= \frac{1}{n} \sum_1^n{}_i \sum_1^n{}_j \sum_1^n{}_k \sum_1^n{}_h \, a_j b_i \gamma_{hik} \gamma_{kjh}.$$

Whence follows $\bar{S} A B = \bar{S} B A$, since the two last members of these two equations are equal, as may be seen by the interchange in either of i and j, and of h and k.

$\mu = n\,S\bar{A} > 0$ independent hypercomplex numbers $\bar{A}_1, \bar{A}_2, \ldots \bar{A}_\mu$, rational in $\mathfrak{K}(R, \bar{e}_i)$, that are idemfaciend with respect to $\bar{A}$; and in terms of these any number idemfaciend to $\bar{A}$ can be expressed linearly. By (4), (6), and (7), $A_1, A_2, \ldots A_\mu$ are independent, rational with respect to $\mathfrak{K}(R, e_i)$, and idemfacient with respect to A. Moreover, there is no number $A_{\mu+1}$, independent of A_1, $A_2, \ldots A_\mu$, that is, idemfacient with respect to A. For, otherwise, by (4) and (7), there are $\mu + 1$ independent numbers of the system idemfaciend to $\bar{A}$, which is contrary to *Theorem I.* Whence it follows that there are

$$n\,\bar{S}A = n\,S\bar{A} = \mu > 0$$

independent numbers, rational in $\mathfrak{K}(R, e_i)$, that are idemfacient to A, in terms of which all numbers of the system idemfacient to A can be expressed linearly. Similarly, we may show that there are $n\,(1 - \bar{S}A)$ independent numbers of the system, rational with respect to $\mathfrak{K}(R, e_i)$, that are nilfacient with respect to A, in terms of which all numbers nilfacient with respect to A can be expressed linearly.

If A is nilpotent, then by (4), $\bar{A}$ is nilpotent, when by *Theorem I,* $S\bar{A}^p = 0$ for any positive integer p; and, therefore, by (1) and (3), $\bar{S}A^p = S\bar{A}^p = 0$. If, conversely, $\bar{S}A^p = 0$ for every positive integer p, then by (1) and (3), $S\bar{A}^p = \bar{S}A^p = 0$ for every positive integer p; and, therefore, by *Theorem I,* $\bar{A}$ is nilpotent, in which case by (4) A is nilpotent.

If there are n independent numbers $A_1, A_2, \ldots A_n$ of the system $e_1, e_2, \ldots e_n$ such that

$$\bar{S}A_1 = \bar{S}A_2 = \ldots = \bar{S}A_n = 0,$$

then, by (1) and (7), there are n independent numbers $\bar{A}_1, \bar{A}_2, \ldots \bar{A}_n$ of the reciprocal system $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$ such that

$$S\bar{A}_1 = S\bar{A}_2 = \ldots = S\bar{A}_n = 0\,;$$

and, therefore, by *Theorem I,* the system $\bar{e}_1, \bar{e}_2, \ldots \bar{e}_n$ is nilpotent, in which case, by (5), $e_1, e_2, \ldots e_n$ is nilpotent.

For a system of $n = m^2$ units ε_{ij} $(i, j = 1, 2, \ldots n)$ whose multiplication table is given by the equations

$$\varepsilon_{ij}\,\varepsilon_{jk} = \varepsilon_{ik}, \quad \varepsilon_{ij}\,\varepsilon_{hk} = 0 \quad (i, j, k, h = 1, 2, \ldots n\,;\ h \neq j),$$

the term "quadrate" has been suggested by Clifford.* A quadrate with $n = m^2$ units may be termed as a *quadrate of order m.* The units of a matrix of order m constitute a quadrate of order m. If we denote by $\bar{\varepsilon}_{ij}$ the units of the reciprocal system, we have

$$\bar{\varepsilon}_{ij} = \varepsilon_{ji} \qquad (i, j = 1, 2, \ldots m).$$

A quadrate is thus equivalent to its reciprocal. For the units of a quadrate we have

$$S\varepsilon_{ii} = \frac{1}{m} = \bar{S}\varepsilon_{ii} \qquad (i = 1, 2, \ldots n),$$

$$S\varepsilon_{ij} = 0 = \bar{S}\varepsilon_{ij} \qquad (i, j = 1, 2, \ldots n;\ i \neq j);\dagger$$

and, thus, for any number $\sum_1^m{}_i \sum_1^m{}_j a_{ij}\varepsilon_{ij}$ of such a system we have

$$SA = \sum_1^m{}_i \sum_1^m{}_j a_{ij} S\varepsilon_{ij} = \sum_1^m{}_i a_{ii} S\varepsilon_{ii} = \frac{1}{m}\sum_1^m{}_i a_{ii},$$

$$\bar{S}A = \sum_1^m{}_i \sum_1^m{}_j a_{ij} \bar{S}\varepsilon_{ij} = \sum_1^m{}_i a_{ii} \bar{S}\varepsilon_{ii} = \frac{1}{m}\sum_1^m{}_i a_{ii}.$$

Therefore,

Theorem II. If ε_{ij}, *for* $i, j = 1, 2, \ldots m$, *are the* $n = m^2$ *units of a quadrate of order* m, *that is, if*

$$\varepsilon_{ij}\varepsilon_{jk} = \varepsilon_{ik}, \quad \varepsilon_{ij}\varepsilon_{hk} = 0 \qquad (i, j, k, h = 1, 2, \ldots m;\ h \neq j),$$

then, if

$$A = \sum_1^m{}_i \sum_1^m{}_j a_{ij}\varepsilon_{ij}$$

is any number of the quadrate,

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* Peirce, loc. cit., p. 217.

† For, for $1 \leqq i \leqq m$, ε_{ii} is idempotent; there are just m independent numbers idemfaciend to ε_{ii}, namely, ε_{ij} $(j = 1, 2, \ldots m)$; and just m independent numbers ε_{ji} $(j = 1, 2, \ldots m)$ idemfacient to ε_{ii}. Moreover, for $1 \leqq i \leqq m$, $1 \leqq j \leqq m$, $i \neq j$, ε_{ij} is nilpotent.

$$\bar{S} A = \frac{1}{m} \sum_{1}^{m} a_{ii} = S A.^{*}$$

Let

$$\Omega(A) \equiv A^{m} + p_1 A^{m-1} + \ldots + p_{m-2} A^{2} + p_{m-1} A = 0$$

be the equation of lowest order in the first and higher powers of

$$A = \sum_{1}^{n}{}_{i}\, a_i e_i ;$$

and let the roots of the scalar equation

$$\Omega(x) \equiv x^{m} + p_1 x^{m-1} + \ldots + p_{m-2} x + p_{m-1} x = 0$$

be $0, x_1, x_2, \ldots x_r$, respectively of multiplicity $v, v_1, v_2, \ldots v_r$. Either $A^m = 0$, or p_1, p_2, etc., are not all zero, in which case $r > 0$. For $A^m \neq 0$, and thus $r > 0$, if we put

$$\mathfrak{f}(x) \equiv 1 - \left(\frac{x^{v} - x_1^{v}}{-x_1^{v}}\right)^{v_1} \left(\frac{x^{v} - x_2^{v}}{-x_2^{v}}\right)^{v_2} \ldots \left(\frac{x^{r} - x_r^{v}}{-x_r^{v}}\right)^{v_r},$$

I have shown in the paper above referred to that the number $\mathfrak{f}(A)$ of the system is idempotent.† Let the polynomial $\Theta(x)$ be defined by the equation

$$x \Theta(x) \equiv \mathfrak{f}(x)\, \mathfrak{f}(x).$$

Then $\Theta(x)$ is linearly in x^{2v-1}, x^{2v}, etc. Therefore, $\Theta(A)$ is a number of the system. Further, since $\mathfrak{f}(A)$ is idempotent,

$$A \Theta(A) = \mathfrak{f}(A)\, \mathfrak{f}(A) = \mathfrak{f}(A).$$

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* In my first extension of the quarternion scalar function to matrices in general, referred to in the note p. 59, I defined the scalar function $S A$ of any number

$$A = \sum_{1}^{m}{}_{i} \sum_{1}^{m}{}_{j}\, a_{ij} \varepsilon_{ij}$$

of the matrix, or quadrate of order m, as above, namely, as equal to

$$\frac{1}{n} \sum_{1}^{m}{}_{i}\, a_{ii}.$$

† Loc. cit., p. 524.

That is, *if A is any non-nilpotent number of the system, there is number $\Theta(A)$, linear in powers of A, whose product as pre-factor, or post-factor into A, is equal to an idempotent number.*

Let now

$$S A e_1 = S A e_2 = \ . \ . \ . \ = S A e_n = 0.$$

Then, for any number

$$B = \sum_1^n b_i e_i$$

of the system, we have

$$S A B = \sum_1^n b_i S A e_i = 0.$$

Therefore, in particular, if A is not nilpotent, we have

$$S A^p \mathfrak{f}(A) = 0$$

for any positive integer p. Whence follows

$$S(\mathfrak{f}(A)\,\mathfrak{f}(A)) = S\mathfrak{f}(A) = 0,$$

which is impossible, since $\mathfrak{f}(A)$ is idempotent; and hence it follows that A is nilpotent. Similarly, if

$$\bar{S} A e_1 = \bar{S} A e_2 = \ . \ . \ . \ = \bar{S} A e_n = 0$$

we may show that A is nilpotent.

Conversely, let A be nilpotent. Then, either

$$S A e_1 = S A e_2 = \ . \ . \ . \ = S A e_n = 0,$$

or for some number

$$B = \sum_1^n b_i e_i$$

of the system $S A B = S B A \neq 0$; in which case neither $A B$ nor $B A$ is nilpotent. But, by the above theorem, if $A B$ is not nilpotent, there is a number C_1 such that $A . B C_1 = A B . C_1$ is idempotent; therefore, a number $B_1 = B C_1$ such that $A B_1$ is idempotent. And, if $B A$ is not nilpotent, there is a number C_2 such that $C_2 B . A = C_2 . B A$ is idempotent; therefore a number $B_2 = C_2 B$ such that $B_2 A$ is idempotent. Similarly, we may show, if A is nilpotent, that, either

$$\bar{S} A e_1 = \bar{S} A e_2 = \ldots = \bar{S} A e_n = 0,$$

or there are two numbers B_1 and B_2 of the system such that $A B_1$ and $B_2 A$ are idempotent.

We have, therefore, the following theorem:

Theorem III. If the number

$$A = \sum_1^n {}_i a_i e_i$$

of the system $e_1, e_2, \ldots e_n$ *satisfies the equations*

$$S e_1 A = S e_2 A = \ldots = S e_n A = 0,$$

or the equations

$$\bar{S} e_1 A = \bar{S} e_2 A = \ldots = \bar{S} e_n A = 0,$$

A is nilpotent. Conversely, if A is nilpotent, it satisfies both systems of equations, or there are two numbers B_1 and B_2 of the system $e_1, e_2, \ldots e_n$ *such that both $A B_1$ and $B_2 A$ are idempotent.*

Clark University, Worcester, Mass.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF IODINE.

SECOND PAPER.

By Gregory Paul Baxter.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
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A REVISION OF THE ATOMIC WEIGHT OF IODINE. SECOND PAPER.

BY GREGORY PAUL BAXTER.

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VERY recently, in an investigation upon the atomic weight of iodine,* the value 126.975 for this constant was obtained as the average from three closely agreeing ratios. Experiments subsequent to the publication of this research have shown that the results of all three methods were subject to slight errors, and that the atomic weight of iodine is even slightly higher than the above value. Following is a description of these experiments, with details where the processes differed from those previously described. For further information the first paper should be consulted.

THE RATIO OF SILVER IODIDE TO SILVER BROMIDE AND SILVER CHLORIDE.

In the previous investigation the ratio AgI : AgCl was determined by heating silver iodide in a current of chlorine in six experiments which yielded an average result of 126.975, calculated upon the basis of silver 107.93 and chlorine 35.467. The latter value for chlorine was a preliminary one, found in an investigation by Richards and Wells then in progress in this laboratory. Later experiments by these chemists have shown that the atomic weight of chlorine is in reality 35.473,† which necessitates a positive correction for the atomic weight of iodine of 0.010. The corrected value, 126.985, agrees very closely with two determinations, made in the course of the earlier investigation, in which silver iodide was heated in a current of carbon dioxide and bromine. Assuming the atomic weight of bromine to be 79.955, the value found by Stas and repeatedly verified by experimenters in this laboratory,* these two determinations give the values 126.987 and 126.985. The results obtained by reference both to chlorine and to bromine thus became almost identical. However, in order to confirm these two determinations with bromine, further experiments were carried out in the same manner as before, as follows:

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* Baxter, These Proceedings, **40**, 419 (1904). Also Zeit. anorg. Chem., **43**, 14; and Jour. Amer. Chem. Soc., **26**, 1577.

† Richards and Wells, Publications of the Carnegie Institution, No. 28 (1905). Also Jour. Amer. Chem. Soc., **27**, 459.

Pure silver iodide was prepared as described in the previous paper, by precipitating pure silver nitrate with a considerable excess of the purest ammonic iodide,† washing the precipitate with one per cent nitric acid and pure water, and drying the iodide, in an air bath, and then by fusion in a tared quartz crucible, first in an atmosphere containing free iodine, and finally in air to expel dissolved iodine. Fusion in an atmosphere containing iodine must have completely converted occluded silver nitrate into silver iodide. The iodide was then heated in a current of air and bromine. The air was purified by being conducted successively over beads moistened with silver nitrate solution, sodium carbonate, and finally concentrated sulphuric acid which had been heated to its boiling point with a small quantity of recrystallized potassium dichromate to eliminate volatile and oxidizable impurities. Four different samples of bromine were employed, each one of which had been thrice distilled from a solution of a bromide, in each distillation the bromide having been made from a portion of the product of the previous distillation. Chlorine must have been completely separated in this way. The presence of iodine in the bromine was of less importance. Nevertheless each sample, while in the form of hydrobromic acid, was freed from iodine by boiling with several small portions of potassic permanganate. The apparatus for purifying the air and saturating the air with bromine was so constructed that the gases came in contact only with glass. The quartz crucibles were always contained in large porcelain crucibles during the heating in bromine, as well as in the initial fusion of the silver iodide with iodine. Volatilization of the silver halides was prevented by heating very gently at first, at a temperature insufficient to fuse the mixture, then, when the greater part of the iodine had been replaced, until the silver bromide barely fused. Furthermore, the crucible was very deep and the current of gases very slow, so that any volatilized silver halides had opportunity to condense upon the cool walls of the crucible. That no loss from volatilization actually took place was certain for two reasons. In the first place, the weight of the bromide in most cases became constant within a few hundredths of a milligram after once being heated, although subsequently the salt was maintained at a temperature slightly above its melting point for at least an hour. In the second place, the perforated cover and hard glass delivery tube for the bromine, when rinsed with ammonia and the solution treated with hydrochloric acid in excess, gave no visible trace of opalescence. Before the salt was allowed to solidify it was maintained above its fusing point in air for a few minutes to eliminate dissolved bromine.

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* Richards, Proc. Amer. Phil. Soc., **43**, 119.

† The method of purifying the iodine is described briefly on page 77 of this paper.

The silver bromide resulting from each analysis was converted into silver chloride by heating in a current of pure dry chlorine in a similar fashion.* Here also constant weight within a very few hundredths of a milligram was easily obtained, and as before the perforated cover and inlet tube were free from weighable amounts of chloride. It has already been shown that silver chloride and silver iodide, when fused in an atmosphere of the corresponding halogens and then in air, retain none of the halogen.† The quartz crucibles remained almost absolutely constant in weight during these experiments. From the ratio between the original silver iodide and the silver chloride the atomic weight of iodine was calculated.

The following vacuum corrections were applied: silver iodide, +0.000071; silver bromide, +0.000046; silver chloride, +0.000075.‡ The analyses of Series II were obtained through the bromide, those of Series III are the results from the previous paper recalculated from the higher atomic weight of chlorine, with the exception of Analysis 18, which was made subsequently. All completed analyses are recorded in the tables. Several were lost, owing to breaking of the crucibles.

THE RATIOS BETWEEN SILVER, IODINE, AND SILVER IODIDE.

In the earlier paper determinations of the ratios of silver to silver iodide and iodine were described. It remained to determine the ratio of iodine to silver iodide. For this purpose iodine was purified as before

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* For preparation and purification of chlorine, see earlier paper.

† See previous paper. Also Richards and Wells, loc. cit., page 59, and Köthner and Aeuer, Liebig's Ann., **337**, 127.

‡ The value for the specific gravity of silver chloride is assumed to be 5.56, as determined by Richards and Stull in this laboratory. In the previous paper this value was incorrectly taken as 5.62.

TABLE I.

Series I. AgI : AgBr.

Number of Analysis.	Sample of Bromine.	Weight of Silver Iodide in Vacuum.	Weight of Silver Bromide in Vacuum.	Atomic Weight of Iodine.
		grams.	grams.	
1	A	13.65457	10.92091	126.985
2	A	17.35528	13.88062	126.987
3	B	9.70100	7.75896	126.982
4	B	10.27105	8.21484	126.983
5	B	9.85688	7.88351	126.986
6	C	8.62870	6.90106	126.991
7	D	11.92405	9.53704	126.981
8	D	7.56933	6.05389	126.987
		Average		126.985

Series II. AgI : AgCl.

Number of Analysis.		Weight of Silver Iodide in Vacuum.	Weight of Silver Chloride in Vacuum.	Atomic Weight of Iodine.
		grams.	grams.	
9		13.65457	8.33538	126.985
10		17.35528	10.59457	126.983
11		10.27105	6.27006	126.980
12		8.62870	5.26735	126.985
13		11.92405	7.27926	126.976
		Average		126.982

Series III. AgI : AgCl

Number of Analysis.		Weight of Silver Iodide in Vacuum.	Weight of Silver Chloride in Vacuum.	Atomic Weight of Iodine.
		grams.	grams.	
14		9.26860	5.65787	126.990
15		6.72061	4.10259	126.984
16		11.31825	6.90912	126.987
17		10.07029	6.14754	126.979
18		13.49229	8.23649	126.980
		Average		126.984

by thrice converting it into hydriodic acid with hydrogen sulphide, and then heating the hydriodic acid with a slight excess of potassium permanganate which had been freed from chlorine by crystallization. The final product of iodine was distilled once with steam, freed from water by suction upon a porcelain Gooch crucible, and dried as far as possible in a desiccator over concentrated sulphuric acid. It was then sublimed once from porcelain boats in a current of pure dry air in a hard glass tube, and then a second time from the hard glass tube into the weighing tube, which was constructed as described on page 428 of the first paper. This weighing tube remained constant in weight within a few hundredths of a milligram in each experiment, and lost in weight in all only one tenth of a milligram in the eight final analyses. Next, the iodine was dissolved in sulphurous acid, and was then precipitated by adding a solution of a slight excess of pure silver to the solution of hydriodic acid in a precipitating flask. The silver iodide clotted together very rapidly in the presence of the nitric acid and excess of silver, and offered none of the difficulties met when the precipitation is carried out with an excess of iodide, for it may be washed indefinitely with pure cold water without showing the least tendency to pass into colloidal solution. The silver iodide was collected upon a Gooch crucible, dried, and weighed. The loss on fusion was determined, and the asbestos shreds in the filtrate were collected. Twelve determinations carried out in this way with solutions varying from tenth to thirtieth normal gave results from 126.93 to 126.98. The variations were finally traced to the carrying down of silver nitrate by the silver iodide; for the more concentrated the solutions during the precipitation and the greater the excess of silver employed, the lower was the observed atomic weight of iodine. The occlusion of silver nitrate by silver halides is well known in the cases of silver bromide and silver chloride, but in these cases it is possible to wash out the occluded salts by oft-repeated treatment with water. The difficulty in removing the silver nitrate in the case of silver iodide is doubtless due to the lesser solubility of this halide. Similar observations have been made by Köthner and Aeuer,* who found that with fifth normal solutions the carrying down of silver nitrate is very considerable, and that an excess of iodide converts this occluded silver nitrate into silver iodide only very slowly if at all. They found, however, that, if the precipitated silver iodide is washed with ammonia, the greater part of the occluded matter may be eliminated, owing possibly to the slight solubility of silver iodide in ammonia. On account of this newly discovered tendency of silver iodide, it seemed probable that the results of the synthesis of silver iodide from known weights of silver in the early part of this research were somewhat too low; for although the precipitations took place in ammoniacal solution and an excess of iodine was employed, the fact that the solutions were always very nearly as strong as tenth normal and that the ammonic iodide was poured into the silver nitrate, made it probable that occlusion had taken place at least to some extent even here. It is noteworthy that three of these syntheses yielded results as high as 126.985. Similar conclusions are to be drawn concerning the results of the titration of silver and iodine, although in this case precipitation took place in the reverse fashion, i. e., by adding the silver nitrate to the iodide.

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* Liebig's Ann., **337**, 123.

Accordingly experiments were carried out for the redetermination of the ratios of silver to silver iodide and to iodine, as well as the new one of iodine to silver iodide, with especial precautions to avoid the error of occlusion by using very dilute solutions and no excess of silver. After the iodine had been weighed, very nearly the exact amount of silver to combine with the iodine was weighed out and dissolved in nitric acid with the usual precautions to avoid spattering. The quantity of iodine used in each analysis was between three and four grams, and the amount of silver consequently about three grams. The solutions of both silver and iodine were diluted to a volume of one liter each, so that the solutions were about thirtieth normal, and then the silver nitrate was added very slowly to the solution of hydriodic acid with constant stirring. The flask in which precipitation was carried out was then shaken for some time, and allowed to stand until the supernatant liquid was clear. This liquid was tested for an excess of iodine or silver in a nephelometer,[*] and if a deficiency of either was found, it was made up and the solution again shaken, until the point was reached where the extremely faint opalescence produced by both hydriodic acid and silver nitrate was equal in both nephelometer tubes. The excess or deficiency of silver was in no case over one tenth of a milligram, and in most cases much less.

Several samples of iodine were employed in these analyses, each one of which had been three times distilled from an iodide as previously described. Three different specimens of silver, purified by different methods, were used. Sample A was a portion of the material used in the first investigation. Sample B was prepared from silver nitrate which had been recrystallized seven times from nitric acid, five times from water, and finally precipitated by ammonium formiate.* Sample C was precipitated as silver chloride once, electrolyzed once, and finally precipitated with formic acid. All three samples were fused in a current of hydrogen on a lime boat. In order to determine whether an even more gradual addition of silver nitrate would influence the extent of the occlusion, in Analysis 23 the silver was introduced by means of a large funnel provided with a fine spout.

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[*] Richards and Wells, Amer. Chem. Jour., **31**, 235.

TABLE II.

SERIES IV. Ag : I.

Number of Analysis.	Sample of Silver.	Weight of Iodine in Vacuum.	Weight of Silver in Vacuum.	Atomic Weight of Iodine.
		grams.	grams.	
19	A	3.29308	2.79897	126.983
20	B	3.70132	3.14584	126.988
21	B	3.75641	3.19258	126.991
22	B	3.24954	2.76186	126.988
23	A	4.12541	3.50639	126.984
24	C	3.53166	3.00165	126.988
25	C	2.99835	2.54842	126.985
26	C	2.00015	1.69991	126.993
		Average		126.987

In one of the foregoing analyses the attempt was made to determine the ratio between iodine and silver iodide by adding an excess of silver nitrate and weighing the precipitate. Although only five milligrams excess of silver nitrate were added, and the total volume of the liquid was over two liters, the precipitate was one milligram heavier than it should have been as calculated from the result of the titration, showing that the occlusion of silver nitrate by the silver iodide may take place even after precipitation. In order to avoid this difficulty, in succeeding analyses the silver iodide was collected without the addition of silver nitrate, after washing with pure water; then the dissolved silver iodide was determined in the filtrate and wash water, by comparing the precipitates produced by the addition of silver nitrate with those formed in standard solutions of hydriodic acid. As a matter of fact, the filtrates were always found to be practically free from silver iodide, while the wash water contained from two to four tenths of a milligram per liter. This difference in solubility of the silver iodide in the filtrate and wash waters is probably due to colloidal solubility of this salt in pure water. The loss on fusion of the silver iodide and the asbestos shreds were determined as usual. The almost absolute lack of opalescence in the filtrate when treated with silver nitrate or hydriodic acid is a good proof of the complete absence of chlorine and bromine, since both silver bromide and silver chloride are considerably more soluble than the iodide.

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* Richards and Wells, Publications of the Carnegie Institution, No. 28, 19.

The vacuum corrections −0.000031 for silver and +0.000102 for iodine were applied. The atomic weight of silver is assumed to be *107.930*, as before.

TABLE III.

SERIES V. I : AgI.

Number of Analysis.	Weight of Iodine in Vacuum.	Corrected Weight of Silver Iodide in Vacuum.	Atomic Weight of Iodine.
	grams.	grams.	
27	3.75641	6.94913	126.987
28	3.24954	6.01137	126.989
29	4.12541	7.63204	126.977
30	3.53166	6.53351	126.979
31	2.99835	5.54682	126.983
	Average		126.983

Finally, in order to determine the ratio of silver to silver iodide, the filtrate and wash waters were evaporated to very small bulk until the greater part of the nitric acid had been expelled, and then the amount of silver in the residue was determined nephelometrically, after dilution to twenty-five cubic centimeters, by adding an excess of hydriodic acid and comparing with standard solutions of silver. This quantity, which was never more than one tenth of a milligram, was subtracted from the original weight of silver, and no correction was applied to the weight of silver iodide for the amount dissolved in the wash water.

TABLE IV.

SERIES VI. Ag : AgI.

Number of Analysis.	Sample of Silver.	Corrected Weight of Silver in Vacuum.	Corrected Weight of Silver Iodide in Vacuum.	Atomic Weight of Iodine.
		grams.	grams.	
32	B	3.19249	6.94877	126.990
33	B	2.76175	6.01110	126.986
34	C	3.00189	6.53399	126.993
35	C	2.54833	5.54659	126.986
		Average		126.989

The close agreement of the averages of the different series makes it certain that no constant error of magnitude still remained undetected in any one of the methods.

In Series I, II, and III, the three chief possible errors have been considered and shown to have no effect. Occluded silver nitrate in the silver iodide was eliminated by fusion in iodine; and it was proved both that no volatilization of the halides actually took place and that no halogen was retained by the solidified salt. In Series IV, V, and VI, the chief possible error, that of occlusion of silver nitrate by silver iodide, was avoided by sufficiently diluting the solutions before precipitation. That the dilution was sufficient was evident from the agreement of the results of the experiments with the larger and smaller quantities of material, the total volume of the solutions being the same in all cases. Furthermore, the weight of the silver iodide obtained in the analyses recorded in Series V was in every case very nearly equal to the sum of the weights of silver and iodine employed (Series IV), which could not have been the case if silver nitrate had been retained by the iodide. One is forced, then, to conclude that the average of all six series, 126.985, represents the atomic weight of iodine within a very few thousandths of a unit, and that the results of the syntheses of silver iodide described in the first paper are slightly too low, owing to occlusion of silver nitrate by the silver iodide. Furthermore, the agreement of the results from silver bromide with those of the other series is an indication that the assumed atomic weight of bromine, 79.955, is very nearly correct. However, from the weights of silver bromide and silver chloride produced in the same analyses, the atomic weight of bromine may be computed.

TABLE V.

SERIES VII. AgBr : AgCl.

Sample of Bromine.	Weight of Silver Bromide in Vacuum.	Weight of Silver Chloride in Vacuum.	Atomic Weight of Bromine.
	grams.	grams.	
A	10.92091	8.33538	79.955
A	13.88062	10 59457	79.951
B	8.21484	6.27006	79.952
B	7.87887	6.01352	79.956
C	6.90106	5.26735	79.951
D	9.53704	7.27926	79.952
		Average	79.953

Stas's value and the value obtained in this laboratory * are thus confirmed. A repetition of these experiments with precipitated fused silver bromide is now under way.

Shortly after the publication of my first paper upon the atomic weight of iodine there appeared the complete paper of Köthner and Aeuer upon the same subject.† These chemists, by determining the ratio AgI : AgCl, obtained the value 126.936 ($Cl = 35.45$; $Ag = 107.93$) as the average of eight closely agreeing analyses. One synthesis, by precipitating a weighed amount of silver with hydriodic acid, gave the value 126.978, and from one synthesis by heating a weighed amount of silver in iodine the result 126.963 was obtained. A second paper ‡ by the same authors contains a recalculation of their results from the higher value of chlorine 35.467, and a critical discussion of their own, Laden-

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* Richards, loc. cit.

† Liebig's Ann., **337**, 123 (1904).

‡ Ibid., **337**, 362 (1904).

burg's,* and my work. In this second paper two main criticisms of my investigation are made. One of these concerns the validity of the conclusion that ordinary iodine does not contain an undiscovered halogen element. This criticism is founded upon a mistaken understanding, however. My experiments were directed to prove the existence or non-existence of an element of *higher* atomic weight than iodine, which, if its properties were those to be expected from the properties of the other members of the halogen family, would have been set free from solutions of its compounds by iodine, and hence would have accumulated in the first fraction of the fractional separation (see page 422 of the first paper). The existence of an unknown halogen of lower atomic weight than iodine was not considered.

Although Köthner assumes the possibility that such an element exists, it is obviously improbable that any halogen of lower atomic weight could have remained in the purified samples of iodine which were employed in my experiments; since those specimens which received even the least purification were thrice distilled from an iodide, the iodide having been made in each distillation from nearly one half the iodine from the previous distillation.

In the second place, it is maintained that volatilization of silver halides takes place when silver iodide is heated in a current of chlorine. This is undoubtedly true, unless precautions are taken to prevent the volatilization. Silver iodide is much more volatile at its fusing temperature than silver chloride; and if the original silver iodide is fused before the current of chlorine or bromide is begun, a loss may take place by volatilization or possibly by *spattering*. In my own work, however, fusion of the salt was always avoided until the greater part of the change had taken place. Proof that no volatilization of silver halides actually occurred in my experiments has already been given (page 75). It may be added that the close agreement of the results of Series I and II is additional evidence in the same direction; for any loss of silver bromide, which is the most volatile of the three halides, would have tended to raise the results of Series II above those of Series I, whereas such slight difference as exists is in the opposite direction.

Köthner's average result in the Series AgI : AgCl, when recalculated upon the basis of chlorine 35.473, becomes 126.974. The slight difference between this value and the final result of the research described in this paper may be explained in several possible ways. A perusal of

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* Ber. d. d. chem. Gesell., **35**, 2275.

Köthner's paper does not make clear whether or not fusion of the silver iodide took place at the beginning of his experiments, so that it is uncertain whether the gain in weight of the coil of glass tubing, which was attached to the reaction tube, was due to volatilized silver halides or to attacking of the glass by the hot halogens. At any rate, it is hard to believe that a meter and a half of glass tubing subjected to the long-continued action of mixed chlorine and iodine at 150° should not have altered its weight somewhat. Furthermore, Köthner himself showed that the tube in which the reaction took place was slightly attacked by the fused silver chloride, but the nature of the correction for this attacking of the glass is uncertain. Finally, although the precaution was taken of fusing the silver iodide before the initial weighing, so that the salt must have been free from moisture, it was not fused in an atmosphere of iodine. Hence it is possible that the iodide still contained traces of occluded silver nitrate (or metallic silver). This deficiency in iodine, as well as the possible gain in weight of the apparatus during the experiment, would have lowered the atomic weight of iodine.

As far as Köthner's syntheses of silver iodide from weighed amount of silver are concerned, it need only be said that, even assuming that occlusion was avoided in the synthesis in the wet way, and that in the synthesis in the dry way a single precipitation of the iodine from solution in an iodide had completely removed such impurities as chlorine and bromine, which would have accumulated in the silver iodide during the experiment, the fact that Richards and Wells* have shown that silver fused in the air or with borax and saltpetre, according to Stas, must contain oxygen, makes it certain that the results of Köthner's syntheses are too low.

I am indebted to the Cyrus M. Warren Fund for Research in Harvard University for platinum vessels, quartz crucibles, and balance.

SUMMARY.

The results of this investigation are, then, as follows: —

1. The atomic weight of iodine is found to be 126.985 (Ag = 107.930) one one-hundredth of a unit higher than the value previously obtained. If silver is assumed to be 107.920 and oxygen 16.000, iodine becomes 126.973.

2. The observation of Köthner and Aeuer that under certain condi-

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* Publications of the Carnegie Institution, No. 28, 62.

tions silver iodide occludes silver nitrate, and that this occluded salt cannot be removed by washing with water, is confirmed.

3. Richards and Wells's value for the atomic weight of chlorine, 35.473, is substantiated.

4. Stas's value for the atomic weight of bromine, 79.955, is shown to be very nearly correct.

Chemical Laboratory of Harvard College,
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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — NO. XLVI.

ON THE COMPOSITION OF PETROLEUM.

BY CHARLES F. MABERY.

THE SULPHUR COMPOUNDS AND UNSATURATED HYDROCARBONS IN CANADIAN PETROLEUM. — BY CHARLES F. MABERY AND WILLIAM O. QUAYLE.

AID IN THE WORK DESCRIBED IN THIS PAPER WAS GIVEN BY THE ACADEMY FROM THE C. M. WARREN FUND FOR CHEMICAL RESEARCH.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — XLVI.

ON THE COMPOSITION OF PETROLEUM.

By Charles F. Mabery.

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THE SULPHUR COMPOUNDS AND UNSATURATED HYDROCARBONS IN CANADIAN PETROLEUM.*

By Charles F. Mabery and William O. Quayle.

In a paper published some time ago by one of us (Mabery) on a preliminary examination of Canadian petroleum (Amer. Chem. Journal, **13**, 89, 1891), it was stated that 225 liters of sulphur oil from Canadian burning oil was in process of distillation for the purpose of separating the individual sulphur constituents. On account of the pressure of other lines of work, study of those distillates was delayed, but it has been resumed at intervals until the results described in this paper were obtained.

On returning to this work it seemed worth while to ascertain first, whether the oil selected really represented constituents of crude petroleum, or whether it consisted to a greater or less extent of decomposition products incident to the process of refining. Since the question of decomposition could be determined in sulphur petroleum from any source, a quantity of crude oil from the Lima (Ohio) field was distilled in vacuo, and the distillate, corresponding to that from which burning oil is prepared, carefully separated, and agitated thoroughly with alcoholic mercuric chloride for the purpose of precipitating the sulphur compounds. The oil was next agitated vigorously with ordinary sulphuric acid, the acid sludge drawn off and diluted with water, carefully avoiding heat so far as possible.

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* The sulphur derivatives of the hydrocarbons described in this paper are evidently members of a new series, differing essentially in composition, physical properties, and chemical relations from any of the well-known series of sulphur compounds. For this new series I suggest the name Thiophane. C. F. M.

As usual, a dark red oil separated above the solution, but it lacked the heavy sulphurous odor characteristics of sludge as it is ordinarily separated from sulphur petroleum. The odor of this oil resembled that of sludge after the sulphur compounds have been removed, and it showed still greater similarity in its behavior toward bromine, with which it readily united with the evolution of heat, and the formation of an addition product heavier than water precisely similar to the behavior of sludge products. The precipitate with mercuric chloride resembled in all respects the characteristic products which are so useful in separating the sulphur compounds.

As further evidence of the presence of unsaturated hydrocarbons in this oil, a portion of the oil precipitated from sulphuric acid was heated with concentrated hydrobromic acid to 100°, and the product fractioned in vacuo to collect in a smaller limit the bromine addition product. It was not intended to prepare an individual bromine derivative sufficient for analysis, since this would have been impossible with the small amount to be obtained in this manner, but only to be sure that an addition product with hydrobromic acid could be formed. The higher vacuum distillate gave an abundant reaction for bromine after heating with lime, showing that addition of the acid had taken place. The results of these experiments are described in full, that there may be no misapprehension concerning the source of the sulphur compounds and unsaturated hydrocarbons which will be described in this paper. They confirm the results of similar experiments described in the former paper.

In continuing the distillation of the 225 liters of crude sulphur oil with the aid of the apparatus elsewhere described (These Proceedings. **13**, 10), the distillates were carried through entire up to 250° under 50mm., at first within 5°, during four distillations. The lower fractions were then continued within single degree fractions until the lower portions could be distilled under atmosphere-pressure. It was not possible to accomplish in this manner more than a general separation, since all the distillates were mixtures, yet the value of the great amount of tedious labor was shown after the separation of the individual constituents, of which the greater portion distilled at approximately the same temperatures as the original distillates. And although it seemed a great expenditure of effort to distil such a large volume of oil in quantities of ten liters, the capacity of the porcelain still employed, and to continue the distillation of the entire quantity, this seemed to be the only available method for the separation of these peculiar bodies from the crude oil. When first collected, all the fractions were nearly colorless, but on standing they turned dark red, especially those with higher boiling points. The fractions collected below 125°, tension 50mm., were found to be exceedingly unstable. On standing from six months to two years, some of these distillates deposited very dark heavy oils, doubtless formed from unstable unsaturated hydrocarbons, terpenes, or sulphur compounds by polymerization.

In separating the constituents of the vacuum distillates, we followed in general the method described in a former paper (Mabery and Smith, These Proceedings, N. S. **17**, 218). Three or four hundred grams of the oil was shaken in a large flask with an excess of an alcoholic solution of mercuric chloride, which gave a heavy precipitate, crystalline in the lower fractions, a thick viscous mass in the less volatile, and a heavy oily deposit in the vacuum distillates above 160°. There invariably separated above the alcohol a lighter oil, which contained no sulphur, or at most, analysis showed less than one half of one per cent. Upon diluting the alcohol more lighter oil collected, which doubtless has a different composition than the other, since, as will appear later, it showed essentially different properties.

In recovering the sulphur compounds from the mercuric chloride precipitates, after thoroughly washing with alcohol and very volatile gasoline, the precipitate was decomposed by hydric sulphide in presence of a considerable volume of alcohol, the mercuric sulphide separated by filtration, washed with alcohol, and the alcohol largely diluted. For the most part the lighter oil separated, some remaining in suspension, which collected after standing a short time. With all possible care in washing the mercuric chloride precipitate, a single precipitation was never sufficient to remove all the sulphur-free oil. The percentages of sulphur in the individual sulphur compounds invariably came from one to three per cent too low, however long they were fractioned, until they were a second time converted into the mercuric chloride compound and again separated by hydric sulphide. Upon diluting a second time, usually another portion of the sulphur-free oil collected, or remained in suspension, and after sufficient fractioning, the percentage of sulphur then corresponded to the value theoretically required.

It has already been explained that the vacuum fractions of the crude sulphur oil, before the separation of its constituents, is a mixture of an unsaturated hydrocarbon, a sulphur compound, and another hydrocarbon of a series not determined. More attention will be given later to the sulphur-free oils. All the portions selected for the separation of these constituents had previously been fractioned from four to twenty times, for the most part in vacuo, and since the sulphur oil from any particular fraction has corresponded in boiling point to that of the fraction before separation, it has been possible to select from the great number of first fractions those which should give the bodies desired, thereby saving much time and unnecessary labor in searching for the individual constituents.

After the second precipitation from the mercuric chloride compound, a few distillations in vacuo, with the aid of a bead column in a long-necked flask, have been sufficient to bring together, for the most part, within one or two degrees, the sulphur compound desired. For the separation without decomposition of the sulphur oils boiling above 130°, distillation in vacuo has been indispensable. Doubtless exclusion of air, as well as the reduction in boiling points, especially of the higher members, is essential.

The most volatile vacuum distillates 40°–50° were subjected to eight additional distillations within limits of two degrees, and 300 c.c. of these distillates was agitated with an excess of alcoholic mercuric chloride. A part of the oil was immediately converted into a crystalline precipitate, and the sulphur-free oil, which formed the greater part of the oil taken, remained entirely in solution and was separated by dilution. After decantation of the alcoholic solution, the crystalline precipitate was well washed with alcohol and light gasoline, and decomposed by hydric sulphide in presence of alcohol.

For complete removal of the sulphur-free oil the mercuric chloride compound was again formed, washed, and the sulphur oil separated by hydric sulphide. Unfortunately, the larger portion of the sulphur oil was accidentally lost, but the remainder was fractioned, so far as possible, until finally it showed a tendency to collect at 125°–130°.

The percentage of sulphur found in this product was 27.12; required for the sulphur compound $C_6H_{12}S$, 27.60. The crystalline condition of this substance is characteristic of the alkyl sulphides; but all the sulphur bodies we have separated from petroleum are viscous oils that could not be solidified, except the more volatile sulphur compounds such as were separated from Ohio petroleum, which were crystalline.

The following series of sulphur compounds which have been identified by analysis, and shown to have the composition of the general formula $C_nH_{2n}S$, do not represent any series known. They are not members of the ethylene sulphide series, as shown by their reactions and very high specific gravity. In empirical composition they correspond to hydrothiophenes, which have not been synthetically prepared.

HEPTYL THIOPHANE.

Fraction 158°–160° (At. Pres.), $C_7H_{14}S$.

The heptyl sulphur compound was identified in the vacuum distillate 71°–73°, 50 mm., seventh distillation. After precipitation with alcoholic mercuric chloride, and thorough washing with alcohol and light gasoline. the thick viscous oil was decomposed with hydric sulphide in the presence of alcohol. The oil that separated on dilution of the alcohol was fractioned in vacuo until it distilled at 74°–76° under 50 mm., and at 158°–160°, 750 mm. A determination of its specific gravity at 20° gave 0.8878. Analysis gave percentages of carbon, hydrogen, and sulphur required for the formula $C_7H_{14}S$. Carbon and hydrogen were determined by combustion in plumbic chromate, the sulphur by combustion in oxygen.

I. 0.1749 gram of the oil gave 0.4141 gram CO_2, and 0.1748 gram H_2O.
II. 0.1666 gram of the oil gave 0.3071 gram BaO_4.

	Calculated for $C_7H_{14}S$.	Found.	
		I	II
C	64.63	64.59	
H	10.76	11.18	
S	24.61		25.31

OCTYL THIOPHANE.

Fraction 167°–169° (At. Pres.), $C_8H_{16}S$.

For the separation of the next sulphur homologue the large quantity of distillate was selected that collected at 79°–81°, 50 mm., after the seventh distillation. 100 c.c. of this, with the treatment described above, yielded 20 c.c. of the sulphur compound, that distilled constant in vacuo at 81°–83°, 50 mm., and at 167°–169° under atmospheric pressure. The specific gravity of this oil was found to be 0.8929 at 20°. Its composition was shown by the following analyses:

I. 0.1666 gram of the oil gave 0.4057 gram CO_2, and 0.1671 gram H_2O.
II. 0.1391 gram of the oil gave 0.2282 gram $BaSO_4$.

	Calculated for $C_8H_{16}S$.	Found.	
		I.	II.
C	66.67	66.46	
H	11.11	11.22	
S	21.22		22.53

ISOOCTYL THIOPHANE.

Fraction 183°–185° (At. Pres.), $C_8H_{16}S$.

A large amount of distillate collected at 97°–99°, 50mm., at the end of the seventh distillation, that gave by analysis the same empirical formula as the sulphur compound previously described. The term isooctyl is applied to this compound to distinguish it from its isomer. Upon treatment with mercuric chloride and second precipitation, a large amount of sulphur-free oil collected on dilution of the alcohol, and analysis showed that this constituent was completely removed by the second treatment. After fractioning in vacuo this product collected to the extent of 15 c.c. at 94°–96°, 50 mm., and distilled with some decomposition at 183°–185°, atmospheric pressure. Specific gravity at 20°, 0.8937.

This oil gave the required percentages of carbon, hydrogen, and sulphur for the formula $C_8H_{16}S$.

I. 0.1458 gram of the oil gave 0.3592 gram CO_2, and 0.1470 gram H_2O.
II. 0.1561 gram of the oil gave 0.3814 gram CO_2, and 0.1575 gram H_2O.
III. 0.2670 gram of the oil gave 0.4183 gram $BaSO_4$.

	Calculated for $C_8H_{16}S$	Found. I.	II.	III
C	66.67	67.19	66.16	
H	11.11	11.27	11.29	
S	22.22			21.52

It will be observed that the last two compounds have the same empirical composition, but differ by 16° or more in boiling points, and differ also in specific gravity. It will also be observed that they agree closely in boiling points with butyl and isobutyl sulphides; but they differ widely from the latter in specific gravity: butyl sulphide, 0.8523 at 0°: isobutyl sulphide, 0.8363 at 10°, and in other properties.

NONYL THIOPHANE.

Fraction 139°–195° (At. Pres.), $C_9H_{18}S$.

A portion of the distillate that collected in considerable quantity at 106°–108°, 50 mm., was treated in the same manner as previously described with two precipitations for the separation of the sulphur compound. The product was fractioned in vacuo until it collected for the most part at 106°–108°, 50 mm. Under atmospheric pressure it boiled at 193°–195°, and gave as its specific gravity at 20°, 0.8997. On analysis it gave percentages of carbon, hydrogen, and sulphur required for the formula $C_9H_{18}S$.

I. 0.1927 gram of the oil gave 0.4852 gram CO_2, and 0.2033 gram H_2O.
II. 0.1547 gram of the oil gave 0.2210 gram $BaSO_4$.

	Calculated for $C_9H_{18}S$.	Found. I.	Found. II.
C	68.35	68.68	
H	11.40	11.80	
S	20.25		19.62

DECYL THIOPHANE.

Fraction 207°-209°, (At. Pres.), $C_{10}H_{20}S$.

From the vacuum distillate 114°-115°, 50 mm., after the seventh distillation, a sulphur oil was prepared by two precipitations with mercuric chloride, that collected after fractioning some time in vacuo for the most part at 114°-116°, 50 mm. Under a tension of 750 mm., it distilled at 207°-209°, and gave as its specific gravity at 20°, 0.9074. The results of analysis correspond to the formula $C_{10}H_{20}S$.

I. 0.2323 gram of oil gave 0.5904 gram CO_2, and 0.2453 gram H_2O.
II. 0.1981 gram of the oil gave 0.5078 gram CO_2, and 0.2094 gram H_2O.
III. 0.1654 gram of the oil gave 0.2240 gram $BaSO_4$.
IV. 0.1760 gram of the oil gave 0.2407 gram $BaSO_4$.

	Calculated for $C_{10}H_{20}S$.	Found. I.	II.	III.	IV.
C	69.77	69.33	69.92		
H	11.62	11.32	11.82		
S	18.60			18.59	18.78

UNDECYL THIOPHANE.

Fraction 128°-130°, 50 mm., $C_{11}H_{22}S$.

The vacuum distillate 135°-145°, 50 mm., was selected for the separation of the next homologous sulphur compound. A thick tarry mass was formed by precipitation with mercuric chloride, which gave the pure sulphur oil by a second precipitation. The viscosity of the compounds with mercuric chloride was found to increase with increase in molecular weights of the sulphur oils. 300 grams of the first distillate 135°-145°, fourth distillation, gave, after the second precipitation with mercuric chloride, 122 grams insoluble in alcohol, 60 grams in the alcoholic solution, and 84 grams of the sulphur oil. Without further purification, the latter gave 16.08 per cent sulphur. After continued fractioning in vacuo the sulphur oil collected for the most part at 128°–130°, 50 mm., and on analysis it gave the percentages of carbon, hydrogen, and sulphur required for the formula $C_{11}H_{22}S$.

I. 0.1793 gram of the oil gave 0.4669 gram CO_2, and 0.1896 gram H_2O.
II. 0.2064 gram of the oil gave 0.5334 gram CO_2, and 0.2122 gram H_2O.
III. 0.1921 gram of the oil gave 0.2411 gram $BaSO_4$.

	Calculated for $C_{11}H_{22}S$.		Found.	
		I.	II.	III.
C	70.98	71.01	70.71	
H	11.82	11.82	11.50	
S	17.20			17.24

The specific gravity of this sulphur oil was found to be 0.9147 at 20°. Particular attention was given to the purification of this product to be certain of the series as shown by the most careful analysis. In addition to the determinations given above that demonstrate its composition, the following additional analyses were made of different preparations: carbon found, 70.73, 70.74; hydrogen in the same combustions, 11.48, 11.95; sulphur, 17.28, 17.05, 17.10.

The portions of the original sludge oil, called sulphur-free oils to distinguish them from the sulphur oils, have contained from 0.1 to 0.3 per cent of sulphur, and it has been assumed that the small proportion of sulphur indicated a little of the sulphur compound not precipitated by mercuric chloride. That the part collecting above the alcohol after precipitation is a different product from the part remaining in solution in alcohol is shown by the difference in specific gravity. For example, the distillate 136°–145° from which the oil $C_{11}H_{22}S$ was obtained gave an oil insoluble in alcohol with a specific gravity 0.8412; but the specific gravity of the portion in solution was found to be 0.8107. Similar differences appeared in oils associated with other thiophanes. Since the specific gravity of any one of the principal hydrocarbons in Canadian petroleum corresponding in boiling point with these bodies is smaller than the specific gravity of the sulphur oils, and since hydrocarbons are sparingly soluble in sulphuric acid and alcohol, the bodies under examination cannot be the principal petroleum hydrocarbons. Then the odors of both the substances soluble and insoluble in alcohol are unlike that of any of the hydrocarbons, — very pungent, resembling the terpenes. Further study will be necessary to identify these peculiar bodies.

QUATDECYL THIOPHANE.

Fraction 266°–268° (At. Pres.), $C_{14}H_{28}S$.

This sulphur oil was separated from the distillate 160°–170°, fourth distillation, 50 mm. After the second precipitation and decomposition by hydric sulphide, 400 grams of the sludge distillate gave 75 c.c. of a yellow oil that collected for the greater part at 168°–170°, 50 mm., after extended fractioning. It collected at 266°–268° under 750 mm., and gave percentages by analysis required for the formula $C_{14}H_{28}S$. Its specific gravity at 20° was found to be 0.9208.

I. 0.1859 gram of the oil gave 0.5010 gram CO_2, and 0.2109 gram H_2O.
II. 0.2290 gram of the oil gave 0.2336 gram $BaSO_4$.

	Required for $C_{14}H_{28}S$.	Found. I.	Found. II.
C	73.68	73.52	
H	12.28	12.69	
S	14.04		14.02

SEXDECYL THIOPHANE.

Fraction 283°–285° (At. Pres.), $C_{16}H_{32}S$.

From 400 c.c. of the first vacuum distillate 170°–180°, 50 mm., after the second precipitation with mercuric chloride and long-continued distillation, 28.5 grams of the sulphur oil collected at 184°–186°, which distilled with some decomposition at 283°–285°, under 750 mm. Analysis gave values required for the formula $C_{16}H_{32}S$. Its specific gravity was found to be 0.9222.

I. 0.2148 gram of the oil gave 0.5934 gram CO_2, and 0.2380 gram H_2O.
II. 0.1850 gram of the oil gave 0.1683 gram $BaSO_4$.

	Calculated for $C_{16}H_{32}S$.	Found. I.	Found. II.
C	75.00	75.36	
H	12.50	12.40	
S	12.50		12.49

OCTODECYL THIOPHANE.

Fraction 290°–295° (At. Pres.), $C_{18}H_{36}S$.

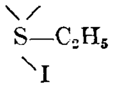
From 350 grams of the vacuum distillate 190°–210°, 50 mm., alcoholic mercuric chloride gave a very thick viscous precipitate, from which was obtained after the second precipitation 50 grams of a dark-colored oil that collected after continued distillation in vacuo, nearly colorless, for the most part at 198°–202°. Under atmospheric pressure this oil distilled with some decomposition at 290°–295°, specific gravity at 20°, 0.9235. The values obtained by analysis corresponded to the formula $C_{18}H_{36}S$.

I. 0.2005 gram of the oil gave 0.5602 gram CO_2, and 0.2214 gram H_2O.
II. 0.1937 gram of the oil gave 0.1634 gram $BaSO_4$.
III. 0.2011 gram of the oil gave 0.1636 gram $BaSO_4$.

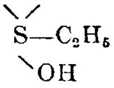
	Calculated for $C_{18}H_{36}S$.	Found. I.	II.	III.
C	76.06	76.22		
H	12.67	12.35		
S	11.27		11.57	11.18

We have made no attempts toward the separation of sulphur compounds with higher molecular weights, but there is no doubt that higher homologues are contained in the less volatile distillates which we have separated from the original oil. It is probable that intermediary members of this series are contained in the distillates examined, and that they might be identified in larger quantities of material.

As has been shown in the method of preparation, the thiophanes combine readily with alcoholic mercuric chloride to form viscous oils. They also form addition products with chlorplatinic acid, which are heavy viscous oils. When heated in a sealed tube with ethyl iodide, addition products are formed which crystallize in small prisms, — $C_nH_{2n}S, C_2H_5I$. These addition products form alkaline hydrates when warmed with silver oxide and water:



$$\underset{\diagdown I}{\overset{\diagdown\diagup}{S}}{-}C_2H_5$$



$$\underset{\diagdown OH}{\overset{\diagdown\diagup}{S}}{-}C_2H_5$$

Bromine exerts a characteristic action on these sulphur compounds. Immediately when they come in contact, combination follows with almost explosive violence, and copious evolution of hydrobromic acid in equivalent amounts to the bromine added. It is wholly unlike the behavior of unsaturated hydrocarbons with bromine. Probably part of the bromine combines with the sulphur, but the evolution of hydrobromic acid begins immediately, doubtless due to the ready replacement of the methylene hydrogen atoms.

The thiophanes still further resemble the well-known series of sulphur derivatives in their behavior toward oxidation. The formation and properties of the sulphones will be described later.

In addition to the analytical data that establish the composition of the thiophanes, we have determined their indices of refraction, and the corresponding molecular refraction verifies the formulas given by analysis.

INDICES OF REFRACTION OF THE THIOPHANES.

Many attempts were made to determine the molecular weights of the sulphur compounds in Canadian petroleum with different solvents, but no concordant values could be obtained, even with the same substance and the same solvent.

Better success, however, was reached in ascertaining the indices of refraction. But since the values for sulphur in organic compounds have not been determined with the same certainty as in the case of carbon, hydrogen, and oxygen, some attention was given to verifying this constant.

The index of refraction found by Nasini for ethyl sulphide was 1.44233; for ethyl mercaptan, 1.43055; and for ethyl disulphide, 1.50633.

Assigning the accepted values to carbon and hydrogen, the calculated value for sulphur becomes, for ethyl sulphide, 7.98; for ethyl mercaptan, 7.81; and for ethyl sulphide, 8.03. In determining the value of sulphur in ethyl mercaptan, we found the angle to be 49° 24′, which corresponds to the index 1.4323, and the calculated value for sulphur is 7.89.

Isobutyl sulphide gave us 1.4585 as its index of refraction, calculated from the observed angle 44° 57′; the corresponding molecular refraction is 46.85. Assigning the accepted values to carbon and hydrogen, the difference is the value of sulphur, 7.98, the same as the value calculated from Nasini's index of ethyl sulphide.

The index assigned to thiophen is 1.5268, which gives as its molecular refraction, 24.12, leaving a value for sulphur 6.53. Thiophen is assumed to have two sets of double bonds. The petroleum sulphur compounds under consideration, although doubtless possessing a ring structure, apparently do not contain carbon atoms connected by double linkage. We have therefore considered it safe to use the value 7.98 for sulphur, and the following molecular refraction for the petroleum sulphur compounds were calculated on the basis of this value.

HEPTYL THIOPHANE.

The fraction 74°–76°, 50mm., which proved to contain heptyl thiophane, gave as its index of refraction 1.468, corresponding to the following molecular refraction:

Calculated for $C_7H_{14}S$.	Found.
40.15	40.82

OCTYL THIOPHANE.

From the fraction 83°–85°, 50mm., was separated octyl thiophane. Its index of refraction was found to be 1.4860, and its molecular refraction:

Calculated for $C_8H_{16}S$.	Found.
44.75	44.91

NONYL THIOPHANE.

Nonyl thiophane was found to be the principal constituent of the fraction 106°–108°, 50mm. It gave as its index of refraction 1.4746, from which was calculated its molecular refraction:

Calculated for $C_9H_{18}S$.	Found.
49.35	49.50

DECYL THIOPHANE, $C_{10}H_{20}S$.

The index of refraction of the distillate 114°–116°, from which was separated decyl thiophane, we found to be 1.4766. Its molecular refraction was calculated as follows:

Calculated for $C_{10}H_{20}S$.	Found.
53.94	53.60

UNDECYL THIOPHANE.

Undecyl thiophane was found in the fraction 130°–135°, 50mm. We found its index of refraction to be 1.480, and its molecular refraction:

Calculated for $C_{11}H_{22}S$.	Found.
58.54	58.53

QUATDECYL THIOPHANE, $C_{14}H_{28}S$.

The fraction 168°–170° gave quatdecyl thiophane. Its index of refraction, 1.4892, corresponded to the molecular refraction :

Calculated for $C_{14}H_{28}S$.	Found.
72.33	71.61

SEXDECYL THIOPHANE.

This sulphur body was found in the fraction 184°–186°, 50mm. Its index of refraction proved to be 1.4903, from which its molecular refraction was calculated :

Calculated for $C_{16}H_{32}S$.	Found.
81.52	80.44

The index of refraction of octodecyl thiophane was found to be 1.4977, but the molecular refraction calculated from this value and its specific gravity 0.9625 at 20° gave a value different by four units from the calculated molecular refraction.

OXIDATION PRODUCTS OF THE THIOPHANES.

Appreciating the importance of oxidation products in ascertaining the composition of the thiophanes, we have tried very thoroughly the action of oxidation agents, nitric acid, chromic acid, and potassium permanganate. These sulphur bodies absorb oxygen with great readiness, and the reaction must be carefully controlled to prevent ultimate oxidation to sulphuric acid. It was not found possible to limit the oxidation to the formation of the sulphoxide, since the sulphone seems to be the more stable condition. Nitric acid readily oxidizes the sulphur with an extremely violent reaction, but apparently also forms nitro-derivatives, since the products contain nitrogen.

Chromic acid oxidizes the sulphur readily, with some danger that the reaction proceeds to ultimate oxidation. Potassium permanganate in either alkaline or acid solution readily effects the change with little danger of the formation of sulphuric acid. After careful study of the conditions, it was found that the largest yield of oxidation product was obtained by adding slowly to the oil one and a half times the calculated weight of permanganate dissolved in thirty times its weight of water and keeping the solution cold. With efficient cooling, the permanganate can be added as rapidly as it is decolorized. After the reaction was complete the precipitated manganic oxide was filtered and the solution distilled with steam, which carried over any unoxidized sulphur compound, leaving behind the oxidation product. When carefully conducted the solution shows no sulphuric acid. From the residue of the steam distillation the oxidation product was extracted with ether and the ether evaporated, leaving a thick heavy oil, which increased in viscosity with increase in molecular weight. In several instances we dissolved the precipitated manganic oxide by saturating with sulphurous oxide and extracted with ether. When the oxidation is complete this is the easier method; but distillation with steam insures the removal of any decomposition products.

The action of oxidizing agents showed a wide difference in deportment between the sulphur compounds from Canadian petroleum and sulphides synthetically prepared. For example, synthetic isoamylsulphide allowed to stand with concentrated nitric acid showed vigorous reaction only after several minutes with evolution of heat. After the reaction ceased heat was applied and water added. The oxidation product collected above the water and partly dissolved. Only a trace of sulphuric acid was found in the solution. On treating the sulphur compounds from Canadian petroleum in the same manner, the reaction was much more vigorous, with immediate evolution of heat. The product formed a thick oil heavier than water, and much sulphuric acid was formed. But the oxidized oil contained much sulphur, showing that it consisted in part of an oxidized sulphur compound. This difference in the action of nitric acid is pretty conclusive proof that the sulphur compounds under consideration are not alkyl sulphides, nor, indeed, members of any other well-known series. It is in accordance with the observation that the hydro-ring compounds, the methylenes, are less stable toward reagents than the aromatic or paraffine compounds.

HEXYL THIOPHANE SULPHONE.

The presence of hexyl thiophane was indicated in the fraction 125°–130° atmospheric pressure, although it was not separated in a pure form. In attempting to prepare the sulphone, the fraction 55°–57°, 50 mm., was treated as described above with potassium permanganate. Oxidation was rapid, even when cooled to 0°. In this instance, after dissolving the manganic oxide in sulphurous acid, the solution was evaporated to dryness and the residue extracted with anhydrous ether. Evaporation of the ether left a thick viscous oil with a sweetish taste and odor, and with no reaction for sulphuric acid. While the analytical values obtained indicated that the hexyl compound was contaminated with a higher homologue, it is evident that hexyl thiophane was the principal constituent.

The values given by analysis were as follows:

C, 49.39; H, 8.47; S, 19.02. Required for $C_6H_{12}SO_2$: C, 48.64; H, 8.11; S, 21.62.

Heptyl Thiophane Sulphone, $C_7H_{14}SO_2$.

The oxidation of heptyl thiophane was conducted as described above. After neutralizing with sodium carbonate and evaporating to dryness, the residue was extracted with anhydrous ether. No sulphuric acid was formed by the oxidation. The following results were obtained by analysis of this product:

I. 0.1992 gram of the substance gave 0.3801 gram CO_2, and 0.1531 gram H_2O.
II. 0.1628 gram of the substance gave 0.2279 gram $BaSO_4$.

	Calculated for $C_7H_{14}SO_2$.	Found.	
		I.	II.
C	51.86	52.05	
H	8.64	8.59	
S	19.75		19.23

This sulphone formed a very thick heavy oil. Its specific gravity was found to be 1.1138 at 20°.

Octyl Thiophane Sulphone, $C_8H_{16}SO_2$.

50 grams of the fraction 87°–89° identified as octyl thiophane was oxidized carefully at freezing temperature. After dissolving the manganic oxide, and distilling with steam, 36 grams of a heavy oil separated with no formation of sulphuric acid. After neutralizing with sodium carbonate, evaporating to dryness, and extracting with ether, this oil gave as its specific gravity at 20°, 1.1142. Its composition was shown by analysis.

I. 0.1697 grams of the substance gave 0.3385 gram CO_2, and 0.1368 gram H_2O.
II. 0.2038 gram of the substance gave 0.2702 gram $BaSO_4$.

	Calculated for $C_8H_{16}SO_2$.	Found. I.	II.
C	54.55	54.41	
H	9.11	9.01	
S	18.18		18.21

NONYL THIOPHANE SULPHONE, $C_9H_{18}SO_2$.

For oxidation of the nonyl sulphur compound, the fraction 106°-108°, 50 mm., was selected. In the first half of the oxidation the potassium permanganate was added slowly in a bath of ice and salt. The reaction was completed at ordinary temperatures, and it proceeded much more slowly toward the end. The required amount of permanganate was consumed in four days, and no sulphuric acid was formed. In all these oxidations, doubtless sulphonic acids were formed to some extent, but their removal was insured by neutralizing with sodium hydrate and extraction of the evaporated residue with anhydrous ether. The specific gravity of the nonyl sulphone was found to be 1.1161 at 20°. Its composition was shown by analysis.

I. 0.1867 gram of the substance gave 0.3874 gram CO_2, and 0.1509 gram H_2O.

II. 8.1679 gram of the substance gave 0.2071 gram $BaSO_4$.

	Calculated for $C_9H_{18}SO_2$.	Found. I.	II.
C	56.83	56.61	
H	9.47	9.04	
S	16.84		16.93

UNDECYL THIOPHANE SULPHONE, $C_{11}H_{22}SO_2$.

We were unable to isolate an oxidation product which corresponded closely in composition to decyl thiophane sulphone from the original fraction, and the amount of the thiophane isolated was not sufficient for oxidation.

Undecyl thiophane was prepared from the undecyl thiophane separated from the fraction 129°-131°, 50 mm. 7.8 grams of the thiophane gave 5 grams of the oxidation product. The heavy oil left after distillation with steam had a pleasant sweetish odor; its specific gravity was found to be 1.1126 at 20°. The following results were given by analysis:

I. 0.1903 gram of the substance gave 0.4176 gram CO_2, and 0.1621 grams H_2O.

II. 0.1470 gram of the substance gave 0.1579 gram $BaSO_4$.

	Calculated for $C_{11}H_{22}SO_2$.	Found. I.	Found. II.
C	60.54	59.84	
H	10.09	9.53	
S	14.68		14.75

Dodecyl Thiophane Sulphone, $C_{12}H_{24}SO_2$.

After treating 7 grams of the distillate 142°–144°, 50 mm., with 9 grams potassium permanganate, the proportions found most suitable, there still remained a slight odor of the thiophane which was completely removed by distillation with steam. The residual oil that remained after extraction of the steam residue with ether gave as its specific gravity at 20°, 1.1372. Its composition was established by analysis.

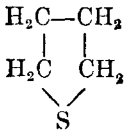
I. 0.1533 gram of the substance gave 0.3483 gram CO_2, and 0.1402 gram H_2O.
II. 0.1645 gram of the substance gave 0.1554 gram $BaSO_4$.

	Calculated for $C_{12}H_{24}SO_2$.	Found. I.	Found. II.
C	62.07	61.98	
H	10.35	10.23	
S	13.79		12.91

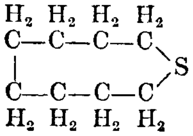
The sulphur compounds in Canadian petroleum have been shown to belong to a series C_nH_{2n}. Their structure must be explained either as unsaturated, like the ethylene sulphides, or by a ring arrangement. That these bodies are not of the ethylene series is shown by the facts that the ethylene sulphides are solids, and that they absorb bromine without evolution of hydrobromic acid. The petroleum sulphur compounds have a very high specific gravity, and they evolve hydrobromic acid copiously when treated with bromine. They resemble the well-known series of sulphur compounds in forming addition products with mercuric chloride, in the formation of sulphones, the formation of addition products with alkyl iodides and alkaline hydrates.

In a former paper by Mabery and Smith it was shown that the sulphur compounds in petroleum are not mercaptans nor thiophenes. The series separated by Mabery and Smith from Ohio petroleum formed crystalline addition products with alcoholic mercuric chloride, and the sulphur oils themselves resembled the alkyl sulphides. But the series contained in the portions of Canadian petroleum with higher boiling points are not alkyl sulphides, mercaptans, thiophenes, nor ethylene sulphides. Their high specific gravity show that they belong to a new series. The hydro-condition of the constituents of Pennsylvania, Ohio, Canadian, Texas, and California petroleum corresponding to the series C_nH_{2n}, C_nH_{2n-2}, C_nH_{2n-4}, etc., has been described in former papers, and the proportions of these series are indicated by the differences in specific gravity. Canadian petroleum is composed for the most part of the series of hydrocarbons poorer in hydrogen. Other constituents of petroleum partake of a similar hydro-condition. The nitrogen compounds, for example, in California petroleum, are tetrahydro- and octohydro-bodies, with structure resembling the hydrochinolines.

Empirically the thiophanes are hydrothiophenes with long side chains, methylene sulphides with numerous or long side chains, or simple ring methylene sulphides. Either class of formulas will satisfy the results of analysis. For example,

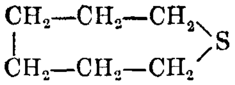


Tetra hydrothiophene or
penta methylene sulphide.

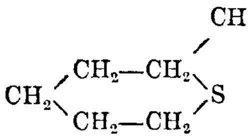


The ready formation of sulphones, addition products with alkyl iodides, and hydrates, is unlike the thiophenes, but the hydrothiophenes may behave differently. It is improbable that rings with a very large number of carbon atoms can exist. A condensed ring structure like chinoline or naptho chinoline with sulphur in place of nitrogen is excluded by the series C_nH_{2n}.

As shown above, the lowest member of the series separated from Canadian petroleum was $C_6H_{12}S$, boiling point 45°–50°, 50mm., which may be hexamethylene sulphide, or methylpentamethylene sulphide. Higher homologues may reasonably be considered as side chain derivatives: —



Hexamethylene sulphide



Methylpentamethylene sulphide

The structure of these bodies will doubtless have to be referred to the methylene ring, yet the precise form of the ring with reference to the number of carbon atoms it contains is still uncertain. For instance, the central group may be a penta-, hexa-, or hepta-methylene ring. It has therefore seemed advisable to adopt a comprehensive name that shall include any possible arrangement, and the term thiophane has been adopted. Without doubt, the structure of these sulphur derivatives will be defined with greater precision when the structure of the hydrocarbons of the series C_nH_{2n}, C_nH_{2n-2}, C_nH_{2n-4}, etc., which constitute the greater part of the constituents of petroleum with high boiling points from any source, are better understood.

UNSATURATED HYDROCARBONS OF THE SERIES C_nH_{2n}.

The need of more accurate information concerning the presence of hydrocarbons of the ethylene series C_nH_{2n} in petroleum was explained in a former paper of this series. Notwithstanding persistent attempts in the numerous treatises on petroleum to show that these bodies have been identified, a critical examination of published results fails to reveal adequate proof of sufficient foundation for these statements. The statement in "Petroleum and its Products," by Redwood, that De la Rue and Mueller proved the presence in preponderating quantities of the ethylene group in Rangoon petroleum is not supported by the results nor by the conclusions of those authors. Concerning the results of Warren, Schorlemmer, and Chandler, it is not true as asserted by Redwood that those chemists found the ethylene series in Pennsylvania petroleum. There are no allusions to this series in Schorlemmer's published papers.

Beyond adopting the general nomenclature then accepted for the ethylene series, Warren evidently left it an open question as to what should be the structural formulae of the products he separated from Pennsylvania petroleum. There are no statements in his papers to show that he made any further preparation of his products for analysis than boiling with sodium, and no further examination to prove structural forms. Certainly it would have been easy to demonstrate the unsaturated condition of those bodies had they contained double bonded carbon, and in view of Warren's clear-sighted and painstaking methods, he could not have overlooked the importance of those tests. In none of his papers on Pennsylvania petroleum does the term olefine or ethylene appear.

The assertion of Hoefer that members of the olefine series, C_2H_4 to $C_{30}H_{60}$ have been found in American petroleum has no foundation. In fact, beside the investigations within recent years in this laboratory leading to a better knowledge of the various series of hydrocarbons with high boiling points, no attempts have been made to discover the true composition of American petroleum. In fractions below 200° from Galician oil, Lachowicz obtained no reaction with bromine, yet so far as they have been examined in this laboratory, distillates below 200° from American oils do not fail to show a considerable bromine absorption. The sulphur compounds in Ohio and Canadian oils have a great affinity for bromine. Substitution may take place in the aromatic hydrocarbons which American oils contain, as well as in the hydrocarbons C_nH_{2n+2}, unless light is excluded. All these bodies, and probably others, are affected in the ordinary determinations of bromine absorption.

At the suggestion of one of us (C. F. M.), in connection with another line of work on petroleum, Mr. T. A. Hicks, a student in this laboratory, undertook a study of bromine absorption in crude oil and in fractions obtained from it after various modes of treatment. Two determinations in the oil before treatment gave as the percentage of bromine absorbed, (I), 8.43, and (II), 8.73. A portion agitated with alcoholic mercuric chloride, washed and dried, gave (I), 9.27, and (II), 9.38, indicating that the crude oil holds in solution to some extent the mercuric chloride compound. After treatment with mercuric chloride, a portion of the oil shaken with common concentrated sulphuric acid gave with bromine, (I), 5.91, and (II), 5.94. The oil was next shaken with fuming sulphuric acid, washed, and dried, when it absorbed bromine equivalent to (I), 3.47, and (II), 3.72. A portion of the oil after the last treatment was distilled under atmospheric pressure, fractions collected at 100°–125°, 125°–200°, and 200°–230°. In the first distillate the bromine absorption was, (I), 4.65, (II), 4.75, in the second, (I), 4.63, (II), 4.58, and in the third, (I), 5.61, (II), 5.17 per cent. Another portion of the treated oil was distilled in vacuo under 50 mm., collecting at 130°–160°, 160°–190°, 190°–260°, and a residue, and bromine absorption determined. Bromine absorption gave in the first distillate, (I), 1.73, (II), 2.00; in the second, (I), 1.96, (II), 2.01; in the third, (I), 3.28, (II), 3.15, and in the residue, 9.52. From these results it is evident that substitution in the less volatile portions of the crude oil or cracking has much to do with the amount of bromine absorbed, especially since in these experiments the reagents employed for the removal of the sulphur compounds should also remove unsaturated hydrocarbons. In this connection Mr. Hicks determined bromine absorption in some distillates from Pennsylvania, Ohio, and Canadian oils, that had been especially purified by long distillation and treatment with reagents, as has been more fully described in another paper. A distillate 194°–196° from Pennsylvania petroleum gave 0.02 per cent bromine absorbed; another 209°–210°, 0.09 per cent; a distillate 209°–210° Ohio purified in the same manner gave 0.03 per cent bromine, and a Canadian distillate, 0.12 per cent. These results may be accepted as showing no absorption of bromine, since this method is not capable of indicating such small percentages. That the quantity of sulphur remaining in the crude oil after treating with fuming sulphuric acid was too small to affect the amount of bromine absorbed is shown by the low percentage of sulphur, 0.09, and in the distillate 200–230 atmospheric pressure, 0.001 per cent.

When the complex composition of American petroleum is borne in mind, and the probability of substitution in the less volatile portions, the difficulties in testing for unsaturated hydrocarbons are evident. Consequently all discussion of this subject by those best informed has been undertaken with reservation, while those of others partake more of the nature of speculation on results which the authors were content merely with presenting, without attempting explanations which the facts were insufficient to justify.

We have had good evidence that unsaturated hydrocarbons are contained in distillates prepared from crude petroleum. Polymerization on standing, the formation of dark colored heavy oils, can only be explained by changes in unsaturated hydrocarbons or unsaturated sulphides. Heptylene was described in another paper as a constituent of Ohio oil.

Since the products described in this paper were contained in the oil extracted with sulphuric acid from burning oil distillate, we have assured ourselves by experiments already described that ordinary concentrated sulphuric acid such as is used in refining burning oil distillates, removes without decomposition, from such distillates, the unsaturated hydrocarbons as well as the sulphur compounds; and consequently that the sludge oil after carefully separating from the acid contained constituents of the crude oil and not products of decomposition.

In fractioning in vacuo as previously explained, a considerable proportion of the distillates from the sludge oil collected below 100°, which could be distilled without decomposition under atmospheric pressure. These portions were therefore carried through ten distillations under atmospheric pressure, and the portions distilling below 150° in vacuo also through ten additional separations in vacuo. On account of great pressure of other work, these distillates stood three years before their study was resumed, although when first obtained they received sufficient attention to show that they contained unsaturated hydrocarbons (Am. Chem. Journ., **16**, 92).

Upon returning to these distillates, it was found that some of them had undergone a marked change. Those collected at certain temperatures, — especially at 44°–45°, seventh distillation, 50 mm.; 70°–71°, twenty-second distillation, atmospheric pressure; 97°–98°, tenth distillation, atmospheric pressure; 118°–119°, tenth distillation, atmospheric pressure, — corresponding in part to the boiling points of well-known hydrocarbons of the ethylene series C_nH_{2n}, had deposited in considerable quantities heavy dark red oils, evidently products of polymerization. Some of the polymerized oils were found to contain as much as 8 per cent of sulphur, although polymerization took place in oils containing not more than 1.5 per cent of sulphur. It was interesting to observe that distillates on either side of those from which the heavy oils separated remained unchanged and were scarcely more discolored than when they were first distilled. That the distillates below 150° contain sulphur compounds with a peculiar odor, attacking the eyes and in other respects different from the series $C_nH_{2n}S$ described above, has been apparent since the first distillation of these oils, although these peculiar compounds are evidently present in much smaller proportions than the others.

All these fractions, as well as those above 150°, united with bromine with explosive violence, and with the evolution of great heat. The bromine addition products, however, were found to be so unstable that it was impossible to purify them by distillation even in vacuo. They decomposed rapidly even on standing with the separation of hydrobromic acid. In ascertaining the composition of these hydrocarbons we have, therefore, relied on the more stable addition product obtained with hydrobromic acid, which had the further advantage that, having no action on the sulphur compounds, the latter need not be removed before heating with the acid.

HEXYLENE.

For the identification of hexylene, the small quantity of the fractions 70°–80°, twenty-second distillation, atmospheric pressure, were put together, and a portion was heated to 120° during five hours with fuming hydrobromic acid. The product was washed, dried, and fractioned in vacuo. On account of the higher boiling point of the bromide it was easy to separate the addition product, and of the small quantity obtained, the greater portion collected at 62°–65°, 50 mm. The boiling point of the bromide under atmospheric pressure could not be determined on account of its instability, but it probably was not far from 135°. The distillate selected for the formation of this bromide was nearly the same in boiling point as that of the hexylenes from various sources, 65°–70°. Upon analysis values were obtained corresponding to those required for monobromhexane;

I. 0.1938 gram of the oil gave 0.3108 gram CO_2, and 0.1283 gram H_2O.
II. 0.2720 gram of the oil gave 0.3045 gram AgBr.

	Required for $C_6H_{13}Br$.	Found. I.	II.
C	43.63	48.73	
H	7.88	7.41	
Br	48.51		47.64

On account of the small amount of this substance it was not possible to purify it more completely.

HEPTYLENE.

The fractions 98°–102°, tenth distillation, atmospheric pressure, more nearly corresponding to the boiling points of the various heptylenes, 90°–100°, were selected more especially because they contained a considerable quantity of the polymerized oil. On either side of these limits the distillates had not changed. After treatment with fuming hydrobromic acid and fractioning in vacuo, the greater portion of the bromide collected at 76°–80°, 50mm. It could not be distilled under atmospheric pressure, but its boiling point was probably not far from 160°. The results of analysis corresponded to the composition of monobromheptane:

I. 0.2000 gram of the oil gave 0.3498 gram CO_2, and 0.1592 gram H_2O.
II. 0.2117 gram of the oil gave 0.2188 gram AgBr.

	Required for $C_7H_{15}Br$.	Found. I.	II.
C	46.97	47.71	
H	8.38	8.90	
Br	44.73	43.99	

The specific gravity of this heptyl bromide was found to be 1.1601.

OCTYLENE.

Octylene was sought for in the fraction 118°–119°, tenth distillation, atmospheric pressure, from which had separated a small quantity of the heavy polymerized oil. After heating with concentrated hydrobromic acid, the oil was fractioned in vacuo until 15 grams collected at 93°–95°, 50mm., corresponding approximately to 180°, atmospheric pressure. The boiling point of the distillate selected is near that of diisopropyl ethylene, 116°–120°, and also that of the octylene, 115°–117°, prepared from octyl chloride by Schorlemmer.

Analyses of the bromide gave values required for monobromoctane:

I. 0.1941 gram of the oil gave 0.3486 gram CO_2, and 0.1613 gram H_2O.
II. 0.1885 gram of the oil gave 0.1851 gram AgBr.

	Required for $C_8H_{17}Br$.	Found.	
		I.	II.
C	49.74	49.00	
H	8.81	9.23	
Br	41.45		41.83

The specific gravity of this octyl bromide was found to be 1.1836.

Nonylene.

The fraction 140°–141°, tenth distillation, atmospheric pressure, was selected for the addition of hydrobromic acid. In fractioning the bromide in vacuo, after the separation of the portion not affected by the acid, the temperature rapidly rose to 110°, and more of the distillate collected at 110°–113° than at any other point in this vicinity. This product was less stable than the lower homologues, but its boiling point under atmospheric pressure would probably be in the vicinity of 200°. Upon analysis it gave values required for bromnonane:

I. 0.1798 gram of the oil gave 0.3481 gram CO_2, and 0.1501 gram H_2O.
II. 0.2117 gram of the oil gave 0.1888 gram AgBr.

	Required for $C_9H_{19}Br$.	Found.	
		I	II.
C	52.17	52.80	
H	9.18	9.33	
Br	38.65		37.96

The specific gravity of this bromnonane was 1.2084.

Higher fractions failed to give us addition products with hydrobromic acid, either because the additive power of the hydrocarbon was too weak, or because if the addition product were formed it was decomposed by distillation in vacuum. The first explanation is probably correct, since we have never noticed the evolution of hydrobromic acid in any considerable quantity. Perhaps it is more reasonable to assume that our distillates contained no higher homologues of this series than those that were identified.

With reference to the solvent power of sulphuric acid for the hydrocarbons described above it is doubtless true that compounds with this acid of the hydrocarbons with high molecular weights are less stable and partake more of the nature of mechanical solutions. It is doubtless true also that the mechanical solvent power of the acid is increased by the other oils that the acid dissolves. If the acid solution separated from the oil be neutralized with calcium hydrate, certain lime salts are formed crystallizing in long needles, and consisting at least in part of nitrogen bases. But sulphonates of the ethylene hydrocarbons and of allied series have not been observed. That unsaturated hydrocarbons may be extracted by sulphuric acid and precipitated by dilution, has been shown in several experiments. We diluted a carboy of sludge acid by pouring slowly into water, and found the precipitated oils to contain these hydrocarbons. As to their relative proportion in the crude oil we have no precise data, but in comparison with the principal constituents it is extremely small.

Since the sulphur compounds described in this paper, as well as the alkyl sulphides, are also readily soluble in sulphuric acid, they should be removed as a part of sludge in refining burning oil distillates.

The results described in this paper may be summarized as follows: The separation from Canadian petroleum of the following series of sulphur compounds: hexyl, heptyl, octyl, nonyl, decyl, undecyl, quartdecyl, sexdecyl, octodecyl thiophanes; the formation of corresponding sulphones by oxidation.

Canadian petroleum contains in minute proportions unsaturated hydrocarbons C_nH_{2n} apparently of the ethylene series. Canadian petroleum contains certain sulphur-free oils soluble in alcohol and different in odor and in other respects from the series hitherto identified in petroleum, possibly terpenes.

We desire to acknowledge our obligations to Messrs. C. A. Lattimer and R. C. McBride, for assistance in the purification and analysis of the sulphur compounds.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF
HARVARD COLLEGE.

THERMAL EXPANSION OF HYDROGEN AND CARBON DIOXIDE UNDER CONSTANT PRESSURE.

By Theodore William Richards and Kenneth Lamartine Mark.

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The Apparatus.

In a former paper, a new apparatus is described for determining the thermal coefficient of expansion of gases under constant pressure. This yielded preliminary determinations of constancy sufficient to warrant the belief that with greater care as to details the method might give extremely accurate results.* The present paper describes further work with this apparatus, proving that the hope was not a vain one. In this new work it was endeavored not only to avoid all conceivable constant errors, but also to introduce a number of technical improvements. These objects were accomplished by improving the methods for the determination of the constants of temperature, pressure, and of volume, and by making the conditions of the experiments more uniform, thus diminishing the number and magnitude of the corrections to be applied.

Many possible sources of error, due to thermal expansion of standards, were at once eliminated by maintaining the room at a constant temperature. A perfectly firm, immovable foundation for the apparatus was also found necessary, both to guard against the accidental moving of the apparatus and to insure freedom from vibration on the part of the various surfaces of mercury.

A room about six meters square, with walls of masonry and floor of concrete, situated in the basement of Boylston Hall, was therefore chosen. The windows were provided with double layers of glass, and a constant temperature of 200°. C. was maintained by means of a gas stove provided with an automatic electrical gas regulator. The electric regulating current was made and broken by a sealed hydrogen barometer,

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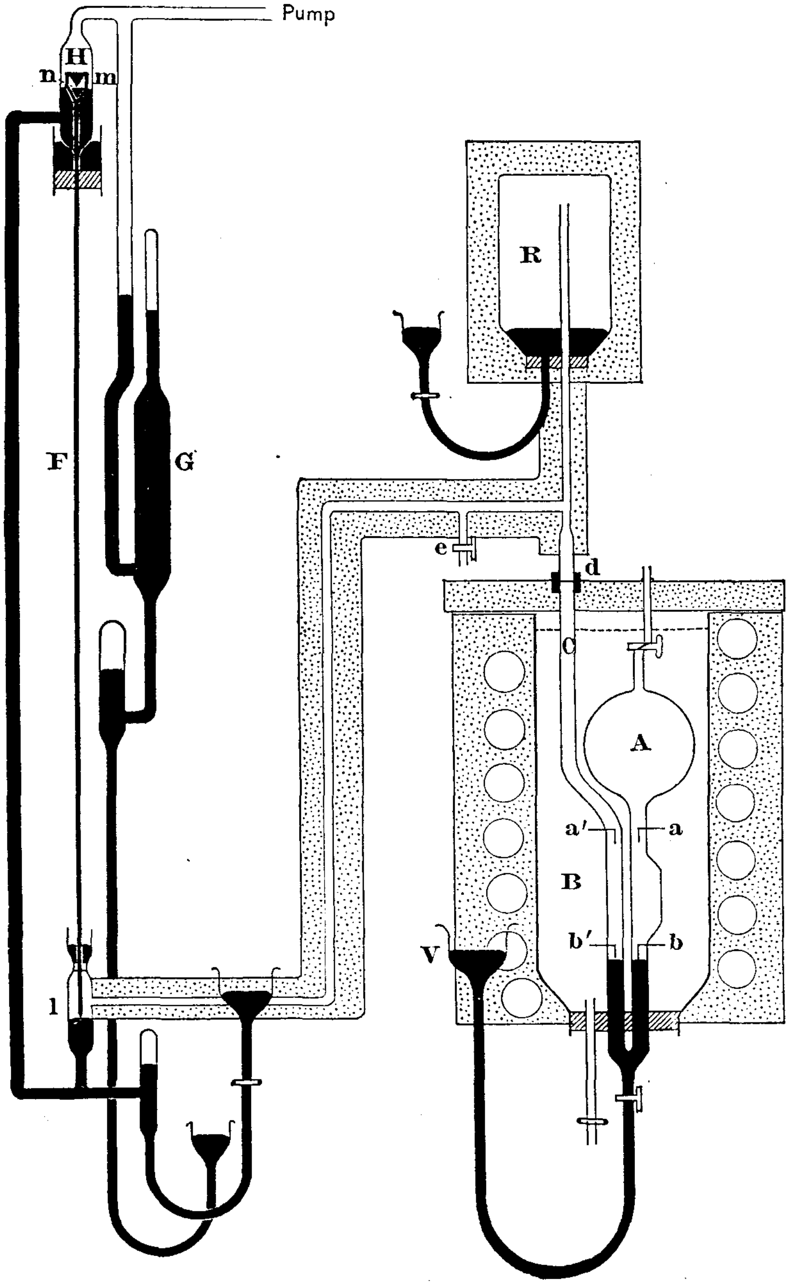
* These Proceedings, **38**, 417 (1902).


Pump
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provided above with a suitably pointed platinum wire to make the current when the pressure of the warming gas reached a certain point.* An electric fan suspended near the ceiling directed a current of air downward, and thus kept all parts of the room at approximately the same temperature. This arrangement was found to give a temperature constant for hours to the tenth of a degree centigrade and seldom to allow a change of more than a few tenths. The great advantage of a constant-temperature room for this work can scarcely be over-emphasized. Not only are the troublesome corrections for temperature in the pressure readings and the errors due to the unequal expansion of the supports of the apparatus eliminated, but all other errors due to thermal effects, both those foreseen and those possibly overlooked, are also thus diminished, if not made quite improbable.

Here was set up, on very firm supports, the apparatus indicated in the diagram, taken from the previous paper. In this apparatus a definite mass of gas was made to expand between two temperatures under almost constant pressure, the slight deviations from perfect constancy being determined with great exactness by a Lord Rayleigh barometer.

The receptacle for containing the gas consisted of two parts, — a large bulb A for holding the total volume of the gas at 0°, and a supplementary bulb (between a and b) for accommodating the increase of volume at 32.383°. These volumes were made as nearly as possible 0.273 liter and 0.0324 liter, proportional to the absolute temperatures used; and each was fixed by a mercury meniscus set by making exact electrical contact with a very finely pointed stout platinum wire firmly fused into the glass.

The bulbs were surrounded by a large vessel, in which were placed in succession the two substances, ice and sodic sulphate, used to fix exactly the lower and upper temperatures.

The framework which supported this vessel or bath was made of stout, well-seasoned pine scantling, mortised and diagonally braced so as to offer great rigidity in all directions. The purpose of this bracing was to prevent any change in the relative heights of the platinum points in the bulb and the side tube. This support was held in place on the floor by four heavy bolts set in cement and imbedded in the ground under the concrete. A bar of two-centimeter channel-iron, similarly bolted down, was forged into the form of a large inverted U extended up and over the middle of the bath support, with which, however, it was not allowed to come in contact. To this bar were firmly wired and cemented with marine glue the tube from the top of the bulb and the upper end of the side tube. Thus the fixed position of the bulb was secured by two supports independent of the building and of each other, — at the bottom by the frame of the bath, and at the top by connection with the iron rod. Both supports were unaffected by temperature changes, because of constancy in the temperature of the room. The frame of the Lord Rayleigh barometer was likewise independently bolted into position.

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* Regaud and Fouillard, Z. wiss. Mikroscop., **20**, 138 (1903); also Journ. Anat. et Phys., **36**, 574 (1900).

The top of the wooden support around the bath was completely covered with a large plate of glass, through a hole in the middle of which rose the upper edge of the bath itself. The joint between the two was filled with marine glue. The cover of the bath was of felt incased in black enamel cloth. By having everything at the mouth of the bath thus capable of being washed, the chance for the introduction of impurities was reduced to a minimum.

A bulb made as nearly as possible like the one used in the preliminary experiments was again employed, and no essential change was made in its arrangement. The lower end of the bulb tube, however, to which the rubber tube leading to the mercury levelling reservoir V had formerly been directly attached, was made to dip into a cup completely filled with mercury and closed at the top by a rubber stopper cemented over with marine glue. This cup, not shown in the diagram, was provided with a side tube near the top to which the rubber tube was attached. This arrangement served as a trap to catch any possible sulphide of mercury formed by the action of the mercury and the rubber and to prevent its passage into the bulb.

In the Lord Rayleigh barometer, which determined the constancy of the pressure throughout the experiment, the position of coincidence of the mercury surface with the lower point (1) of the iron rod (F) was now found by the making of electrical contact instead of by observation through a small lens as before. This electrical method was made possible by cutting the iron rod in two parts and by slipping a very thin sheet of mica for insulation between them. The parts were firmly fastened together again by a collar made of a closely fitting glass tube held in sealing-wax. This change is not indicated in the diagram. The lower end of the iron rod was provided with a sharp platinum tip, which allowed greater accuracy in setting.

In order to read accurately the position of the upper meniscus with respect to the upper iron point, a background, black above and white below, was always adjusted so that the dividing line between the black and white surfaces was exactly tangent to the mercury meniscus, as viewed by the carefully levelled telescope. In this way an exceedingly sharp definition and reading may be obtained, as is well known. Inadequate precautions concerning this point undoubtedly vitiated to some extent the preliminary determinations, but the readings obtained in the determinations discussed below are believed to have been very exact.

A few preliminary trials with the new apparatus under its improved conditions served to show that the accuracy of the determinations of the pressure of the gas, as observed by means of the Lord Rayleigh barometer, had been increased nearly tenfold. Since this measurement now overstepped in accuracy all the others involved in the experiment, it became necessary to improve all the others to an equal degree of precision.

It is obvious that of the three variables — volume, pressure, and temperature — temperature alone requires reference to some generally accepted standard in order that the results of this investigation may be compared with those previously obtained and in order that they may readily be made use of in other calculations. Since changes in volume and pressure are purely relative, the use of the same standards throughout is all that is required; but for the sake of convenience and consistency the international standards were used throughout.

The Measurements of Volume.

A volume is most conveniently measured by weighing the water or mercury which fills it, and making the appropriate corrections for temperature and pressure. For this reason the standard of volume, the liter, is fixed as the space occupied by a kilogram of water at 4° corrected for the weight of the displaced air, but not for the contraction of 0.000043 produced by the pressure of the atmosphere. This standard is used throughout the present work.

It becomes now a matter of importance to decide upon the liquid to be used in the actual measurement of the volume. Into this decision several factors enter.

In the first place, the internal and external pressures upon the rather thin large bulb must be during the measurement as nearly as possible those under which it is actually used. For this reason mercury was rejected as the standard liquid in this case, because its weight would inevitably distort the bulb. Another objection to the use of mercury is the fact that in order thoroughly to expel the air when a large bulb is being filled, the mercury must be boiled in the bulb. This involves the heating of the bulb to a high temperature, — an operation inevitably causing an increase of volume which only slowly readjusts itself, as in a thermometer thus heated. Regnault continually adopted this practice, but it cannot but be considered as dangerous when great accuracy is desired. These objections are less applicable to the use of mercury in the case of the smaller supplementary bulb, for this was so stout as to be scarcely susceptible of deformation by the slight pressure of the small column of mercury which it contained, and could easily be freed from air by exhaustion.

In the next place, the conditions of measurement also must as nearly as possible approach those of the actual expansion-experiment. Especially, because the gas volume is always in contact with mercury, the meniscus used to determine the volume must in each case assume the same shape, and therefore be measured at the same temperature and be set in the same fashion. This condition decides that mercury must be used in measuring the volume of the supplementary bulb, and demands that the temperature at the upper setting should be zero, and that at the lower setting 32.38°, the actual temperatures used in the final experiments. Fortunately, because the volumes of mercury are small, this demand is not inconsistent with adequate accuracy as to the other attendant conditions.

One other point proved to be at least equally important; namely, the increase of volume of the large bulb upon warming from 0° to 32.38°. This increase, of course, adds to the volume of the auxiliary bulb, constituting the total gain in volume with change of temperature. Two alternatives were open, — either the coefficient of expansion of the glass could be calculated from the weight of water filling the bulb at 32.38°, or else the linear coefficient of expansion might be measured directly.

After a number of time-consuming preliminary experiments it was decided to adopt the latter course, because the expansion of water over the range in question is six times as great as that of glass, and hence any error in the assumed value of the former or in our measurements would cause a sixfold percentage error in the coefficient of expansion sought.

In view of these considerations, the following means of measurement were adopted: The bulb and supplementary bulb were cut off from the rest of the apparatus somewhat below the lower pointed platinum wire (b), and a glass stopcock was attached at the junction. The total volume between the stopcocks was now found by weighing the bulb full of water at 0°; the volume from the lower stopcock to the upper contact point (a) was found by weighing the mercury which filled this at 0°, and the volume between the lower stopcock and the lower point (b) was found by weighing the mercury which filled this at a higher temperature. The linear coefficient of expansion of the glass was then determined with great care.

From these data the volumes sought may be calculated with as great accuracy as is possible under the circumstances. The details of the measurements are given below.

The bulb, filled with air dried by passage over potassic hydroxide, was balanced against a sealed counterpoise of approximately the same volume. Observations of temperature and atmospheric pressure necessary for the calculation of the weight of air contained in the bulb were also made. The weight of water of the corresponding volume at 0° C. was next obtained as follows: A levelling bulb was connected with the temporary stopcock at the lower end of the bulb by a rubber tube. Through this the bulb was filled with pure water to the upper stopcock. The lower stopcock was then closed and, in order to remove the dissolved air, the water was boiled in vacuo in the bulb. After the amount of water had again been adjusted by opening the lower stopcock so that it filled the bulb to the upper cock, this latter cock was closed and all was packed in ice for one and a half hours. In a special experiment this time had been found amply long enough for the required degree of accuracy in temperature. The lower stopcock was then closed (the upper one still remaining open), the bulb and projecting tube were cleaned and dried, and the levelling bulb was removed. When the same temperature as that within the balance case had been obtained, the bulb and water were weighed and the volume from stopcock to stopcock was calculated.

These weighings of the bulb filled with water indicated 391.900, 391.904, and 391.903 grams; average, 391.902 grams. The weight of the bulb filled with pure dry air under the same conditions of temperature and pressure (20° and 752.9 mm.) was 85.192, and the air which it contained must have weighed 0.366 gram. Therefore the water which it contained must have weighed 391.902 + 0.367 − 85.192 = 307.077 grams. If the density of water at 0° is taken as 0.999868 *, the total volume of the bulbs is found to be 307.118 milliliters ± 0.001. This measurement was made when the bulb was full of water, but in other respects under exactly the same conditions of temperature and pressure as in an actual gas measurement. The average pressure of the column of water inside during the measurement of volume was less than 8 millimeters of mercury, a pressure which would produce an increase of volume in the globe of less than 0.001 milliliter,* hence this distention during measurement may be neglected. If mercury had been used instead of water it would have been a very serious possible cause of error.

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* Chappuis, Trav. et Mém. du Bureau des Poids et Mes., **13** (1904).

In order to obtain actually the volume occupied by the gas at 0° the volume below the upper platinum point must be determined and subtracted from 307.118. Therefore mercury was next run into the bulb from below to the upper platinum point, the air bubbles were removed by exhaustion, and after equilibrium of temperature had been reached in the ice bath, the exact adjustment of the mercury to the level of the platinum point was accomplished by a fine screw and indicated by electrical contact. The stopcock was then closed, the rubber tube removed as before, and the mercury inclosed in the bulb run out and weighed. Three successive experiments gave 472.21, 472.14, and 472.19 grams; in average, 472.18 grams. If mercury has a density of 13.5956 at 0°, this volume is then 34.730 milliliters ± 0.001. The mercury was so shallow and this bulb so stout, that it could not have been appreciably distended during measurement. The actual volume occupied by the gas at 0° must therefore have been $307.118 - 34.730 = 272.388$ milliliters, ± 0.002.

To ascertain the total volume of the supplementary bulb and the large bulb taken together, — namely, the volume from the upper stopcock to the lower platinum point, — the small volume between the lower stopcock and the *lower* platinum point was found and subtracted from the total volume 307.118. Instead of measuring this small volume at 0° C., it was done at a higher temperature, in order that the shape of the meniscus at this point might be more nearly that which it assumed during the performance of the main experiment.† After dividing by the density of mercury and making correction for the contraction of the glass, the volume at 0° was found to be in five determinations, — 1.944, 1.941, 1.946, 1.944, 1.943; average, 1.944 ± 0.0008 milliliters.

Thus the volume of the large bulb and the supplementary bulb together at 0° was 307.118 − 1.944 = 305.174 milliliters. This volume was sought at 32.38°, however, not at 0°, therefore the coefficient of expansion of the glass must be determined and the appropriate correction applied. It is easily calculated that the slight difference between the expansion of the projecting platinum points and the glass is wholly negligible, causing a possible error in the volume of only about 0.00002 milliliter. Therefore the coefficient of expansion of glass alone need be considered.

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* Lord Rayleigh found in a 2 liter globe a distention of 0.6 milliliters, and Cooke and Richards found in a 5 liter globe a distention of 1.7 milliliters for each atmosphere of internal pressure. Our bulb was at least as stout in proportion as these, hence the above conclusion.

† This determination should have been made at 32.4°. It was, as a matter of fact, through some oversight, made at 18.3°, where the average amount of mercury found was 26.350 grams. The breaking of the tube made a redetermination at 32.4° impossible, but the above value can hardly be essentially wrong.

For the sake of simplicity of manipulation the measurement of the coefficient of expansion of glass was made upon the side tube which had been cut off as already explained. The assumption here made that the glass of the bulb and of the side tube was of the same quality seemed admissible both because of the homogeneous appearance of the glass itself at the line of welding and because of the consideration that such a union could be made only between pieces of glass of approximately the same coefficient of expansion.

After a cross had been scratched with a diamond a few centimeters from each end, the tube was mounted on a firm support the height of which could be regulated by a finely threaded screw. Connections were made so that a rapid stream of water from a large reservoir could circulate through the tube and a thermometer for reading the temperature of this water was inserted into the middle of the tube. The tube as well as the connection to the reservoir were wrapped in cotton-wool except for the spaces about the crosses and the position opposite the thermometer scale.

The reservoir was now filled with ice and water, and after the temperature of the stream of water had come to equilibrium, the lower cross on the tube was adjusted by means of the fine screw of the support to the same level as the cross-hair in the eyepiece of the cathetometer telescope. The position of this cross-hair was kept fixed throughout the experiment, and was made the permanent point from which changes in height were measured. The position of coincidence of the cross-hair of a second telescope with the image of the upper cross on the tube was read off on the dial of the micrometer which bore this cross-hair. When this position had been determined by the average of several settings, the tube was readjusted in respect to the lower cross and the height of the upper cross was again read. This performance was repeated several times until the tube until the exact distance between the upper cross and the lower cross was well established.

Warm water was then substituted for the ice-water and a similar set of adjustments were made. Thus, since the lower cross was always set at the height of the permanent cross-hair in the lower telescope, the whole expansion of the tube was observed by the movement of the micrometer which bore the cross-hair in the upper telescope. The temperature of the stream of water was noted at the time of reading, and also the temperature of the cathetometer was taken in order to correct for the thermal expansion of this instrument, in case there had been a slight change.

The total distances between the two crosses was determined by direct reference through the telescopes to the brass meter stick to be 611.3 mm. In the telescopes a magnification was used greater than the standard adopted for all other determinations. It was necessary therefore to refer the micrometer dial to the brass meter stick. One dial division was found to equal 0.0886 mm.

Two series of determinations of the expansion of the tube were made — the first in the manner described above; but in the second the cotton-wool insulation was replaced by a glass jacket which was filled successively with melting ice and with warm water. In this determination the temperature of the tube was taken as the average of that of the stream within and of the water of the jacket without. The two were of course very near together. The latter measurements are rather the more satisfactory, because the temperature was more certain.

EXPANSION OF GLASS.

	Expansion of Tube (Corrected).	Change in Temperature of Tube.	Linear Coefficient of Expansion of Glass.
	mm.	° C.	
I.	0.186	36.0	0.00000846
	0.182	35.9	0.00000832
	0.183	35.9	0.00000836
III.	0.185	35.9	0.00000844
	0.182	35.9	0.00000832
Average coefficient of linear expansion .			0.00000838 ± 0.00000002
Coefficient of cubical expansion			= 0.00002524 ± 0.00000006

This is very near the average value 0.000025 found by others with soft glass of this kind. Therefore the volume of the two bulbs together at 32.38° becomes

$$305.174\ (1 + 0.0000252\, t^\circ) =$$
$$305.174\ (1 + 0.000817) = 305.423 \text{ milliliters.}$$

It is well known that glass when heated assumes an increase of volume which is not at once given up on cooling. Thus the zero point of a thermometer made of hard French glass is lowered for a time by 0.03° after heating to 33°.* This corresponds to an increase of volume of only one in two hundred thousand, and is therefore on the limit of the accuracy of the above volume. Error from this source, slight as it might be, was partly if not wholly eliminated by determining the volume under conditions as nearly as possible like those used in the actual expansion experiments. The small temperature interval of 32.4° used in the present experiments is a very favorable circumstance, as regards this correction, because the error increases greatly with rising temperature. In a research aiming at greater precision than the present one, the point should receive careful consideration.

MEASUREMENTS OF TEMPERATURE.

The necessity of recording the temperature of the bath in units of the international standard has already been pointed out. The temperature of melting ice under atmospheric pressure is one of the bases of this standard, and the position on it of the transition temperature of sodic sulphate has been accurately determined by Richards and Wells as 32.383° on the hydrogen scale.† The temperature of the gas becomes, then, a question of the purity of the substances used in the bath.

The bath was filled with very clear pure washed commercial ice and distilled water. After standing half an hour the water was replaced with freshly distilled water which had been cooled to nearly 0° C. in Jena glass. By this means very little impurity from melting ice was introduced. As a matter of fact, careful comparison of the melting point of this ice thus measured was found to be within a thousandth of a degree of that of the purest ice made from boiled distilled water, wholly in platinum. Moreover, the higher temperature was read in such a way as to eliminate any error from this source, so that the results cannot be affected by any impurity in the ice.

Because ten kilograms of sodic sulphate was needed to keep the bath at 32.38°, the preparation of the salt in a perfectly pure state was a serious task. Therefore, instead of always using sodic sulphate of the greatest possible purity, only moderate precautions were used, and the presence of slight impurities was corrected for by observing the temperature of the melting mass on the standardized Baudin thermometer number 9390. A very small amount of impurity did not affect the constancy of the point, but only its absolute position. The zero point of this thermometer was determined immediately afterwards in each case by plunging it into ice, which had been prepared similarly to that for the bath itself; thus was the actual interval over which the gas had been heated recorded on the thermometer.

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* Notice sur les Thermom. à mercur. Bureau Internat. Poids et Mes. (1896), p. 9.

† Proc. Amer. Acad., **38**, 431 (1902).

In order to determine the exact value of the temperature interval thus found on the international scale, the thermometer was afterwards standardized by reading it in the purest sodic sulphate many times recrystallized and then in purest ice. The following changes in temperature were observed: 32.580 — 0.103 = 32.477; 32.583 — 0.107 = 32.476; 32.579 — 0.101 = 32.478; average, 32.477.

Because in these tests exactly as much of the thermometer was exposed to air at 20° as in the actual measurement, the reading 32.477 signified the transition temperature of pure sodic sulphate, a temperature of 32.383°. In order to obtain the true temperature, 0.094° must be subtracted from the reading of the thermometer. This was done in each case.

In a few cases the temperature is recorded as 32.384°, or 0.001°, above the maximum temperature given by the purest sodic sulphate. This difference was probably due to a slight error in thermometry; but since it may have represented the real interval, — the excess being due to a trace of impurity in the ice, — the reading is recorded as found.

Measurements of Pressure.

The standard of length, in terms of which pressure was measured, was kindly furnished us in terms of the international meter by the National Bureau of Standards in Washington, by their calibration of our Geneva brass meter-bar with silvered scale, at 20.5° C. The scale was found to be amply accurate enough for our purpose, no single centimeter varying more than 3 microns from the mean, and very few varying more than 1 micron. No length of 20 centimeters on the scale varied from the average value by more than 0.01 millimeter. The actual readings, however, were recorded in terms of the dial divisions of the micrometer screw which carried the cross-hair in the eyepiece of the telescope.

The evaluation of these divisions was made by repeatedly noting the number of divisions passed through in changing the setting of the cross-hair from coincidence with one mark of the meter-bar to coincidence with another. It was found that the distance from the telescope to the object observed must not vary more than half of one millimeter in order to preserve this value to the required accuracy. The distance from the upper iron point of the Lord Rayleigh barometer to the object lens of the telescope — 775.0 mm. as given by concordant measurements — was adopted as standard whenever the telescope was used. The following table contains the comparisons of dial divisions with the length of separate successive lengths of two millimeters each. The measurements were made from two independent adjustments of the focus, with very careful measurement of the distance between the meter-bar and the telescope. Each of these figures represents the mean of these measurements.

Designation of Distance Measured.	Value in Micrometer-Dial Divisions.
31.0–31.2	162.2
31.2–31.4	162.7
31.4–31.6	162.0
31.6–31.8	162.5
31.8–32.0	162.1
32.0–32.2	162.0
32.2–32.4	162.7
32.4–32.6	162.6
32.6–32.8	162.7
32.8–33.0	162.1
31.4–31.6	162.1
Average, 2 mm. of scale . .	162.34 ± .06

The slight differences between these measurements are due in part to necessary experimental error, and in part to the slight inaccuracies of the divisions measured; but the probable error of the average is so small as to be negligible. According to the report of the National Bureau of Standards, a millimeter in this part of the scale has the average value of 1.0004; hence a single dial division has the value in millimeters of $\frac{2.0008}{162.34} = 0.012325 \pm 0.000005$. This value was used throughout the work.

Before proceeding to the measurement of the pressures exerted by the gas during the experiment, that constant correction due to the difference in the distance apart of the platinum points in the bulb and in the side tube was evaluated. As in the preliminary experiments, the bulb was placed at the arbitrary standard distance from the telescope and the difference in level of the upper pair of points was observed by successive settings of the cross-hair of the telescope upon each point. The telescope was then lowered and a similar set of observations of the lower points was made. Then the position of the upper points was again determined. This first series showed the points in the bulb to be higher than those in the side tube — for the upper pair by 0.749 mm., 0.750 mm., 0.752 mm.; 0.751 mm., average 0.751; and for the lower pair by 0.167 mm., 0.169 mm., average 0.168 mm.

These showed as the average the distance apart of the points in the bulb to be 0.583 mm. greater than the corresponding distance in the side tube.

The second series of measurements was made after the bulb had been readjusted in respect to its distance from the telescope. The further precaution was also taken of viewing the points from the opposite side of the bulb in order to eliminate any error due to possible irregular refraction of the glass.

In the second series, the heights of the bulb points exceeded those of the side points, in the case of the upper pair by 0.753 mm., 0.754 mm., 0.752 mm., average 0.753; and in the case of the lower pair by 0.167 mm., 0.169 mm., 0.170 mm., average 0.169.

These gave the average greater distance apart of the bulb points as 0.584 mm. (agreeing as nearly as is reasonable with the proceeding), and so for the total average of all determinations 0.5835. These measurements were made at 20°, while the tube was used at 0° and 32.4°, but the effect of the corresponding dilation is negligible. The difference of mercury level at 32.4° must nevertheless be reduced to 20° (at which the main barometer was read), before it is added to the reading of the main barometer. The value then becomes 0.582, which is used as K in the calculation on page 135.

Although the *change* in pressure of the gas with change in temperature must be determined with the utmost accuracy, the actual pressure which the gas exerts need be known only approximately. An inspection of the formula for calculation on p. 135, where this value P enters both the numerator and the denominator, makes this fact obvious. An accuracy to within half a millimeter was shown by calculation to be quite sufficient. Since the pressure of the gas is measured on the Lord Rayleigh barometer by the height of the mercury meniscus (m) above the lower point (1), and since the distance from the meniscus to the upper point (n) is determined during the course of the experiment, it only remains to find the distance between the two points. In making this measurement, the cross-hair of the telescope was set upon one of the points in question. The telescope standard was then revolved so that on a vertical axis this height could be directly read on the silvered brass meter-bar, which had been placed at the proper distance from the object glass. The height of the other point was determined similarly. Two determinations, giving 761.33 and 761.35 when corrected to the international meter, agreed in showing the distance between these points to be 761.34 mm. This measurement, like all the other measurements concerning the pressure, was made at 20°.

The actual pressure of the gas in the tubes leading to the bulb was of course indicated by the barometric column thus read, plus the very small pressure (usually 0.03 mm.) of the residual gas in the Torricellian vacuum. This latter was kept as constant as possible by a good Sprengel pump, and read from time to time with a McLeod gauge.

The McLeod gauge was so made that a difference of pressure read in the fine tube was one hundred times as great as the real pressure being determined. It was only necessary, therefore, to read this gauge to an accuracy of 0.1 mm. Since, moreover, the only requisite is that this pressure shall remain about constant within a small known range throughout the course of an experiment, a possible inaccuracy in registration of the gauge due to adsorption * would make no difference, applying alike to all measurements, and cancelling from the final result.

The manner of making the pressure readings was the same as that previously used, except for the improvement in the background already described. The following typical day's record shows the constancy of reading attained and the usual magnitude of the corrections which were applied.

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* Ramsay and Baly, Phil. Mag. [5], **38**, 314 (1894).

February 28, 1902. Ice in Bath at 12.30 p.m. Pure previously Boiled Freezing Water run in in Place of Ice-Water at 2.00 p.m. Meniscus above the Point.

		Successive Readings.				
		1	2	3	4	5
Hour	. . . . .	2.15	2.40	3.05	3.50	4.20
Temperature of barometer	°C (upper) °C (lower)	20.0 20.0	20.0 20.0	20.0 20.0	20.0 19.9	20.0 20.0
McLeod gauge with corresponding pressures . .	dial divisions millimeters	329 0.041	308 0.038	267 0.033	256 0.032	250 0.031
Reading of cross-hair set on meniscus	dial number	32.6 32.6 32.5	48.4 48.3 48.5	38.7 38.9 38.9	44.3 44.1 44.5	4.0 3.8 4.0
Revolutions of dial* . .	. . . . .	2	2	2	2	2
Reading of cross-hair set on point	dial number	35.4 35.6 35.6	0.9 1.2 1.2	42.3 42.5 42.7	47.5 47.8 47.5	7.7 7.7 7.8
Distance between meniscus and point	dial divisions millimeters	103.0 1.269	102.5 1.263	103.7 1.278	103.3 1.273	103.8 1.279
Corrected excess of pressure of gas over height of iron rod	millimeters	1.310	1.301	1.311	1.305	1.310
Average		1.307 ± 0.001 mm.				

Because the iron rod was 761.34 mm. long, the total height of mercury at 20° supported by the gas pressure was 762.647 millimeters. The points a and a^1 being essentially on the same level, this is also essentially the pressure of the gas in the bulb. Each determination of the coefficient of expansion yielded a corresponding table at 0°, and another similar one at 32.48°. The difference between the corrected values for the two temperatures is recorded in the tables on pp. 136, 139 as p.

A change in atmospheric pressure affects equally the levels of the mercury in the lower and in the upper part of the Lord Rayleigh barometer. Provided, however, that the pressure on the lower meniscus, fixed by the pressure in the side tube of the bulb, did not alter, a readjustment of the lower level into contact with the lower point (1) of the barometer would bring the upper level also back to its proper place. In practice the reading of the upper mercury level was taken within a minute after the adjustment was made. Under ordinary atmospheric conditions, no barometric change sufficient to invalidate the setting takes place in so short a time. Thus atmospheric changes could be neglected, unless the atmospheric barometer was changing with unusual rapidity, in which case measurements had to be suspended.

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* One revolution of the dial equals fifty dial divisions.

The temperature correction of the Lord Rayleigh barometer is based on the difference between the change in length of the mercury column, due to change in the density of the mercury, and the change in the length of the iron rod. This amounts to 0.136 mm. per degree centigrade, which is to be subtracted from the height of the mercury meniscus when the temperature of the barometer is above the standard (20° C.). The temperature of the barometer was taken as the mean of that recorded by a standardized thermometer at the bottom and another at the top. In most cases no correction at all was applied, because the room was kept at constant temperature and the air in it was thoroughly stirred by a large electric fan. Nevertheless it is probable that slight fluctuations of a few hundredths of a degree often occurred, and it would have been better to surround the iron stem with a very accurate constant temperature bath. The accuracy was increased by taking many observations over several hours, the average of which would of course be more precise than a single setting, because it would represent an average temperature; but it is probable that this point was the weakest one in the present series of measurements, and accounts for the most of the variations in the preceding table as well as for those in the final tables of results. This conclusion is based upon the evidence found in the foregoing table that the agreement between successive readings of pressure at a given time was closer than between the averages at different times. These differences could hardly be due to the electrical contact arrangement, since the levelling bulbs were adjusted by means of very finely threaded screws and the current used to show the contact was of too low strength and too small voltage to allow perceptible sparking. Moreover, the tubes were so wide and the platinum points were so sharp that no difference in position was noted between the making and breaking of contact. It may be noted that the variation between the readings, undesirably large as it is, does not exceed that observed by Chappuis in his admirable work on the constant-volume hydrogen thermometer. The measurement of pressure more accurately than this is an exceedingly difficult problem.

In preparing the bulb for the introduction of the gas, it is of great importance to remove as far as possible all other adsorbed gases, of which water vapor probably forms the most dangerous component. After dry air had been passed through the bulb for several hours, it was exhausted by means of a Sprengel pump and the outside was heated with hot water to about 70° C. Dry gas was next admitted, and then the bulb was again exhausted. This process was repeated three times, after which dry gas was allowed to stream through the bulb from two to three hours. The mercury used in the bulb was dried by heating it to 150° C., and dry air was drawn through the rubber tube which connected the lower part of the bulb with the levelling bulb for five hours.

CALCULATION OF RESULTS.

The value sought in the work is the coefficient of expansion of each gas under constant pressure; but because it was impracticable to keep the pressure absolutely constant a slight correction had to be applied for changes. The consideration below shows that Boyle's law may safely be used in referring these changes to changing volume. The actual variation of pressure in the case of hydrogen was about 2.5 mm., and Regnault found that for this gas the product of pressure and volume varied from constancy by less than one part in two thousand for an increase of an atmosphere in pressure. This correction would amount, then, to an error in volume of not over 0.0005 c.c. for a change of $\frac{1}{300}$ atmosphere in these experiments, a quantity which is beyond their limit of accuracy. Hence, for this very small range of pressure Boyle's law may be used as exact.

Accordingly, in the previous paper, the equation used for calculating the results was as follows:

$$\alpha = \frac{(P + p - K)\,(V + v) - PV}{PVt}$$

in which α equals the expansion per degree Centigrade of unit volume at 0° C.; V equals the volume at 0° C.; v, the increase in volume; t, the difference in temperature between the two baths; P, the original pressure at 0°; p, the small increase of pressure in the gauge on the reading at the higher temperature; and K, the constant value to be subtracted from p due to the greater distance apart of the platinum points in the side tube over the distance apart of those in the bulb.

In this equation no account is taken of the fact that the mercury enclosing the gas has a higher vapor tension at 32.4° than at 0°. This correction was then omitted because it was distinctly beyond the probable accuracy of the work. In the present case, however, the vapor tension of mercury at 32.4° (about 0.003 mm., according to Morley)* must be considered, because it is larger than the probable error of the pressure readings. The pressure at 0° is undoubtedly less than 0.001 mm. and therefore negligible even here. It is of course not quite certain that the gas is wholly saturated with mercury vapor, but since several hours were allowed to pass before the final readings were taken, the saturation was probably almost complete. The equation giving the desired result is therefore

$$a_p = \frac{(P + p - K - 0.003)\,(V + v) - P\,V}{P\,V t}$$

Here $V = 0.272388$ liter, $V + v = 0.305423$ liter, and $K = -\,0.582$ mm., as recounted in the preceding pages. The other quantities in the equation are given below for each experiment.

The Coefficient of Expansion of Hydrogen.

Hydrogen was made electrolytically from pure hydrochloric acid, using a platinized platinum cathode and a zinc amalgam anode. It was purified by passage over platinized asbestos, through two tall drying towers filled with solid potash sticks and over phosphoric oxide. The hydrogen entered the bulb from the top, and "dead ends" were avoided as much as possible in the supply system. The method of enclosing within the bulb an amount of gas suitable for the experiment was not modified from that used in the preliminary work.

In the following table are contained the eight last consecutive and most satisfactory determinations made with pure hydrogen gas. Numbers 1–5 inclusive are measurements on the same hydrogen, and numbers 6–8 inclusive are determinations made after a complete repetition of the process of drying and filling the bulb.

The "probable error" of these results, computed according to the method of least squares, is ±0.00000003. This does not mean, however, that the expansion of hydrogen is known to an accuracy of three in the sixth significant figure, because at least one quantity entered into the calculation of this coefficient which has a possible error of one in the fifth significant figure; but it does show that accidental errors of reading tem-

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* Morley, Phil. Mag. (1904), June, 663.

HYDROGEN.

No.	P.	p.	t°.	α_p.
	mm.	mm.	°C.	
1	762.6	−2.438	32.384	0.0036607
2	"	−2.437	32.384	0.0036608
3	"	−2.437	32.384	0.0036609
4	"	−2.437	32.382	0.0036610
5	"	−2.437	32.382	0.0036610
6	762.4	−2.440	32.382	0.0036608
7	"	−2.443	32.378	0.0036611
8	"	−2.443	32.378	0.0036611
		Average		= 0.00366092

perature and pressure and errors of manipulation arising during the performance of the experiment have a "probable error" of only one in a hundred thousand. It seems plain, therefore, that farther advance in accuracy can be attained only by improvements in the methods of measuring volumes and the expansion of the various solid and liquid substances connected therewith, and in the measurement of temperature. These difficulties must enter equally into any other investigation with the same object. When these are improved, the method will give yet better results.

The difference between the final result, 0.0036609, and that found in the preliminary work, 0.003659, is no greater than the known experimental error of the preliminary work, which made no pretensions to absolute accuracy, but was intended only to outline the capabilities of a new arrangement of apparatus.

The comparison of this result with the work of others is much complicated by the fact that no one else has worked under similar conditions. Nearly every one has chosen a wider range of temperature, namely, from 0° to 100°; and, moreover, very few have worked upon the volume-increase, but have chosen rather the pressure-increase in constant volume.

The old result of Regnault,* 0.0036613, is indeed not very far from the new one, but has too large a possibility of error to afford important evidence. More recently, the coefficient of expansion at constant pressure has been studied very carefully by Chappuis,† who finds that the volume-change per degree under constant pressure is 0.0036600, a value noticeably lower than ours. The pressure was, however, different from ours, and the range of temperature greater.

The value corresponding to tension-increase in constant volume has been found by three separate experimenters recently to be about 0.00366256,‡ as follows:

Chappuis	0.00366256
Travers	0.00366255
Onnes	0.00366257
	0.00366256

There can be no doubt that the value for constant pressure is less than this, because $p\,v$ with hydrogen undoubtedly increases as the volume diminishes. According to Regnault, if $p\,v$ at a pressure of one meter of mercury is taken as 1.00000, that at 2 meters is 1.00043, that at 4 meters is 1.00153, and that at 8 meters is 1.00424. By plotting these figures as ordinates and the pressures as abscissae, there is obtained, as is well known, a curve which is almost linear, and is therefore capable of accurate use in graphic calculation. The application of this curve to the present case is now to be discussed.

Imagine a mass of gas at the temperature 32.38° under atmospheric pressure compressed further until it assumes the volume occupied by the same mass of gas at 0° and atmospheric pressure. Its tension will now be 1.12 atmospheres. For this increased tension, by a slight extrapolation of Regnault's curve, it is easy to find that the value for $p\,v$ would be about 1.000044§ or $\frac{0.0044}{0.12} = 0.037$ per cent of the increase in pressure. Thus it would appear that the pressure-coefficient in constant volume should be about 0.037 per cent greater than the volume coefficient under constant pressure, or the latter would be $0.00366256 - 0.00000134 = 0.0036612$, a value but little above the value 0.00366092 found in the present series of experiments. An objection to this comparison lies in the fact that Regnault made his measurements of the change of pv at 4°, and not at 32°; but probably this difference would not affect very seriously the results.

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* Regnault, Mémoirs de l'Académie des Sciences, **21**, 116, 426 (1847).

† Chappuis, Rapports du Congrès International de Physique, **1**, 133 (1900). Trav. et Mém du Bur. Int. Poids et Mes., **13** (1903.) See also Travers, The Experimental Study of Gases, p. 149 (Macmillan, 1901).

‡ Travers, Chem. News., **86**, 61 (1901 ?).

§ Various graphic calculations gave figures varying from 1.000042 to 1.000046.

Lord Rayleigh, in his recent careful determination of the compressibility of hydrogen, has found that if the value of pv at one atmosphere's pressure is taken as 1.00000, at half an atmosphere's pressure it will be 0.99974.* The results of Regnault given above show that this change is nearly linear with the pressure, hence the slight extrapolation giving the value 1.000062 for 1.12 atmospheres is permissible. Now $\frac{0.000062}{0.12}$ is 0.052 per cent, or the percentage difference between the coefficients of expansion in constant volume and that under constant pressure; therefore on the basis of the results of Travers and others just recorded, the latter value α_p should equal 0.00366065, while we find 0.0036609. The agreement is as good as could reasonably be expected, especially considering the fact that Lord Rayleigh's experiments were made at 11°, while ours demand results for 32°.

Thus if the usual value for the tension increase in constant volume is assumed as correct, the application of Regnault's value for the relation of pressure and volume would make our results appear slightly too low, while the application of Lord Rayleigh's value would make it appear slightly too high. While neither application is strictly precise, in any case it appears that the new value for the coefficient of expansion is not without support in other work, and may be not far from the truth.

Further work upon hydrogen seemed to be aimless unless much time was spent in the redetermination of the essential dimensions involved, — a colossal task, involving several separate investigations. Therefore another gas was studied.

THE COEFFICIENT OF EXPANSION OF CARBON DIOXIDE.

Carbon dioxide, made by the action of hydrochloric acid on marble and purified by passing through two wash bottles containing acid sodic carbonate solution, an Emmerling tower containing the same solution, two towers with sulphuric acid and over phosphoric oxide, gave the following results :

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* Phil. Trans., A **204**, 351 (1905).

CARBON DIOXIDE.

No.	P.	p.	t°.	α_p.
		mm.	°C.	
1	761.5	–0.983	32.376	0.0037277
2	"	–0.984	32.376	0.0037276
3	"	–0.974	32.376	0.0037282
4	"	–0.971	32.372	0.0037288
5	"	–0.971	32.372	0.0037288
		Average		0.0037282

The final value, it will be noticed, is in accord with that obtained in the preliminary work, 0.003727, having been increased by about the same small amount as has the figure for hydrogen. Except for the possibility of some error dependent on the nature of the gas itself, such as adsorption, the relative rates of expansion of hydrogen and carbon dioxide seem to be well established.

As has been suggested in the last paragraph, the value obtained from these experiments, as well as all those previously found by others, may contain a constant error of uncertain magnitude caused by the different amount of adsorption of the gas by the glass at the two different temperatures employed. In the case of hydrogen and glass this correction is probably small. A similar error would, of course, enter equally into the determinations of increased tension at constant volume, but the ratio of the results from the two methods might nevertheless be changed, since the constant volume measurements were made between 0° C and 100° C. Possibly the platinum-iridium bulb used by Chappuis may have exerted a slightly adsorbing action upon his hydrogen, and, in the case of carbon dioxide, the adsorption is probably of some importance. It is hoped that determinations of the actual amount of this correction will soon be made in this laboratory.

The American Academy of Acts and Sciences has given valuable pecuniary assistance to this investigation through a grant from the Rumford Fund, for which it is a pleasure to acknowledge our indebtedness.

SUMMARY.

Improvements in the surroundings, the stability, and the manipulation of the apparatus previously described for the determination of the coefficient of expansion of gases by heat under constant pressure are set forth. The possibility of measuring the pressure of a gas to an accuracy of one thousandth of a millimeter of mercury is shown, and some of the difficulties of the determination of volume and temperature to a corresponding degree of accuracy are pointed out.

Measurements of the coefficient of expansion of hydrogen (0.0036609) and of carbon dioxide (0.0037282) under constant pressure as far as five significant figures are recorded, and their probable reliability is discussed. The importance of knowledge of the amount of adsorption of gases by glass is mentioned in this connection. The results must not be considered as final measurements, but their consistency shows that the apparatus is capable of attaining a high degree of precision.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XXX.

A REVISION OF THE GENUS ZEXMENIA.

By W. W. Jones.

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A REVISION OF THE GENUS ZEXMENIA.

By W. W. Jones.

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The genus *Zexmenia*, belonging to the helianthoid *Compositae* and restricted to tropical and subtropical America, was originally described by La Llave and Lexarza[1] in 1824 and founded upon a single Mexican species, *Z. serrata.* The name *Zexmenia* was not taken up by De Candolle in his Prodromus, nor by his immediate successors in cosmopolitan classification, as, for instance, Endlicher (Gen. Pl. 1838) and Steudel (Nomencl. Bot. 1840). This neglect was doubtless due to the fact that La Llave's plant had not been rediscovered and the type, even if preserved, had not been seen by any of these authors. It is probable from a reference by D. Don[2] that La Llave sent him a specimen of his *Z. serrata*, but even if that were the case, it was probably deposited in the great Lambert Herbarium, which was sold in sections, and it would now be difficult if not impossible to trace the specimen in question.

In the Prodromus, v. 610, De Candolle described under the name *Lipochaeta* a genus of nine species, five of which came from the Sandwich Islands and the remaining four from Mexico, La Llave's *Zexmenia serrata* being added as doubtfully belonging to this genus.

In 1852 Gray[3] restored the name *Zexmenia*, not only for its still unknown type, but for *Lipochaeta strigosa*, DC., and several of his own new species. Gray states that these are not congeneric with De Candolle's Hawaiian species, to which, in his opinion, the name *Lipochaeta* should be restricted. He also refers *Lasianthaea helianthoides*, DC. to *Zexmenia* and makes it the basis of the sectional group *Lasianthaea*, later taken up by Bentham and Hooker, f. Gray, however, did not transfer specifically the five American species of *Lipochaeta* to *Zexmenia*, but this was done later by C. H. Schultz.[1] In 1858 Mueller[2] collated briefly nine species of *Zexmenia*, not including those previously referred to *Lipochaeta*.

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[1] Nov. Veg. Desc. fasc. i. 13.

[2] Trans. Linn. Soc. xvi. 170.

[3] Pl. Wright. i. 113.

At the time of Bentham and Hooker's Genera Plantarum *Zexmenia* had been extended to include about twenty-five species, and was divided into three sections: *Wedelioides*, *Lipochaeta*, and *Lasianthaea*. In the present treatment the achenial characters alone have been made the basis of the sectional division, and it has seemed undesirable to maintain *Lipochaeta* and *Lasianthaea* as sections of the genus, for their species are often so nearly alike habitally that it is difficult to distinguish them, and in respect to the achenial characters these two sections form a very homogeneous group, which may be very fitly regarded as *Zexmenia* proper as by Hoffmann[3] (*Euzexmenia*). In this portion of the genus the achenes are all strongly angled, triangular in the ray-flowers, and much flattened (except in *Z. Salvinii*) in the disk-flowers. True wings (developing after anthesis) are never present, but the angles are often produced into wing-like longitudinally striate margins. The awns are generally longer in the disk-achenes than in the ray-achenes. In the former they frequently exceed the length of the body of the achene, although they are sometimes quite short. On the ray-achenes they are sometimes reduced to short stoutish teeth. When the intermediate pappus scales (squamellae) are developed to the length of 1 mm. they are always united to form a small cup or short tube at the summit of the achene. The entire pappus is continuous with the margins of the achene, there being no constriction or pappus-disk at the summit, as will be found in *Auchenocarpa*, to be described later. The inflorescence is of a terminal cymose character, but often appears umbellate or fasciculate by a shortening of the internodes. In a few species peduncles arise from several of the upper nodes. This section includes about twenty-five species.

The § *Wedelioides*, proposed by Bentham and Hooker, f.[4] was used by its authors to include an obvious mixture of plants not very nearly related. The name was probably suggested by the Wedelia-like habit of such species as *Z. reticulata*, *Z. caracasana*, and *Z. helianthoides*, Benth. & Hook. f., which can no longer be referred to *Zexmenia*. *Wedelioides* as a section of *Zexmenia* would, therefore, better be abandoned. The portion of the § *Wedelioides*, Benth. & Hook. f., which it seems best to retain in *Zexmenia*, together with several more recently described species, may well be called § *Auchenocarpa*, a name which indicates the achenial character common to all these species, namely, a more or less marked constriction of the achene near its summit, forming a sort of neck, which separates the insertion of the pappus from the body of the achene. The awns in this section are generally fragile and pubescent. Squamellae are present in most of the species but are usually quite distinct. The body of the achene varies in the different species from the type found in § *Euzexmenia* on the one hand to 4-angled in the ray-flowers and 3-angled in the disk-flowers, or somewhat rounded in both. In none of the species of § *Auchenocarpa* do we find the rather dense inflorescence of many species of § *Euzexmenia*, the heads being commonly solitary on long leafy stems, or in few-headed cymes or more rarely somewhat corymbose.

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[1] Sch. Bip. in Seem. Bot. Voy. Herald, 305–306.

[2] Mueller in Walp. Ann. v. 225–226.

[3] Hoffm. in Engl. & Prantl. Nat. Pflanzenf. iv. Ab. 5, 238.

[4] Gen. ii. 373 (1873).

So far as known *Zexmenia* is strictly a New World genus and is confined to the tropical and subtropical regions of the mainland. Three species are found in the southwestern part of the United States and four in South America. The greater number, however, are confined to Mexico and Central America.

This revision has been prepared at the Gray Herbarium, under the direction of Dr. B. L. Robinson, where it has been possible to study what is probably the largest collection of this particular group. The material examined, however, has also included that of the United States National Museum, the herbarium of Mr. John Donnell Smith, and of the Missouri Botanical Garden. Acknowledgment is due to Mr. F. V. Coville and Dr. J. N. Rose of the National Museum, Mr. Smith of Baltimore, and Professor Trelease of the Missouri Botanical Garden, for their obliging loan of these specimens. Through the kindness of Sir W. T. Thiselton Dyer and Mr. W. Botting Hemsley of the Royal Gardens at Kew, and Mr. Casimir de Candolle of Geneva in sending type-material and tracings, several species have been identified which would otherwise have remained indefinite. Special thanks are due to Dr. B. L. Robinson, Dr. J. M. Greenman, and Miss Mary A. Day for their assistance in the preparation of the paper, the revision of the manuscript, and the proof-reading.

§ 1. EUZEXMENIA. Perennial herbs or shrubs, with simple or branching stems, generally pubescent: leaves all opposite, more or less serrate, short-petioled or sessile: peduncles solitary or the heads corymbose or borne in umbelliform clusters; heads never more than 2 cm. long; involucre subglobose, campanulate, or short-cylindric, the scales 1–5-seriate: ligules usually conspicuous, yellow (except in *Z. zinnioides*);

disk-corollas mostly exceeding the involucre, numerous: achenes smooth (i. e. not warty), glabrous or glabrate, from fusiform to cuneate, those of the ray-flowers 3-angled and 3-awned, of the disk-flowers strongly flattened, 2-awned (in *Z. aurea* generally 3-awned and somewhat 3-angled); body of the achene 3 to 6 mm. long, 1 to 2.5 mm. wide, the angles never definitely winged but in several species produced into thin longitudinally striate margins; pappus of 2 to 3 short tooth-like or longer rigid awns continuous with the edges of the achene, a series of very short intermediate squamellae often present.

* Intermediate scales (squamellae) of the pappus on the disk-achenes at least 0.5 mm. long.

+ Involucral bracts in 2 series.

++ Involucral bracts recurved at tip.

1. Z. SERRATA, La Llave. Shrub, 4 m. or more high, pubescent, much-branched: leaves lanceolate, serrate, petiolate, stellate-pubescent on both sides: *heads corymbose; involucre of 10 to 12 ovate recurved* scales: ligules 10 to 12, ovate, emarginate. — Nov. Veg. Descript. 13 (1824); Hemsl. Biol. Cent.-Am. Bot. ii. 174. *Lipochaeta? serrata*, DC. Prodr. v. 611 (1836). — Southern Mexico. Type collected by La Llave at San José del Corral. No specimens examined, the above description compiled.

++ ++ Involucral bracts not recurved.

2. Z. MONOCEPHALA, Sch. Bip. Shrub: leaves ovate-lanceolate, 2 to 10 cm. long, 0.5 to 5 cm. wide, acuminate at the apex, rounded or somewhat acuminate at the base, serrate, scabrous above, very sparsely strigillose beneath; petioles 5 to 13 mm. long, very villous on the upper side: peduncles solitary, slender, monocephalous, shorter than the upper leaves; heads 15 mm. high; involucre campanulate, 10 to 15 mm. broad, the scales ovate, submembranaceous, about 2-seriate: ray-flowers 4 or 5: achenes margined; squamellae well developed. — Sch. Bip. in Seemann, Bot. Voy. Herald, 306 (1852–57); Hemsl. Biol. Cent.-Am. Bot. ii. 173. *Lipochaeta monocephala*, DC. Prodr. v. 610 (1836). — Southern Mexico. STATE OF VERA CRUZ: Mirador, *Sartorius*. Of this rare and imperfectly known species I have seen only a tracing with a leaf and some immature achenes from the De Candollean Herbarium in Geneva.

3. Z. SCABERRIMA, Benth. & Hook. f. Shrub (?): leaves short-petioled, ovate-lanceolate, acute at the apex, cuneate at the base, remotely serrulate, very scabrous on both sides, 5-nerved: heads in a lax corymb;

outer involucral bracts obtuse; the inner acute: one awn of the ray-achenes about as long as the body of the achene; squamellae very small. — Gen. ii. 373 (1873). *Lipochaeta scaberrima*, Benth. in Hook. Journ. Bot. ii. 43 (1840). — South America. No specimens seen. Type collected by Schomburgk in the Roraima Mountains of Guiana, 1840. A doubtful species.

+ + Involucral bracts in 3 to 5 series.

++ Awns of the disk-achenes as long as the body.

4. Z. SEEMANNII, Gray. Shrub, usually much-branched: leaves linear-oblong to oblong-lanceolate, 2 to 10 cm. long, 7 to 15 mm. wide, acute to rounded at the apex, acute or acuminate at the base, subentire, shining, hispid-scabrous above, glabrate beneath; petioles 3 to 10 mm. long: peduncles commonly fascicled or cymose in umbelliform clusters in the upper axils, 3 to 13 mm. long; heads about 15 mm. high; involucre cylindrical, 5 to 8 mm. broad; the involucral bracts about 5-seriate, lax at the apex and variously toothed, the outer series very short; the inner much longer: ligules 6 to 8, about 1 cm. long: margined angles of the achenes very prominent and confluent with the awns; body of the disk-achenes 4 mm. long, 1.5 mm. wide; squamellae about 1 mm. long, more or less united. — Pl. Wright. i. 114 (1852); Hemsl. Biol. Cent.-Am. Bot. ii. 174. — Northern Mexico. *Specimens examined*, — Cerro de Pinal, *Seemann*, no. 1476 (type, in herb. Gray). SINALOA: Sierra de Choix, *Goldman*, no. 249. TEPIC: *Palmer*, no. 1857, coll. of 1892.

5. Z. HELIANTHOIDES, Gray. Shrub or lignescent herb, 6 to 9 dm. high; branches stout: leaves 2 to 10 cm. long, ovate, subcordate, serrate or subentire, reticulate-veiny, above warty-strigillose, beneath velvety-villous; petioles less than 1 cm. in length: peduncles 1 to 3 cm. long, very stout, fascicled or occasionally solitary at the ends of the branches; heads about 2 cm. long; involucre 10 to 15 mm. broad, its bracts about 4-seriate, ovate, appressed, entire, rounded at the apex, the outer series considerably shorter than the inner: ligules 8 to 16, rather small; disk-corollas long and slender, much exceeding the involucre: achenes 4 to 6 mm. long, with margined angles much developed and confluent with the awns, these at least in the disk-achenes longer than the body of the achene. — Pl. Wright. i. 113 (1852); Hemsl. Biol. Cent.-Am. Bot. ii. 172; Sch. Bip. in Seemann, Bot. Voy. Herald, 305. *Z. ovata*, Benth. & Hook. f. Gen. ii. 373 (1873). *Lasianthus helianthoides*, Zucc. in Herb. Acad. Monac. according to Gray, l. c. *Lasianthaea helianthoides*, DC. Prodr. v. 608 (1836). *Tithonia ovata*, Hook. Bot. Mag. t. 3901

(1842). — Southern Mexico. *Specimens examined,* — MORELOS: near Cuernavaca, 1520 m. alt., *Pringle*, no. 6225; *Ghiesbreght*, no. 379 (type, in herb. Gray).

+ + Awns of the disk-achenes usually about half as long as the body.

6. Z. GREGGII, Gray. Shrub or lignescent herb, 2 to 3 m. high: leaves large on the lower part of the stem, 1.5 dm. long, 8 cm. wide, smaller above, sessile or nearly so, ovate to lanceolate, serrate, obtuse to acuminate, cordate or subcordate, reticulate-veiny, scabrous: peduncles 2 to 25 cm. long, stout, naked or bearing 1 or 2 scale-like bractlets; heads about 15 mm. high; involucre 1.5 to 2.5 cm. broad, its bracts 3-seriate, ovate or lanceolate, the outer attenuate, appressed, several-nerved, the inner rounded and chartaceous at the summit: ligules rather small, commonly not exceeding the disk; disk-corollas with long slender tubes, generally surpassing the involucre: achenes 5 to 6 mm. long, narrowly fusiform, the angles only slightly margined; squamellae about 1 mm. in length. — Pl. Wright. i. 113 (1852); Hemsl. Biol. Cent.-Am. Bot. ii. 172. — Southern Mexico. *Specimens examined,* — JALISCO: near Guadalajara, *Pringle*, no. 1769; Tequila, *Palmer*, no. 356, coll. of 1886; between the city of Mexico and Mazatlan, *Gregg*, no. 1003 (type, in herb. Gray).

* * Squamellae minute (less than 0.5 mm. long or entirely wanting).

+ Heads few, in terminal 3-headed cymes or solitary (in *Z. fruticosa*, the peduncles commonly in several of the upper axils).

++ Herbs, 3 to 9 dm. high, from tuberiform roots or thickened lignescent base.

= Peduncles very long, usually not less than 1 dm. in length.

7. Z. PODOCEPHALA, Gray. Herbaceous, 6 to 9 dm. high, somewhat branched from a lignescent base; roots much thickened, fusiform; stems spreading-hirsute, bearing about 5 pairs of leaves and 1 to 3 long-peduncled heads: leaves subsessile, ovate-oblong to oblong-lanceolate, acute at the apex, subcordate, rounded or abruptly narrowed to a subcuneate base, serrate, 6 to 8 cm. long, 15 to 32 mm. wide, coarsely pubescent on both sides: heads about 1 cm. high; involucre cylindrical or campanulate, about 1 cm. broad, 2-seriate; outer bracts oblong-ovate, subacute, hirsute: ray-flowers about 12; ligules 13 mm. long; disk-corollas scarcely exceeding the involucre: achenes 3 mm. long, widest at or near the summit; awns shorter than the body, widened at the base, those of the ray-achenes usually reduced to strong teeth; minute squamellae sometimes present. — Syn. Fl. i. pt. 2, 286 (1884); Rose, Contrib. U. S. Nat. Herb. i. 103; Britton & Kearney, Trans. N. Y. Acad. Sci. xiv. 44. *Verbesina*

podocephala, Gray, Pl. Wright. ii. 92 (1853); Rothrock in Wheeler, Rep. 164; Torr. Bot. Mex. Bound. 92; Hemsl. Biol. Cent.-Am. Bot. ii. 190. — Rocky hills of southwestern United States to Guatemala. *Specimens examined*, — ARIZONA: Chiricahua Mountains, *Rothrock*, no. 517; near Fort Huachuca, *Wilcox*, no. 319; Huachuca Mountains, *Lemmon*, no. 374; Santa Rita, *Pringle*, no. 57. CHIHUAHUA: near the City, *Pringle*, no. 349; Colonia Garcia, *Townsend & Barbour*, no. 219; in the southwestern part, *Palmer*, no. 11, coll. of 1885. SONORA: Santa Cruz, *Wright*, no. 1239 (type, in herb. Gray); Alamos, *Palmer*, no. 362, coll. of 1890.

8. **Z. Palmeri**, GREENMAN, n. sp. "Stem erect, 5 to 6 dm. high, apparently from a woody base, hirsute-pubescent: leaves showing a tendency to alternation, short-petioled, elliptical-lanceolate or oblong, 5 to 10 cm. in length, 2 to 4 cm. broad, acute or acuminate, serrate-dentate except toward the base, hirsute-hispid on both sides: heads 1 to 2 cm. high; peduncles solitary, terminal and axillary, 1 to 2 dm. long, naked or sparingly bracteolate, appressed-pubescent; involucre 3-seriate, the outer scales longer than the inner, lance-oblong, 12 to 18 mm. in length, acute or obtuse, externally hirsute-hispid: ray-flowers about 15; disk-flowers numerous: achenes narrowly margined on the inner side. — Southern Mexico. *Specimens examined*, — JALISCO: Rio Blanco, *Palmer*, no. 50, coll. of 1886 (type, in herb. Gray). The specimen here proposed as a new species was doubtfully referred by the late Dr. Sereno Watson to *Z. podocephala*, Gray, and its affinity certainly seems to be with that species; but it is readily distinguished by the distinctly although shortly petiolate leaves, and by their lanceolate rather than ovate form."

= = Peduncles less than 1 dm. long.

a. Pappus of the disk-achenes consisting of 2 short teeth connected by a shallow more or less continuous border (connate squamellae): ligules purple.

9. Z. ROSEI, Greenman. Herbs, 3 to 4 dm. high; roots several, fusiform-thickened; stem terete, leafy chiefly below the middle, covered by a dense spreading grayish villous pubescence, also finely and obscurely puberulent: leaves opposite, membranaceous, oblanceolate-oblong, acutish, short-petioled, subremotely serrate, scabrous-pubescent above with thick-based hairs, grayish tomentose and reticulate-veiny beneath, 6 to 16 cm. long, 2 to 5 cm. wide: inflorescence a rather contracted terminal cyme subtended by 1 or 2 pairs of large foliaceous bracts; heads 12 mm. high; involucre campanulate, 2–3-seriate; the outer scales subfoliaceous, oblanceolate, coarsely pubescent, obtusish:

ligules dark purple, 12 to 15 mm. long, a third as broad: achenes narrowly wing-margined; pappus reduced to short teeth more or less united into a shallow cup. — Proc. Am. Acad. xl. 42 (1904). — Western Mexico. TEPIC: in the foot-hills of the Sierra Madre between Pedro Paulo and San Blacisto, *Rose*, no. 1987 (type, in herb. U. S. Nat. Mus.; fragments and tracing in herb. Gray).

b. Pappus of the disk-achenes consisting of 2 to 3 awns nearly as long as the body of the achene: ligules yellow.

10. Z. AUREA, Benth. & Hook. f. Stem 1.5 to 3 dm. high, strict and simple or sparingly branched from the base, bearing 3 to 5 pairs of leaves, hirsute-pubescent, generally terminated by a 3-headed cyme: leaves ovate-oblong to elliptical or rhomboid-oval, dentate-serrate, rather densely pilose or hirsute along the veins beneath, densely strigose above: heads about 1 cm. high; involucre campanulate, commonly 1 cm. broad; outer bracts 5, ovate-oblong, densely hirsute: ray-flowers about 10; ligules bright yellow, considerably exceeding the disk; disk-flowers numerous, their corollas tubular-funnel-shaped, the teeth papillose on the inner surface: disk-achenes 3 mm. long, somewhat 3-angled but strongly compressed, usually bearing 3 rigid awns no longer than the body of the achene; awns of the ray-achenes none or reduced to 3 short teeth. — Gen. ii. 371 (1873); Hemsl. Biol. Cent.-Am. Bot. ii. 172. *Verbesina aurea*, DC. Prodr. v. 613 (1836). *V. tuberosa*, Klatt, Ann. Naturh. Hofmus. Wien, ix. 361 (1894). *Wedelia? aurea*, D. Don, Bot. Mag. t. 3384 (1835). — Prairies in Central Mexico. *Specimens examined,* — STATE OF MEXICO: *Pringle*, no. 3239. JALISCO: Henjuquilla, *Rose*, no. 2549. Mexico without precise locality, *Schmitz*, no. 248, and *Schaffner*, no. 56.

11. Z. **xylopoda**, n. sp. Herb, with a small cluster of rather slender stems 3 to 6 dm. high, from a lignescent base: leaves 1 to 7 cm. long, 7 to 25 mm. wide, ovate or spatulate to elliptic-linear, short-petioled or sessile, from cuneate to rounded at the base, generally acute at the apex, dentate-serrate to subentire, ciliate, sparingly pubescent or glabrous below, strigose above with bases of the hairs enlarged: peduncles from 1 to 9 (generally about 5) cm. long, terminal and solitary or in cymes of 3, strictly glabrous; heads 10 to 15 mm. high; involucre 5 to 10 mm. broad, 3-seriate; outer bracts ovate, attenuate, ciliate: ligules bright yellow, 1 to 1.5 cm. long, 5 mm. wide; disk-corollas scarcely exceeding the involucre, with short proper tube, long funnel-shaped throat, and short somewhat fimbriate limb: disk-achenes 2 mm. broad, 5 mm. long,

2-awned, the awns shorter than the body; ray-achenes somewhat margined and the margins produced upward into 3 coarse teeth. — Mexico. *Specimens examined,* — JALISCO: Rio Blanco, *Palmer,* no. 757, coll. of 1886 (type, in herb. Gray). This species has been confused with *Z. aurea,* but it is certainly quite distinct, being well marked by its thickened lignescent base, the shape and pubescence of its leaves, long and ciliated involucral scales, and large more or less margined ray-achenes.

++ ++ Suffruticose, fruticose, or arborescent.

= Rays purple.

12. Z. ZINNIOIDES, Hemsl. Much-branched; branches long, slender, scabrous-pubescent, ferrugineous: leaves sessile or shortly petioled, rigid, ovate-oblong, 5 to 9 cm. long, acute or obtuse, remotely serrulate or subentire, sparsely strigillose-pubescent on both surfaces, shining above: peduncles 1.5 to 2.5 cm. long, commonly solitary, pubescent; heads about 1 cm. high; involucre campanulate, usually 1 cm. broad; bracts about 16, appearing somewhat decussate, scabrous, the outer rather broad, acute, the inner oblong or linear, glabrous, colored at the tip: ligules 10 to 12, purple, broadly oblong; disk-corollas purple, funnelformed, papillose-hirsute on the limb: achenes margined; awns about as long as the body, rigid. — Biol. Cent.-Am. Bot. ii. 175 (1881). — Northern Mexico. *Specimens examined,* — A portion of the type, *Seemann's* no. 1464, collected on the Cerro de Pinal, and obligingly sent from herb. Kew.

= = Rays yellow.

a. Bracts unequal, in several series, the outer much the shortest.

1. Scales of the involucre recurved at the tip.

13. **Z. squarrosa,** GREENMAN, n. sp. "Shrub; stems and branches appressed sericeous-pubescent, often copiously dotted with lenticels: leaves broadly ovate, 3 to 7 cm. long, 2 to 6 cm. wide, somewhat acuminate, dentate, cuneate or rounded at the base, tuberculate-hispid above, paler and hirsute-pubescent on the prominently reticulated veins beneath, 3-nerved from the base; petioles 0.5 to 1.5 cm. long: peduncles terminal, 5 to 20 mm. long, cymosely arranged; heads 1 to 1.5 cm. high; involucre cylindrical or slightly turbinate, 5–6-seriate; bracts ovate to oblong, acute to rounded at the apex, conspicuously ciliate, the outermost shorter ones pubescent on the dorsal surface and with subfoliaceous squarrose tips: ray-flowers about 5, the ligules usually less than 1 cm. long, lemon-yellow; disk-flowers numerous, considerably exceeding the in-

volucre: achenes 6 to 7 mm. long, narrowly margined on the inner edge; squamellae minute or none. — Mexico. *Specimens examined,* — GUERRERO: mountains near Iguala, alt. 1230 m., *Pringle,* no. 8411 (type, in herb. Gray)."

2. Scales of the involucre not recurved at the apex.

14. Z. GHIESBREGHTII, Gray. Shrubby or arborescent: leaves ovate-lanceolate, acuminate, reticulate, submembranaceous, hirtellous-scabrous, subcordate: peduncles terminal, solitary or in cymes of 3, frequently thickened upward, several times longer than the upper leaves; heads about 15 mm. high; involucre hemispherical, 1 to 2 cm. in diameter, 3-seriate; outer bracts orbicular-ovate, appressed, considerably shorter than the others; the inner long, narrower, membranaceous at the tip: ligules orange-yellow, only slightly exceeding the disk; disk-flowers numerous, generally exceeding the involucre: achenes somewhat margined, about 5 mm. long, 1 to 2 mm. wide; awns, in the disk-achenes, more than half as long as the body, in the ray-achenes shorter; squamellae minute. — Pl. Wright. i. 113 (1852), as *Ghiesbrechtii;* Hemsl. Biol. Cent.-Am. Bot. ii. 172. — Mexico. *Specimens examined,* — Without precise locality, *Ghiesbreght,* no. 385 (type, in herb. Gray). COLIMA: *Palmer,* no. 1241, coll. of 1891. GUERRERO: Acapulco, *Palmer,* no. 491, coll. of 1895; between Juchitango and Omelepec, *Nelson,* no. 2312. TEPIC: *Palmer,* Jan. 5 to Feb. 6, 1892. Some of these specimens have been distributed as *Z. Greggii,* a species which should be readily distinguished by its prominent squamellae, very narrowly fusiform achenes, and sessile leaves.

15. Z. MACROCEPHALA, Hemsl. Suffruticose or fruticose, densely pubescent above; leaves firm, subsessile, lanceolate to elliptical, attenuate, serrate, short-pilose and more or less scabrous with tuberculate hairs above, strongly reticulate-veiny and densely hirsute beneath; peduncles longer than the upper leaves, terminal, usually in cymes of 3; heads 1 to 2 cm. high; involucre 10 to 15 mm. broad; bracts somewhat decussately arranged, ovate, from acute to rounded at the apex, outer pair quite small: achenes rather long and slender (as in *Z. Greggii*), those of the disk with slender awns less than half as long as the body, those of the ray with tooth-like awns; squamellae none. — Biol. Cent.-Am. Bot. ii. 173 (1881). *Lipochaeta macrocephala,* Hook. & Arn. Bot. Beech. Voy. 436. — Southern Mexico. *Specimens examined,* — GUERRERO: Acapulco, *Hinds,* no. 1841 (a portion of the type material sent from herb. Kew).

b. Bracts of the involucre subequal or the outer longest, in few series.

1. Outer bracts the longest, not foliaceous.

16. Z. CROCEA, Gray. Much-branched shrub: leaves oblong-ovate, subcordate, acuminate, short-petiolate, serrulate, reticulate-veiny, scabrous above, pilose beneath: peduncles often 12 cm. long, generally thickened upward, hirsute, terminal and solitary or in a cyme of 3; heads 1 to 2 cm. high; involucre 10 to 15 mm. broad, 2-seriate; outer bracts oblong-ovate to obovate or spatulate, acute or obtuse, strigose to densely hirsute, much longer than the inner: ligules orange-yellow, little or not at all exceeding the disk; disk-corollas with long slender tubes: achenes of the ray-flowers usually with 1 long awn and 2 short teeth; achenes of the disk-flowers about 5 mm. long, 1 to 2 mm. wide, their awns unequal and shorter than the body. — Pl. Wright. i. 114 (1852); Hemsl. Biol. Cent.-Am. Bot. ii. 172; Seemann, Bot. Voy. Herald, 305. *Z. Stenantha*, Hemsl. l. c. 174 (1881). — Mexico. *Specimens examined*, — MORELOS: near Cuernavaca, *Pringle*, no. 6187; near Yautepec, *Pringle*, no. 8701; *Bourgeau*, no. 1204. OAXACA: Huajuapam, *Nelson*, no. 1978.

17. Z. SALVINII, Hemsl. Shrub, diffusely branched, with large leaves and marked pubescence: leaves broadly-ovate, shortly petiolate, acute or acuminate, serrate, scabrous above, pilose beneath: peduncles about 1 to 2 cm. long, pilose, bearing a few small bracteal leaves and a solitary head; heads few, 10 to 15 mm. high; involucre about 1 cm. broad; bracts apparently decussate, the outer 4 strigose, ovate-oblong or spatulate, brownish: ligules conspicuous, yellow, exceeding the disk and involucre: achenes of the ray-flowers 3-angled, with 3 stout awl-shaped awns, those of the disk-flowers 2-angled, cuneate, 4 mm. long, with 2 awns. — Biol. Cent.-Am. Bot. ii. 173 (1881). — Central America. *Specimens examined*, — GUATEMALA: Santa Rosa, *Heyde & Lux*, nos. 3358 and 3797 of J. D. Smith's distrib.

2. Outer bracts of the involucre not longer than the inner.

18. Z. FRUTICOSA, Rose. Diffusely branched shrub: leaves lanceolate or ovate, obtuse to subcordate at the base, acute or acuminate at the apex, serrate, scabrous above, hispid-pubescent beneath, sessile or shortly petioled: peduncles slender, 2 to 4 cm. long, somewhat exceeding the upper leaves, chiefly terminal, solitary or in cymes of 3; heads about 1 cm. high; involucre campanulate, 2–3-seriate, 5 to 7 mm. broad; outer bracts lax, ovate or lanceolate, hispid, ciliate, acute, somewhat shorter than the inner: ray-flowers 5 to 10; ligules lemon-yellow, surpassing

the disk; disk-corollas considerably longer than the involucre: awns of the disk-achenes 2, longer than the body of the achene, much exceeding the involucre; ray-achenes slightly margined, with 2 tooth-like awns and one slender and bristle-like. — Contrib. U. S. Nat. Herb. i. 103 (1891). — Western Mexico. *Specimens examined*, — SONORA: Alamos, *Palmer*, no. 645, coll. of 1890 (co-type, in herb. Gray).

+ + Heads numerous; peduncles fasciculate, terminal and subterminal.

++ Involucral bracts 2 mm. or more broad, imbricated in several series, commonly appressed; heads small; involucre generally less than 5 mm. broad.

= Achenes 4 to 5 mm. long, those of the ray-flowers 3-awned, the awns on the outer angles short, tooth-like, on the inner long and bristle-formed; achenes of the disk-flowers with 2 awns as long as the body.

1. Some of the outer bracts of the involucre reflexed: petioles generally over 1 cm. long.

19. **Z. gracilis**, n. sp. Shrub with long very slender branches: leaves ovate to lanceolate, 5 to 10 cm. long, 2 to 4 cm. wide, acuminate to rounded at the base, very acute or acuminate at the apex, serrate except near the ends, strigose above, the hairs with swollen bases, the lower surface sparsely short-strigose; petioles 5 to 15 mm. long: peduncles 4 to 8 cm. long, slender, commonly in 3 or 4 of the upper axils, superposed; heads 1 to 1.5 cm. high; involucre 4 to 7 mm. broad, cylindrical, 2–3-seriate; outer bracts considerably the shortest, generally reflexed, ciliate: ligules orange-yellow: achenes as in the preceding species. — Western Mexico. *Specimens examined*, — COLIMA: Colima, *Palmer*, no. 163, coll. of 1897 (type, in herb. U. S. Nat. Mus.).

2. Outer bracts of the involucre not recurved or reflexed: petioles less than 1 cm. long.

20. Z. CEANOTHIFOLIA, Sch. Bip. Shrub 2 to 3 m. high; branches striate, roughened by lenticels; branchlets pubescent: leaves from broadly ovate to narrowly oblong-lanceolate, usually ovate-lanceolate, acute or acuminate at the apex, acute to rounded at the base, serrulate, reticulate-veiny, sparsely scabrous-pubescent to densely strigose above, strigose to coarsely and densely pubescent beneath; petioles 5 to 10 mm. long: peduncles 1 to 3 cm. long, in terminal or subterminal fascicles; heads 5 to (occasionally) 10 mm. high; involucre 3–5-seriate, 4 to 5 mm. broad; outer bracts ovate, very obtuse to abruptly acuminate, much shorter than the inner: ligules 5 to 7, oblong, emarginate, orange-yellow; disk-flowers numerous: ray-achenes 4 mm. long, about 1 mm. wide, linear-ovate, with narrow margins, their pappus of 2 short teeth and one

awn shorter than the body; disk-achenes slightly narrower and longer, their 2 awns subequal, longer than the body and at maturity much exceeding the involucre; squamellae none or minute. — Sch. Bip. in Seemann, Bot. Voy. Herald, 305 (1852–57); Hemsl. Biol. Cent.-Am. Bot. ii. 172. *Verbesina ceanothifolia*, Willd. Sp. Pl. iii. 2225 (1804); DC. Prodr. v. 613 (1836). *Lipochaeta umbellata*, DC. l. c. 610 (1836). — Mexico. *Specimens examined,* — VERA CRUZ: Orizaba, *Botteri*, nos. 800 (in part), 477, and 487; *Thomas*, coll. in 1864. MORELOS: Cuernavaca, *Berlandier*, no. 1065 (co-type, in herb. Gray); *Pringle*, no. 9163; *Holway*, no. 3530; *Bourgeau*, no. 1219. OAXACA: Sierra de San Felipe, *Pringle*, no. 5783; *Conzatti & González*, no. 993; *Baites*, coll. in 1846. JALISCO: near Guadalajara, *Pringle*, nos. 2167 and 2301; near Colotlan, *Rose*, no. 2814. TEPIC: *Palmer*, presumably coll. of 1892.

Var. **conferta**, Gray in herb. Leaves lanceolate to elliptical, 1 to 2.5 cm. wide, attenuate to subtruncate at the apex, finely appressed-pubescent on both sides, subsessile: peduncles few to numerous, generally very short; involucre about 3-seriate; bracts acute: ligules apparently lemon-yellow. — *Lipochaeta umbellata*, var. *conferta*, DC. Prodr. v. 610 (1836). — Mexico: *Specimens examined,* — MORELOS: Cuernavaca, *Berlandier*, no. 1053 (type, fragment in herb. Gray). Without exact locality, *Gregg*, no. 1070. TEPIC: Acaponeta, *Lamb*, no. 519.

21. Z. MICROCEPHALA, Hemsl. Shrub with scabrous stems: leaves ovate-lanceolate, 4 to 10 cm. long, minutely serrulate, obtuse, short-petioled, rugose and strigillose above, strigillose, whitish, and with veins prominent beneath: peduncles slender, fasciculate-corymbose, about 15 mm. long; heads about 1 cm. high; involucre 4 to 6 mm. wide; bracts subcordate, oblong or ovate, rounded at the apex: ray-flowers about 4; ligules oblong-elliptical; disk-flowers about 20; teeth of the corollas hirsute: achenes glabrous, with 2 or 3 wing-like margins upwardly adnate to the awns; squamellae none. — Biol. Cent.-Am. Bot. ii. 173 (1881). — Western Mexico. *Specimens examined,* — TEPIC: San Blas, *Sinclair* (a leaf and head from herb. Kew).

= = Achenes about 3 mm. long, those of the ray-flowers obovoid, rounded above and with one long awn and two short teeth, those of the disk-flowers somewhat cuneate, scarcely 1 mm. wide at the summit and with two slender unequal awns shorter than the body.

22. Z. IMBRICATA, Sch. Bip. Suffruticose, canescent; stem glabrate: leaves 1 cm. long, subcordate, ovate, acute, irregularly dentate, scabrous; petioles about 2 cm. long: heads in the upper axils; involucre hemi-

spherical, turbinate, 5-seriate; bracts narrowly ovate, obtuse, scabrous: achenes ovoid, margined on the angles; squamellae minute. — Sch. Bip. in Seemann, Bot. Voy. Herald, 306 (1852–57). — Mexico. No specimen of this species has been seen by the writer and its place here in the genus is somewhat inferential.

23. Z. FASCICULATA, Sch. Bip. A tall much branched shrub; branches roughened by lenticels: leaves oblong-lanceolate to broadly ovate, acute or acuminate at the apex, acute to rounded at the base, serrate or subentire, appressed-strigose above (the hairs frequently with swollen bases), sparsely pubescent on the veins to densely pilose beneath: peduncles 1 to 5 cm. long in terminal and subterminal fascicles; heads 5 to 8 mm. high; involucre narrowly campanulate to cylindrical, 3 to 6 mm. broad; bracts about 3-seriate, ovate, obtuse to abruptly acuminate, the outer the shortest: rays conspicuous; disk-corollas elongated and rather narrowly tubular. — Sch. Bip. in Seemann, Bot. Voy. Herald, 306 (1852–57); Hemsl. Biol. Cent.-Am. Bot. ii. 173. *Lipochaeta fasciculata*, DC. Prodr. v. 610 (1836). — Mexico. *Specimens examined*, — CHIHUAHUA: southwestern part, *Palmer*, no. 155. TAMAULIPAS: Tula, *Berlandier*, nos. 2134, 717 (co-type, in herb. Gray). SAN LUIS POTOSI: Canoas, *Pringle*, no. 3753; between San Luis Potosi and Tampico, *Palmer*, no. 1578. A variable species, which with further knowledge may prove divisible. Palmer's no. 1578 and Townsend & Barbour's no. 384 are doubtfully placed here.

++ ++ Bracts about 1 mm. broad, in 3 series, lanceolate.

24. **Z. Pittieri**, Greenman, n. sp. "Stems striate, finely strigillose: leaves remote, elliptic-oblong, 10 to 15 cm. long, 6 to 8 cm. wide, short-acuminate, rounded or obtuse at the base, remotely denticulate, strongly 3-nerved from near the base to the apex, usually with 2 pairs of less conspicuous marginal veins, prominently reticulate-veiny beneath, hirtellous-pubescent on both surfaces; petioles 10 to 15 mm. long: heads 6 to 8 mm. high, borne in a terminal corymbose panicle; involucre 3-seriate, about 5 mm. broad; bracts ovate-lanceolate, 3 to 4 mm. long, appressed-hirtellous-pubescent, the outer somewhat the smallest: ray-flowers 6 to 9; ligules small and inconspicuous; disk-flowers about 35, exceeding the involucre: achenes narrowed below, glabrous, inconspicuously ciliate, those of the ray-flowers with 3 paleaceous awns longer than the body, those of the disk-flowers with 2 awns and several minute squamellae more or less united. — Central America. *Specimens examined*, — COSTA RICA: in woods, Tsâki, Talamanca, altitude 200 m., *Pittier*, no. 9565 (type, in herb. Gray)."

"In the Gray Herbarium there is another specimen from the same locality as the Pittier specimen here cited and with the same number, but said to have been collected by Tonduz. This was rightly referred by Dr. Klatt to *Salmea Eupatoria*, DC. (*S. scandens*, DC.). Notwithstanding a close habital similarity the plant of Pittier exhibits, by the presence of fertile ray-flowers with triangular achenes as well as by the pappus both in the ray- and disk-flowers, the technical characters of *Zexmenia*."

+ + + Bracts in few series, subequal.

= Heads on short peduncles (10 to 25 mm. long) in close clusters; bracts of the involucre ovate, rather long-acuminate.

25. Z. **elegans**, Sch. Bip. Leaves oblong-lanceolate to ovate, acute to somewhat rounded at the base, acute or acuminate at the apex, serrate, strigose or scabrous, beneath sparsely pubescent on the veins; petioles less than 1 cm. long, pilose: peduncles several in a cluster, terminating the branches, 10 to 25 mm. long, usually pilose; involucre subglobose or campanulate, 7 to 10 mm. wide, 2–3-seriate; outer bracts ovate, caudate-acuminate, pubescent: ligules about 7 mm. long, somewhat exceeding the disk; disk-corollas slightly surpassing the involucre: achenes 4 to 5 mm. long, those of the ray-flowers usually with 1 long awn and 2 short teeth, those of the disk-flowers rounded at the summit, cuneate toward the base, with 2 unequal awns shorter than the body. — Sch. Bip. acc. to Benth. & Hook. f. Gen. ii. 373 (1873), without description — Southern Mexico. *Specimens examined*, — VERA CRUZ: Mirador, *Liebmann*, no. 378 (type, in herb. Gray); *Sartorius* (without data); Orizaba, *Pringle*, no. 5911, *Gray*, May, 1885, *Thomas*, Coll. of 1864, *Botteri*, no. 800 (in part). This species, although long recognized, seems never to have been described.

26. Z. COSTARICENSIS, Benth. Branching shrub or lignescent herb: leaves 5 to 16 cm. long, 4 to 11 cm. wide, generally broadly ovate, with acuminate or rounded base, and very acuminate apex, subentire to serrate, above scabrous to densely strigose, beneath strigillose to densely pubescent; petioles 1 to 3 cm. long, puberulent to villous: peduncles fasciculate in terminal or subterminal clusters, 1 to 5 cm. long, puberulent to villous; heads 6 to 12 mm. high; involucre campanulate to subcylindrical, 5 to 10 mm. broad, about 2-seriate, outer bracts villous, from oblong to obovate or slightly panduriform, acute or occasionally rounded; ligules oblong, about 1 cm. in length: disk-corollas with short tubes: achenes about 4 mm. long, those of the disk-flowers somewhat margined

on the inner angle, the awns about as long as the body; ray-achenes narrowly margined and with the 2 outer awns usually tooth-like, the inner one commonly long and slender. — Benth. in Oersted, Vidensk-Meddel. 1852, p. 95. *Z. nicaraguensis*, C. Muell. in Walp. Ann. v. 226 (1858). — Central America. *Specimens examined,* — GUATEMALA: Santa Rosa, J. D. Smith's sets nos. 3786, 4196; Alta Verapaz, J. D. Smith's sets, nos. 1412, 7446; *Heyde*, no. 424; near Neuton, *Nelson*, no. 3545. HONDURAS: San Pedro Sula, *Thieme*, no. 5320. COSTA RICA: Llanuros de Santa Clara, J. D. Smith's sets, nos. 6617, 6618; also the following numbers of *Pittier* and assistants (*Tonduz* and *Biolley*) 1556, 3134, 4138, 4359, 4522, 4912, 4921, 5823, 7111, 7165, 7428, 8991, 12,911, 13,617. ? VERA CRUZ (Mexico): Cordoba, *Kerber*, no. 59 (not characteristic).

§ 2. AUCHENOCARPA. Leaves opposite or the uppermost alternate: mature achenes winged (except in *Z. aurantiaca*, *Z. lantanifolia*, and *Z. strigosa*), more or less constricted into a short neck bearing the pappus, or the summit of the body rounded and bearing the awns directly without any intervening neck; the awns 1 to 3, long and slender (or reduced to short teeth), not continuous with the margins of the wings.

* Leaves small, the cauline 2 to 3 or rarely 4 cm. long: outer involucral scales squarrose, loose and enlarged, or the heads subtended by 2 or 3 leaf-like bracts.

+ Achenes not winged.

27. Z. LANTANIFOLIA, Sch. Bip. Stem woody, branched, striate, scabrous-hirtellous or subcanescent-pubescent above: leaves ovate, acute at the apex, acuminate to subcordate at the base, 2 to 4 cm. long, irregularly serrate or serrate-dentate, above papillose-strigose, beneath strigose-hirsute, the veins prominent on the under surface; petioles 5 to 10 mm. long: peduncles solitary, 3 to 10 cm. long; heads 10 to 15 mm. high; involucre 10 to 12 mm. broad, 3–5-seriate, subtended by 4 leaves, 2 of these being scarcely larger than the ovate acute or abruptly acuminate proper involucral scales: ligules very showy, considerably exceeding the disk-flowers: achenes pubescent, those of the ray-flowers 2 to 3 mm. long, 1 mm. wide, abruptly rounded or subtruncate at the summit, with usually 1 very fragile awn and several minute squamellae; achenes of the disk-flowers ovate, 3 to 4 mm. long, 1 to 2 mm. wide, with 2 unequal awns (one of these about as long as the body) and usually a few minute squamellae. — Sch. Bip. in Seemann, Bot. Voy. Herald, 306 (1852–57); Hemsl. Biol. Cent.-Am. Bot. ii. 173. *Lipochaeta lantanifolia*, Sch. Bip. Linnaea, xix. 729 (1847). — Central Mexico. *Specimens examined,* —

San Luis Potosi: San José Pass, *Pringle*, no. 3062. This plant seems to have no very close affinities with the other known species, but in respect to the achenes it most nearly resembles *Z. Pringlei*.

+ + Achenes winged.

++ Squamellae minute or wanting.

28. Z. Pringlei, Greenman. Tall, slender-branched, suffruticose or fruticose, arachnoid-pubescent except on the upper surface of the leaves: leaves opposite or the uppermost alternate, ovate, generally less than 3 cm. long, subcordate at the base, obtuse or acute at the apex, above rugose and appressed-tuberculate-hispid, beneath densely tomentose; petioles 4 to 7 mm. long, narrowly margined: peduncles solitary, long, slender; heads 10 to 15 mm. high; involucre 1 to 1.5 cm. broad, about 3-seriate and subtended by several leaf-like bracts; the inner scales oblong to oblong-lanceolate: ligules oblong, about 12 mm. in length, usually half as broad: achenes much as in the preceding species but slightly larger and those of the ray-flowers verrucose, those of the disk-flowers winged. — Proc. Am. Acad. xxxiii. 489 (1898). — Southern Mexico. *Specimens examined,* — Puebla: near Tehuacan, alt. 2166 m., *Pringle*, no. 6768 (type, in herb. Gray).

++ ++ Squamellae evident.

29. Z. gnaphalioides, Gray. Slender-branched floccose-lanate shrub: leaves deltoid-ovate to cordate-lanceolate, acute at the apex, entire, revolute-margined, above shortly sericeous-hispidulous, beneath tomentose; petioles very short: peduncles long, slender, solitary, tomentose or floccose-lanate; heads and involucre much as in the preceding but the subtending foliaceous bracts much smaller: achenes short-necked, sparsely pubescent, rugose; those of the ray-flowers 3 mm. long, narrow, with 1 to 3 fragile awns; those of the disk-flowers with 2 unequal awns and narrow wings; squamellae well developed. — Proc. Am. Acad. xv. 36 (1879). — Central Mexico. *Specimens examined,* — San Luis Potosi: between the city and Tampico, *Palmer*, no. 1106, coll. of 1878–79 (type, in herb. Gray).

30. Z. brevifolia, Gray. Fruticose or suffruticose, much-branched, about 1 m. high: leaves ovate or oval, less than 2 cm. long, 1 cm. or less in width, entire, obtuse at the apex, attenuate to subcordate at the base, above strigose, beneath papillose-scabrous, the uppermost alternate: peduncles long, slender, terminal, solitary, densely strigose; heads 10 to 15 mm. high; involucre lax, about 3-seriate, 1 cm. broad; the outer bracts ovate, foliaceous at the apex: ray-flowers 5 to 8; ligules small; disk-corollas

somewhat exceeding the involucre: achenes 4 to 6 mm. long, slightly rugose, those of the disk-flowers narrowly, of the ray-flowers broadly winged; squamellae well developed, usually united at the base to form a short cup on the summit of the achene, the neck evident at least in the achenes of the disk-flowers; awns shorter than the body, rather rigid.— Pl. Wright. i. 112 (1852); Hemsl. Biol. Cent.-Am. Bot. ii. 172.— Southwestern United States and northern Mexico. *Specimens examined*, — TEXAS: hills near Eagle Pass, alt. 250 m., *Pringle*, no. 9039; *Palmer* no. 625 (coll. of 1880); in the valley of the Rio Grande below Doñana, Mexican Boundary Survey, no. 591; limestone rocks 112 km. below El Paso, *Parry* (coll. of 1852); *Wright*, no. 353; Chisos Mountains, *Havard*, no. 10. CHIHUAHUA: near the city, *Pringle*, nos. 41, 349, 998. NUEVO LEON: Soledad, *Palmer*, no. 625 (coll. of 1880). COAHUILA: Saltillo, *Palmer*, no. 803; Mesillas, *Gregg*, no. 534 (type, in herb. Gray). SAN LUIS POTOSI: near the city, alt. 2000 to 2500 m., *Parry* & *Palmer*, no. 451.

* * Leaves larger, the cauline, 5 cm. or more in length (sometimes shorter in *Z. hispida*).

+ Achenes angular, only slightly constricted at the summit (except in *Z. virgulta*); awns generally about as long as the body (but short and tooth-like in *Z. goyazensis*).

++ Awns short and tooth-like; ray-achenes scarcely 3 mm. in length.

31. Z. GOYAZENSIS, Benth. & Hook. f. Stem woody, terete, scabrous: leaves 10 to 15 cm. long, 5 to 7 cm. broad, ovate or ovate-lanceolate, acute, subcordate or cuneate-attenuate at the base, serrate-dentate, above somewhat scabrous, beneath sparsely tomentose; petioles about 3 cm. long: peduncles terminal, usually 2.5 cm. long; heads 1.5 cm. high; involucre about 1 cm. broad, 2-seriate; outer bracts obtuse, foliaceous, scabrous, commonly equalling the disk-flowers: ray-flowers about 10; ligules oblong: achenes with short tooth-like awns and a short crown of paleaceous squamellae; achenes of the ray-flowers wingless, obpyramidal, 3-angled, about 3 mm. long; those of the disk-flowers more or less winged, 4 to 5 mm. long, flattened.— Gen. ii. 373 (1873). *Lipochaeta goyazensis*, Gardn. in Hook. Lond. Jour. Bot. vii. 406 (1848); Walp. Ann. ii. 867. — South America. *Specimens examined*, — BRAZIL: near San Domingos, Province of Goyaz, *Gardner*, no. 4235 (co-type, in herb. Gray). This is the most southern species of *Zexmenia* and its achenial characters are very unlike all others.

++ Awns long, bristle-like; body of the achene usually exceeding 3 mm. in length.

= Achenes wingless.

a. Squamellae distinct.

32. Z. AURANTIACA, Klatt. Stem woody, branching, scabrous: leaves ovate, serrulate, acuminate at both ends, above scabrous or strigose, beneath somewhat pilose; petioles margined, clasping, about 2 cm. long: peduncles 4 to 12 cm. long, terminal, commonly solitary or in a cyme of 3; heads 10 to 15 mm. high; involucre cylindrical, about 1 cm. broad, 2-seriate; outer bracts herbaceous, generally reflexed at the apex, ovate-lanceolate, densely pubescent: ray-flowers about 15; ligules broadly ovate, tridentate: pales trifid, purple or violet at the apex: achenes 4 to 5 mm. long, slender, very slightly constricted, with very fragile plumose awns, generally only one as long as the body, and several unequal bristle-like squamellae; achenes of the ray-flowers 4-, of the disk-flowers, 3-angled. — Bull. Soc. Bot. Belg. xxxv. 296 (1896). — Central America. *Specimens examined,* — COSTA RICA: thickets on the banks of Virilla River, *Tonduz,* no. 9836 (type, in herb. Gray). This species may not belong to *Zexmenia.* The shape of the achene is exceptional and the pappus somewhat resembles that of a *Perymenium.*

b. Squamellae united; awns rather rigid, subequal.

33. Z. STRIGOSA, Sch. Bip. Suffruticose: leaves short-petioled, ovate-lanceolate, very acute, serrulate, above sparsely strigose, beneath appressed villous: peduncles terminal, monocephalous; involucre 2-seriate; bracts lanceolate, the outer foliaceous, strigose: achenes 5 to 7 mm. long, slender, very slightly constricted above, wingless, 2–3-awned; squamellae united between the awns but free from them, about 1 mm. long. — Sch. Bip. in Seemann, Bot. Voy. Herald, 306 (1852–57). *Lipochaeta strigosa,* DC. Prodr. v. 610 (1836). — Southern Mexico. *Specimens examined,* — OAXACA: *Andrieux,* no. 313, fragments lent from the De Candollean Herbarium.

= = Achenes of the ray-flowers winged; those of the disk-flowers wingless.

34. Z. GUATEMALENSIS, J. D. Smith. Fruticose, somewhat scabrous, covered with a whitish pilosity: leaves ovate-lanceolate, muriculate-scabrous above, pilose beneath, mucronulately serrulate, acute or acuminate at the apex, obtuse or rounded at the base; petioles margined, less than 1 cm. long: peduncles canescent-hirsute, slender, 0.5 to several cm. long, subumbellate; heads about 1 cm. high; involucre 2-seriate, campanulate; outer bracts slightly the longest, densely pubescent: ray-flowers

8; ligules 2-toothed, 10 to 15 mm. long; disk-flowers somewhat exceeding the involucre, about equalling the produced tips of the pales; limb of the corollas exceeding the tube, the lobes coarsely tomentose: ray-achenes obovoid, 3 mm. long, broadly winged; wings confluent with the awns and squamellae; disk-achenes oblong-turbinate, subtriquetrous, costate, with very caducous awns. — Bot. Gaz. xiii. 188 (1888). — Central America. *Specimens examined,* — GUATEMALA: Alta Verapaz, Coban, alt. 1400 m., *von Tuecrkheim,* no. 853 of J. D. Smith's distrib.

= = = Achenes of both the ray- and disk-flowers winged, or occasionally wings wanting in the ray-flowers.

a. Achenes long, narrow, not strongly constricted; wings very narrow. South American species.

1. Outer involucral bracts long, foliaceous: ligules wanting.

35. Z. FOLIOSA, Rusby in herb. Nearly herbaceous, leafy-stemmed: leaves commonly 1 to 2 dm. long, about half as broad, ovate, acuminate at the apex, subcordate or somewhat attenuate at the base, serrate, above strigose, beneath scabrous-puberulent; petioles margined, 1 to 4 cm. long: peduncles 1 to 3 cm. long, slender, pubescent, usually terminal and solitary or in cymes of 3; heads 1 to 2 cm. high, subtended by many long spreading foliaceous bracts; involucral bracts caudate-attenuate, ciliate or somewhat hirsute: rays none; disk-corollas exceeding the pales, the black anthers exserted: achenes narrowly winged, constricted, with short distinct squamellae; the marginal achenes 3-angled, 3-awned; those of the interior of the head compressed, 2-awned. — South America. *Specimens examined,* — BOLIVIA: Guanai-Tapuani, *Bang,* no. 1340 (type). Dr. Rusby (Mem. Torr. Bot. Club, vi. pt. 1, p. 63) mentions this plant (without specific name or characterization) and expresses a doubt whether it should be included in *Zexmenia* on account of the foliaceous bracts of the involucre and the absence of rays. However, it is closely related in achenial characters to *Z. wedelioides,* and in general habit to several other species of the genus.

2. Outer bracts not long and foliaceous: ligules present.

36. Z. WEDELIOIDES, Klatt. Stem woody, velvety-villous: leaves elliptical or ovate, 5 to 8 cm. long, about 4 cm. wide, remotely serrulate, above scabrous, beneath villous, acute or acuminate at the apex, obtuse or rounded at the base: peduncles subumbellately clustered, short, 1 to 2 cm. long; heads generally less than 1 cm. high; involucre subglobose; bracts oblong, acute, scabrous: achenes at maturity black, those of the ray-flowers 3-, of the disk-flowers, 2-awned; awns fragile, unequal, about

as long as the body; squamellae numerous, short, distinct or somewhat connate: — Abh. Naturf. Gesellsch. Halle, xv. 328 (1881). — South America. *Specimens examined*,— PERU: *Matthews*, no. 1383 (fragments of the type, in herb. Gray).

b. Achenes rather short and broadly winged, or strongly constricted at the summit. Mexican species.

1. Involucral scales subequal.

37. Z. SCANDENS, Hemsl. Shrub, climbing or reclining; branches rusty-pubescent: leaves thick or leathery but varying to thinner and submembranaceous, from ovate to oblong-lanceolate, subcordate to somewhat attenuate at the base, acuminate to obtuse at the apex, serrate or subentire, above scabrous or strigose, beneath villous-hirsute, 7 to 13 cm. long: peduncles 1 to 4 cm. long, corymbosely arranged, slender; heads rarely over 1 cm. high; involucre 3–4-seriate, about 1 cm. broad, subcylindrical or campanulate; bracts from appressed and closely imbricated to rather loosely spreading: disk-flowers exceeding the involucre: achenes of the ray-flowers from strongly 3-winged to narrowly 2-winged (the third angle wingless); wings generally prolonged beyond the neck but not confluent with the awns; awns about as long as the body, enlarged below, subequal, pubescent; squamellae distinct or somewhat connate; achenes of the disk-flowers 2-winged, 2-awned. — Biol. Cent.-Am. Bot. ii. 174 (1881). *Z. trachylepis*, Hemsl. l. c. 175 (1881). *Z. dulcis*, Coulter, Bot. Gaz. XVI. 99 (1891). — Southern Mexico and Central America. *Specimens examined*, — VERA CRUZ: Valley of Cordova, *Bourgeau*, no. 1930 (type); Cordova, *Gray*, coll. of 1885; Fortin, near Orizaba, *Kerber*, no. 285; Coatzacoalcos, *C. L. Smith*, no. 976. CHIAPAS: Tuxtla, *C. & E. Seler*, no. 1897. YUCATAN and TABASCO: *Johnson*, no. 24. GUATEMALA: Rio Dulce, *J. D. Smith*, no. 1607; eastern portion of Vera Paz and Chiquimula, *Watson*, no. 16. This species is highly variable and perhaps composite, but constant characters for its division have not been found.

2. Outer involucral scales longest, spreading and foliaceous.

38. Z. VIRGULTA, Klatt. Shrub with fuscous-hirsute branches: leaves ovate-lanceolate, 5 to 15 cm. long, acuminate at the apex, acute to rounded at the base, subentire to serrate, somewhat reticulate-veiny, above strigose, beneath villous; petioles 1 to 2 cm. long: peduncles 1 to 5 cm. long, cormybosely arranged, slender, pubescent; heads about 1 cm. high; involucre campanulate, 1 to 2 cm. wide (including the spreading bracts); outer bracts long, linear-lanceolate, foliaceous, spreading or re-

flexed: ray-flowers about 12; ligules small, narrow; disk-flowers much exceeding the involucre: achenes winged, pubescent, constricted at the summit; awns about as long as the body; squamellae small, numerous. — Bull. Soc. Bot. Belg. xxxi. 203 (1892). — *Z. dulcis*, Coulter in J. D. Smith, Enum. Pl. Guat. iv. 86 (1895), not Bot. Gaz. xvi. 99 (1891). — Central America. *Specimens examined*, — COSTA RICA: near San Mateo, *Pittier*, no. 7027; San José, *Tonduz*, no. 12,967; Aragon, Rio Turrialba, *Pittier*, no. 13,241 (no. 7504 of J. D. Smith's distrib.); San Miguel, *Biolley*, no. 7423; Rio Zhorquin, *Tonduz*, no. 8532; Talamanca, *Pittier*, no. 8676; Boca de Zhorquin, in thickets, *Tonduz*, no. 8699; Atirro, Carthago, *J. D. Smith*, no. 4872; Rio Turrialba, Carthago, *J. D. Smith*, no. 6610. GUATEMALA: Rio Samalá, Retalhuleu, *J. D. Smith*, no. 2854.

+ + Body of the achenes rounded (as in *Wedelia*) not strongly angled, but winged, strongly constricted near the summit; awns usually short.

++ Shrubs or perennial herbs.

39. Z. HISPIDA, Gray. Branching shrub, about 1 m. high, strigose-hispid: leaves rhomboid-lanceolate or ovate-lanceolate, acute at the base, sessile or nearly so, sparingly dentate or subhastately lobed, generally strigose on both sides: peduncles terminal, solitary or occasionally in a cyme of 3, very long; heads about 1 cm. high; involucre 2-seriate, cylindraceous-campanulate, generally less than 1 cm. wide; outer bracts lanceolate, subfoliaceous: ligules broad, conspicuous: achenes variable as to wing and size; those of the ray-flowers commonly 3-angled, with 2 or occasionally 3 wings; achenes of the disk-flowers broadly 2-winged or with the wings reduced to 2 upwardly directed auricles; pappus separated from the body of the achene by a strong constriction or neck bearing a somewhat expanded row of very short squamellae and 2 or 3 short awns. — Proc. Am. Acad. xix. 10 (1883). *Z. texana*, Gray, Pl. Wright. ii. 112 (1852). *Stemmodontia scaberrima*, Cass. Dict. Sci. Nat. xlvi. 407. ? *Wedelia hispida*, HBK. Nov. Gen. et Spec. iv. 215 (1820). *Wirtgenia texana*, Sch. Bip. in Seemann, Bot. Voy. Herald, 304 (1852–57). *Viguiera longipes*, Coulter, Contrib. U. S. Nat. Herb. i. 41 (1890). — Southwestern United States to Central Mexico. *Specimens examined*, — TEXAS: Austin, *Hall*, no. 341; Crocket County, *Reverchon*, no. 1542; southwestern part of the state, *Wright*, no. 354, *Palmer*, no. 630; Rio de Medina, *Berlandier*, no. 2460; Arroyo de Colorado, *Berlandier*, no. 2553; Nueces County, *Heller*, no. 1479; New Braunfels, *Lindheimer*, no. 436, *Mex. Bound. Surv.* no. 590; Kerr County, *Bray*, no. 199;

Burnet County, *Hill*, no. 22; Tom Green County, *Tweedy*, no. 324; Corpus Christi County, *Nealley*, no. 95 (type of *Viguiera longipes*, Coulter); Gillespie County, *Jermy*, no. 646; Upper Guadaloupe River, *Lindheimer*, no. 345. TAMAULIPAS: near Victoria, *Berlandier*, Nov. 1830; Matamoras, *Berlandier*, no. 3010. NUEVO LEON: Monterey, *Palmer*, no. 631, *Pringle*, nos. 2264, 11,907. COAHUILA: Saltillo, *Parry*, no. 24, *Palmer*, no. 328. SAN LUIS POTOSI: *Parry & Palmer*, no. 449. HIDALGO: Tula, *Pringle*, no. 6568. STATE OF MEXICO: Santa Fé, *Bourgeau*, no. 594; Tacubaya, *Schmitz*, no. 1029, *Schaffner*, no. 45. VERA CRUZ (?): near Tantoyuca, *Ervendberg*, no. 89.

Var. RAMOSISSIMA, Greenman. Stems 2 to 3 m. high, much-branched: peduncles rather slender, 1.5 to 6 cm. long. — Field Col. Mus. Bot. ser. iii. 127 (1904) — Southern Mexico. *Specimens examined*, — OAXACA: Tomellin Cañon, *Pringle*, no. 6283. YUCATAN: *Gaumer*, no. 826. "This variety in habit, more particularly on account of the much branched character of the stem and the numerous heads, seems, at first sight, specifically distinct from the typical *Z. hispida*, but a considerable series of specimens indicates that it is only an extreme variation. Forms connecting the species and the variety are shown by Pringle's no. 2264 and Palmer's no. 631 from Soledad." J. M. G.

40. Z. LONGIPES, Benth. Fruticose or suffruticose, much-branched, erect: leaves broadly ovate, 2 to 15 cm. long, 1 to 7 cm. wide, acuminate, serrate, at base somewhat attentuate, not lobed, above strigose, below hirsute-villous; petioles 0.2 to 2 cm. long: peduncles 1 to 6 cm. long, corymbosely arranged, densely hirsute; heads about 1 cm. high; involucre 2–3-seriate, cylindrical or somewhat campanulate, about 1 cm. broad; bracts lanceolate, hirsute: ligules conspicuous: achenes brownish, strongly 2-winged: wings projecting upwards in ear-like lobes, not confluent with the awns but attached to the neck of the achene; ray-achenes glabrous or nearly so and usually with 3 short awns and whitish wings not markedly streaked; disk-achenes hirsute, 2-awned and with wings streaked with brown. — Benth. in Oerst. Vidensk. Meddel. 1852, p. 95. *Aspilia costaricensis*, Klatt, Bull. Soc. Bot. Belg. xxxiv. 286 (1896), not xxxi. 201 (1892). — Central America. *Specimens examined*, — COSTA RICA: Guanacaste, *Pittier*, no. 2801 bis; near San Mateo, *Biolley*, no. 7016; near San José, *Pittier*, no. 3135; Rio Teliri, *Pittier*, no. 7194; between Mano and Tigre, *Pittier*, no. 4683; Nicoya, *Tonduz*, no. 13,616.

41. Z. PHYLLOSTEGIA, Klatt. Branches slender: leaves linear-lanceolate, 4 to 8 cm. long, 10 to 15 mm. wide, attenuate into a very short

petiole, acuminate, entire, above scabrous, beneath tomentose-canescent: peduncles solitary, terminal; involucre several-seriate; bracts ovate, obtuse; ray-flowers about 20; ligules exceeding the disk, dentate at the apex. — Leopoldina, xxv. 105 (1889). — Central America. *Specimens examined*, — GUATEMALA: *Bernouilli*, no. 1077 (tracing, in herb. Gray).

++ ++ Annuals.

42. Z. RUDIS, Baker. A much-branched herb, 6 to 12 dm. high; branches densely hispid: leaves broadly ovate to ovate-lanceolate, acuminate usually at both ends, irregularly serrate-dentate, short-petioled, strongly hispid-pubescent on both surfaces: peduncles 0.5 to several cm. long, borne in a lax corymb; involucre campanulate, 5 to 8 mm. broad, about 2-seriate; bracts lanceolate, with foliaceous tips, acute: ray-flowers about 10; ligules oblanceolate, 1 cm. long: achenes 2 to 3 mm. long, laterally compressed, pubescent; awns 2 to 3, short: squamellae united into a short cup, confluent with the awns.—Bak. in Mart. Fl. Bras. vi. pt. 3, 188 (1884). *Wedelia rudis*, Benth. in herb. ex Bak. l. c. — South America. *Specimens examined*,— BRAZIL: Falls of the Madeira, *Rusby*, 2142.

DOUBTFUL AND EXCLUDED SPECIES.

Zexmenia (?) in sched. pl. Rusby, no. 2138. *Calea*, Britton, Bull. Torr. Bot. Club, xix. 151 (1892). This plant, if one may judge by the specimen received under the number at the Gray Herbarium, is certainly neither a *Zexmenia* nor a *Calea*. It has sterile ray-flowers and the pappus of a *Viguiera*.

Zexmenia in sched. pl. Bang, no. 546, is *Oyedaea Pearcei*, Rusby, Mem. Torr. Bot. Club, iii. pt. 3, p. 59 (1893).

Z. caracasana, Benth. & Hook. f. Gen. ii. 371 (1873), is better treated as *Wedelia caracasana*, DC.

Z. caracasana, Klatt, Bull. Soc. Bot. Belg. xxxi. 202 (1892), is a mixture. Pittier's no. 3737 is apparently a *Wedelia*, while his no. 7019 is *Baltimora scolospermum*, Steetz.

Z. dulcis, Coulter, Bot. Gaz. xvi. 99 (1891), is not clearly distinguishable from *Z. scandens*, Hemsl., but Mr. J. D. Smith's nos. 2854 and 6610, referred to *Z. dulcis* by Coulter in J. D. Smith, Enum. Pl. Guat. iv. 86 (1895), seems better placed in *Z. virgulta*, Klatt.

Z. fasciculata, Coulter in J. D. Smith, Enum. Pl. Guat. iv. 86 (1895), as to no. 4244, is *Perymenium grande*, Hemsl., var. *strigillosum*, Robinson & Greenman.

Z. helianthoides, Benth. & Hook. f. Gen. ii. 371 (1873), is better restored to *Wedelia helianthoides*, HBK.

Z. Lindenii, Sch. Bip. in Seemann, Bot. Voy. Herald, 306 (1852–57). *Lasianthaea Lindenii*, Sch. Bip. l. c. "Foliis alternis ab omnibus differt, 3½ poll. longis, ¾–1 poll. latis, elliptico-lanceolatis, utrinque attenuatis, petiolatis, serratis, penninerviis, supra asperis, infra tomentosis, corymbo polycephalo (floribus flavis: Linden)." This inadequately characterized plant has not been identified among the specimens examined.

Z. longipes, Klatt, Bull. Soc. Bot. Belg. xxxi. 202 (1892) and xxxv. 287 (1896), as to Pittier's nos. 4365, 6961, and 6991, is referred with some doubt to *Wedelia filipes*, Hemsl.

Z. ovata, Benth. & Hook. f. Gen. ii. 373 (1873). *Tithonia ovata*, Hook. Bot. Mag. t. 3901 (1842). This plant is exactly *Z. helianthoides*, Gray.

Z. reticulata, Benth. & Hook. f. Gen. ii. 373 (1873), is better restored to *Wedelia*.

Z. scandens, Coulter in J. D. Smith's Enum. Pl. Guat. iv. 86 (1895), is probably *Perymenium Nelsonii*, Robinson & Greenman.

Z. stenantha, Hemsl. Biol. Cent.-Am. Bot. ii. 174 (1881), is *Z. crocea*, Gray.

Z. tagetiflora, D. Don ex Sweet, Hort. Brit. ed. 2, p. 309 (1830), is a name only.

Z. tequilana, Gray, Proc. Am. Acad. xxii. 425 (1887), is *Otopappus acuminatus*, Wats.

Z. trachylepis, Hemsl. Biol. Cent.-Am. Bot. ii. 175 (1881), is not satisfactorily distinguishable from *Z. scandens*, Hemsl. l. c. 174.

Z. villosa, Polak. Linnaea, xli. 579 (1877), was based on a Costa Rican plant, described as having the involucre 2 cm. broad. It is believed that a part of the type-material was deposited in the herbarium of the Royal Botanical Museum at Berlin, but Professor Urban, who has kindly made a search for it, writes that it is not to be found under *Zexmenia*. It is probable, therefore, that it has been found to belong elsewhere and has been removed to some other genus.

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TEMPORAL CLAUSES IN HERODOTUS.

By Haven D. Brackett.

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Introduction.

The purpose of this study is in general twofold; first, to discover and record the facts, so far as they can be ascertained, concerning the use of temporal clauses in Herodotus; and secondly, upon the basis of these facts, to determine as far as possible the general principles of Herodotus's usage, and to apply these principles, where it may be possible, in determining the true reading of the text.

The relation of time is expressed in Greek in a variety of ways: we are concerned here however with those sentences only in which this relation is expressed by a temporal clause which is introduced by a temporal conjunction or a similar expression.

The temporal relation of one clause to another may be that of antecedence, or of contemporaneity, or of subsequence. But not all temporal clauses can be ranged under any single one of these classes. Furthermore, there may arise certain combinations of these relations. Antecedence may be combined with contemporaneity, as in Her. 9, 117*: ἐπεὶ δὲ πολιορκεομένοισί σφι φθινόπωρον ἐπεγίνετο, ἤσχαλλον οἱ Ἀθηναῖοι, κ.τ.λ., where the action of ἐπεγίνετο had not ceased when that of ἤσχαλλον began. Contemporaneity may also be combined with subsequence, as in Thuc. 8, 45, 1: ἐν δὲ τούτῳ καὶ ἔτι πρότερον, πρὶν ἐς τὴν Ῥόδον αὐτοὺς ἀναστῆναι, τάδε ἐπράσσετο.†

All the moods of the Greek verb express the character of the action. There are three kinds of action which are expressed by the Greek verb: (1) action evolving itself or in process of development; (2) action at a point, simply attained or brought to pass; (3) action completed, to which is often added the idea of a permanent result. The terminology of the first two of these kinds of action has varied widely; they will be designated here as extensive and aoristic, respectively.

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* Passages in Herodotus are cited according to the edition of Dietsch, revised by Kallenberg: Herodoti Historiarum Libri ix, Lipsiae, 1898.

† Cf. Warren, Conjunctional Temporal Clauses in Thuc., Berlin, 1897, p. 37.

In expressing the relation of antecedence, if a verb of extensive action be used in the temporal clause it is evident that, as far as the form of the verb is concerned, there is opportunity for at least partial coincidence in time. Cf. Her. 9, 117 (quoted above, p. 171). Hence, for the definite and unequivocal expression of pure antecedence, the aoristic aspect is to be expected in the temporal clause. It should be well noted, however, that this need is not absolute, even theoretically, since there is nothing in either the present or the imperfect tense that expresses absolute duration of time. Cf. Her. 7, 188: *ὁ δὲ δὴ ναυτικὸς στρατὸς ἐπεἰτε ὁρμηθεὶς ἔπλεε . . ., αἱ μὲν δὴ πρῶται τῶν νεῶν ὅρμεον πρὸς γῇ, κ.τ.λ.*, where the idea of overlapping action is wholly excluded.

In clauses of contemporaneity two relations are possible, coincidence and insertion; in other words, one action may be coextensive with another, or may be inserted in it. In the relation of coincidence both actions may be viewed as extensive or as aoristic.* Again a clause of contemporaneity may coincide in time with the action of the main clause and at the same time mark its limits.

In cases where the action of one clause is inserted in that of another, the action of one clause is generally viewed as extensive, while that of the other may be either extensive or aoristic. Warren,† speaking of these clauses, says, "In the relation of insertion one action must be viewed as extensive," etc. But if our reasoning above was correct, the action into which the other is inserted may be in the aorist, if that aorist has complexive force.

In clauses of subsequence also, if the main verb is of extensive aspect, the main action may overlap that of the subordinate clause, as in Thuc., 1, 118, 2: *οἱ δὲ Λακεδαιμόνιοι. αἰσθόμενοι οὔτε ἐκώλυον, εἰ μὴ ἐπὶ βραχύ, ἡσύχαζόν τε τὸ πλέον τοῦ χρόνου, . . . τὸ δέ τι καὶ πολέμοις οἰκείοις ἐξειργόμενοι, πρὶν δὴ ἡ δύναμις τῶν Ἀθηναίων σαφῶς ἤρετο καὶ τῆς συμμαχίας αὐτῶν ἥπτοντο.* Hence to express pure subsequence a verb of aoristic action is to be expected in the main clause. The subordinate verb may be either extensive or aoristic.

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* Warren (op. sup. cit., p. 5) quotes Il., Λ, 671–2:

ὡς ὁπότ' Ἠλείοισι καὶ ἡμῖν νεῖκος ἐτύχθη
ἀμφὶ βοηλασίῃ, ὅτ' ἐγὼ κτάνον Ἰτυμονῆα, κ.τ.λ.,

as an example of aoristic coincidence. But it can hardly be supposed that the action described by the words *νεῖκος ἐτύχθη* continued no longer than the action of *κτάνον.* On the contrary *ἐτύχθη* here is the constative or complexive aorist. The action of *κτάνον* is only a part of the whole *νεῖκος* and the relation is that of insertion.

† Op. sup. cit., p. 6.

When a clause of subsequence defines the posterior limit of the progress of an action, the action whose limit is to be stated is regularly expressed by a verb of extensive action; the defining action is regularly of the aoristic aspect, less often extensive. The action to be defined may either in part or wholly * precede the defining action.

The forms of the verb which are to be expected in the various kinds of temporal sentences may be indicated as follows:

Action	Main Verb	Subord. Verb
Purely antecedent	extensive aoristic	[extensive]† aoristic
Antecedent overlapping	extensive aoristic	extensive
Contemporaneous	extensive	extensive
	aoristic	aoristic
Inserted, extensive in extensive	extensive	extensive
Aoristic in extensive	extensive	aoristic
	aoristic	extensive
[Aoristic in aoristic]‡	aoristic	aoristic
Subsequent overlapping	extensive	aoristic extensive
Purely subsequent	aoristic [extensive]§	extensive aoristic
Subsequent expressing limit	extensive	aoristic [extensive]§

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* Warren (op. sup. cit., p. 7) argues that the action to be defined can never wholly precede the defining action. Is this so? The true temporal relation in these clauses can best be tested in clauses of limit with ἕως, ἄχρι, etc., after affirmative main clauses. Now the progress of an action may be limited in either of two ways: (1) it may be limited loosely by a verb which expresses an action as in progress; or, (2) it may be limited exactly by a verb of aoristic aspect. In the former case the main action may overlap the subordinate action, as in Thuc., 1, 30, 3: τοὺς τῶν Κορινθίων ξυμμάχους ἐπιπλέοντες ἔφθειρον, μέχρι οὗ Κορίνθιοι . . . ἐστρατοπεδεύοντο ἐπὶ Ἀκτίῳ, κ.τ.λ. Cf. also Thuc., 1, 118, 2 (πρίν) and Xen. Cyrop., 3, 3, 18 (ἕως). In the latter case it may happen that the action of the main verb ceases at the exact instant when that of the subordinate verb begins. In the sentence, "The pencil moves until it touches the table," it is clear that the action of the main verb ceases at exactly the same time that the subordinate action begins. Here, therefore, there is no relation of contemporaneity whatever.

† See below, pp. 189 ff.

‡ See above, p. 172.

§ See below, pp. 202 f.

‖ See below, pp. 205 ff.

Temporal clauses may be divided into two classes, prior and posterior, according as they do or do not logically precede the main clause.[*] Prior clauses regularly are placed before, posterior clauses after, the main clause. In clauses of contemporaneity when the relation is that of coincidence, the temporal clause regularly precedes.

In the case of insertion, the temporal clause is prior if the main action is inserted, and posterior if the subordinate action is inserted.

The appearance in temporal clauses of moods and tenses is not as regular as the foregoing statement of general principles might seem to imply. The Greek verbal system is far from complete. Again it is coming to be generally recognized that in the study of syntactical problems word-meaning is of no little importance.[†] Many verbs, because of their special meaning, have a natural affinity for a certain tense or a certain kind of action. Moreover the influence of negatives is not to be overlooked. The tense of sharp definitive negation is the aorist; a negative with the imperfect expresses disappointment or resistance to pressure.[‡]

The temporal clauses in Herodotus will be treated in this discussion from five points of view: mood, tense, the character of the action as expressed by the tense-stem, conjunctions, and order of clauses.

I. Mood.

A. *Antecedent and Contemporaneous Action.*

Particular action in accordance with fact is expressed as a rule both in the main and the subordinate clause by the indicative mood. The main verb, as regards its mood, is under no restriction arising from the temporal clause, and therefore may take any form which is possible for an independent sentence.

In accordance with the regular law of indirect discourse a secondary tense of the indicative in the clauses now under discussion is almost never changed to the optative. Such a change is made in Herodotus only once, 2, 121ϵ: *ἐσελθόντα δὲ ὡς τοῦ βασιλέος τὴν θυγατέρα καὶ εἰρωτώμενον τά περ καὶ οἱ ἄλλοι, ἀπηγήσασθαι ὡς ἀνοσιώτατον μὲν εἴη ἐργασμένος ὅτε τοῦ ἀδελφεοῦ . . . ἀποτάμοι τὴν κεφαλήν. κ.τ.λ.* *ὅτε* is the reading of all MSS.

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[*] See Delbrück, Syntak. Forsch., Halle, 1871, vol. 1, p. 35.

[†] Cf. E. P. Morris, Principles and Methods in Latin Syntax, p. 47: "syntax can be understood only when it is studied in the closest connection with word-meaning," etc.

[‡] Cf. Gildersleeve, Syntax of Classical Greek, 216 and 245; Brugmann, Griech. Gram.³, 544, 1, Anm.

and of all editors except Stein in his 2d ed. (1884) and van Herwerden. Madvig * would change ὅτε to ὅτι for two reasons: (1) because he thinks it strange that in parallel clauses ὅτι is used in one case and ὅτε in the other; (2) because he believes that in a clause with ὅτε a secondary tense of the indicative could not be changed to the optative. It does seem improbable that Herodotus used different words in introducing two clauses which are so exactly parallel. But should ὅτε be changed? Undoubtedly the reason why the secondary tenses of the indicative are in general not changed in indirect discourse is to avoid obscurity. But here, as Goodwin has said,† no obscurity whatever is caused by using the optative, and further, it is to be noticed that an optative (εἴη ἐξεργασμένος) has preceded. Should we not rather read ὅτε for the following ὅτι? ὅτι καταλύσειε can hardly mean "because I took down," as Goodwin says (l. s. c.), but rather "in that," or "therein that." ‡ But this use of ὅτι seems not to be common in Herodotus; Madvig does not cite another instance.§ In 9, 75, moreover, in a passage exactly parallel to this, ὅτε is found in MSS. R and S, and was conjectured by Krüger. In 7, 20, where all MSS. give ὅτε, it is used in the same sense as here. ὅτε should, I think, be retained in our passage. If any change is to be made I would change the following ὅτι to ὅτε.

A primary tense of the indicative in a temporal clause is changed to the optative only once, 3, 27, where παρείη stands for πάρεστι. The subjunctive occurs twice in a clause with ὅτε after a secondary tense (5, 30; 6, 78) and in both cases the subjunctive is retained. In clauses of contemporaneity introduced by relative expressions a secondary tense of the indicative (2, 137; 5, 84), a primary tense of the indicative (1, 164), and a subjunctive (1, 21; 4, 201) are found in a secondary sequence in indirect discourse. In 5, 63 the optative with ὅκως stands in indirect discourse for the subjunctive with ἄν.

In two passages the conclusion of the temporal clause is lost sight of in a long rambling parenthesis. In 1, 137, the clause ἐπείτε . . . ἐξεβλήθησαν finds its conclusion only at the end of that section in the words τοὺς δὲ οὕτω δὴ . . . Λῆμνον. Of the passage in 9, 84, ἐπείτε . . . ἠφάνιστο, κ.τ λ., Stein says, "locus et lacunosus et corruptus." But it seems

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* Advers. Crit, III, 26, note.

† Moods and Tenses, 693.

‡ Cf. Madvig, Synt. of the Greek Lang., trans. by Browne, London, 1880, 3d ed., p. 113, n. 1, b.

§ I have found ὅτι used in this sense in Herodotus but once, apart from the present passage, i. e., 7, 137, but my search for such a use was by no means exhaustive.

hardly necessary to assume textual corruption here. As in 1, 137, the conclusion of the temporal clause is not definitely expressed, but is loosely indicated by what follows. It is a good example of the fine disregard of rigorous logical structure, which is thoroughly characteristic of Herodotus.

Customary or repeated action is expressed in the main clause by the present or imperfect indicative, or an equivalent expression, in the temporal clause by the subjunctive after a primary tense, and the optative after a secondary. The adverb ἄν is regularly used with the subjunctive.

A noteworthy example of a clause of generic action with ἦμος is found in 4, 28: βρονταί τε ἦμος τῇ ἄλλῃ γίνονται, τηνικαῦτα μὲν οὐ γίνονται, θέρεος δὲ ἀμφιλαφέες. Here the action of γίνονται is general and the subjunctive with ἄν would be expected in the protasis. But in this case the indicative is used because the action described by βρονταὶ γίνονται is regarded as sure to take place and the time, though not expressed, is nevertheless felt as definite. A similar usage is found in clauses with ἐς ὅ in 3, 104: ἐπὶ μᾶλλον ψύχει, ἐς ὃ ἐπὶ δυσμῇσι ἐὼν καὶ τὸ κάρτα ψύχει, and 4, 181.*

Future action is expressed in the main clause by any form which expresses future time; in the subordinate clause it is invariably expressed by the subjunctive. When such clauses stand in indirect discourse Herodotus always uses the principle of *repraesentatio*. But in 9, 51 the subjunctive of a clause of purpose, though it is in the same sequence as a clause with ἐπεὰν and the subjunctive, is changed to the optative (ἐπεὰν . . . ᾖ . . . ὡς ἂν μὴ ἰδοίατο). A similar case is found in 8, 61.

B. *Subsequent Action.*

The law stated above for antecedent and contemporaneous action holds true in general for clauses of subsequence; but clauses introduced by πρίν and πρὶν ἤ present certain marked peculiarities. After negative clauses πρίν and πρὶν ἤ are followed by a finite verb except in 1, 71, and 1, 165. In 1, 71: Πέρσησι γάρ, πρὶν Λυδοὺς καταστρέψασθαι, ἦν οὔτε ἁβρὸν οὔτε ἀγαθὸν οὐδέν, the position of the clause with πρίν † and the context show that the idea of "before" was more prominent in the writer's mind than that of "until."[*] For 1, 165, see below, p. 180.

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* ἦμος is found in a similar passage in Od., 4, 400:

ἦμος δ' ἠέλιος μέσον οὐρανον ἀμφιβεβήκει,

(ἀμφιβεβήκῃ MS. Q and La Roche). On the analogy of the passages cited from Herodotus the indicative would be expected since the action of the temporal clause is in no real sense contingent and the time is not indefinite but definite. Furthermore, ἦμος is elsewhere always found with the indicative except in later Greek. Cf. Merry and Riddell, Homer's Odyssey, 2d ed., Oxford, 1886, note ad loc.

† πρίν with a finite verb stands before the main clause in Herodotus only once.

The reading in 4, 9, μιχθῇ, which is given by Kallenberg, Holder, and van Herwerden (μειχθῇ), is surely right.

After affirmative clauses, temporal clauses with πρίν always take the infinitive. πρίν used with the infinitive after an affirmative clause has the idea of "until" as well as "before" in these passages: 4, 1; 6, 119; 9, 13, 101. πρότερον ἤ, like πρίν and πρὶν ἤ, meaning "before," and referring to simple past action, takes the infinitive.

In clauses of subsequence Herodotus nowhere changes the indicative or the subjunctive to the optative after a secondary tense in indirect discourse.

Future action is expressed in the temporal clause by the subjunctive; the future indicative is found but once, 9, 58: διωκτέοι εἰσὶ ἐς ὃ καταλαμφθέντες δώσουσι ἡμῖν . . . δίκας. Fuchs[†] thinks that the use of the future here expresses Mardonius's confidence that he will punish the Greeks. Gildersleeve[‡] calls attention rightly to the presence of the aorist participle καταλαμφθέντες, and sees in the expression not confidence but grimness. It is certainly not impossible to recognize in the sentence both of these ideas.

An interesting passage is found in 8, 7: ταῦτα βουλευσάμενοι ἀπέπεμπον τῶν νεῶν τὰς ταχθείσας, αὐτοὶ οὐκ ἐν νόῳ ἔχοντες ταύτης τῆς ἡμέρης τοῖσι Ἕλλησι ἐπιθήσεσθαι, οὐδὲ πρότερον ἢ τὸ σύνθημά σφι ἔμελλε φανήσεσθαι παρὰ τῶν περιπλεόντων ὡς ἡκόντων. There is here a confusion of direct and indirect discourse. The idea contained in πρότερον . . . ἔμελλε is really a part of the original thought of the Persians who despatched the ships. But Herodotus, instead of incorporating this in the indirect discourse, writes from the viewpoint of the time when he speaks and says: "They were not going to attack until the signal, which *was going to be shown*, was displayed to them." A close parallel to this passage is found in Xen. Anab., 3, 1, 2: ἐν πολλῇ ἀπορίᾳ ἦσαν . . . ἐννοούμενοι μὲν ὅτι ἀγορὰν δὲ οὐδεὶς ἔτι παρέξειν ἔμελλεν, κ.τ.λ. Goodwin translates[§]: "nor did they mean to attack until the signal was ready to be given," and adds, "with πρίν we should probably have had πρὶν ἂν μέλλῃ." But the context shows that the Persians were not going to attack until the signal was actually given; for how could they know when it was ready to be given? If *πρίν* had been used the form would probably have been *πρὶν ἂν φανῇ*.

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[*] Cf. Goodwin, Moods and Tenses, 627; Sturm, in Schanz's Beiträge. Bd. 1, p. 293.

[†] Die Temporalsätze mit den Konjunktionen "bis" und "so lange als," in Schanz's Beiträge, Heft 14, 1902, pp. 71 ff.

[‡] Am. Jour. Phil., **24** (1903), p. 401.

[§] Moods and Tenses, 653.

One of the clearly marked and distinctive characteristics of Herodotus's style is his frequent use of the infinitive in subordinate clauses in oratio obliqua. The general fact has long been known, but an examination of the temporal clauses in Herodotus shows that this usage is surprisingly frequent. The infinitive is found in a temporal clause 46 times, as follows*: *ἐπεί* — 4, 10, 33; 5, 84; 7, 150. *ἐπειδή* — 3, 26. *ἐπείτε* — 1, 94; 2, 42, 118; 6, 84, 137; 8, 135. *ἐς ὅ* — 1, 94, 202; 2, 102; 5, 85, 86. *ἔστε* — 7, 171. *ἕως* — [4, 42].† *ὅκως* — 2, 140. *ἐν τῷ* — 3, 105. *πρίν* — 1, 165. *ὡς* — 1, 24, 86, 94, 141; 2, 99, 107, 121β, $121\beta_2$, $121\beta_3$, 121δ, $121\delta_2$, 121ε, 140, 162; 3, 35, 87; 4, 8, 9, 11, 179; 6, 137; 7, 148, 170; 8, 94, 94_2, 118.

In order to find out the proportion between the cases where the infinitive was used and those where the finite mood was retained, I collected the passages where a finite verb in a temporal clause in oratio obliqua *might, according to the principles in the cases above enumerated*, have been changed to the infinitive but was not so changed. There are 65 such cases. Out of 111 cases, then, where it was possible for Herodotus to make this change, in 46 cases he used the infinitive, in 65 he did not. The infinitive so used stands for an indicative 43 times, for an optative once (2, 140), and for a subjunctive twice (1, 165, 202).

In 2, 140: *ὅκως γάρ οἱ φοιτᾶν . . . Αἰγυπτίων ὡς ἑκάστοισι προστετάχθαι . . .*, *ἐς τὴν δωρεὴν κελεύειν σφέας καὶ σποδὸν κομίζειν*, *φοιτᾶν* clearly stands for *φοιτῷεν* of the oratio recta. In 1, 202: *πλεῦνος δὲ ἐπιβαλλομένου τοῦ καρποῦ μᾶλλον μεθύσκεσθαι, ἐς ὃ ἐς ὄρχησίν τε ἀνίστασθαι καὶ ἐς ἀοιδὴν ἀπικνέεσθαι*, the main verb expresses generic action, and we should expect after *ἐς ὅ* the aorist subjunctive with *ἄν*.‡ In 4, 43 (*ἐς ὃ ἂν ἀπίκηται*) and 8, 108 (*ἐς ὃ ἔλθῃ*), *ἐς ὅ* with the subjunctive stands in oratio obliqua: but the subjunctive is not, as here, changed to the infinitive. One reason for the change in 1, 202, is this: *ἐς ὅ* has here dropped its distinct temporal force and is virtually equivalent to a mere co-ordinating conjunction. Cf. 4, 203 (*ἐς ὃ . . . μετεμέλησε*), where no main verb is expressed. The meaning of our passage is this: "as more of the fruit is thrown on they drink more and more, *and finally* they begin to dance and sing."

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* The list of Heilmann, De Infinitivi Syntaxi Herodotea, Gissae, 1879, pp. 49 f., is not entirely accurate.

† See below, pp. 179 f.

‡ Goodwin, Moods and Tenses, 613, 5.

In 7, 171, this passage occurs: *ἀντὶ τούτων δέ σφι . . . λιμόν τε καὶ λοιμὸν γενέσθαι, ἔστε τὸ δεύτερον ἐρημωθείσης Κρήτης . . . τρίτους αὐτὴν νῦν νέμεσθαι Κρῆτας.* Liddell and Scott, s. v. *ἔστε*, I, d, say that the infinitive with *ἔστε* stands here for the optative. This is impossible; past generic action is out of the question here, and there was no subjunctive in the oratio recta, since the clause *ἔστε . . . νέμεσθαι* does not refer to future time at all. The structure of the entire sentence is loose and brachyological; the thought fully expressed would be this: "famine and pestilence befell them [and continued] till Crete was devastated a second time; and now," etc.[*]

A noteworthy passage is found in 4, 42: *ἀπέπεμψε Φοίνικας ἄνδρας πλοίοισι, ἐντειλάμενος ἐς τὸ ὀπίσω δι' Ἡρακλέων στηλέων διεκπλέειν ἕως ἐς τὴν βορηΐην θάλασσαν καὶ οὕτω ἐς Αἴγυπτον ἀπικνέεσθαι.* Nearly all editors and commentators regard *ἕως* here as the conjunction meaning "until." and take *ἀπικνέεσθαι* as an infinitive used on the principle of oratio obliqua. The original form of the thought was this: *διεκπλέετε ἕως ἂν . . . ἀπικνέησθε*, and if any change was to be made, we should expect here *ἕως . . . ἀπικνέοιντο*.[†] The use of the infinitive here is in the highest degree strange. Fuchs compares this passage with the *ἐς ὅ* clauses which have the infinitive,[‡] but he overlooks the fact that in the *ἐς ὅ* clauses the verb nowhere refers to future time. There are the following objections to taking *ἕως* here as "until." First, there is no other passage in classical Greek where *ἕως* is used with the infinitive. Nowhere, in Herodotus or in any other classical writer, is the infinitive certainly used in oratio obliqua for a finite mood in a temporal clause which refers distinctly to future time. Secondly, the tense of *ἀπικνέεσθαι*, if it be connected with *ἕως*, has not been satisfactorily explained.[§] We should expect here the aorist.[‖] As to Herodotus's usage elsewhere, *ἕως* is found meaning "until" in two other passages, 7, 23, and 100; in both cases the verb is *ἀπικνέεσθαι* and is in the aorist indicative. Thirdly, in a sentence in 4, 43, which is exactly parallel to ours: *Λιβυὴν γάρ οἱ ἀνάγκην ἔσεσθαι περιπλέειν, ἐς ὃ ἂν ἀπίκηται περιπλέων, κ.τ.λ.*, where an infinitive in oratio obliqua precedes the temporal clause, the subjunctive is retained and the tense is the aorist. Lastly, in 4, 42, contrary to Herodotus's regular usage,* the whole passage in oratio obliqua is very short and no infinitive in oratio obliqua has preceded.

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[*] Cf. Fuchs, op. cit., p. 76; Gildersleeve, Am. Jour. Phil., **24**, p. 401.

[†] Goodwin, Moods and Tenses, 695.

[‡] Op. cit., p. 75. Fuchs's statement on p. 77 ("indirekte Rede — Beziehung auf Thatsächliches") is incorrect.

[§] Gildersleeve, Am. Jour. Phil., **24** (1903), p. 401, calls attention to the plural, πλοίοισι, but this can hardly be considered a certain and satisfactory explanation.

[‖] See below, p. 206.

These reasons are, I think, sufficient to make it doubtful whether the current interpretation of this passage is correct. All these difficulties can be avoided however by a change in the interpretation of one word. Is not ἕως to be taken closely with ἐς, in the sense of the Latin *usque ad?* ἀπικνέεσθαι will then be co-ordinate with διεκπλέειν, and we may translate: "Necos despatched the Phœnicians in boats, commanding them to sail, on their homeward voyage, through the Pillars of Hercules *clear to* the Northern Sea, and in that way to get back to Egypt." It should be noted that a few lines later, Herodotus, writing of this same event, says: ὥστε . . . τρίτῳ ἔτει κάμψαντες Ἡρακλέας στήλας ἀπίκοντο ἐς Αἴγυπτον. ἕως is used with εἰς by Polybius (1, 11, 14; 34, 4, *et al.*) and in the Septuagint (Num., 17, 13; Mac., 1, 2, 58). I am aware that no examples are given of ἕως used in this sense, as early as Herodotus. But Xenophon uses ἄχρι (Anab., 5, 5, 4), μέχρι (Anab., 6, 4, 26), and ἔστε (Anab., 4, 5, 6) with this meaning, and I see no *a priori* reason why Herodotus may not have used ἕως in the same sense. The difficulties in taking ἕως = *usque* are, to my mind, much fewer than those involved in the common interpretation.†

The only other case where the infinitive may be thought to stand for the subjunctive in oratio obliqua is 1, 165: ὤμοσαν μὴ πρὶν ἐς Φώκαιαν ἥξειν πρὶν ἢ τὸν μύδρον τοῦτον ἀναφανῆναι (MSS. ἀναφῆναι). The use of the infinitive here is very strange. First, if we exclude the passage in 4, 42, it is the only case in Herodotus where the infinitive is used in oratio obliqua in a temporal clause which refers to future time. Exclusive of this passage, after a negative main clause in a clause referring to the future, πρὶν ἤ is used six times (1, 19; 4, 9; 6, 133; 7, 8 β; 9, 93, 117). Curiously, in all but one of these cases (7, 8 β) the clause is in oratio obliqua, but in each case the subjunctive is retained. Secondly, as we saw above (p. 176), this is the only case where πρίν, meaning "until," takes the infinitive after a negative main clause.

Assuming that the infinitive should be read here, why was it used? No satisfactory explanation has yet been given. Sturm ‡ regards the passage as analogous to certain passages in Homer, and thinks further that Herodotus may have used here the exact words of the original oath. It is true that Homer frequently has *πρίν* as an adverb in the main clause, but he does not use *πρὶν ἤ* with the infinitive referring to future time. Furthermore, Sturm does not give any evidence to show that the Phocaeans about 545 B.C. would have been any more likely to use the infinitive in this sentence than the subjunctive. If then we accept the reading *ἀναφανῆναι* we must admit that the infinitive is wholly exceptional for Herodotus.

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* See below, p. 181.

† Helbing, Die Präpositionen bei Herodotos und andern Historikern, in Schanz's Beiträge, Heft 16, p. 44, takes ἕως here with ἐς. I may state that the above interpretation occurred to me before I saw the article of Helbing.

‡ Schanz's Beiträge, vol. 1, pp. 293 ff.

But should we not read here the subjunctive? Heilmann's suggestion *πρίν τις . . . ἀναφήνῃ* has little to recommend it. It has occurred to me that perhaps Herodotus wrote here *μύδρον σιδήρεον κατεπόντωσαν καὶ ὤμοσαν μὴ πρὶν ἐς Φώκαιαν ἥξειν πρὶν ἢ ἀναφανῇ. τὸν μύδρον τοῦτον* would then be a marginal or interlinear note which was inserted in the text. The change from *ἀνα*ΦΑΝΗΙ to *ἀνα*ΦΗΝΑΙ of the MSS. is hardly more difficult than that from *ἀναφανῆναι* to *ἀναφῆναι*; and, as Sturm has already remarked, the repetition after *μύδρον σιδήρεον* of *τὸν μύδρον τοῦτον* is rather strange.

As regards the principles which govern the use of the infinitive in oratio obliqua in the temporal clause, no general law can be laid down. It should, however, be noted that in every case but two (1, 165; 5, 84) the infinitive so used occurs in a passage of considerable length in oratio obliqua; and further that in every case but two (3, 26; 6, 137), one or more infinitives in oratio obliqua has preceded. But, on the other hand, under exactly the same conditions as those just described, a finite mood is frequently not changed to the infinitive.

THE USE OF *ἄν*.

As stated above, *ἄν* is, as a rule, used with the subjunctive in all temporal clauses. Certain exceptions however seem to occur, which demand separate treatment. For convenience I will first consider the use of *ἄν* in clauses of antecedence and contemporaneity. Herodotus's use of *ἄν* in these clauses is surprisingly uniform. In clauses of generic action with the subjunctive, *ἄν* is omitted but twice; in clauses which refer to the future, *ἄν* is omitted according to the MSS. but once, i. e., 8 22: *Θεμιστοκλέης δὲ ταῦτα ἔγραφε, . . ., ἵνα ἢ λαθόντα τὰ γράμματα βασιλέα Ἴωνας ποιήσῃ μεταβαλεῖν . . ., ἢ ἐπείτε ἀνενειχθῇ καὶ διαβληθῇ πρὸς Ξέρξην, κ.τ.λ.* The omission in this case is the more striking since Herodotus everywhere else in such a clause uses *ἄν*; and secondly, in the other two cases in Herodotus of *ἐπείτε* with the subjunctive (1, 200, and 202), *ἄν* is present in both. Conditional relative sentences are found in Homer without ἄν or κέ, but no case is cited by writers on this subject of such an omission with a temporal clause of antecedence which refers to future time.* There is no such case in Thucydides. As regards the use of the tragedians, in the case of relative conditions that have the subjunctive without ἄν, exclusive of clauses introduced by a word meaning "until," "the generic meaning preponderates over the future in the overwhelming proportion of 33 to 2."† Furthermore, out of 52 cases in the tragedians of the omission of ἄν with the subjunctive introduced by relative words, there is not one case with ἐπεί or ἐπειδή in a clause of future meaning, and only two cases in clauses of generic meaning (both with ἐπεί). Hartung ‡ cites besides Od., 15, 453, only Anacr., 29, 8 and 29, and the passage under discussion. Hartung refers apparently to Anacreontica, 16 (Bergk), v. 8: ἄφες, ὡς θέλωσι, κεῖσθαι, and v. 42: λαβὲ μισθὸν ὅσσον εἴπῃς. These passages are however much too late to have much weight in justifying the omission of ἄν in the passage from Herodotus. In the Greek lyric poets ἄν is omitted in clauses of antecedence but twice, Mimn., 1, 5, and 11, 4 (ed. of Hiller-Crusius, 1897); in both of these cases the action is generic. The case in Herodotus seems therefore to be the sole case in classical Greek of such an omission of ἄν, and as such is in the highest degree suspicious.

Krüger § in a note ad loc. wrote: ἐπείτε, ἐπεάν? but did not explain how ἐπεάν could have become ἐπείτε. I believe that Herodotus wrote here ἐπείτε ἂν ἀνενειχθῇ. If ἄν had been written it would necessarily have followed ἐπείτε (cf. 1, 200, and 202), and could have easily been lost before ἀνενειχθῇ by haplography. A striking parallel to such a loss of ἄν is found in Thuc., 7, 77, 5: μὴ ἄλλο τι ἡγησάμενος ἕκαστος ἢ ἐν ᾧ ἂν ἀναγκασθῇ χωρίῳ μάχεσθαι, κ.τ.λ., where ἄν, which is read by all recent editors, is found in only one MS., the Vaticanus, being omitted in A, C, E, F, G, and M.*

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* ἄν is omitted in Homer in clauses with ἐπεί but twice, and in each case the main verb expresses generic action. Cf. Schaub, De Usu Coniunctivi et Optativi in Enuntiatis Lyricorum Graecorum Secundariis, Liestal, 1889, p. 15.

† Clapp, in Trans. Am. Phil. Assoc., **22** (1891), p. 89.

‡ Cf. Hartung, Partikeln der gr. Sprache, Erlangae, 1833, vol. 2, p. 296, who says, speaking of the use of ἄν in relative clauses which express particular action: "Sie (i. e., ἄν and κέ) sind hier dermassen an ihrem Platze dass sie schon bei Homer nur höchst selten weggelassen werden; denn es ist mir keine Stelle ausser Od. ο, 453, gegenwärtig." But in this passage: ὅπῃ περάσητε, Monro regards περάσητε as post-Homeric both in form and syntax, and reads περάσαιτε. See Monro, Homer's Odyssey, Books 13–24, Oxford, 1901, crit. note on 15, 453.

§ Herodotus, 1856.

I am aware that the doctrine has been advanced that ἄν is sometimes omitted owing to the presence of the same sound in another word in the sentence.† But until this theory is more firmly established for prose than it is at present, it cannot have much weight in determining the usage of prose writers.‡ For the repetition of the same sound in Herodotus, cf. e. g., 3, 117: *διὰ διασφάγος ἀγόμενος ἑκάστης ἑκάστοισι.*

ὡς occurs with the subjunctive in Herodotus twice: 1, 132, and 4, 172; in both cases the clause expresses generic action and ἄν is omitted. The examples follow. 1, 132: *τῶν δὲ ὡς ἑκάστῳ θύειν θέλῃ, . . . καλέει τὸν θεόν, κ.τ.λ.*; 4, 172: *τῶν δὲ ὡς ἕκαστός οἱ μιχθῇ, διδοῖ δῶρον τὸ ἂν ἔχῃ φερόμενος ἐξ οἴκου.*

According to Herodotus's usage elsewhere and according to the general law of Greek grammar we should expect ἄν in both of these clauses with ὡς. Cf. with 4, 172 a passage in 1, 199, *ἐπεὰν δὲ μιχθῇ, κ.τ.λ.* Bähr § attempts to explain the passage in 4, 172, by saying that he would take ὡς closely with *ἕκαστος*, but he does not make it plain how he would translate ὡς. It is clear that ὡς is here a temporal conjunction meaning "after," Latin *simulac*, which is the opinion of Sauppe, Hartung, and Hermann.¶

As regards the usage of other Greek authors, as has already been pointed out,** ὡς is almost never used in classical Greek with the subjunctive or optative. The only other examples seem to be Sappho, 2, 7: *ὥς σε γὰρ Fίδω, βροχέως με φώνας οὖδεν ἔτ' εἴκει*, and Cebes, Tab., 4, 3, (ed. of Drosihn, 1871): *προστάττει δὲ τοῖς εἰσπορευομένοις, τί δεῖ αὐτοὺς ποιεῖν, ὡς ἂν εἰσέλθωσιν εἰς τὸν βίον.* The authorship of the Tablet of Cebes is not fully determined, but the work probably does not belong to the classical period.

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* For the loss of ἂν by haplography cf. also Ant., 1, 10: *ἀναγκάζοι τὰ γεγονότα κατηγορεῖν*, where Gildersleeve would, correctly, I think, insert ἄν before *ἀναγκάζοι*. Cf. Gildersleeve, Syntax, 450.

† Wright, Harv. Stud. in Class. Phil., **12** (1901), 145 ff.; Gildersleeve, Syntax, 450, Am. Jour. Phil., **12** (1891), p. 387.

‡ Hermann, Opusc., **4**, 103, attempts to justify the omission of ἄν in our passage, but his argument is entirely refuted by Bäumlein, Unters. über die gr. Modi, p. 237.

§ Herodoti Musae, ed. alt., J. C. F. Bähr, Lipsiae, 1857, note ad loc.

¶ Sauppe, Epist. Crit. ad Herm., p. 89; Hartung, Partikeln der gr. Sprache, vol. 2, 296; Hermann, Annot. in Vigerum, Lipsiae, 1834, p. 941.

** Cf. Klotz, Devarius, Lipsiae, 1840, vol. 2, p. 759; Gildersleeve, Am. Jour. Phil., **7** (1886), pp. 167 and 543.

In pure relative clauses ἄν is omitted with the subjunctive in all MSS. of Herodotus in four places: 1, 216; 2, 85; 4, 46, 66; but in 4, 66, ἄν is doubtless to be read, with Schweighäuser, and in 2, 85, ἄν can easily be supplied from the preceding clause, thus leaving two places where ἄν seems to have been wholly omitted. In view of these facts, although (if 8, 22, be excepted) these are the only cases (excepting the "until" clauses) where Herodotus omits ἄν with the subjunctive in a temporal clause, there does not seem to be sufficient evidence to warrant the insertion of ἄν in these passages. Possibly the omission in these cases is due in part to the relative origin of ὡς, and also to the fact that it is less sharp and distinct in its meaning than the other conjunctions of antecedence.

A very remarkable usage of ἄν, if the text be correct, is found in 1, 196: *ὡς ἂν αἱ παρθένοι γινοίατο* (Aldine ed. *γενοίατο*) *γάμων ὡραῖαι, ταύτας ὅκως συναγάγοιεν πάσας, ἐς ἓν χωρίον ἐσάγεσκον ἀλέας, κ.τ.λ.* *ὡς ἄν* is given in all MSS., and is read by Kallenberg, Holder, Stein, Abicht, Bähr; Krüger brackets ἄν; Stein proposed *ὅσαι αἰεὶ παρθένοι γινοίατο*, and this is adopted by van Herwerden. Bähr (note ad loc.) attempts to defend ὡς ἄν, but the passages which he cites are by no means parallel to our passage. Kühner (Gr. Gram.², II, 2, 567, 7) gives two passages where the optative with ἄν in temporal clauses has the same meaning as in independent sentences (Xen., Oec., 11, 14; Dem., 4, 31), but neither of them is analogous to the passage from Herodotus.* The solitary other example of such a usage is that cited by Thomas † and Goodwin,‡ Il., 9, 525:

> *οὕτω καὶ τῶν πρόσθεν ἐπευθόμεθα κλέα ἀνδρῶν*
> *ἡρώων, ὅτε κέν τιν' ἐπιζάφελος χόλος ἵκοι.*

But here there is a special reason for the addition of κέν. First there is a certain degree of contingency about the action of ἵκοι χόλος, i. e., whenever, *as it chanced*, any one became angry. Furthermore, Monro § shows that in conditional temporal clauses with ὅτε, expressing generic action, the pure optative is generally used, and that the use of *κέν* here may be accounted for by the change from the plural to the singular. Cf. further Od., 21, 293, and Monro, Hom. Gram., 283, c.

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* Richard Horton-Smith (Cond. Sent. in Greek and Latin, 1894, p. 438, cites our passage as a case where the optative with ἄν "represents after a historic or quasi-historic tense what would after a present tense have been a present subjunctive similarly placed," i. e., with ἄν. But surely it cannot be said that in our passage the optative with ἄν represents anything but the original idea as it first occurred to Herodotus

† Am. Jour. Phil., **7** (1886), p. 543.

‡ Moods and Tenses, 542.

§ Hom. Gram., 308, 1, d.

But in our passage there is no element of contingency whatever; all the maidens, whether handsome or homely, necessarily at some time became *γάμων ὡραῖαι*. Herodotus uses *ὡς* with the optative in general past conditions four times, and *ὅκως* 51 times, and nowhere (except in 1, 196) is *ἄν* used with the optative; and it cannot be shown that there is any more reason for an *ἄν* here than in any of the 54 other cases. There is, therefore, every reason to believe that the text here is corrupt. Stein's suggestion gives a very apt meaning, but does not account well for the MS. reading, *ὡς ἂν αἱ*.

The expression *ὅσος δή* is used by Herodotus in the sense of "a certain number or quantity," as in 1, 160: *οἱ δὲ Μυτιληναῖοι . . . ἐκδιδόναι τὸν Πακτύην παρεσκευάζοντο ἐπὶ μισθῷ ὅσῳ δή*, "they were preparing to give up Paktyas *for a certain price*." *ὅσος δή* means in Herodotus, as Schweighäuser says, "quantitatem aliquam per se definitam sed quam is qui loquitur exacte definire non potest aut non vult;" *ὅσος δή* is used in this sense also in 1, 157; 2, 103; 3, 52, 159; 4, 151. It seems quite possible that Herodotus wrote here: *ὡς ὅσαι δὴ παρθένοι γινοίατο γάμων ὡραῖαι*, "whenever a certain number of girls became of marriageable age," etc., which number he was doubtless unable to state exactly. This meaning is exactly appropriate here. For *ὅσος δή* placed before its noun, cf. 4, 151: *σιτία καταλιπόντες ὅσων δὴ μηνῶν*. The reading of the MSS. may then have arisen thus: first *δή* (ΔΗ) may have been changed to *ἄν* (AN); that this mistake is a common one in MSS. is well known.* Then *ὡς ὅσαι ἄν* was changed to *ὡς ἂν ὅσαι*, and later *ὅς-* was dropped.

I pass now to consider the use of *ἄν* in clauses of subsequence. The statistics of the use of *ἄν* with *πρίν* have been given by Sturm in Schanz's Beiträge, Bd. 1, pp. 296 ff. The subjunctive with *πρίν* occurs in clauses of generic action six times (1, 140, 197, 198; 3, 109; 4, 117, 196); in five cases all MSS. give *ἄν*; and in 3, 109, where the MSS. differ, *ἄν* is doubtless to be read. *πρίν* with the subjunctive referring to future time is found eight times; in three cases all MSS. give *ἄν* (1, 32, 159; 5, 106); in three cases all MSS. omit *ἄν* (4, 157; 6, 82; 7, 220), but 7,

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* Cf. Gildersleeve, Syntax, 450; Voemel, Dem. Contiones, p. 228, and crit. notes on De Cherson., 43, and Phil., 4, 12; Porson, Tracts and Miscellaneous Criticism, London, 1826, p. 182; and especially, H. Richards, Class. Rev., **6** (1892), pp. 340–342.

220, does not count, as the sentence is part of an oracle in hexameter verse; and in two places (1, 32, 82) the MS. authority is divided.

As to the usage of other authors, the three tragic poets omit ἄν with πρίν and the subjunctive ten times, three times in generic action, seven times in particular.* Thucydides omits ἄν with πρίν five times.† ἄν is omitted with πρίν in the lyric poets once (Simonid. Amorg., 1, 12). Excluding, then, the three places where the MSS. disagree, and 7, 220, ἄν is found in all MSS. eight times, and omitted in all MSS. twice. So far, then, this furnishes a supposition in favor of the use of ἄν. In 1, 32 (πρὶν ἄν, A^2Rbdz; πρίν, ABCP), and 1, 82 (πρὶν ἄν, A^2Rdz; πρίν, ABCP) I would read ἄν with Sturm. It is harder to account for the insertion of ἄν than its omission. Moreover, not infrequently in Herodotus the correct reading is found in only a small number of MSS.‡

It now remains to consider the passages where ἄν is omitted in all MSS., 4, 157: οὐ γὰρ δή σφεας ἀπίει ὁ θεὸς . . ., πρὶν δὴ ἀπίκωνται ἐς αὐτὴν Λιβύην; and 6, 82: πρὸς ὧν ταῦτα οὐ δικαιοῦν πειρᾶν τῆς πόλιος, πρίν γε δὴ ἱροῖσι χρήσηται, κ.τ.λ. Sturm would read πρὶν ἄν in both of these places. His statement, however, "die Partikeln δή und γὲ δή . . . finden sich in Verbindung mit diesem Modus bei keinem Schriftsteller vor," is not accurate. δή is found with the subjunctive in Il., 5, 684 (μὴ δὴ με . . . ἐάσῃς) and Il., 7, 24 (ἵνα δὴ . . . δῷς), and in Herodotus 1, 29 (ἵνα δὴ μὴ . . . ἀναγκασθῇ), and in 1, 32 (ἵνα δὴ αἱ ὧραι συμβαίνωσι). For the repetition of δή within a short space, which Sturm calls "störend," cf. Her., 1, 43: ἔνθα δὴ ὁ ξεῖνος, οὗτος δὴ ὁ καθαρθεὶς τὸν φόνον, κ.τ.λ., and 6, 83: ἐκ τούτου δὴ πόλεμός σφι ἦν ἐπὶ χρόνον συχνόν, ἐς ὃ δὴ μόγις. κ.τ.λ. It is worth noting that both in 4, 157, and 6, 82, the πρίν clause follows a past tense, and the subjunctive might, on the principle of oratio obliqua, have been changed to the optative. The meaning of δή in 4, 157, seems to me to be entirely appropriate: "for in fact the god would not let them off till they actually had come to Libya itself." Although the change of δή to ἄν may be right and is tempting, the evidence is not sufficient to warrant us in making the alteration.

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* Of these seven cases, five are in Euripides. Cf. Clapp, Trans. Am. Phil. Ass., **22** (1891), p. 91.

† The use of ἄν in Thucydides is discussed by Warren, Temp. Clauses in Thuc., p. 19, who concludes that the omission of ἄν with πρίν in Thucydides must for the present be admitted.

‡ It is a little difficult to understand upon what principle Holder, in 3, 109, writes "ἄν β, om. α," and reads ἄν, while in 1, 32, he writes "πρὶν ἄν β, πρίν α," but omits ἄν.

Sturm's change of *πρίν γε δή* to *πρὶν ἄν* in 6, 82, has much less to recommend it. His note (op. cit., p. 298) shows that he is aware that the reading *πρὶν ἄν* does not explain the origin of the *γέ* at all. The conjecture of Lüth on Ant., 1, 29, by which Sturm tries to support his proposal, is not approved by Blass, and not mentioned by Jernstedt. The meaning of *γὲ δή* here is not less appropriate than with the indicative in 6, 79: *οὐκ ὥρων . . . ὅ τι ἔπρησσον, πρίν γε δὴ αὐτῶν τις ἀναβὰς ἐπὶ δένδρος κατεῖδε τὸ ποιεύμενον.* Cf. also *γὲ δή* with the optative, 2, 11: *κοῦ γε δὴ ἐν τῷ προαναισιμωμένῳ χρόνῳ πρότερον ἢ ἐμὲ γενέσθαι, οὐκ ἂν χωσθείη κόλπος, κ.τ.λ.* The MS. reading in the passage under discussion should be retained.

ἐς ὅ is used with the subjunctive in Herodotus nine times, — three times of future action (2, 115; 4, 43; 8, 108), six times of generic action (1, 93; 3, 31, 82; 4, 30, 196; 6, 86 *γ*). *ἄν* is everywhere used with the subjunctive except in 3, 31: *οἱ δὲ βασιλήιοι δικασταὶ κεκριμένοι ἄνδρες γίνονται Περσέων, ἐς ὁ ἀποθάνωσι, κ.τ.λ.*; and 8, 108: [*Εὐριβιάδης δὲ τὴν ἐναντίην γνώμην ἐτίθετο*] *ἐατέον ὢν εἶναι φεύγειν ἐς ὁ ἔλθῃ φεύγων ἐς τὴν ἑωϋτοῦ.*

Let us first consider the passage in 8, 108. The MSS. read here as follows: *ἔλθῃ*. PRs, and all editors except van Herwerden; *ἔλθοι*, ABC, and van Herwerden.* The omission of *ἄν* with *ἐς ὅ* is a different matter from its omission with other clauses meaning "until," and for this reason: outside of Herodotus, *ἐς ὅ* occurs as a temporal conjunction in classical Greek only in Homer (50 times), Hesiod (1), and Hom. Hymns (1), and in each of these cases it is accompanied by *κέ*.† The use of *ἐς ὅ* in Herodotus seems to be a literary convention. If now the usage whereby *κέ* is required has become so firmly settled in the time of the composition of the Iliad and the Odyssey that it was never omitted, it seems strange, to say the least, if Herodotus was able to omit it at will. It can of course be said that *ἄν* may have been omitted with *ἐς ὅ* on the analogy of *ἄχρι οὗ* (1, 117), *μέχρι* (4, 119), and possibly *πρίν* (see above); but inasmuch as *ἄν* is used with *ἐς ὅ* much oftener than it is omitted with the conjunctions just mentioned, this does not seem likely.

As regards the reading *ἔλθοι*, it gives a wholly appropriate meaning, and is grammatically above reproach, representing *ἐς ὁ ἂν ἔλθῃ* of the oratio recta. To be sure, as we have seen above,‡ Herodotus does not often change the subjunctive to the optative in oratio obliqua, but he does do this in 5, 63 (*ὅκως ἔλθοιεν*), and there is no cogent reason why he should not have done the same in our passage. How now shall we account for the ἔλθῃ of MSS. PRs? It may, of course, have arisen by a mistake of a scribe, but since this is the only place in classical Greek where ἐς ὃ occurs with the optative, it seems to me more probable that some scribe or corrector arbitrarily changed ἔλθοι to ἔλθῃ. Van Herwerden and Stein are, I think, unquestionably right in reading ἔλθοι.

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* Stein in his school edition (5th ed., 1893) reads *ἔλθοι*.

† Cf. Fuchs, op. cit., pp. 129–30.

‡ See p. 175.

In 3, 31, for the reasons given above, it does not seem probable that ἄν was omitted. That the passage in which the sentence occurs is somewhat corrupted in some MSS. is shown by the ἥν σφι of ABP and by the variation in the MSS. between ἐς ὃ and ἐς οὗ. It would not have been difficult for ἄν to be lost when the sentence was written thus: ΕϹ ΟΑΝΑΠΟΘΑΝΩϹΙ. I am inclined to think that ἄν should be inserted.

ἄν is omitted with ἄχρι οὗ (1, 117) and μέχρι (4, 119). The omission of ἄν with πρὶν ἤ (eight times) and πρότερον ἤ has been discussed by Sturm, op. cit., p. 296.

II. Tense System.

A. Clauses of Antecedence.

Under the head of tense system I will consider first clauses of antecedence, both those of purely antecedent and those of overlapping action. In examining these sentences it should be noted that the writer could often represent an action either as partly antecedent and partly contemporaneous, or as wholly contemporaneous; in other words the mode of conception was often within the writer's choice, as in 1, 183: καὶ καταγίζουσι λιβανωτοῦ χίλια τάλαντα οἱ Χαλδαῖαι τότε ἐπεὰν τὴν ὁρτὴν ἄγωσι τῷ θεῷ τούτῳ. Here it was possible for Herodotus to represent the action of ἄγωσι as wholly contemporaneous, using ὅταν, or as antecedent overlapping. Cf. also 2, 149 (ἐπεὰν μὲν ἐκρέῃ), where he might equally well have written ἕως ἂν ἐκρέῃ. Secondly, it should be observed that the English word "when," which is often used in translating the Greek temporal conjunctions of antecedence, is far from being an exact word, since it really means both "at," "during," and "after the time when"; the accurate English equivalent for the Greek conjunctions is "after that" (more commonly "after").

In clauses of antecedence the main verb, as regards its tense, is under no limitation arising from the temporal relation and does not require treatment. In describing an action which is wholly antecedent we find, in accordance with the law stated above (p. 172), a verb of aoristic action almost always used in the temporal clause. How neatly and exactly the present and the aorist tense stems are sometimes differentiated is seen in such examples as 2, 173: τὰ τόξα οἱ ἐκτημένοι, ἐπεὰν μὲν δέωνται χρᾶσθαι, ἐντανύουσι. ἐπεὰν δὲ χρήσωνται. ἐκλύουσι, where the action of δέωνται is not yet terminated at the time of the action of ἐντανύουσι, but the action of χρήσωνται has ceased before that of ἐκλύουσι begins; and 1. 9: ἐπεὰν δὲ . . . στείχῃ ἐπὶ τὴν εὐνὴν, κατὰ νώτου τε αὐτῆς γένῃ. κ.τ.λ., where the action of στείχῃ has not ceased, but that of γένῃ is finished, at the time of the beginning of the action of the main verb.

The number of cases where a verb of extensive action is used in the temporal clause when the idea of overlapping action is wholly excluded, is, relative to the whole number of clauses with a verb of extensive action, very small. In most of these cases a reason for the departure from the normal usage can be given; occasionally there is ground for believing that the current reading of the text is wrong. I will consider these cases briefly under the several conjunctions, reserving the passages where forms of γίνομαι occur for separate treatment. In 7, 188: ὁ δὲ δὴ ναυτικὸς στρατὸς ἐπείτε ὁρμηθεὶς ἔπλεε καὶ κατέσχε . . ., αἱ μὲν δὴ πρῶται τῶν νεῶν ὅρμεον πρὸς γῇ. κ.τ.λ., the action of ἔπλεε had of course ceased when that of ὅρμεον began; but note that the aspect of the action is recalled from extensive to aoristic by κατέσχε. and further that πλέω has a special fondness for the imperfect.* In 2, 45: ἐπεὶ δὲ αὐτοῦ πρὸς τῷ βωμῷ κατάρχοντο. ἐς ἀλκὴν τραπόμενον πάντας σφέας καταφονεῦσαι, it is clear that the action of κατάρχοντο cannot overlap that of καταφονεῦσαι; it may, however, partly coincide with the action of τραπόμενον; besides, Herodotus desires to represent the slaying as a sudden and unpleasant interruption of the proposed sacrifice, and the imperfect was therefore needed in the main clause. We naturally suppose in 2, 29: στρατεύονται δέ, ἐπεάν σφεας ὁ θεὸς οὗτος κελεύῃ, κ.τ.λ., that the action of κελεύῃ was finished before that of στρατεύονται began. But note here the καὶ τῇ ἂν κελεύῃ immediately following, and also the fact that κελεύω in Polybius, Thucydides, and Demosthenes prefers the imperfect rather than the aorist.†

An interesting example is found in 2, 120: εἰ δέ τοι καὶ ἐν τοῖσι πρώτοισι χρόνοισι ταῦτα ἐγίνωσκον, ἐπεὶ πολλοὶ μὲν τῶν ἄλλων Τρώων . . . ἀπώλλυντο. . . . τούτων δὲ τοιούτων συμβαινόντων, ἐγὼ μὲν ἔλπομαι . . . ἀποδοῦναι ἂν αὐτὴν τοῖσι Ἀχαιοῖσι, κ.τ.λ. Since ἀποδοῦναι ἂν = "[Priam] would have given

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* Cf. Hultsch, Die Erzählenden Zeitformen bei Polybius, in the Abh. d. k. sächs. Gesellsch. d. Wissen., **13** (1893), pp 89 ff.; Miller, Am. Jour. Phil., **16** (1895), p. 161, n. 1.

† Cf. Hultsch, op. cit., pp. 103 ff; Miller, Am. Jour. Phil., **16** (1895), p. 163; Blass, Rhein. Mus., **44** (1889), pp. 410 ff.

Helen back *and thus stopped the slaughter*," the action of ἐπεὶ ἀπώλλυντο is wholly antecedent. But here there is an unmistakable demand for continued action in the temporal clause; the imperfect here gives a strong causal coloring with indication of the spectator (i. e., when Priam *saw that* his sons were perishing), and there is at the same time the idea of the interruption of a state of affairs previously existing.*

To pass now to clauses with ὡς, we read in 1, 47: ἐν δὲ Δελφοῖσι ὡς ἐσῆλθον τάχιστα ἐς τὸ μέγαρον . . . καὶ ἐπειρώτων, ἡ Πυθίη . . . λέγει τάδε. Surely we must suppose that the Lydians had ceased to speak, when the priestess began to give the answer of the oracle. As to the reason for this imperfect, no aorist of the simple verb εἰρωτάω (Att. ἐρωτάω) seems to occur in Ionic Greek.† This is a case where we should expect an aorist, if an aorist of the desired meaning were in common use. The fact that εἴροντο is not used here makes somewhat against the view of Kallenberg, who believes that εἰρόμενος and εἰμέσθαι (for so he would write it) are not presents, as they have usually been regarded, but aorists.‡

A noteworthy case is found in 7, 146: ὡς δὲ περιεόντας αὐτοὺς κατέλαβον καὶ ἦγον ἐς ὄψιν τὴν βασιλέος, τὸ ἐνθεῦτεν πυθόμενος . . . ἐκέλευσε, κ τ.λ. It is clear that the action of ἦγον must have been completed before that of either πυθόμενος or ἐκέλευσε began. The readings of the MSS. and the editors here is as follows: ἦγον, ABCP, edd. Kall., Stein, Krüg.; ἤγαγον, RS, edd. v. Herw., Hold. The aorist is surely needed here; cf. Miller,§ on Polybius: "The aorist . . . expresses the successful conduct to a goal, or the successful transference from one state to another." ἤγαγον is certainly to be preferred. In the passage 2, 121 β: ὡς δὲ αὐτῷ καὶ δὶς καὶ τρὶς ἀνοίξαντι αἰεὶ ἐλάσσω φαίνεσθαι τὰ χρήματα, ποιῆσαί μιν τάδε, the action of the temporal clause is shown by the context to be wholly antecedent. The present tense (= imperfect) is due to the idea of repetition in the protasis, to the presence of αἰεί, and also to the distinct causal coloring: "after he saw that his money all the time was growing less." 1, 65: ὡς ἐσήιε ἐς τὸ μέγαρον, ἰθὺς ἡ Πυθίη λέγει τάδε. Here again we must suppose that the action of ἐσήιε was finished before that of λέγει began. Cf. 1, 47, ὡς ἐσῆλθον (above), in an exactly similar sentence. Does the imperfect ἐσήιε express here extensive or aoristic action? According to

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* Cf. Gildersleeve, Synt., 223 and 208.

† Cf. Veitch, Greek Verbs Irregular and Defective^4, Oxford, 1878, s. v.

‡ Jahresb. d. Philolog. Vereins zu Berlin, **28** (1902), p 96. Cf. Smyth, Ionic Dial., 224, 3; Hoffmann, Der ion. Dial., 167, 12.

§ Am. Jour. Phil., **16** (1895), p. 166.

L. Meyer[*] and Gildersleeve[†]. the preterite ᾖα and its forms are always aoristic in Homer, but Delbrück shows conclusively that they are sometimes extensive as well. He shows further that the present stem of εἶμι can express both extensive and aoristic action.[‡] As regards the meaning of these preterite forms in Herodotus, in this passage at least it is probable that ἐσῄιε is used aoristically.[§] In 9, 119: *οἱ δὲ ἀμφὶ τὸν Ἀρταΰκτην ὕστεροι ὁρμηθέντες φεύγειν, ὡς κατελαμβάνοντο ὀλίγον ἐόντες ὑπὲρ Αἰγὸς Ποταμῶν, ἀλεξόμενοι χρόνον ἐπὶ συχνὸν οἱ μὲν ἀπέθανον, οἱ δὲ ζῶντες ἐλάμφθησαν*, it is certain that the action of *κατελαμβάνοντο* was antecedent to that of *ἀπέθανον* and *ἐλάμφθησαν*. Here the action of *ἀλεξόμενοι* was prominent in the author's mind; and since several persons were engaged in the flight, Herodotus represents the action of *κατελαμβάνοντο* not as taking place at once but as continuing; "Artayktes and his men, who had set out to flee later than the others, after they were being (had begun to be) overtaken." etc. The participle *καταλαμβανόμενοι* is used in exactly the same sense, 7, 211: *αἱ δ' ἂν καταλαμβανόμενοι ὑπέστρεφον, κ.τ.λ.*

Let us next examine 8, 70: *ἐπεὶ δὲ παρήγγελλον ἀναπλέειν, ἀνῆγον τὰς νέας ἐπὶ τὴν Σαλαμῖνα, κ.τ.λ.* *παρήγγελλον*, A²BRP; *παρήγγελον*, A¹, v; *παρήγγειλον*, C. The temporal relation here demands the aorist, and inasmuch as a 2nd aorist ἤγγελον is doubtless to be rejected for Herodotus,* we must choose between παρήγγελλον and παρήγγειλαν. Herodotus uses elsewhere both the imperfect (9, 53) and the aorist (3, 147; 4, 89, partic.) of παραγγέλλω. In pseud.-Dem., 50, 19, we have what we should expect in our passage: ἐπειδὴ ὅ τε Εὐκτήμων ἧκεν καὶ ὁ στρατηγὸς παρήγγειλεν ἀνάγεσθαι, κ.τ.λ. It should however be noted that here the subject is singular, while in our passage it is plural. In view of the passage in pseud.-Demosthenes, and the παρήγγειλον of C, the change to παρήγγειλαν is tempting†; but since all the MSS. readings point to an ending in -ον, I would not urge the change. The imperfect, if it be read, is probably to be explained as descriptive or panoramic.

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[*] Griechische Aoriste, Berlin, 1879, p. 98.

[†] Am. Jour. Phil., **4** (1883), p. 161, fn.

[‡] Grundriss der vergl. Gram. der indog. Sprachen, 4, 69 ff.; and cf. Brugmann, Gr. Gram.³, 536.

[§] Cf. Brugmann, op. cit., 544, 1: "Das Imperfekt versetzte die Aktion des Praesensstamms in die Vergangenheit. War diese Aktion punktuell, so besagte dieses Augmentpräteritum dasselbe wie der Aorist." This statement, if correct, is somewhat startling. For if this be true, in the case of verbs whose present stem expresses aoristic ("punktuell") action, there is no appreciable distinction between the aorist and imperfect, and hence no ground for discriminating between them in the narration of past actions. Brugmann's words seem to imply that a verb of aoristic meaning is just as likely to appear in the imperfect as in the aorist. But he himself virtually says elsewhere (op. cit, 533, 541) that a purely aoristic stem does not as a rule appear in the imperfect, nor a purely non-aoristic stem ("nicht-punktuell") in the aorist. Certainly we do not expect to see the stem of *ὁράω* in the aorist, nor that of *εἶδον* in the imperfect. *ἔφην* and *ἵτην*, *ἵσαν* the examples by which Brugmann supports this theory, are not entirely normal in their use; cf. Gildersleeve, Synt., 212; L. Meyer, Gr. Aor., pp. 95 ff. As regards the further example *νεόμην*, in Od., 4, 585, I regard it as by no means certain that the verb is aoristic: *ταῦτα τελευτήσας νεόμην, δίδοσαν δέ μοι οὖρον ἀθάνατοι, κ.τ.λ.* Why not "I proceeded on my way"? Observe too the imperfect *δίδοσαν*. These examples then are certainly insufficient to establish such a general law as Brugmann here lays down.

The conjunction ὅκως is found with the present optative where overlapping action is excluded in 1, 11 (καλέοι) and 1, 171 (δέοιτο). In 6, 77: ὅκως ὁ Σπαρτιάτης κῆρυξ προσημαίνοι τι Λακεδαμονίοισι, ἐποίευν καὶ οἱ Ἀργεῖοι τὠυτὸ τοῦτο, προσημαίνοι is the reading of all MSS. except R, which gives προσημαίνει. It is beyond question here that the action of προσημαίνοι was wholly antecedent. Furthermore a few words after this passage the same verb occurs twice in the same temporal relation, and in both cases the aorist stem is used: ὁκοῖόν τι . . . σημήνειε and ὅταν σημήνῃ. As to the possibility of corruption from προσημήνειε to προσημαίνει, note that for σημήνειε in 6, 78, MS. *d* has σημήνει, and for σημήνῃ the same MS. has σημαίνῃ. Cf. also 7, 8δ: ἐπεὰν ὑμῖν σημήνω, κ.τ.λ.; 8, 18, 80; 9, 1. I would read here σημήνειε.

In 2, 108: ὅκως τε ἀπίοι ὁ ποταμός, σπανίζοντες ὑδάτων, κ.τ.λ., and 2, 125: ὅκως δὲ ἀνίοι ὁ λίθος ἐπ' αὐτόν, κ.τ.λ., where the subordinate action is wholly antecedent, observe first, that this verb had no aorist in use, and secondly, that the root ει- expresses both aoristic and extensive action.‡ 2, 140: ὅκως γὰρ οἱ φοιτᾶν σῖτον ἄγοντας . . ., ἐς τὴν δωρεὴν κελεύειν σφέας καὶ σποδὸν κομίζειν. Here φοιτᾶν is for φοιτῷεν of the oratio recta. The present stem is used because this verb, as we should expect from its meaning, was not used in the aorist.§

In regard to verbs of perception, both sensuous and intellectual, the usage of Herodotus is surprisingly regular, and this is the more noteworthy, because in verbs of this kind the mode of conception was, to a larger extent than in the case of other verbs, within the writer's choice. Out of 23 examples of αἰσθάνομαι, μανθάνω, πυνθάνομαι, in clauses with

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* Cf. Veitch, Greek Verbs^2, p. 5; Kühner-Blass, Gr. Gram., Teil 1, Band 2, p. 344.

† Polybius uses 37 aorists to 9 imperfects of παραγγέλλω. See Am. Jour. Phil., **16** (1895), p. 164.

‡ Cf. above, p. 191.

§ Cf. Veitch, Gr. Verbs^2, s. v.

'πεί (ἐπεάν), ἐπειδή and ἐπείτε (1, 27, 54, 62, 70, 83; 3, 69, 117; 4, 3, 43, 112, 165; 5, 34, 115; 6, 17; 7, 28, 163, 172, 220; 8, 108, 144; 9, 7β, 12, 76), the aorist is everywhere used in accordance with the principles above stated, except in one passage. 1, 70: αὐτοὶ δὲ Σάμιοι λέγουσι ὡς ἐπείτε ὑστέρησαν οἱ ἄγοντες τῶν Λακεδαιμονίων τὸν κρατῆρα, 'πυνθάνοντο δὲ Σάρδις τε καὶ Κροῖσον ἡλωκέναι, ἀπέδοντο τὸν κράτῆρα, κ.τ.λ. Here, although we might have expected the aorist, the imperfect was used because the action of ἐπυνθάνοντο was regarded as not entirely finished when they sold the bowl; not, therefore, "when they had learned," but rather, "after the report began to reach them," or, "was coming to their ears." Cf. 1, 35: ἐπυνθάνετο ὁκόθεν τε καὶ τίς εἴη, κ.τ.λ. αἰσθάνομαι does not occur with ὡς at all. μανθάνω is used with ὡς temporal 10 times (1, 80, 160; 2, 107; 3, 78; 7, 189; 8, 97, 109, 128; 9, 19, 47), and πυνθάνομαι 30 times (1, 122, 127, 128; 2, 118, 181; 3, 22, 64; 4, 113, 120; 5, 15, 79, 92 γ, 104, 105, 122; 6, 23, 85, 103; 7, 35, 119, 146, 147, 173, 177, 192, 225; 8, 21; 9, 5, 58, 98). The imperfect of μανθάνω does not occur in these examples, the imperfect of πυνθάνομαι but twice. In 2, 118: ἐλοῦσι δὲ τὸ τεῖχος ὡς οὐκ ἐφαίνετο ἡ Ἑλένη, ἀλλὰ τὸν αὐτὸν λόγον τῷ προτέρῳ ἐπυνθάνοντο, κ.τ.λ., the causal meaning is strong, probably stronger than the temporal; and in 5, 79: ὡς ἐπυνθάνοντο δὲ λεγόντων αὐτῶν τῶν ἄγχιστα δέεσθαι, εἶπαν οἱ Θηβαῖοι ἀκούσαντες τούτων, as is shown by ἀκούσαντες τούτων, which is a reiteration, with somewhat more exactness, of the temporal clause, Herodotus here was writing loosely, without regard for the strict temporal relation.

ἀκούω is used with ὡς 21 times, with ὅκως once (1, 66, 67, 90, 119, 141, 158, 160; 3, 25, 128, 156; 4, 136; 5, 32, 89; 6, 80; 7, 14, 102, 169; 8, 137; 9, 8, 111, 115; ὅκως, 4, 129). In 21 cases the aorist is found in all MSS. 3, 25: οἷα δὲ ἐμμανής τε ἐὼν καὶ οὐ φρενήρης, ὡς ἤκουε τῶν Ἰχθυοφάγων, ἐστρατεύετο, κ.τ.λ.; ἤκουε, ABCP; ἤκουσε, R s; ἤκουσε is edited by van Herwerden and Holder. ἤκουσε is almost certainly the right reading. The clear supposition is that the action of ἤκουε (ἤκουσε) was purely antecedent and, secondly, Herodotus nowhere else uses the imperfect of this verb with ὡς.

ὁράω is used with ὡς in a temporal clause 30 times, with ὅκως four times. The imperfect occurs only four times (1, 87; 4, 134; 5, 91; 7, 45), and in each case the overlapping character of the action is clear.

As regards clauses of antecedence in which the action of the verb is negatived, a rather surprising state of affairs confronts us. The action of the verb in a clause of antecedence is negatived 46 times: ἐπεί — 4, 33; 5, 67; 7, 211; 9, 13; ἐπείτε — 1, 94, 165; 2, 150, 181, 181_2;

3, 36, 78, 131; 4, 151; 5, 36, 44, 62; 6, 91; 8, 121, 125; 9, 118; ὡς — 1, 10, 67, 77, 84, 112, 163, 205; 2, 2, 118, 121 δ; 3, 56; 4, 124, 126; 5, 92 δ, 104; 6, 87, 90, 107; 7, 107, 210, 212; 8, 4; 9, 20, 38, 49, 108. In every one of these cases the verb is in the imperfect. Is this merely accidental?

Clearly, the function of a clause of antecedence is, by naming some event or describing some condition, to give either a point or a period of time after which, either immediately or with some interval, another action took place. This being the case, it is necessary that the time-defining action should be of a nature sufficiently definite and recognizable to furnish such a point of departure for reckoning. Let us now apply this to negative clauses.

An action may be negatived in either of two ways: first, simply and unemphatically, and secondly, with more stress on the negation, as in the case of the failure of an attempt, a wish, or a hope. Thus a man who was not present at a certain public address may say, "I did not hear that address;" whereas a man who was present but was unable to hear the speaker says, "I did not hear that address," but with a different connotation, i. e., "I *could not* hear it." It will be evident that a negation of the former type is not sufficiently strong, it does not, so to speak, partake sufficiently of the nature of an event, to give a point of time from which to reckon. But, as has been recognized for some time,[*] we have in Greek two different forms of expression corresponding to these two kinds of negation, i. e., the aorist with the negative for the simple negation, and the imperfect for the other. According to this principle we shall expect the imperfect tense to appear in negative clauses of antecedence. The resistance to the action attempted, desired, or hoped for, causes, so to speak, a ripple on the surface of time, while the simple absence of an event, or its failure without resistance, leaves that surface unruffled.

The facts of Herodotus's usage seem amply to support this principle. As we have seen, the imperfect is used in every case. In 36 cases the idea of disappointment or failure is unmistakable. A clear sign of this meaning is given by the frequent use of verbs which refer specifically, when negatived, to failing in something, as e. g., *οἷος τε γίνεσθαι* (1, 67; 2, 181; 6, 91; 8, 121); *δύνασθαι* (1, 10; 2, 2; 3, 131; 7, 211; 9, 108); *ἔχειν* (3, 36); *προχωρέειν* (1, 84, 205; 5, 44, 62); *προκόπτεσθαι* (3, 56). Cf.

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[*] This principle was first distinctly stated, so far as I have seen, by Gildersleeve, Am. Jour. Phil., **2** (1881), pp. 466 ff.

9, 13: ἐπεὶ δὲ οὐκ ἔπειθε, κ.τ.λ., "when he could not (saw that he could not) persuade them," etc., and the same expression, 1, 112, 163; 2, 121 δ; 5, 36, 104; 8, 4. The imperfect thus used is often best rendered in English by a periphrasis, as 8, 125: ἐπείτε οὐκ ἐπαύετο λέγων, κ.τ.λ., "When he would not cease," or, as Macaulay well renders it, "When he ceaselessly continued to repeat this."* The use of the imperfect in negative clauses of antecedence furnishes, it seems to me, a striking confirmation of the doctrine mentioned above, regarding the imperfect with a negative.

On Certain Forms of γίνομαι.

The occurrence in the MSS. and editions of Herodotus of certain forms of the verb γίνομαι shows such aberrations from Herodotus's usage elsewhere that it will be convenient to treat these forms separately. I will first consider these forms in clauses with ἐπεί, ἐπεάν, ἐπειδή and ἐπείτε. Some form of the imperfect or the aorist (ἐγινόμην, ἐγενόμην) indicative, or of the present or aorist subjunctive (γίνωμαι, γένωμαι), occurs in these clauses 60 times: 1, 9, 45, 58, 70, 84, 105, 105_2, 182, 182_2, 189, 190, 216; 2, 60, 78, 89, 92, 93, 97, 103, 107, 181, 181_2; 3, 15, 25, 26, 45, 69, 78, 80, 117; 4, 9, 23, 80, 173, 180, 187; 5, 24, 33, 50; 6, 19, 43, 58, 91, 118 [122]; 7, 23, 44, 44_2, 77, 207; 8, 37, 76, 76_2, 121, 137; 9, 4, 69, 96, 98, 117 (ἐπι-); one of these examples is in a passage which is doubtless spurious (6, 122). Out of these 60 clauses, in 51 cases where the MS. authority is either entirely or nearly unanimous, the usage of the aorist or of the present stem conforms wholly to the principles above laid down for antecedent and overlapping action. In examining the cases which show divergence from normal usage, the following facts should be kept in mind. First, I assume as a fundamental proposition that the imperfect and the aorist of γίνομαι cannot under any circumstances have *exactly the same* meaning and force†; it is admitted at once that there may be places where it is possible to use either tense, but whichever tense is chosen carries a shade of meaning at least slightly different from that of the other. Many examples could be given to show that this distinction is preserved in Herodotus; I quote but one, in which the distinction is clear, 8, 64: ἡμέρη τε ἐγίνετο καὶ ἅμα τῷ ἡλίῳ ἀνιόντι σεισμὸς ἐγένετο ἐν τε τῇ γῇ καὶ τῇ θαλάσσῃ. Secondly, since the forms ἐγενόμην and ἐγινόμην differ from each other only in one letter, confusion between them, and especially the change from aorist to imperfect, was easy, especially in capitals (ϵ to ɪ); indeed the much more difficult interchange between ἐγιγνόμην and ἐγενόμην was not unheard of in Greek MSS.[*] That these words were actually confused in Herodotus is proved by the fact that out of the 60 cases cited above, the MSS. disagree in 18 cases. Thirdly, in view of the recognized shortcomings of the MSS. of Herodotus the method of determining the text in the passages under discussion by a mere numerical count does not commend itself.[†]

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* G. C. Macaulay, The History of Herodotus, trans. into English, 1890. I should like here to call attention to the great accuracy with which this translator has rendered the tenses of the original, an accuracy as difficult to attain as it is refreshing in a work of this kind.

† Cf. Kühner, Gr. Gram., II, 1, 383, 3: "Eine Vertäuschung der Zeitformen anzunehmen ist durchaus unstatthaft." Gildersleeve, Syntax, 212: "An actual interchange of tenses is not to be admitted except in the case of a few old preterites such as ἦν and ἔφην."

There is a variation among the principal MSS., ABCPR s v[‡] in these places: 1, 105; 2, 93, 103, 107, 181; 3, 45, 78, 80; 4, 9, 80; 5, 33; 6, 58 [122]; 7, 23, 207; 8, 76, 121; 9, 117. I will first consider briefly the cases where the MSS. disagree, and where I would depart from Kallenberg's reading. 1, 105: οἱ δὲ ἐπείτε ἀναχωρέοντες ὀπίσω ἐγίνοντο τῆς Συρίης ἐν Ἀσκάλωνι πόλι. τῶν πλεόνων Σκυθέων παρεξελθόντων ἀσινέων, ὀλίγοι τινὲς αὐτῶν ὑπολειφθέντες ἐσύλησαν τῆς οὐρανίης Ἀφροδίτης τὸ ἱρόν. ἐγίνοντο, AB, Kal.,[§] H.[‖]; ἐγένοντο. CPR, St.,[¶] Kr.,[**] Ab.,[††] vH.[‡‡] This passage raises the question of the use in Herodotus of γίνομαι with a preposition (κατά, ἐν, ἐπί, πρός), meaning to "come to (or opposite) a place," "to arrive at a place." Such a combination occurs in the clauses now under discussion 19 times; 10 times all MSS. give the aorist stem (1, 9, 105; 2, 60; 3, 25; 6, 19, 43, 118; 7, 77; 9, 4, 96); six times MSS. differ (1, 105_2; 2, 103, 107; 3, 45, 80; 5, 33); and three times all MSS. give the imperfect (1, 70, 189; 4, 173). In 3, 45, 80 and 5, 33, the MSS. evidence is overwhelmingly in favor of the aorist; in the other three cases where the MSS. disagree and in the three cases where all MSS. give the imperfect, the aorist is to be expected, as I shall attempt to show below.

As to 1, 105, the meaning expected here is simply, "after they came to (reached) Askalon." ἐγίνοντο can mean here nothing but "after that they *were arriving* at Askalon." Now παρεξελθόντων shows that most of the soldiers had actually passed by the city before the time of ἐσύλησαν; hence it is a fair inference that the ὀλίγοι τινές *had already arrived* at the city before they sacked the temple. Indirect evidence upon the question of the tense in such a case as this can justly be drawn from the usage of Herodotus in synonymous words or expressions. Now γίνεσθαι ἐν in such a passage as this is practically equivalent to ἀπικνέεσθαι. ἀπικνέεσθαι occurs, with and without a preposition, in clauses of antecedence 67 times, and *in every case* the tense used is the aorist. Palæographically too, the change from ε to ι by the loss of the transverse stroke is easier than the opposite change. ἐγένοντο is surely to be preferred in this passage.

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[*] Cf. Thuc., 1, 49, 7: ἐγίγνετο C, ἐγένετο ABEFM; 1, 119: γίγνεσθαι CG; γενέσθαι ABEFM.

[†] Cf. Smyth, Ionic Dialect, pref., p. x, §§ 88 and 91.

[‡] I follow the nomenclature of Stein in designating the MSS.

[§] Kallenberg.

[‖] Holder.

[¶] Stein.

[**] Krüger.

[††] Abicht.

[‡‡] van Herwerden.

2, 107: ἐπείτε ἐγίνετο ἀνακομιζόμενος ἐν Δάφνησι . . ., τὸν ἀδελφεὸν ἑωυτοῦ, . . . τοῦτον ἐπὶ ξείνια αὐτὸν καλέσαντα καὶ πρὸς αὐτῷ τοὺς παῖδας περινῆσαι ἔξωθεν τὴν οἰκίην ὕλῃ, περινήσαντα δὲ ὑποπρῆσαι.

ἐγίνετο, ABCP, Kal., H., St.; ἐγένετο, Rz, Krüg. vH. Here, again, we have γίνεσθαι with a preposition, in the sense of "arriving." In this case the context shows that the action of περινῆσαι could not take place while Sesostris was still arriving (i. e., while that act was going on), and did not take place until he actually arrived at Daphnae. According to Herodotus's usage with other verbs, ἐγίνετο is here impossible.

2, 103: καὶ ἐπείτε ἐγίνετο ἐπὶ Φάσι ποταμῷ, οὐκ ἔχω . . . εἰπεῖν εἴτε αὐτὸς ὁ βασιλεὺς Σέσωστρις ἀποδασάμενος τῆς ἑωυτοῦ στρατιῆς μόριον ὅσον δὴ αὐτοῦ κατέλιπε . . ., εἴτε τῶν τινες στρατιωτέων τῇ πλάνῃ αὐτοῦ ἀχθεσθέντες περὶ Φάσιν ποταμὸν κατέμειναν.

ἐγίνετο, ABR, St., Kal., H.; ἐγένετο, CPz, vH., Ab., Kr. Here the simple and obvious meaning is, as Macaulay translates, "when he came to the river Phasis." Moreover, the action of both κατέλιπε and κατέμειναν could not well take place until the army had actually arrived at the river. To express this meaning the aorist is much to be preferred.

In five passages, where all the MSS. agree, the appearance of the imperfect is such as to occasion surprise.

1, 70: οἱ μὲν Λακεδαιμόνιοι λέγουσι ὡς ἐπείτε ἀγόμενος ἐς τὰς Σάρδις ὁ κρητὴρ ἐγίνετο κατὰ τὴν Σαμίην, πυθόμενοι Σάμιοι ἀπελοίατο αὐτὸν νηυσὶ μακρῇσι ἐπιπλώσαντες.

The imperfect must mean here, "after they *were getting* (coming) opposite the land of Samos"; the aorist would mean, "after they came opposite." There is no discernible reason here why Herodotus should emphasize the idea of continuance, as ἐγίνετο necessarily would. For this reason, and for the reasons given above under 1, 105, I regard it as more probable that Herodotus wrote ἐγένετο, though the imperfect is not impossible.

1, 189: *ἐπείτε δὲ ὁ Κῦρος πορευόμενος . . . ἐγίνετο ἐπὶ Γύνδῃ ποταμῷ, . . . τοῦτον δὴ τὸν Γύνδην ποταμὸν ὡς διαβαίνειν ἐπειρᾶτο ὁ Κῦρος, . . . ἐνθαῦτά οἱ τῶν τις ἱρῶν ἵππων . . . ὑπὸ ὕβριος ἐσβὰς ἐς τὸν ποταμὸν διαβαίνειν ἐπειρᾶτο, κ.τ.λ.*

The context makes the imperfect *ἐγίνετο* here absurd. It is self-evident that the action both of *ὡς . . . ἐπειρᾶτο* and *ἐνθαῦτα . . . ἐπειρᾶτο* did not take place until Cyrus had reached the river. The meaning we require is, "After Cyrus came to the Gyndes river." Unless we admit an inaccuracy of expression such as Herodotus is guilty of with no other verb, we must believe that the true reading is *ἐγένετο*.

3, 69: *ἐπείτε αὐτῆς μέρος ἐγίνετο τῆς ἀπίξιος παρὰ τὸν μάγον (ἐν περιτροπῇ γὰρ δὴ αἱ γυναῖκες φοιτῶσι τοῖσι Πέρσῃσι), ἐλθοῦσα παρ' αὐτὸν ηὗδε, κ τ λ.*

Here it is difficult or impossible to elicit a satisfactory meaning from the imperfect, "after that it was becoming," etc. The real meaning can only be, "after her turn had come," and to express this *ἐγένετο* is demanded.

4, 173: *καὶ ἐπείτε ἐγίνοντο ἐν τῇ ψάμμῳ, πνεύσας ὁ νότος κατέχωσέ σφεας.*

Here the imperfect is not wholly impossible, but, for reasons similar to those given above, the aorist is to be preferred. Cf. 3, 25: *ἐπεὶ δὲ ἐς τὴν ψάμμον ἀπίκοντο, δεινὸν ἔργον αὐτῶν τινες ἐργάσαντο.*

8, 37: *οἱ δὲ βάρβαροι ἐπειδὴ ἐγίνοντο ἐπειγόμενοι κατὰ τὸ ἱρὸν τῆς Προνηΐης Ἀθηναίης, ἐπιγίνεταί σφι τέρεα ἔτι μέζονα, κ.τ.λ.*

If this sentence be considered by itself, the case is similar to 1, 70 (above), and the aorist, though not necessary, would be more natural. But a little later in the same chapter we have *ἐπεὶ γὰρ δὴ ἦσαν ἐπιόντες οἱ βάρβαροι κατὰ τὸ ἱρὸν τῆς Προν. Ἀθ., κ.τ.λ.*, referring to the same event as the preceding clause, *ἐπειδὴ ἐγίνοντο*. Strictly speaking, *ἐγίνοντο* and *ἦσαν* cannot be exactly equivalent. But if we read *ἐγένοντο* in the first clause the meaning of the two clauses becomes the same, and the required sense is thus satisfied.

It remains to speak of the use of *γίνομαι* with *ὡς* and *ὅκως*. The present or the aorist stem of *γίνομαι* occurs with *ὡς* in these passages: 1, 10, 50, 67, 108, 113, 196; 2, 104, 113, 119, 121 γ, 121 δ; 3, 16, 41, 85, 86; 4, 126, 154, 179; 5, 12, 18, 32, 33, 56, 63, 65; 6, 14, 77, 90, 109, 110, 112, 129, 129_2, 140; 7, 18, 119, 146, 166, 197; 8, 13, 14, 44, 94, 94_2, 108, 126, 132, 132_2; 9, 39, 62, 118; *ὅκως* — 2, 133; 9, 66. In 17 of these 53 places there is a greater or less degree of variation in the reading of the MSS. In nine passages, where most of the MSS. agree, the readings as given in Kallenberg are doubtless correct. In two passages where the MSS. differ, I would depart from Kallenberg's reading. 4, 154: *παραλαβὼν* τὴν παῖδα ἀπέπλεε, ὡς δὲ ἐγίνετο ἐν τῷ πελάγεϊ . . . αὐτὴν διαδήσας κατῆκε ἐς τὸ πέλαγος, κ.τ.λ. ἐγίνετο, ABCPR, St., Kal., H., Ab., Krüg.; ἐγένετο, sz. ἐγίνετο, if read here, must mean, "after he was coming to (getting to) the open sea." The meaning which we expect is, "when he got out into the open sea," as Macaulay translates. It is obvious that Themison could not let the girl down into the πέλαγος until he had actually reached it, and the natural and normal tense by which to express this idea is the aorist.

7, 197: Ξέρξης δὲ ταῦτα ἀκούσας, ὡς κατὰ τὸ ἄλσος ἐγίνετο, αὐτός τε ἔργετο, κ.τ.λ.

ἐγίνετο, ABC, Kal., vH., St.; ἐγένετο, PRsvz, H., Ab., Kr. The MS. authority is here about equally divided. There is certainly little aptness of meaning in saying, "after he *was coming* near the sacred grove he kept away from it." What we want here is, "after he came" ("had come"), and hence the aorist is to be preferred.

In six passages (5. 18; 6. 129; 7. 119; 8, 14, 108, 126) the context does not give any strong indication whether the imperfect or aorist is to be preferred. The best that can be done therefore is, in the light of Herodotus's general usage, to be guided by the weight of MS. authority. Where the MS. evidence is about evenly divided, and the context is neutral, I should myself prefer the aorist for reasons given above.

In two places the present stem, though it is given in all MSS., seems to be inappropriate: 1, 196: ὡς ἂν αἱ παρθένοι γινοίατο γάμων ὡραῖαι, ταύτας . . . ἐς ἓν χωρίον ἐσάγεσκον ἀλέας, κ.τ.λ. This can mean only "after the maidens *were becoming* of marriageable age." But we may surely assume that they were not thus sold off until they *had already become* old enough to be married; and, to express this, γενοίατο is necessary, which was read by Aldus and more recently by Krüger.

6, 110: μετὰ δὲ οἱ στρατηγοὶ τῶν ἡ γνώμη ἔφερε συμβάλλειν, ὡς ἑκάστου αὐτῶν ἐγίνετο πρυτανηίη τῆς ἡμέρης, Μιλτιάδῃ παρεδίδοσαν. All MSS. give ἐγίνετο. But observe what follows: ὁ δὲ δεκόμενος οὔτι κω συμβολὴν ἐποιέετο, πρίν γε δὴ αὐτοῦ πρυτανηίη ἐγένετο. ὡς δὲ ἐς ἐκεῖνον περιῆλθε, ἐνθαῦτα δή, κ.τ.λ. It cannot be denied that ἐγένετο and περιῆλθε describe an event exactly like the one described by the preceding ἐγίνετο. What we need here is ἐγένετο, "after the turn of each to command for the day had come."

B. Clauses of Contemporaneity.

Logically, clauses of contemporaneity may be divided into two classes: clauses of coincidence, and clauses of insertion. In the clauses of these kinds, the use of tense-stems conforms in nearly all cases to the principles stated above (p. 173). Two passages only demand notice. 1, 164: ἐν ᾧ ὢν ὁ Ἅρπαγος ἀπὸ τοῦ τείχεος ἀπήγαγε τὴν στρατιήν, οἱ Φωκαιέες ἐν τούτῳ . . . ἔπλεον ἐπὶ Χίου. The use of ἐν ᾧ with the aorist is strange, and yet no editor seems to have commented upon it. The phrase ἐν ᾧ, used in a temporal clause, everywhere else in Herodotus means "while," and we should naturally expect the meaning here to be, "While Harpagus was leading away his army," etc. But it is impossible in the light of Herodotus's usage elsewhere to believe that the action of the imperfect ἔπλεον is inserted in that of the aorist ἀπήγαγε. Nothing like it is found elsewhere in Herodotus. The explanation is this. By a confusion of ideas — another example of Herodotus's "genial *laissez aller*" — ἐν ᾧ was used with a verb with which properly a conjunction meaning "after" should have been used. ἐν ᾧ stands here therefore in a clause which is strictly one of prior action. Cf. Thuc., 7, 29, 4: τὸ γὰρ γένος τὸ τῶν Θρᾳκῶν . . . ἐν ᾧ ἂν θαρσήσῃ, φονικώτατόν ἐστιν. The only other explanation that can be given is that the true reading here is ἀπῆγε, but this is not probable.

In Herodotus, 3, 150: ἐν ὅσῳ γὰρ ὅ τε μάγος ἦρχε καὶ οἱ ἑπτὰ ἐπανέστησαν, ἐν τούτῳ παντὶ τῷ χρόνῳ καὶ τῇ ταραχῇ ἐς τὴν πολιορκίην παρεσκευάδατο, it is clear that we have an example of coincident extension. As to παρεσκευάδατο, MSS. and editors read thus: παρεσκευάδατο, CP, Kr., Ab., vH., Kal.; παρεσκευάζοντο, ABR, St., H., παρεσκευάσαντο, d z. The pluperfect is here impossible. If we keep the pluperfect, the meaning must be that during all the time indicated by the temporal clause the preparations were *already completed*. But we know from the context that during the rule of the magus and the insurrection of the seven the preparations were *going on*. Stein and Holder are undoubtedly right in adopting the imperfect. Bähr's citations (5, 34, and 7, 218) have no bearing on this question, except as to the possibility of the form παρεσκευάδατο.

Sometimes in clauses which formally are temporal clauses of contemporaneity the temporal relation is loose and inexact, and cannot with accuracy be called either that of coincidence or that of insertion. Such a case is found in 6, 106: τότε δὲ πεμφθεὶς ὑπὸ τῶν στρατηγῶν ὁ Φειδιππίδης οὗτος, ὅτε πέρ οἱ ἔφη καὶ τὸν Πᾶνα φανῆναι, κ.τ.λ. It is obvious that the temporal clause defines the time of πεμφθείς; but it is not less obvious that the action neither of ἔφη nor φανῆναι is either coincident with that of the main verb or inserted in it; neither is the temporal clause one of antecedence. The fact is that by stating in the temporal clause a fact presumably well known to his hearers, the author thus identifies the occasion upon which the main action took place. ὅτε here is equivalent to "on the occasion when." A similar usage is found with ὅτε in 6, 65, and 2, 156.

In 7, 20, the clause *ὅτε Σκύθαι . . . ἐνέμοντο* is epexegetic of the expression *τὸν Σκυθικὸν* [*στόλον*].

The perfect stem occurs in clauses of contemporaneity but rarely, as would be expected. In Herodotus, 7, 158: *αὐτοὶ δὲ ἐμεῦ πρότερον δεηθέντος βαρβαρικοῦ στρατοῦ συνεπάψασθαι, ὅτε μοι πρὸς Καρχηδονίους νεῖκος συνῆπτο, . . . οὔτε . . . ἤλθετε, κ.τ.λ.*, the pluperfect *συνῆπτο* expresses the completion of the action, together with the resulting state, in the extensive aspect. Cf. Thuc., 2, 95, 2,[*] and Herodotus, 9, 8. In 9, 39: *ἡμέραι δέ σφι ἀντικατημένοισι ἤδη ἐγεγόνεσαν ὀκτώ, ὅτε ταῦτα ἐκεῖνος συνεβούλευε Μαρδονίῳ*, we have an example of inversion. The pluperfect expresses an action not contemporaneous with that of *συνεβούλευε*, but completed at the time of that verb.

The periphrastic form of the perfect middle or passive occurs twice, 2, 121ϵ (*εἴη ἐργασμένος*), and 9, 75 (*ἔστι . . . ἐξεργασμένον*). In both cases the *ὅτε* clause is epexegetic; in 9, 75, the aorist would have been expected, but *ἔστι* is emphatic; in English: "there is another famous deed which was done by Sophanes."

The historical present is found in the clauses now under discussion, but once, 3, 136: *ἐν ᾧ δὲ οὗτοι ταῦτα ἔπασχον, ὁ Δημοκήδης ἐς τὴν Κρότωνα ἀπικνέεται*, and here *ἀπικνέεται* is clearly aoristic.

In Herodotus, as in Thucydides, the action of the temporal clause is only rarely inserted into the action of the main verb. But two cases of this kind occur in clauses with *ὅτε*, 5, 30: *ὁ γὰρ Ἱστιαῖος . . . ἐτύγχανε τοῦτον τὸν χρονον ἐὼν ἐν Σούσοισι ὅτε οἱ Νάξιοι ἦλθον, κ.τ.λ.*; and 9, 8: *ὅτε δὲ Ἀλέξανδρος ἀπίκετο ἐς τὴν Ἀττικήν, οὔκω ἀπετετείχιστο, κ.τ.λ.* In 1, 21: *αὐτίκα ἔπεμπε κήρυκα . . . βουλόμενος σπονδὰς ποιήσασθαι . . . χρόνον ὅσον ἂν τὸν νηὸν οἰκοδομέῃ*, the expression is elliptical and the verb upon which the temporal clause depends is not formally expressed. Cf. 4, 201: *ἔστ' ἂν ἡ γῆ αὕτη οὕτω ἔχῃ, μένειν τὸ ὅρκιον, κ.τ.λ.*

C. CLAUSES OF SUBSEQUENCE.

1. *Main Clause.*

As we have seen above, clauses of subsequent action can be divided into two classes: clauses of mere subsequence, in which cases the introducing conjunction (*πρίν, πρὶν ἤ, πρότερον ἤ*) means "before"; and, secondly, clauses that define the posterior limit of the main action, where the temporal conjunction (*πρίν, ἕως, μέχρι, μέχρι οὗ, ἄχρι οὗ, ἐς ὅ*) means "until."

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[*] Cf. Warren, Temp. Clauses in Thuc., p. 35.

In clauses of mere subsequence, if the action of the main verb is purely antecedent, Herodotus almost invariably uses the aorist stem in the main clause; if the action of the main verb is overlapping, he generally uses a tense of extensive action. In a few cases, however, the present or imperfect is found in the main clause, though the action described by it is purely antecedent.

As would be expected, if the main verb expresses customary or generic action, the present or imperfect appears in the main clause; as 4, 180: ὁτέοισι δὲ τὸ πάλαι ἐκόσμεον τὰς παρθένους πρὶν ἢ σφι Ἕλληνας παροικισθῆναι, οὐκ ἔχω εἰπεῖν, κ.τ.λ.

In a few cases where the main action is wholly antecedent, the imperfect is used to express some shade of meaning which could not be expressed by the aorist, as in 4, 134: νῦν ὦν μοι δοκέει, ἐπεὰν τάχιστα νὺξ ἐπέλθῃ . . . [ἡμᾶς] ἀπαλλάσσεσθαι. πρὶν ἢ καὶ ἐπὶ τὸν Ἴστρον ἰθῦσαι Σκύθας λύσοντας τὴν γέφυραν, κ.τ.λ. It is quite evident that the speaker conceives of the action of ἀπαλλάσσεσθαι as taking place before the Scythians set out to destroy the bridge, and one might expect the aorist ἀπαλλαχθῆναι. But the present means not, "to depart," "to go away," but rather, "to set about getting away," "to try to get away." As a matter of fact, the army is in something of a strait, and is by no means sure that it can get away. The meaning of the aorist is very aptly illustrated by the passage 6, 45, where it is said of an army which had suffered many disasters and much slaughter: οὗτος μέν νυν ὁ στόλος αἰσχρῶς ἀγωνισάμενος ἀπηλλάχθη ἐς τὴν Ἀσίην, i. e., actually made good its return. So in 4, 167: πρὶν δὲ ἢ ἀποστεῖλαι τὴν στρατιήν, ὁ Ἀρυάνδης πέμψας ἐς τὴν Βάρκην κήρυκα, ἐπυνθάνετο τίς εἴη. κ.τ λ., the verb ἐπυνθάνετο means "he proceeded to find out" (*quærebat*). In 9, 13: πυθόμενος πάντα λόγον, πρὶν ἢ τοὺς μετὰ Παυσανίεω ἐς τὸν Ἰσθμὸν ἐσβαλεῖν. ὑπεξεχώρεε. κ.τ.λ., the imperfect is rendered correctly by Macaulay: "he began to retire." And in 9, 68: δηλοῖ τέ μοι ὅτι πάντα τὰ πρήγματα τῶν βαρβάρων ἤρτητο ἐκ Περσέων. εἰ καὶ τότε οὗτοι πρὶν ἢ καὶ συμμῖξαι τοῖσι πολεμίοισι ἔφευγον, κ τ.λ.. ἔφευγον means not "they made good their escape" (*effugerunt*), but "they betook themselves to flight," "turned to flee."

Sometimes the fondness of a verb for the present stem outweighs the demand of the temporal relation, as, e. g., in 6, 22: ἐδόκεε δὲ . . . πρὶν ἢ . . . ἀπικέσθαι τὸν τύραννον . . . ἐς ἀποικίην ἐκπλέειν, κ.τ.λ. Cf. Thuc., 6, 29, 1, and 61, 1.* πειράομαι, which is used in 1, 76: πρὶν δὲ ἐξελαύνειν . . . ἐπειρᾶτό σφεας ἀπὸ Κροίσου ἀπιστάναι, as a verb of effort, naturally takes the imperfect.[*] In the following passages the imperfect expresses continued action: 9, 13: *πρὶν μέν νυν ἢ πυθέσθαι, ἀνεκώχευε, κ.τ.λ.*; 4, 1: *οὗτοι γὰρ πρὶν ἢ Σκύθας ἀπικέσθαι, ἦρχον τῆς Ἀσίης.* In 6, 119: *βασιλεὺς δὲ Δαρεῖος, πρὶν μὲν αἰχμαλώτους γενέσθαι τοὺς Ἐρετριέας. ἐνεῖχέ σφι δεινὸν χόλον, κ.τ.λ.*, the periphrastic expression, *ἐνεῖχε χόλον*, requires the imperfect, since *ἐνέσχε χόλον* would have signified, "he conceived heavy anger." It is worthy of note that in all of the passages just mentioned, except two, the temporal clause is in the abnormal order, i. e., before the main clause.[†]

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* Cf. Warren, op. cit., p. 41.

The perfect stem is found in the main clause three times: 1, 92: *ὃς οἱ πρὶν ἢ βασιλεῦσαι, ἀντιστασιώτης κατεστήκεε, κ.τ.λ.* : *κατεστήκεε* is here equivalent to an imperfect, and expresses extensive action. 1, 159: *ἡμεῖς δὲ . . . τὸν ἱκέτην ἐς τόδε οὐ τετόλμηκαμεν ἐκδιδόναι, πρὶν ἂν τὸ ἀπὸ σεῦ δηλωθῇ, κ.τ.λ.*; *τετολμήκαμεν* is clearly extensive, and is used of action begun in the past and continuing in the present; it is the perfect of "unity of time."[‡] 3, 25: *πρὶν δὲ τῆς ὁδοῦ τὸ πέμπτον μέρος διεληλυθέναι τὴν στρατιήν, αὐτίκα πάντα αὐτοὺς τὰ εἶχον σιτίων ἐχόμενα ἐπελελοίπεε, κ.τ.λ.* The pluperfect describes in the extensive aspect the condition resulting from the action of the verb, i. e., all their food *was gone.*[§]

The historical present is found in the main clause once, 4, 93 (*αἱρέει*); here the verb is aoristic.[‖]

As was stated above, if the action of the main verb was not terminated before the subordinate action began, a verb of extensive action is used in the main clause. Such a case is seen in 4, 147: *οὗτοι ἐνέμοντο τὴν Καλλίστην καλεομένην ἐπὶ γενεάς, πρὶν ἢ Θήραν ἐλθεῖν ἐκ Λακεδαίμονος. ὀκτὼ ἀνδρῶν.* Here we know from the context that the people who lived on the island of Calliste before Theras came, inhabited it after he came as well. The following examples of the same usage need no comment: 2, 2: *οἱ δὲ Αἰγύπτιοι, πρὶν μὲν ἢ Ψαμμέτιχον σφέων βασιλεῦσαι. ἐνόμιζον ἑαυτοὺς πρώτους γενέσθαι πάντων ἀνθρώπων*: 7, 94: *καὶ πρὶν ἢ Δαναόν τε καὶ Ξοῦθον ἀπικέσθαι . . . ἐκαλέοντο Πελασγοί, κ.τ.λ.*; 8, 59: *πρὶν ἢ τὸν Εὐρυβιάδην προθεῖναι τὸν λόγον τῶν εἵνεκα συνήγαγε τοὺς στρατηγούς, πολλὸς ἦν ὁ Θεμιστοκλέης, κ.τ.λ.*

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[*] Cf. Hultsch, op. cit., pp. 37 ff.; Am. Jour. Phil., **16** (1895), p. 155. Thucydides does not use the aorist of *πειράομαι*.

[†] See below, p. 221.

[‡] Cf. Gildersleeve, Syntax, 203.

[§] Cf. Heilmann, De Infin. Syntaxi Herodotea, Gissae, 1879, p. 54.

[‖] Cf. 2, 44: *καὶ ταῦτα καὶ πέντε γενεῇσι ἀνδρῶν πρότερά ἐστι ἢ τὸν Ἀμφιτρύωνος Ἡρακλέα . . . γενέσθαι.*

We now pass to clauses of subsequence which express limit. In these clauses the main verb is generally of extensive aspect. First, as to clauses with *πρίν* and *πρὶν ἤ*, followed by a finite verb. In all these sentences the main verb is formally or virtually negative. This negative introduces a disturbing factor in determining what tense stem shall be used, and as a result no general law can be laid down, In the clauses of this kind which refer to past time the imperfect is found four times (1, 14; 6, 79, 211; 7, 239), and the aorist three times (6, 45; 7, 137; 8, 8). In the clauses of limit introduced by *ἕως, μέχρι, μέχρι οὗ, ἄχρι οὗ*, a verb of extensive action is used in the main clause in every case but one, 5, 28: [ἡ Μίλητος] *κατύπερθε δὲ τούτων ἐπὶ δύο γενεὰς ἀνδρῶν νοσήσασα ἐς τὰ μάλιστα στάσι, μέχρι οὗ μιν Πάριοι κατήρτισαν.* Here the temporal clause depends upon and limits the participle *νοήσασα*; but that the action of *νοήσασα* extended over a considerable time is shown by the *ἐπὶ δύο γενεὰς ἀνδρῶν.**

In clauses with *ἐς ὅ* where a choice between the imperfect and aorist was possible the aorist is found in the main clause only eight times † (1, 158; 2, 143, 143_2; 4, 105, 160, 201; 6, 24, 83). In four of these places, 6, 83 (*ἔσχον . . . ἄρχοντες*); 2, 143 (*ἀπεδείκνυσαν . . . διεξίοντες*); 2, 143_2 (*ἀντεγενεηλόγησαν . . . φάμενοι*); 1, 158 (*ἔσχε . . . δοκέων καὶ ἀπιστέων*), though the verb is aorist, the notion of extensive action is conveyed by the present participle. In 4, 160: *ὅς βασιλεύσας πρῶτα τοῖσι ἑωυτοῦ ἀδελφεοῖσι ἐστασίασε, ἐς ὅ μιν οὗτοι ἀπολιπόντες οἴχοντο, κ.τ.λ.*, we should rather expect the imperfect. but the expression is condensed and almost elliptical; the meaning fully expressed would be, "he first raised a quarrel with his own brothers, *and this continued until*," etc. Another explanation would be that *ἐς ὅ* is used here in a loose sense, meaning little more than "and finally." Cf. above, p. 178. 4, 105: *ὄφιας γάρ σφι πολλοὺς μὲν ἡ χώρη ἀνέφαινε. οἱ δὲ πλεῦνες ἄνωθέν σφι ἐκ τῶν ἐρήμων ἐπέπεσον, ἐς ὃ πιεζόμενοι οἴκησαν μετὰ Βουδίνων, κ τ.λ.* Here the temporal clause refers both to *ἀνέφαινε* and to *ἐπέπεσον*, but it cannot be denied that the temporal relation is loose and of secondary importance. The aorists in the main clause, 4, 201 (*ὑπήκουσαν*) and 6, 24 (*ἀπίκετο*) are correctly explained by Fuchs (op. cit., p. 70).

The future tense stem is found in the main clause, 7, 158: *ἔστ' ἂν δια-*

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* Hence correct the statement of Fuchs (Schanz's Beiträge, Heft **14**, p. 78), who says that in clauses with *μέχρι οὗ* in Herodotus the imperfect is always found in the main clause.

† Fuchs (op. cit., p. 70) seems to be in error in saying that the aorist occurs five times in these circumstances.

πολεμήσωμεν, ὑποδέκομαι παρέξειν. Here the use of *παρέξειν.* where the general law leads us to expect a verb of extensive aspect, squares with the now accepted doctrine that *ἕξειν* is extensive or durative, and *σχήσειν* aoristic.[*] The historical present is found in the main clause, 8, 58: *ἐνθαῦτα ὁ Θεμιστοκλέης παριζόμενός οἱ καταλέγει ἐκεῖνά τε πάντα τὰ ἤκουσε Μνησιφίλου, ἑωυτοῦ ποιεύμενος, . . . ἐς ὃ ἀνέγνωσε, κ.τ.λ.* *καταλέγει* is plainly extensive. The verb of the main clause is a gnomic aorist in 4, 196: *ἄλλον πρὸς ὦν ἔθηκαν χρυσόν, ἐς ὃ ἂν πείθωσι.*

2. *Subordinate Clause.*

In clauses which express plain subsequence, the verb, as regards its tense-stem, is under no restriction as far as the temporal relation is concerned and does not need to be considered. As a matter of fact, however, the conjunctions *πρίν*, *πρὶν ἤ*, and *πρότερον ἤ*, on account of the negative idea which they contain, generally take the aorist.[†] The present infinitive is found after *πρίν* once (8, 144); after *πρὶν ἤ* twice (6, 116; 8, 3). In 8, 144: *πρὶν ὦν παρεῖναι ἐκεῖνον ἐς τὴν Ἀττικήν, ἡμέας καιρός ἐστι προβοηθῆσαι ἐς τὴν Βοιωτίην*, the present *παρεῖναι* is equivalent to an aorist, e. g., *ἀπικέσθαι.* The present, *ἥκειν*, 6, 116: *Ἀθηναῖοι δὲ . . . ἔφθησάν τε ἀπικόμενοι πρὶν ἢ τοὺς βαρβάρους ἥκειν*, has generally been regarded as equivalent to a perfect [‡]; but if this be true we naturally expect it to express the extensive aspect, and this is not in point. But since the imperfect of *ἥκειν* is often used aoristically [§], why not take *ἥκειν* here as aoristic and render, with Macaulay, "and they arrived there before the barbarians came"? *πέμπειν*, in 8, 3, has been sufficiently explained.[‖]

After *πρότερυν ἤ* the present infinitive is found once, 7, 228: *τοῖσι πρότερον τελευτήσασι ἢ ὑπὸ Λεονίδεω ἀποπεμφθέντας οἴχεσθαι. κ.τ.λ.*, where I regard *οἴχεσθαι* as aoristic rather than as a perfect, as does Sturm.

In connection with clauses of limit, the aorist is almost always found in the temporal clause. In clauses with *πρίν* and the indicative or subjunctive the aorist is found everywhere except in 1, 197: *σιγῇ δὲ παρεξελθεῖν τὸν κάμνοντα οὔ σφι ἔξεστι, πρὶν ἂν ἐπείρηται ἥντινα νοῦσον ἔχει.* Observe that, according to the generally accepted opinion, *ἐπείρομαι* has no aorist in use in Ionic.* It is pretty clear that the action of ἐπείρηται is here aoristic.

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[*] Cf. Brugmann, Gr. Gram.^{3}, 540, and the writers there cited.

[†] Cf. Gildersleeve, Am. Jour. Phil., **2** (1881), pp. 466 ff.

[‡] Cf. Gildersleeve, Am. Jour. Phil., **2** (1881), p. 478; Sturm, Schanz's Beiträge, Heft 1, p. 295.

[§] Cf. Gildersleeve, Syntax, 225.

[‖] Cf. Gildersleeve, Am. Jour. Phil., **2** (1881), p. 477; Heilman, op. cit., p. 53; Sturm, op. cit., p. 295; Stein, note ad locum.

A very remarkable use of the present infinitive after ἕως, if it really is after ἕως, has been cited above, p. 179. Assuming that ἕως is here the temporal conjunction, we should certainly expect the aorist. As was mentioned above, Gildersleeve calls attention to the plural πλοίοισι. But cf. 2, 179: τὰ φορτία ἔδεε περιάγειν ἐν βάρισι περὶ τὸ Δέλτα, μέχρι οὗ ἀπίκοιτο ἐς Ναύκρατιν; and 2, 102: τὸν ἔλεγον οἱ ἱρέες πρῶτον μὲν πλοίοισι μακροῖσι ὁρμηθέντα . . . τοὺς . . . κατοικημένους καταστρέφεσθαι, ἐς ὃ πλέοντά μιν πρόσω ἀπικέσθαι ἐς θάλασσαν, κ.τ.λ. In this connection it is not without significance that, in 67 cases of ἀπικνέεσθαι in clauses of antecedence, the aorist is used in *every case.* As a matter of fact the imperfect of this verb is very rare in Herodotus. Schweighäuser does not cite it at all.† The difficulty, if not the impossibility, of explaining the present, is a strong indication that ἕως is here not temporal at all.

The use of νέμεσθαι in 7, 171, after ἔστε: λιμόν τε καὶ λοιμὸν γενέσθαι . . . , ἔστε τὸ δεύτερον ἐρημωθείσης Κρήτης μετὰ τῶν ὑπολοίπων τρίτους αὐτὴν νῦν νέμεσθαι Κρῆτας, has been explained by Gildersleeve ‡ and Fuchs.§ ἐρημωθείσης forms the virtual conclusion of ἔστε, and the νῦν . . . νέμεσθαι looks very much like a sudden and unannounced shift from a subordinate to a main construction.

The imperfect is found in clauses with ἐς ὅ eight times ‖ (1, 158, 196; 3, 48; 4, 160, 160_2; 6, 113; 9, 55, 94). In 3, 48 (οἴχοντο ἀπολιπόντες), and 4, 160 (ἀπολιπόντες οἴχοντο), an aorist participle occurs, and οἴχοντο is itself aoristic. 1, 158: Ἀριστόδικος . . . ἔσχε μὴ ποιῆσαι ταῦτα Κυμαίους, . . . ἐς ὃ τὸ δεύτερον . . . ἐπειρησόμενοι ἤισαν ἄλλοι θεοπρόποι, κ.τ.λ. Note that εἶμι has no aorist; furthermore, this passage seems to give evidence, in addition to the conclusions reached above (p. 191), that the preterite of εἶμι is in Herodotus sometimes, at least, aoristic. 1, 196: καὶ ταύτην ἀνεκήρυσσε . . . ἐς ὃ τῷ τὸ ἐλάχιστον ὑπισταμένῳ προσέκειτο. The verb here used has no aorist. In 6, 113, an aorist participle is used with the imperfect.

In 9, 94: [Εὐηνίῳ] παρίζοντο καὶ λόγους ἄλλους ἐποιεῦντο ἐς ὃ κατέβαινον συλλυπεύμενοι τῷ πάθει, Gildersleeve¶ says there is overlapping action. But it is not easy to see how in this case overlapping was possible. Whatever the words are in the Greek, the bare fact described is simply this: certain men sat down beside the blinded Euenius and talked with him, first about matters in general, and secondly (*κατέβαινε*) about his misfortune. These facts certainly preclude the possibility of overlapping action. As a matter of fact the temporal relation here is loose and secondary. Of what importance is it how long they talked about other matters? Just as in another case mentioned above, *ἐς ὅ* is loosely used and means little more than "and finally." * Furthermore, it is worthy of note that the use of the imperfect seems to be almost idiomatic when *καταβαίνω* is used in a past tense with a present participle in the sense which it has in this passage. The imperfect is thus used 1, 90 (*παραιτεόμενος*), 1, 116 (*κελεύων*), 1, 118 (*λέγων*).

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* Cf. Veitch, Greek Verbs, 1887, s. v. εἴρομαι: "aorist late, if correct." Kallenberg would, I suppose, regard this as an aorist. See above, p. 190.

† I have found the imperfect in Herodotus but once (4, 125), though I have not made an exhaustive search.

‡ Am. Jour. Phil., **3** (1882), p. 515.

§ Op. cit., p. 76.

‖ Fuchs, op. cit., p. 69, cites only five of these cases.

¶ Am. Jour. Phil., **24** (1903), p. 400.

In 4, 160: *ὁ δὲ Ἀρκεσίλεως εἵπετο φεύγουσι ἐς ὃ ἐν Λεύκωνι . . . ἐγίνετο ἐπιδιώκων καὶ ἔδοξε τοῖσι Λίβυσι ἐπιθέσθαι οἱ*, Fuchs † would change *ἐγίνετο* which is found in all MSS., to *ἐγένετο*. Gildersleeve says ‡: "No change is necessary. Note the shift of subject." It seems to me that the question here is not what is necessary, but what is, in the light of all the evidence, probable. The fact that there is a shift of subject here has no direct bearing upon the question whether we ought to have here the imperfect or the aorist; the question is whether the meaning and general grammatical usage makes probable the one or the other tense. Now, if Herodotus meant to say here, "he followed them until he *was arriving* at Leucon," then *ἐγίνετο* is right; but if he wishes to say, "he followed them until he arrived at Leucon," *ἐγένετο* is the only natural form to use. I see no good reason why Herodotus should here make such a fine distinction as is implied in the former expression. Also the facts which we have observed above regarding *γίνομαι* in clauses of antecedence, both with prepositions (= to arrive) and without, justify us in demanding the credentials of any *ἐγίνετο* in a temporal clause which violates normal usage. I think that on the score of intrinsic probability, Fuchs's change is right.

As to *ἐς ὅ* with the subjunctive, the present is found but twice. In 2, 115: *ἀλλ' αὐτὰ ἐγὼ τῷ Ἕλληνι ξείνῳ φυλάξω, ἐς ὃ ἂν αὐτὸς ἐλθὼν ἐκεῖνος ἀπαγάγεσθαι ἐθέλῃ*, the action of *ἐθέλῃ* is conceived as a state rather than an act, and the limit falls within the continuance of this state. Observe, too, the aorist participle *ἐλθών*. The other example is 4, 196: *οἱ δὲ προσελθόντες ἄλλον πρὸς ὦν ἔθηκαν χρυσόν, ἐς ὃ ἂν πείθωσι*. The use of the present *πείθωσι* is very strange. In all other clauses with *ἐς ὅ* which express generic action the aorist is used. The present is here all the more surprising because of the well-recognized difference in meaning between the present and aorist stems of *πείθω*. The aorist stem means "to persuade;" the imperfect, and by inference the present, in cases where a distinction of tense stems is possible, express the attempt to persuade.* In this case it is absolutely essential that the form of the verb express the successful completion of the action; for this we unquestionably expect the aorist. Cf. 5, 97: *οὐδὲν ὅ τι οὐκ ὑπίσχετο . . . ἐς ὃ ἀνέπεισέ σφεας.* In capitals the lunar sigma C and Θ could easily be confused. I would read here *πείσωσι*.

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* Macaulay translates, I think, with entire accuracy when he says: "they came and sat by him and conversed about other matters *and at last* they came to sympathizing with him in his misfortune."

† Op. cit., p. 69.

‡ Am. Jour. Phil., **24** (1903), p. 400.

The reason for the two present infinitives in 1, 202, has been explained above (p. 178), The present indicative is used of generic action after *ἐς ὅ* twice, — 3, 104 (quoted above, p. 176), and 4, 181. No distinction of stems was here possible.

The perfect stem is found in all clauses of subsequence but three times. In 3, 25 (quoted above, p. 203), the perfect infinitive *διεληλυθέναι*, seems to emphasize the idea of completion. 5, 51: *προέβαινε τοῖσι χρήμασι ὑπερβάλλων ὁ Ἀρισταγόρης, ἐς ὃ πεντήκοντά τε τάλαντα ὑπεδέδεκτο καὶ τὸ παιδίον ηὐδάξατο, κ.τ.λ.* Here the pluperfect seems to emphasize the idea of the completion of the action at a point in past time: "until he had received them and *had them*." The third case is in 9, 55: *ὡς γὰρ δὴ παρηγόρεον τὸν Ἀμομφάρετον . . . μὴ κινδυνεύειν μένοντας μούνους Λακεδαιμονίων, οὔ κως ἔπειθον, ἐς ὃ ἐς νείκεά τε συμπεσόντες ἀπίκατο καὶ ὁ κῆρυξ . . . παρίστατό σφι ἀπιγμένος.* This case is interesting as an example where indisputably *ἐς ὅ* does not have its strict subordinating temporal force. The meaning is not, "they could not persuade him until losing their temper they had begun to quarrel," for the context plainly shows that A. was *not* persuaded even then. If *ἐς ὅ* meant here strictly "until," then the sentence should mean that as soon as the quarrel began A. was persuaded.† *ἐς ὅ* is here virtually a co-ordinating conjunction; and here, again, Macaulay translates with admirable accuracy: "they did not at all persuade him, and at last they had come to downright strife."

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* Cf. Brugmann, Gr. Gram.^{3}, 544, 1, Anm.; and Miller, Am. Jour. Phil., **16** (1895), p. 177.

† Goodwin (M. and T., 611) says: "The idea of a clause with 'until' is that the action (or negation) of the leading clause continues to a time *at which* that of the dependent clause takes place. That the former action then *ceases* is an inference generally made, but not positively implied in the language, and not necessary." As regards the English word "until" (and this I understand the writer is

The historical present is found in a clause with πρὶν once, — 9, 22: *τύπτοντες . . . ἐποίευν οὐδέν, πρίν γε δὴ μαθών τις τὸ ποιεύμενον παίει μιν ἐς τὸν ὀφθαλμόν.*

III. TENSE.

I shall here treat the subject of tense principally in regard to its function in expressing the sphere of time, including also a few other matters which naturally range themselves under this head.

The historical present is found in the main clause very frequently with clauses of antecedence, very rarely with clauses of contemporaneity and subsequence. The conditions of its appearance do not differ from those under which it occurs in other sentences. It occurs as follows: *ἐπεί ἐπειδή, ἐπείτε* — 1, 34, 45, 62, 105; 2, 30, 119, 181; 3, 11, 72, 78; 4, 76, 118, 146, 148, 151, 154; 5, 12, 62, 100; 6, 18; 7, 61, 163, 172; 8, 27, 27_2, 37, 37_2, 129; 9, 4, 12, 107. *ὡς* — 1, 47, 62, 65, 112, 117, 160, 160_2, 213; 2, 2, 113, 152, 160; 3, 1, 29, 41, 128, 140; 5, 12, 17, 20, 41, 63, 92γ, 112; 6, 5, 16, 29, 29_2, 31, 35, 135; 7, 7, 18, 136, 140, 142, 210; 8, 4, 106; 9, 20, 39, 60, 88, 99, 108, 109. *ἐν ᾧ* — 3, 136. *πρίν* — 4, 93. *πρότερα ἤ* — 2, 44. *ἐς ὅ* — 8, 58.

The historical present is found in the subordinate clause only rarely. It occurs with *ἐπεί* twice (5, 55; 9, 2); with *ὡς* once (6, 5); with *πρίν* once (9, 22); and with *ἐς ὅ* twice (6, 75; 1, 98).

In connection with clauses of antecedence, a temporal clause which refers to the past cannot be used with a main clause which refers solely to the present or future, for the reason that there is not a sufficient point of contact between the two clauses. When such a combination seems to exist, the subordinate clause is not temporal but causal, as in 3, 71: *ἐπείτε δὲ συνήνεικε ὥστε καὶ ὑμέας εἰδέναι καὶ μὴ μοῦνον ἐμέ, ποιέειν αὐτίκα μοι δοκέει, κ.τ.λ.* Cf. 4, 98, and 7, 39. When, however, the present is used of action which began in the past, but continues in the present (present of "unity of time"), it is possible for a past tense in the subordinate

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here speaking of), this is true without doubt after a positive main clause, but it is not true, it seems to me, after a negative main clause. E. g., in the sentence, "He did not see the man until he reached the village," it certainly is positively implied in the language that he *did see* the man. Indeed, if this be not true, then there is no basis for a distinction between "before" and "until" in such a sentence. If we say, however, "He did not see the man before he reached the village," it is not implied whether he saw him at all or not.

clause to be used with a present in the main clause, as in 2, 2: ἐπειδὴ δὲ Ψαμμήτιχος ἠθέλησε εἰδέναι οἵτινες γενοίατο πρῶτοι, ἀπὸ τούτου νομίζουσι Φρύγας προτέρους γενέσθαι ἑωυτῶν, κ.τ.λ.; i. e., "since the time when . . . they *have believed.*" The present is thus used in the main clause in 1, 27, 58, 196; 2, 2; 3, 49, 117, 117_2; 4, 118. In 2, 15, the infinitive εἶναι is thus used. In two passages a present of this kind is used both in the main and the subordinate clauses; 3, 117: ἐπείτε δὲ Πέρσαι ἔχουσι τὸ κράτος, ἔστι [τοῦτο τὸ πεδίον] τοῦ βασιλέος; and 2, 98: τοῦτο δὲ γίνεται ἐξ ὅσου ὑπὸ Πέρσῃσί ἐστι Αἴγυπτος.

An interesting passage is found in 7, 8*a*: ὡς γὰρ ἐγὼ πυνθάνομαι τῶν πρεσβυτέρων, οὐδαμά κω ἠτρεμίσαμεν, ἐπείτε παρελάβομεν τὴν ἡγεμονίην τήνδε . . . ἐγὼ δὲ ἐπείτε παρέλαβον τὸν θρόνον τοῦτον, ἐφρόντιζον ὅκως μὴ λείψομαι, κ.τ.λ. The first ἐπείτε here means "since" (*quo ex tempore*), and the aorist ἠτρεμίσαμεν* is used in the same way as the present in the sentences just quoted. The second ἐπείτε also means "since," and ἐφρόντιζον likewise expresses unity of time, i. e., "since I took the throne I have not ceased to consider," etc. The perfect of unity of time is found in 4, 7 (γεγόνασι). †

A glance at the statistical summary below will show that the present and the perfect indicative, used of particular action, and expressing the present sphere of time only, occur but infrequently. They are found only in these places: present, ἐν ᾧ, 1, 164; 4, 139; ἕως, 3, 134; ὅτε, 9, 122; ἔστε, 7, 171; perfect, ἐπεί, 5, 84 (infinitive); ἐπειδή, 4, 118; 7, 16β, 158. A moment's reflection will show that, as was pointed out by Hultsch,‡ the perfect is not likely to occur in strictly historical narrative; its proper place is in direct address, either of a character in the narrative or of the author direct to his readers. The perfect indicative (or infinitive in oratio obliqua = indicative) occurs in temporal clauses in the usage above stated, only in clauses of antecedence; in each of these cases it is found in a speech reported either directly, 7, 16β (τέτραψαι), and 7, 158 (περιελήλυθε and ἀπῖκται); or indirectly, 4, 118 (κατέστραπται), and 5, 84 (ἐστερῆσθαι). In each case also there is a distinct tone of remonstrance or displeasure.

If we exclude νέμεσθαι after ἔστε, 7, 171, where the sentence structure is not normal,§ the present indicative, used in the manner above described, occurs only in clauses of contemporaneity, as the list above shows. In such clauses, if there is only partial co-extension, we expect generally, as Gildersleeve has shown,[*] a causal connotation. In 3, 134: νῦν γὰρ ἄν τι καὶ ἀποδέξαιο ἔργον, ἕως νέος εἶς ἡλικίην, the causal idea is clear. In 9, 122: κότε γὰρ δὴ καὶ παρέξει κάλλιον ἢ ὅτε γε ἀνθρώπων τε πολλῶν ἄρχομεν πάσης τε τῆς Ἀσίης, the idea of cause, though not marked, is discernible. In 1, 164: ἐν ᾧ δὲ βουλεύονται αὐτοί, ἀπαγαγεῖν ἐνεῖνον ἐκέλευον τὴν στρατιὴν ἀπὸ τοῦ τείχεος, the real meaning is, "they bade him lead his army from the wall *and keep it away* while they were deliberating." βουλεύονται here expresses time partly future as well as present, and there is virtually complete coextension.

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* The perfect of ἀτρεμίζω seems not to be found in classical Greek. Cf. Gildersleeve, Syntax, 248, 1.

† Cf. Gildersleeve, Syntax, 203. ‡ Op. cit., p. 458. § Cf. above, p. 206.

As to the future sphere in the temporal clause, it is with one exception (9, 58) expressed by the subjunctive. Occasionally in the main clause the future sphere is expressed by a verb which is formally present. So in 9, 98: ἐπεὰν συμμίσγωμεν, μεμνῆσθαί τινα χρὴ ἐλευθερίης, κ.τ.λ., χρή expresses the future sphere. There is a notable variation between present and future in 7, 8 δ: ἐπεὰν ὑμῖν σημήνω τὸν χρόνον ἐς τὸν ἥκειν δεῖ, προθύμως πάντα τινὰ ὑμέων χρήσει παρεῖναι, where δεῖ and χρήσει refer to the same sphere of time. Cf. 3, 146: εἰδώς τε ἑωυτῷ ἀσφαλέα ἔκδυσιν ἐοῦσαν ἐκ τῆς νήσου τότε ἐπεὰν αὐτὸς βούληται, where ἐοῦσαν must express future time. In 3, 57:

> Ἀλλ' ὅταν ἐν Σίφνῳ πρυτανήια λευκὰ γένηται
> Λευκοφρύς τ' ἀγορή, τότε δὴ δεῖ φράδμονος ἄνδρος. κ.τ.λ.,

δεῖ may be an example of the so-called "oracular" present.[†]

By far the most frequent tense in clauses of pure antecedence is the aorist. The pluperfect occurs with ἐπεί, ἐπειδή, and ἐπείτε 13 times, with ὡς 44 times. Out of these 57 examples, in 35 passages the action of the verb of the subordinate clause has been previously described.[‡] Very often one or more words of the previous description are repeated, as 1, 185: τὸν δὲ ὀρυσσόμενον χοῦν ἐκ τούτου τοῦ ὀρύγματος ἀναισίμου παρὰ τὰ χείλεα τοῦ ποταμοῦ παραχέουσι. ἐπείτε δὲ οἱ ὀρώρυκτο, κ.τ.λ. Out of the 28 passages with ὡς the same verb is repeated 12 times; in 16 cases the previous action is recapitulated not by the same word or words. In 8, 6 (ἐπείτε . . . ἐς τὰς Ἀφετάς), the preceding expression is separated by a considerable distance from the pluperfect. The word thus used in the pluperfect is often a word of general meaning, as ἐπεποίητο (7, 55), and παρεσκεύαστο (9, 100).

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[*] Am. Jour. Phil. **24** (1903), pp. 389, 394.

[†] Cf. Brugmann, Gr. Gram.^{3}, 543, 7; Gildersleeve, Syntax, 194.

[‡] ἐπεί, ἐπειδή, ἐπείτε — 1, 116, 185; 7, 175; 8, 6, 27, 53; 9, 17. ὡς — 1, 27, 80, 118, 141, 165, 186, 201, 214; 2, 100; 3, 142; 5, 20; 6, 9, 112; 7, 37, 55, 100, 157, 183; 8, 5, 96, 110, 113, 126; 9, 17, 33, 37, 76, 100.

Causal force and indication of the spectator is found occasionally, as in 7, 232: [λέγεται] νοστήσαντα δὲ τοῦτον ἐς Σπάρτην, ὡς ἠτίμωτο, ἀπάγξασθαι, i. e., "when (and because) he found himself dishonored." Cf. 5, 34 (ὡς . . . κατεδεδαπάνητο . . . καὶ . . . προσανισίμωτο). In some cases the pluperfect is used to emphasize the completion of an act in past time, especially when the beginning of the action has been mentioned, as in 9, 102: [συν]επισπόμενοι συνεσέπιπτον ἐς τὸ τεῖχος. ὡς δὲ καὶ τὸ τεῖχος ἀραίρητο, κ.τ.λ. Cf. 9, 52, and 9, 44. With verbs whose meaning allows it the perfect stem emphasizes the resulting state or condition as in 7, 193; ὡς ἐπαύσατό τε ὁ ἄνεμος καὶ τὸ κῦμα ἔστρωτο, κ.τ.λ. Cf. also 9, 62.

The nouns ἡμέρη, νύξ, and εὐφρόνη are used with the pluperfect ἐγεγόνεε four times (1, 11; 3, 69; 8, 12; 9, 11); with the aorist ἐγένετο eight times (1, 113; 2, 121 γ; 5, 33, 56; 6, 129; 7, 18; 9, 39, 118). A comparison of these passages does not show any marked difference in meaning or usage. Cf. 3, 69, and 7, 18; 1, 11, and 5, 56.

The pluperfect is used with ἡμέρη and a numeral, 9, 41: ὡς δὲ ἑνδεκάτη ἐγεγόνεε ἡμέρη ἀντικατημένοισι ἐν Πλαταιῇσι, κ.τ.λ., and in 3, 56. Cf. also 2, 2: ὡς γὰρ διέτης χρόνος ἐγεγόνεε ταῦτα τῷ ποιμένι πρήσσοντι, κ.τ.λ., and 8, 129: Ἀρταβάζῳ δὲ ἐπειδὴ πολιορκέοντι ἐγεγόνεσαν τρεῖς μῆνες, κ.τ.λ.

IV. CONJUNCTIONS.

A. CONJUNCTIONS OF ANTECEDENCE.

ἐπεί, ἐπειδή, ἐπείτε.

It is universally admitted that these conjunctions are used to express action either wholly or partly antecedent.* There is no case in Herodotus where it is necessary that any of these three words mean, "at or during the time when," and that only.

No sharp line of distinction can be drawn between ἐπεί. ἐπειδή and ἐπείτε. In 5, 99, ἐπειδή is repeated after a parenthesis by ἐπείτε. Gregorius Corinthius is right when he says: Ἡρόδοτος "ἐπείτε ἐγένοντο ταῦτα" ἀντὶ τοῦ "ἐπεὶ ἐγένοντο."† Nowhere in Herodotus is ἔπειτε so used that the τέ can be regarded as having connective force. For some reason ἐπειδή seems to have been a favorite word after the relative pronoun, as in

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* Cf. Kühner, Gr. Gram.², II, 2, 566, 1, b. (p. 949).

† Gregorius Corinthius, De Dialecto Ionica, 26, ed. Schaeffer, Lipsiae, 1811, p. 412

5, 98 : ὃς ἐπειδὴ ἀπίκετο ἐς τοὺς Παίονας, κ.τ.λ. ἐπειδή is also found so placed in 7, 107, 181 ; 8, 110, 114 ; 9, 72, 76.

ἐπεάν [ἐπειδάν], ἐπείτε ἄν.

The form ἐπειδάν, which is nowhere printed by Kallenberg, is found in one or more of the five principal MSS. in the following passages : 1, 193 ; 3, 21, 117 ; 4, 22, 61 ; and in 8, 144, it is given by all MSS. ἐπειδάν is read by Stein in 1, 193, and 4, 61, and by Stein and van Herwerden in 8, 144.* An examination of the use of ἐπεάν and ἐπειδή shows that it is difficult to discover any principle of usage which would argue either for or against ἐπειδάν in any of the above passages. As regards 8, 144 : ἀλλ' ἐπεὰν (ἐπειδὰν, codd.) τάχιστα πύθηται τὴν ἀγγελίην, κ.τ.λ., τάχιστα is used with ἐπεάν three times (4, 134, 98 ; 7, 129) ; it is not used with ἐπειδή at all. This might furnish a slight presumption that ἐπειδάν was not used with τάχιστα in 8, 144 ; but this inference is weakened by the fact that with ἐπεί also τάχιστα does not occur at all.

ὡς.

The temporal use of ὡς was developed, as is well known, from its use as an adverb of comparison. A transition stage may be seen in such a sentence as 1, 79 : ὡς δέ οἱ ταῦτα ἔδοξε, καὶ ἐποίεε κατὰ τάχος. Cf. also 7, 128, and 8, 64. The particle ὡς in its temporal meaning is always used by Herodotus in a clause in which the action is either partly or wholly antecedent. We may, therefore, conclude that in Herodotus ὡς is always a conjunction of antecedence.†

ὅκως.

ὅκως is used by Herodotus with more freedom than any other conjunction of antecedence. Most frequently it is used with the optative in past general conditions. Once (5, 63) it is used with the optative of the future sphere, representing the subjunctive with ἄν of the oratio recta. ὅκως is confined almost entirely to the optative ; it occurs with the indicative but twice (7, 229 ; 9, 66).‡

In at least one passage (3, 125) ὅκως is used where it seems necessary to regard the temporal relation as that of co-extension, ὅκως being equivalent to ὁκότε: ἐλοῦτο μὲν γὰρ ὑπὸ τοῦ Διός, ὅκως ὕοι, κ.τ.λ.

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* Cf. Smyth, Ionic Dialect, p. 606.

† ὡς as a temporal conjunction in Thucydides has been discussed by Warren (op. cit., pp. 46 ff.), who concludes that in that author ὡς is a conjunction of antecedence.

‡ Stein is in error (note on 7, 229) in citing 1, 68, as an example of ὅκως = ὡς or ἐπείτε.

ὁσάκις.

ὁσάκις is used but once (1, 198), and expresses purely antecedent action.

ἀπ' ἧς, ἀπ' οὗ, ἐξ οὗ, ἐξ ὅτευ, ἐξ ὅσου.

All of these expressions, except ἀπ' ἧς. are used absolutely, and in each case the meaning is "since." ἀπ' ἧς always has ἡμέρη as an antecedent.

[ὥσπερ.]

ὥσπερ is given by Kühner (Gram.², II, p. 948) and Liddell & Scott (Lex., s. v. ὥσπερ) as a temporal conjunction. Kühner says expressly that ὥσπερ is used as a temporal conjunction in Herodotus; the only place to which he can refer is 6, 41: καὶ ὥσπερ ὁρμήθη [ὁ Μιλτιάδης] ἐκ Καρδίης πόλιος, (Kallenberg and most editors punctuate so) ἔπλεε διὰ τοῦ Μέλανος κόλπου· παραμείβετό τε τὴν Χερσόνησον καὶ οἱ Φοίνικές οἱ περιπίπτουσι τῇσι νηυσί. This is generally interpreted, I suppose, as Macaulay translates it: "and having set out from the city of Cardia, he was sailing," etc. But does ὥσπερ mean "after" here? Leaving aside this passage, no sure case of ὥσπερ as a temporal conjunction seems to occur in classical Greek. In Aristoph., Pax, 24, quoted by L. & S., ὥσπερ is clearly not temporal, but modal. In Soph., Oed. Col., 1361, ὥσπερ, if it be read, is not temporal but modal.*

A close parallel to our passage is found in Thuc., 8, 23, 1: Ἀστύοχος, . . . ὥσπερ ὥρμητο, πλέων ἐκ τῶν Κεγχρειῶν ἀφικνεῖται ἐς Χίον, which Jowett translates correctly, I think, "Astyochus set out from Cenchreae, *as he intended*, and arrived at Chios." Compare Herodotus, 7, 1: καὶ μᾶλλον ὥρμητο [Δαρεῖος] στρατεύεσθαι ἐπὶ τὴν Ἑλλάδα; and 5, 50: ὁ δὲ ὑπαρπάσας τὸν ἐπίλοιπον λόγον τὸν ὁ Ἀρισταγόρης ὥρμητο λέγειν. κ.τ.λ.; and Od., 4, 282:

νῶι μὲν ἀμφοτέρω μενεήναμεν ὁρμηθέντε
ἢ ἐξελθέμεναι ἢ ἔνδοθεν αἶψ' ὑπακοῦσαι.

If in our passage we take ὥσπερ in the sense of "as," and punctuate, not after πόλιος, but after ὁρμήθη, we may translate thus: "and, as he intended, he sailed from the city of Kardia through the gulf of Melas," etc. In view of the evidence, it seems much better to take ὥσπερ here as comparative rather than temporal.

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* Cf. Jebb, note ad loc.: "with ὥσπερ the sense is 'however I may live,'—whether my remaining life be less, or even more, wretched than now."

B. Conjunctions of Contemporaneity.

I. *Conjunctions of Simple Contemporaneity.*

῞τε, ὁπότε.

ὅτε is generally a conjunction of contemporaneity, but occasionally it is used in a clause of pure antecedence, as e. g., 4, 78: ὅτε δὲ διατρίψειε μῆνα ἢ πλέον τούτου, ἀπαλλάσσετο, κ.τ λ.; and similarly in 1, 191; 3, 57: 4, 110; 5, 16, 30, 92η; 6, 69, 77, 78. Furthermore, ὅτε is the conjunction which is used when the time of the main action is defined, not exactly but loosely, by stating some event which was connected with the action of the main verb. Cf. above, p. 200. In range and variety of its meanings ὅτε is the nearest equivalent of the English "when." ὅτε is used with the optative of generic action but once (4, 78), and there it is equivalent not to ὁκότε but to ὅκως.

MS. variation in one place should be noticed, 9, 75: ἔστι δέ καὶ ἕτερον Σωφάνεϊ λαμπρὸν ἔργον ἐξεργασμένον, ὅτε . . . ἄνδρα πεντάεθλον, ἐκ προκλήσιος ἐφόνευσε. ὅτε, R, Krüg., Kal.; ὅτι, ABCP, St., Ab., Baehr. For reasons similar to those given above, sp. 175 f.. on 2, 121ϵ, ὅτε is probably the right reading. Cf. 5, 92η, where ὅτι is read by P d z, but ὅτε is undoubtedly correct.

ὅταν.

εὖτε.

εὖτε like ὅτε is generally used of contemporaneous action, but in 4, 78: εὖτε ἀγάγοι τὴν στρατιὴν τὴν Σκυθέων ἐς τὸ . . . ἄστυ, . . . ἐς τούτους ὅκως ἔλθοι ὁ Σκύλης, κ.τ.λ., it is used of pure antecedence, as is shown by ὅκως ἔλθοι, which merely repeats the preceding clause.

ἦμος.

ἦμος, in the single place where it occurs, has as its correlative τηνικαῦτα.

Forms of ὅς.

The dative with ἐν (ἐν ᾧ), and the accusative (τόν) are used to introduce clauses of contemporaneity. If the reading ἀπήγαγε is right in 1, 164 (cf. above, p. 200), ἐν ᾧ is used once of antecedent action.

ἕως.

See below.

II. *Conjunctions of Limit.*

ἕως.

ἕως is used by Herodotus in clauses of contemporaneity 10 times: in nine cases it expresses limit, and means "as long as"; once only (3, 134) it expresses partial coextension and means "while."*

The particle δέ is found in the apodosis, after a clause with ἕως, in four passages: 1, 173; 4, 165; 9, 6, 70.† I quote one of these examples, 1, 173: ἕως μὲν δὴ αὐτῶν Ξαρπηδῶν ἦρχε, οἱ δὲ ἐκαλέοντο . . . Τερμιλαι· ὡς δὲ ἐξ Ἀθηνέων Λύκος . . . ἀπίκετο . . ., οὕτω δή, κ.τ.λ. Fuchs (l. i. c.) explains the μέν in these passages as having reference both to the δέ of the main clause and to the δέ used with the following temporal conjunction. As a matter of fact, there is almost no opposition in meaning between the μέν and the first δέ; the real contrast is between the μέν and the following temporal clause. The first δέ is inserted partly on account of the demonstrative οἱ (ἡ), which is used in each case, and partly on account of the tendency of Herodotus to introduce δέ in the apodosis where there is no true opposition. Buttmann ‡ cites a case exactly similar to ours, in 4, 126: εἰ μὲν γὰρ . . . , σὺ δὲ . . . , εἰ δὲ . . . σὺ δὲ καὶ οὕτω. κ.τ.λ., and says: "Inter particulam μέν igitur et primum δέ nulla hic est relatio; sed μέν spectat ad veram oppositionem quae est inter utrumque totius sermonis membrum referturque ad secundum δέ quod est in altera protasi."

ἔστε.

ἔστε is used, meaning "as long as," six times. Fuchs is clearly wrong in saying that ἔστε, 3, 53, means "until." It always refers in Herodotus to the future sphere, and in each case it is in oratio obliqua.

Forms of ὅσος.

The dative of ὅσος with ἐν, and the accusative of extent are used in clauses of contemporaneity to express limit. ἐν ὅσῳ is used absolutely once, 3, 150; ὅσον is used with χρόνον eight times, and alone only once, 7, 161.

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* Fuchs, op. cit., p. 74, is in error in saying that ἕως, 3, 134, means "as long as"; and has omitted examples of ἕως in 2, 57; 4, 165; and 9, 70.

† Cf. Sagawe, δέ im Nachsatz bei Herodot, Breslau, 1893, p. 9; and Fuchs, op. cit., p. 74.

‡ Demosthenis in Midian., Berolini, 1833, ed. alt., p. 155.

μέχρι, μέχρι ὅσου.

μέχρι is used, meaning "as long as," once, 4, 3 : *μέχρι μὲν γὰρ ὥρων ἡμέας ὅπλα ἔχοντας, οἱ δὲ ἐνόμιζον ὁμοῖοί τε καὶ ἐξ ὁμοίων ἡμῖν εἶναι· ἐπεὰν δὲ, κ.τ.λ.* The *μέν* here refers to the *δέ* after *ἐπεάν*; *δέ* is inserted after *οἱ* for the same reason as above, p. 216.

C. CONJUNCTIONS OF SUBSEQUENCE.

I. *Conjunctions of Subsequence, not expressing limit.*

πρίν, πρίν ἤ.

πρίν after an affirmative clause generally means "before"; rarely it means "until," as well as "before" (4, 1; 6, 119; 9, 13, 101).

The following correlative expressions are found with *πρίν* and *πρὶν ἤ*: *πρότερον* is used with *πρίν* three times (1, 82; 5, 106; 6, 110); with *πρὶν ἤ* four times (6, 45; 7, 8β; 8, 8; 9, 93); *οὔκω* (*οὔτι κω, μήκω*) occurs with *πρίν* three times (1, 32, 32_2; 6, 110); *ἐς τόδε*, once: *προ-βοηθῆσαι* is used before *πρίν*, 8, 144.

The following correlatives are found with *πρὶν ἤ*: *πρίν* (1, 165); *τὸ πάλαι* (4, 180); *προ-γεγονότες* (7, 3); *τότε* (9, 68); *φθάνω* (6, 116; 9, 70).

πρότερον ἤ.

πρότερον ἤ after an affirmative main clause expresses plain subsequence ("before"); after a negative main clause it expresses limit ("until"). *πρότερον ἤ* is preceded by *φθάνω* in 6, 91, and by *προ-αναισιμωμένῳ* in 2, 11.

II. *Conjunctions of Limit.*

ἄχρι οὗ.

ἐς ὅ.

ἐς ὅ is by far the favorite Herodotean expression for "until" after an affirmative main clause. In four passages, however, all MSS. give *ἐς οὗ* (4, 160, 181, 196; 5, 51), and once *ἕως οὗ* (2, 43); but in these places most editors prefer *ἐς ὅ*. In seven other places (1, 67; 3, 31; 4, 12, 30, 105, 166; 5, 86) *ἐς οὗ* is found in one or more MSS. Holder reads *ἐς οὗ* in only one place (1, 67)*; Kallenberg, van Herwerden, and Abicht read *ἐς ὅ* everywhere. Stein printed *ἐς ὅ* everywhere in 1869, but in his second text edition (1884) reads *ἐς οὗ* in 10 places, and *ἕως οὗ* in 2, 143.* Brugmann thinks there is no valid reason for departing from the MSS. where they all give ἐς οὗ or ἕως οὗ.† It is, of course, possible that ἐς οὗ arose from analogy with μέχρι οὗ and ἄχρι οὗ ‡; or ἐς οὗ may be related to ἐς ὅ, as εἰς Ἅιδου to εἰς Ἅιδην.§

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* Why Holder reads *ἐς οὗ* in 1, 67 where one MS. gives *ἐς ὅ*, but reads *ἐς ὅ* in all the places where all MSS. give *ἐς οὗ*, it is not easy to understand.

But yet it seems strange that after ἐς ὅ became fully established, meaning "until," even in Homer, another form, ἐς οὗ, should have been able to make its way against it, especially as ἐς ὅ was used by early writers far oftener than ἄχρι οὗ or μέχρι οὗ. εἰς ὅ, "until," is found in Homer 48 times; in Hesiod once; in the Homeric Hymns once; in Herodotus 53 times in all MSS. μέχρι οὗ is not found before Herodotus, and is used by him but seven times; ἄχρι οὗ is not found at all before Herodotus, and is used by him but once.‖ No occurrence of ἐς οὗ is quoted from any other classical writer, nor is it quoted in any Ionic inscription. Furthermore, it seems to me that it is a somewhat suspicious circumstance that of the 11 cases of the reading ἐς οὗ, seven are in Book 4, and two are among the first occurrences of ἐς ὅ and ἐς οὗ in Book 5. We know that ἕως οὗ became common in post-classical times,¶ and survives in modern Greek as ὥσου (or ὡσοῦ). Gregorius Corinthius, writing about 1150 A. D., says **: ἀντὶ τοῦ ἕως οὗ, ἐς ὃ λέγουσιν. Ἡρόδοτος· ἐς ὃ ἐξεῖλον; from which we may infer either that the expression ἕως οὗ was in common use in the living language when Gregorius wrote, or that it had been in common use at some time between the classical period and the time of Gregorius. From these facts we may perhaps infer that some one or other, to whom the form ἕως οὗ was well known, either carelessly or deliberately changed the form ἐς ὅ into ἐς οὗ (ἕως οὗ, 2, 143). Of course it is impossible to reach any certain conclusion, but on the basis of the evidence at hand it seems to me that ἐς ὅ is to be preferred.

ἔστε.

ἕως.

ἕως is used, meaning "until," three times. (See list below.)

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* See Stein, Herodotos, 5th ed., 1883, note on 1, 67, line 21. Cf. Smyth, Ionic Dialect, p. 453, footnote 4.

† Brugmann, Gr. Gram.³, p. 563.

‡ Cf. Smyth, op. cit., p. 608; Brugmann, op. cit., p. 563.

§ Cf. Brugmann, op. cit., p. 395.

‖ See Fuchs, op. cit., p. 130.

¶ Cf. Jannaris, Hist. Greek Gram., § 1785.

** Gregorius Cor., ed. Schaefer, § 63, p. 472.

μέχρι, μέχρι οὗ.

πρίν, πρὶν ἤ.

πρίν and πρὶν ἤ after a negative main clause express limit.

πρότερον ἤ.

See above.

V. Order of Clauses.

The general principles regarding the relative order of the main and the temporal clause have been stated above (p. 174). I shall give under this topic merely a statement of the facts with regard to the divergences in Herodotus from the normal order, and, where possible, the reason for such divergences.

A. *Antecedence.*

Generally the temporal clause precedes the main clause. Very frequently, however, some word or words, not the predicate verb, belonging to the main clause, are placed before the temporal conjunction. The word so placed is frequently the subject either of both clauses (1, 10, 76; 3, 76), or of the main clause only (1, 21); the object either of both verbs (4, 168), or of the main verb only (6, 19; 7, 77); or a dative case belonging to the main clause (1, 103). A genitive absolute sometimes precedes (1, 65) and sometimes follows (1, 12) the temporal clause. ὡς is placed with the verb at the very end of the temporal clause five times (3, 13, 54; 5, 18, 101; 8, 134).*

In clauses of antecedence the temporal clause stands after the verb of the main clause in the following passages: ὡς: 1, 12, 103; 3, 13, 21, 54, 76, 156; 6, 135; 7, 173; 8, 137; 9, 13, 115; ὅκως: 1, 186; 6, 121; 7, 41, 128; ἐπεί: 1, 60; 7, 176; 9, 58; ἐπεάν: 1, 182, 183; 2, 19, 29, 73, 97, 132, 153; 3, 72, 151; 4, 23, 68, 69, 72; 8, 144; ἐπείτε: 1, 14, 56; 2, 57; 3, 7, 156; 4, 43, 155; 5, 44; 6, 101; 7, 8*a*, 59; 9, 26, 107; forms of ὅς (ὅσος): 2, 15, 44, 44_2, 98.

Examination of these clauses shows that there are certain general principles which in most cases govern the deviation from the regular order. When the temporal clause specifically *defines* the time of the main clause, the action whose time is thus defined naturally stands first (2, 29, 73, 132; 3, 72, 151; 4, 68; 7, 173; 8, 144); and this relation is specially clear when the temporal conjunction or expression means *ex quo tempore* (1, 60; 2, 15, 44, 44_2, 98; 3, 7; 7, 8a, 59; 9, 26, 107).

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* Thomas, De Particulae ὡς Usu Herodoteo (1888), p. 51, has omitted, 3, 13; 5, 18; 8, 134.

When for *any* reason the main clause has special emphasis, it naturally takes precedence over the temporal clause (1, 14, 182; 2, 19; 3, 72, 156 (ὡς), 156 (ἐπείτε); 4, 153; 5, 44; 6, 101; 7, 137; 9, 13, 58, 115).

In some cases the reason for the less common order is stylistic. In 2, 153: ἐποίησε . . . προπύλαια . . . αὐλήν τε τῷ Ἄπι, ἐν τῇ τρέφεται ἐπεὰν φανῇ ὁ Ἆπις, κ.τ.λ. the temporal clause stands in a dependent relative clause. Sometimes there is parallel structure in main clauses, as in 4, 69: πολλοὶ μὲν δὴ συγκατακαίονται τοῖσι μάντισι βόες, πολλοὶ δὲ περικεκαυμένοι ἀποφεύγουσι, ἐπεὰν αὐτῶν ὁ ῥυμὸς κατακαυθῇ. Cf. also 2, 56; 3, 72; 6, 135.

In some cases the temporal meaning is wholly incidental. In 4, 23 (ἐπεὰν . . . περικαλύψῃ), the temporal clause really expresses the manner more than the time, as is shown by the corresponding phrase, ἄνευ πίλου. A noteworthy case is found in 4, 72 (ἐπεὰν . . . διελάσωσι), where the adverb ὧδε is explained by the temporal clause. In 1, 182, and 4, 23, there is ellipsis of the main verb. Occasionally, as would be expected, no special reason can be given for the irregular position, as in 4, 43.

B. *Contemporaneity.*

Irregular order is found in clauses of contemporaneity in the following passages: ὅτε: 1, 160; 2, 121ε, 156, 165, 182; 3, 131; 6, 41, 65; 7, 215; 8, 115; 9, 8, 75; εὖτε: 7, 193, 209; ἕως: 3, 134; ἔστε: 3, 53; 8, 142; μέχρι ὅσου: 8, 3; forms of ὅς (ὅσος): 1, 21; 3, 105; 7, 73.

On the whole we find in Herodotus a high degree of uniformity and regularity in regard to the order of clauses of contemporaneity. In this respect he differs sharply from Thucydides.* Out of 30 clauses of contemporaneity introduced by relatives (ὅς, ὅσος), only three show modification of the regular order. In the case of ὅτε deviation is more frequent.

The causes which bring about modification are in general the same as in clauses of antecedence. The temporal clause is epexegetic in 1, 160; 2, 121ε; 7, 144; 9, 75. It is regular for the temporal clause to be placed after a participle whose time it defines (2, 165, 166; 7, 193; 8, 115).

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* Cf. Warren, op. cit., pp. 53 ff.

C. *Subsequence.*

Clauses of subsequence show abnormal order in the following passages: *πρίν*, with infin.: 1, 46, 71, 76; 3, 25; 4, 93, 180; 6, 87, 119; 8, 144; *πρίν*, with subj.: 1, 32; *πρίν ἤ*, with infin.: 1, 92, 170; 2, 1; 4, 1, 167; 6, 22; 8, 12, 59; 9, 13, 13_2, 48, 68; *πρὶν ἤ*, with subj.: 1, 136; *ἔστε*: 7, 158; *ἐς ὅ*: 1, 93; *μέχρι*: 4, 119.

Out of 43 cases of *πρίν* or *πρὶν ἤ* with the infinitive, the temporal clause precedes the main clause 21 times.* The reasons which cause the infinitive clause to be placed before the main clause, or within it, but before the leading verb, are in general the same as those given above. The temporal clause precedes, if it is emphatic (1, 46, 71; 4, 93; 6, 22, 119; 8, 144; 9, 13, 48, 68), or if it is closely connected in thought with the preceding sentence (1, 76, 78, 92; 3, 127; 4, 1, 167). Sometimes the *πρίν* clause is put first to avoid ambiguity (2, 2); sometimes to bring *τάδε* or *τοιόνδε* at the end of a sentence (4, 180; 6, 87). In 8, 12, and 6, 119, the order is probably due to the parallel arrangement of clauses.

Clauses with *πρότερον ἤ* always stand after the main clause. In 7, 158: *σῖτόν τε ἀπάσῃ τῇ Ἑλλήνων στρατιῇ ἔστ᾽ ἂν διαπολεμήσωμεν, ὑποδέκομαι παρέξειν*, the clause with *ἔστε* is connected closely with *σῖτον*, and is virtually a modifying expression co-ordinate with *τῇ Ἑλ. στρατιῇ*. Clauses with *ἐς ὅ* follow the main clause except in 1, 93: *τοῦ γὰρ δὴ Λυδῶν δήμου αἱ θυγατέρες πορνεύονται πᾶσαι, συλλέγουσαι σφίσι φερνάς, ἐς ὃ ἂν συνοικήσωσι τοῦτο ποιέουσαι*, but here the participle *ποιέουσαι* is little more than a repetition of *πορνεύονται*. In 4, 119, the clause with *μέχρι* is emphatic.

VI.

Below is given a classified list of the temporal clauses in Herodotus. This list is based upon Kallenberg's revision of Dietsch's Herodotus, Leipzig, imprint of 1898, although in some cases, as has been stated above, I should prefer a different reading. In some cases it is impossible to draw a sharp line of demarcation between temporal and causal clauses; not infrequently both ideas are present in the same clause. In making this list I have aimed to include all clauses in which the temporal idea is clearly discernible.

Since the verb *εἰμί* cannot be classed as exclusively either extensive or aoristic, forms of the imperfect indicative and the present subjunctive and optative have been specified where they occur.

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* Sturm's statement (op. cit., pp. 295 f.), "Schliesslich bemerken wir hinsichtlich der Stellung des Infinitivsatzes nach *πρίν* und *πρὶν ἤ*, dass derselbe seinem Hauptsatze meistens vorangeht," is clearly wrong.

ἄχρι οὗ.

Future sphere. Subordinate verb:

(*a*) Subjunctive aorist: 1, 117.

ἐπεί.

Past sphere. Subordinate verb:

(*a*) Indicative, historical present: 5, 55; 9, 2.

(*b*) Indicative imperfect: 1, 10; 2, 45, 52, 120; 3, 41, 53,* 57; 4, 146; 5, 67; 6, 14; 7, 44,† 53,‡ 168, 211; 8, 4, 37,§ 37_2,‖ 67, 68, 70, 107,‖ 137; 9, 13, 18, 26, 56, 58, 98,¶ 102, 117.

(*c*) Indicative aorist: 1, 5, 60, 110, 190 209; 2, 121β; 3, 9, 25, 70, 78; 4, 110, 112, 113, 124; 5, 34, 92γ; 6, 96, 137; 7, 1, 44, 44_2,** 61,†† 100,†† 128,†† 164,* 176, 223, 239; 8, 42, 69, 89, 108, 116, 127; 9, 4, 45, 48, 96, 107, 108,†† 116.

(*d*) Indicative pluperfect: 1, 116; 3, 53**; 7, 164†; 8, 53; 9, 17.

(*e*) Optative present: 3, 27.‡‡

(*f*) Infinitive present: 4, 33; 7, 150.

(*g*) Infinitive aorist: 4, 10.

(*h*) Infinitive perfect: 5, 84.

ἐπεάν.

(1) Present sphere. Subordinate verb:

(*a*) Subjunctive present: 1, 183, 199; 2, 19, 29, 41,† 60, 93, 93_2, 132. 149, 149_2, 173; 3, 72, 108, 109,†† 117; 4, 30, 33, 68, 190; 5, 18; 6, 84; 7, 85, 209, 226; 8, 104.

(*b*) Subjunctive aorist: 1, 74, 132, 182, 182_2, 193, 194, 194_2††, 194_3, 199, 216; 2, 14,†† 14_2, 37, 40, 40_2, 41,§§ 46, 47, 60, 60_2, 62, 66, 68,†† 70, 70_2, 73, 73_2, 78, 85, 86, 86_2, 87, 87_2, 89, 89_2, 91, 92,†† 92_2, 92_3, 92_4,†† 93, 93_2, 93_3, 94, 96, 97, 97_2, 123, 153, 173; 3, 24, 27, 96, 98,†† 105, 110. 117; 4, 2, 2_2, 22, 23, 23_2, 26, 61, 61_2, 62. 64, 67, 67_2, 68, 69,†† 69_2, 71, 71_2, 71_3, 72, 72_2, 73, 73_2, 75, 168, 172, 172_2, 180, 187, 188, 195, 196.†† 198; 5, 5; 6, 58, 58_2, 58_3, 58_4; 7, 9β, 10α, 10ϵ, 129.

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* Also the pluperfect.

† Also the aorist.

‡ *παρῆσαν.*

§ Twice, once *ἦσαν.*

‖ *ἦσαν.*

¶ Three times.

** Also the imperfect.

†† Twice.

‡‡ *παρείη.*

§§ Also the present.

(2) Future sphere. Subordinate verb:

(*a*) Subjunctive present: 1, 9*; 3, 21, 69,* 134, 146; 7, 146†; 8, 22, 76; 9, 51,‡ 98.

(*b*) Subjunctive aorist: 1, 9,§ 21, 209; 3, 69,§ 151, 153; 4, 3, 9, 9_2, 89, 98, 111, 134; 5, 23, 35, 107; 6, 19, 139; 7, 8δ, 77; 8, 60, 80, 144.

ἐπειδάν.

(1) Future sphere. Subordinate verb:

(*a*) Subjunctive aorist: [8, 144].‖

ἐπειδή.

(1) Present sphere. Subordinate verb:

(*a*) Indicative perfect: 4, 118; 7, 16β, 158.¶

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 1, 67; 2, 119**; 3, 72 [6, 122]††; 7, 107, 160, 181; 8, 37, 76; 9, 72, 90.

(*b*) Indicative aorist: 1, 84; 2, 2; 3. 142; 5, 22, 42,¶ 98, 99; 6, 16; 7, 36, 183, 207; 8, 31, 66, 87, 110, 110_2, 114, 118; 9, 76.

(*c*) Indicative pluperfect: 4, 83; 8, 129.

(*d*) Infinitive present and aorist: 3, 26.

ἐπείτε.

(1) Present sphere. Subordinate verb:

(*a*) Subjunctive aorist: 1, 200, 202.¶

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 1, 50, 70, 70_2,* 105, 115,‡‡ 119,‡‡ 126,§§ 165, 186, 189; 2, 57, 103, 107, 150, 162, 181, 181_2; 3, 11, 26, 36, 50, 58, 69, 78, 131, 133**; 4, 43, 76, 79,* 123, 139, 148, 151,** 173; 5, 20, 36, 44, 62,¶ 86, 91, 99*; 6, 18, 23, 91, 101; 7, 23, 59, 169, 188,* 211; 8, 65, 121, 125; 9, 26, 69, 118.

(*b*) Indicative aorist: 1, 14, 27, 34,¶ 35, 43, 45, 48, 54, 55, 58, 62, 70,‖‖ 74, 75, 76, 83, 92, 96, 100,¶ 105, 111, 114, 117, 117_2, 166, 168, 178, 196¶; 2, 30, 125, 134,¶ 162, 169, 174; 3, 3,¶ 7, 13, 15, 16, 16_2, 20, 25, 45, 46, 49, 50, 68, 71, 77, 80,¶ 85, 119, 135, 150, 157, 159; 4, 3, 42, 43,

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* Also the aorist.

† ἔωσι.

‡ ᾖ

§ Also the present.

‖ See above, p. 213.

¶ Twice.

** ἦν.

†† Most editors regard this passage as spurious.

‡‡ παρῆσαν.

§§ ἦσαν.

‖‖ Also the imperfect.

79, 79_2,* 80, 85, 115, 118, 146, 147, 153, 154, 155, 155_2, 165, 179; 4, 202; 5, 12, 12_2, 18, 24,† 33, 34, 38, 46, 50,† 63, 67, 91, 92 ζ, 93, 96, 98, 99,* 100, 115; 6, 13, 17, 43, 76, 82, 112, 118, 119, 137; 7, 8*a*, $8a_2$, 26, 28, 29, 38, 43, 56, 156, 163, 172, 188,* 220, 239; 8, 27, 64, 111; 138; 9, 7β, 12, 12_2, 18, 22, 46, 47, 93, 106, 107.

(*c*) Indicative pluperfect: 1, 185; 4, 200; 7, 175; 8, 6, 27; 9, 84.

(*d*) Optative present: 2, 181.

(*e*) Infinitive present: 1, 94,† 2, 42; 8, 135.‡

(*f*) Infinitive aorist: 2, 118; 6, 84, 137; 8, 135.

(3) Past and present spheres combined. Subordinate verb:

(*a*) Indicative present: 3, 117, 117_2.

(*b*) Indicative perfect: 4, 7.

(4) Future sphere. Subordinate verb:

(*a*) Subjunctive aorist: 8, 22.

ἐς ὅ.

(1) Present sphere. Subordinate verb:

(*a*) Indicative present: 3, 104; 4, 181.†

(*b*) Subjunctive present: 4, 196.

(*c*) Subjunctive aorist: 1, 93; 3, 31,† 82; 4, 30; 6, 86 γ.§

(*d*) Infinitive present: 1, 202.†

(2) Past sphere. Subordinate verb:

(*a*) Indicative aorist: 1, 67, 102, 115, 130, 191; 2, 103, 118, 143, 143_2, 150, 157; 3, 136, 158; 4, 12, 105, 122, 123, 166, 201, 203, 203_2; 5, 51,‖ 92 γ, 92 ζ, 97; 6, 15, 24, 40,† 75,† 83, 83_2, 99, 140; 7, 59, 113, 181, 225 †; 8, 58; 9, 62.

(*b*) Indicative imperfect: 1, 158, 196; 3, 48; 4, 160, 160_2 ¶; 6, 113 †; 9, 55.‖

(*c*) Indicative pluperfect: 5, 51 ‡; 9, 55.*

(*d*) Indicative historical present: 1, 98; 6, 75.

(*e*) Infinitive aorist: 1, 94; 2, 102; 5, 85, 86.

(3) Future sphere. Subordinate verb:

(*a*) Indicative future: 9, 58.

(*b*) Subjunctive present: 2, 115.

(*c*) Subjunctive aorist: 4, 43; 8, 108.

---

* Also the imperfect.

‡ Also the aorist.

‖ Also the pluperfect.

† Twice.

§ *εἰς ὅ*; in hexameter verse.

¶ *ἐγίνετο*, but see above, p. 207.

ἔστε.

A. "Until." (1) Future sphere. Subordinate verb:

(*a*) Subjunctive aorist: 7, 141, 158; 8, 4.

(2) Present and past spheres combined. Subordinate verb:

(*a*) Infinitive present: 7, 171.

B. "As long as." (1) Future sphere. Subordinate verb:

(*a*) Subjunctive present: 3, 53; 4, 201; 7, 61, 143, 144.

(*b*) Subjunctive perfect: 8, 142.*

εἶτε.

(1) Present sphere. Subordinate verb:

(*a*) Subjunctive present: 2, 63; 6, 27.

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 7, 193, 209.

(*b*) Optative aorist: 4, 78.

ἕως.

A. "Until." (1) Past sphere. Subordinate verb:

(*a*) Indicative aorist: 7, 23, 100.

(2) Future sphere. Subordinate verb:

(*a*) Infinitive present: 4, 42.

B. "As long as," or "while." (1) Present sphere. Subordinate verb:

(*a*) Indicative present: 3, 134.

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 1, 62, 173; 2, 57; 3, 25; 4, 165; 9, 6, 23, 70, 102.

ἦμος.

Present sphere. Subordinate verb:

Indicative present: 4, 28.

μέχρι.

A. "Until." (1) Future sphere. Subordinate verb:

(*a*) Subjunctive aorist: 4, 119.

B. "As long as." (1) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 4, 3.

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* συνεστήκῃ.

μέχρι ὅσου.

Past sphere. Subordinate verb:
(*a*) Indicative imperfect : 8, 3.

μέχρι οὗ.

A. "Until." (1) Past sphere. Subordinate verb:
(*a*) Indicative aorist: 1, 187; 5, 28; 7, 60, 225; 8, 100.
(*b*) Optative aorist: 2, 179.

ὁκότε.

Past sphere. Subordinate verb :
Optative present: 2, 120.

ὅκως.

(1) Past sphere. Subordinate verb:
(*a*) Indicative aorist: 7, 229.
(*b*) Indicative imperfect: 9. 66.
(*c*) Optative present: 1, 11, 17, 68, 171, 175, 186; 2, 108, 125, 133, 174; 3, 125; 4, 42, 43; 6, 77; 7, 41, 128.
(*d*) Optative aorist: 1, 17, 100, 162, 186, 196, 196_2; 2, 13, 137; 3, 51, 148, 148_2; 4, 78, 78_2,* 129, 130, 130_2; 5, 29, 92ζ; 6, 31, 61, 75, 121, 137; 7, 6, 119, 119_2, 211; 8, 52, 90, 91, 105, 128; 9, 74, 116.
(*e*) Infinitive present: 2, 140.
(2) Future sphere. Subordinate verb:
Optative aorist : 5, 63.

Forms of ὅς (ὅστις).

ἀπ' ἧς. (1) Past sphere. Subordinate verb:
Indicative aorist: 1, 1; 3, 14; 7, 192.
(2) Future sphere. Subordinate verb:
Subjunctive aorist : 3, 155.
ἀπ' οὗ. Present and past sphere. Subordinate verb:
Indicative present: 2, 44.
ἐξ οὗ: (1) Past sphere. Subordinate verb:
(*a*) Indicative aorist: 2, 15; 6, 109.
(*b*) Indicative perfect: 2, 44.
ἐν ᾧ. (1) Present sphere. Subordinate verb:

---

* Twice.

Indicative present: 1, 164; 4, 139.

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 1, 186; 3, 19, 74, 136; 4, 95, 124; 5, 108*; 6, 97; 7, 26; 9, 23, 102.

(*b*) Indicative aorist: 1, 164; 5, 108.†

(*c*) Infinitive present: 3, 105.

τόν. Present sphere. Subordinate verb:

Subjunctive present: 1, 67.

ἐν τῇ. Present sphere. Subordinate verb:

Subjunctive aorist: 2, 47.

ἐξ ὅτευ. Past sphere. Subordinate verb:

Indicative aorist: 3, 63.

Forms of ὅσος.

ἐξ ὅσου. Past and present spheres. Subordinate verb:

Indicative present: 2, 98.

ὅσον. (1) Past sphere. Subordinate verb:

Indicative imperfect: 3, 48, 133‡; 5, 84; 6, 128; 7, 73,§ 94, 161; 8, 29; 9, 63.‖

(2) Future sphere. Subordinate verb:

Subjunctive present: 1, 21; 4, 201.

ἐν ὅσῳ. Past sphere. Subordinate verb:

Indicative imperfect: 1, 174; 3, 150.¶

ὁσάκις.

Present sphere. Subordinate verb:

Subjunctive aorist: 1, 198.

ὅταν.

(1) Present sphere. Subordinate verb:

Subjunctive aorist: 5, 16.

(2) Future sphere. Subordinate verb:

(*a*) Subjunctive present: 5, 93; 7, 10 δ; 8, 20.

(*b*) Subjunctive aorist: 1, 55, 191; 3, 57; 5, 30; 6, 77,** 78; 8, 77; 9, 43.

---

* Twice; also the aorist.

† Also the imperfect twice.

‡ ἦν.

§ ἦσαν.

‖ περιῆν.

¶ Also the aorist.

** Twice.

ὅτε.

(1) Present sphere. Subordinate verb:

Indicative present: 9, 122.

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 1, 114,* 145, 160†; 2, 13, 174,* 182; 3, 57, [131]‡; 4, 15; 5, 99; 6, 41, 106; 7, 8β, 20, 39, 144, 215§; 8, 115; 9, 39.

(*b*) Indicative aorist: 1, 103; 2, 165, 166; 5, 30, 76, 76_2, 76_3, 92η; 6, 46, 65, 69; 7, 10γ; 9, 8, 75.

(*c*) Indicative pluperfect: 7, 158.

(*d*) Optative aorist: 2, 121ε; 4, 78.

πρίν.

A. Followed by the infinitive.

Subordinate verb:

(*a*) Infinitive aorist: 1, 46, 71, 76; 2, 121ε; 3, 127; 4, 93, 117, 179, 180; 6, 87, 109, 119; 9, 9, 101.

(*b*) Infinitive present: 8, 144.

(*c*) Infinitive perfect: 3, 25.

B. Followed by a finite verb.

(1) Present sphere. Subordinate verb:

(*a*) Subjunctive aorist: 1, 140, 198; 3, 109; 4, 117, 196.

(*b*) Subjunctive present: 1, 197.

(2) Past sphere. Subordinate verb:

(*a*) Indicative, historical present: 9, 22.

(*b*) Indicative aorist: 1, 14; 6, 79, 111; 7, 239.

(3) Future sphere. Subordinate verb:

Subjunctive aorist: 1, 32, 32_2, 82, 159; 4, 157; 5, 106; 6, 82‖; 7, 220.

πρὶν ἤ.

A. Followed by the infinitive.

Subjunctive verb:

(*a*) Infinitive aorist: 1, 78, 79, 92, 165, 170; 2, 2; 4, 1, 125, 134,‖ 147, 167, 180; 5, 65,‖ 72; 6, 22; 7, 3, 94, 226; 8, 12, 59; 9, 13, 13_2, 48,‖ 68, 70.

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* ἦν.

† Three times.

‡ This passage is bracketed by Kallenberg and Abicht.

§ ἦσαν.

‖ Twice.

(*b*) Infinitive present: 6, 116; 8, 3.

B. Followed by the finite verb.

(1) Present sphere. Subordinate verb:

Subjunctive aorist: 1, 136; 7, 10η.

(2) Past sphere. Subordinate verb:

Indicative aorist: 6, 45; 7, 137; 8, 8.

(3) Future sphere. Subordinate verb:

Subjunctive aorist: 1, 19; 4, 9; 6. 133; 7, 8β*; 9, 93, 117.*

πρότερον ἤ.

A. Followed by the infinitive.

(*a*) Infinitive present: 7, 228.

(*b*) Infinitive aorist: 1, 72; 2, 11. 44†; 5, 118; 6, 91; 7, 2. 150; 9, 16.

B. Followed by a finite verb.

(1) Present sphere. Subordinate verb:

Subjunctive aorist: 1, 199; 4, 196.

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 8, 7.

(*b*) Indicative aorist: 8, 93*; 7. 175.

(3) Future sphere. Subordinate verb:

Subjunctive aorist: 7, 54; 9. 86. 87.*

ὥς.

(1) Present sphere. Subordinate verb:

(*a*) Subjunctive present: 1, 132.

(*b*) Subjunctive aorist: 4, 172.

(2) Past sphere. Subordinate verb:

(*a*) Indicative imperfect: 1, 10, 13,‡ 47,§ 48,‖ 62, 65. 67. 77, 80, 84, 86, 87, 112, 117,¶ 119, 126.‖ 127, 163, 186,** 189, 190,§ 205. 209; 2, 2. 2_2,†† 118,* 121δ,* 121δ_2,†† 121δ_3, 121δ_4, 160, 162; 3, 1, 14, 25, 56.‡‡ 83, 85, 86, 154; 4, 1,§§ 80, 95. 124, 126,* 134, 154, 155; 5, 1, 12, 18, 34,‖‖ 63, 67, 72, 79, 91,§ 92 δ, 92 η, 104, 112; 6, 9,¶ 14,* 16, 29, 29_2,* 35, 50, 77, 86, 87, 90, 107, 109,* 110, 112,‡‡ 129; 7, 8. 16, 17, 37,‡‡ 38, 44, 45, 54, 107,¶¶ 115, 140, 142, 146,§ 166, 166_2, 173, 181,*** 197,

---

* Twice.

† πρότερα.

‡ Twice, once ἦσαν.

§ Also the aorist.

‖ παρῆσαν.

¶ παρῆν.

** ἦσαν; also the pluperfect.

†† ἦν.

‡‡ Also the pluperfect.

§§ ἀπῆσαν.

‖‖ Also the pluperfect twice.

¶¶ ἐνῆν.

*** ἦν; also the aorist.

210,* 211, 212, 218; 8, 3, 4, 13, 14, 16, 41, 44, 67, 75, 75_2, 94, 101, 106, 108, 123, 126,† 129 ‡; 9, 16,§ 19, 20, 34, 34_2, 38, 39, 44,‡ 47, 49, 52,‡ 55, 60, 73,‖ 82,¶ 82_2, 91,¶ 108,* 119.

(*b*) Indicative aorist: 1, 10, 11, 12, 19, 21, 22, 31, 31_2,* 47,** 48, 50, 53, 56, 62, 65, 66, 67, 68, 75, 76,* 77, 79, 80, 82_2, 80_3, 85, 90, 98, 103, 108, 109, 111, 111_2, 112, 113, 117, 119, 119_2, 122, 127, 128, 141, 152, 154, 158, 160, 160_2, 162, 164, 169, 172, 173, 173_2, 176, 190,** 211, 213,* 213_2,* 214; 2, 30, 102, 104, 111, 113, 119, 121*a*, 121ϵ $121\epsilon_2$, 121ζ, 122, 135, 138, 151, 152, 152_2. 181 *; 3, 3, 13, 14, [14_2],††, 16, 19, 21, 22,* 29, 41, 42,* 42_2, 54, 64, 64_2, 66, 66_2, 74, 75, 76, 78, 83, 127, 128, 129, 130, 134, 140,‡‡ 143, 145, 145_2, 153,* 156; 4, 76, 76_2, 87, 89, 91, 92, 96, 97, 113, 120, 122, 122_2, 125, 125_2, 133, 136, 143, 205; 5, 2, 11, 12, 13, 15, 17, 18, 23, 29, 29_2, 32, 32_2, 33, 35, 38, 41, 46, 56, 65, 69, 72, 89, 91,** 92γ, $92\gamma_2$, 92η, 101, 103, 104, 105, 105_2, 112, 116, 122; 6, 23, 30, 30_2, 31, 32, 43, 43_2,* 69, 69_2, 73, 80, 85, 95, 99, 99_2, 103, 111, 112, 118, 119, 129, 129_2, 135, 138, 140; 7, 7, 8*a*, 14, 18, 18_2, 31, 35, 39, 43, 57, 101,* 102, 115, 119, 122,* 122_2, 127, 128, 136,* 136_2, 146,* 146_2, 146_3, 146_4,** 147, 155, 155_2, 156, 167, 169, 173, 173_2, 177, 181,** 182, 189, 192, 193,‡ 194, 208, 210, 218, 225, 233, 233_2; 8, 4, 8, 11, 14, 18,* 21, 24, 32, 49, 53, 55, 56, 59, 64, 67, 71, 71_2, 79, 87,* 90, 92, 97, 101, 106, 106_2, 106_3, 107, 109, 117, 125, 128, 130, 132, 132_2, 134, 137, 138, 140, 142; 9, 1, 5, 7, 8, 13, 17, 18, 19, 19_2, 20, 23, 25,* 31, 39, 47, 49, 52, 54, 55, 58,* 61, 62, 63,* 65, 67, 70, 70_2, 73,** 85, 86, 86_2 88, 88_2, 93, 93_2, 96, 98, 99, 101, 103, 109, 111, 115, 118, 120.

(*c*) Indicative pluperfect: 1, 11, 27, 80, 118, 141, 165, 186,** 186_2*; 201, 214; 2, 2, 100; 3, 56,** 69, 142, 157; 5, 20, 34 §§; 6, 9,** 112 **; 7, 37,** 55, 100, 157, 183, 193,‖ 232; 8, 5, 12, 96, 110, 113, 126,* 129 **; 9, 11, 17, 33. 37, 41, 44,** 52,** 62, 76, 100, 102.

(*d*) Optative present; 1, 196; 7, 119.

(*e*) Optative aorist: 1, 17, 196.

(*f*) Infinitive present: 1, 24, 94; 2, 121β, 121 δ,‖‖ 121 ϵ, 140; 3, 87; 8, 94.

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* Twice.

† Twice; once ἦν.

‡ Also the pluperfect.

§ ἦσαν.

‖ Also the aorist.

¶ ἦν.

** Also the imperfect.

†† The MSS. read here: καὶ ταῦτα ὡς ἀπενειχθέντα ὑπὸ τούτου. Abicht's addition of ἤκουσε is probably right.

‡‡ Three times.

§§ Twice; also the imperfect.

‖‖ Also the aorist twice.

(*g*) Infinitive aorist: 1, 86, 141; 2, 107, 121 β, 121 β_2, 121 δ, 121 δ_2,* 162; 3, 35; 4, 8, 9, 11, 179; 6, 137; 7, 148, 170; 8, 94, 118.

(*h*) Infinitive perfect: 2, 99.

Statistical Summary.

Below will be found a statistical summary of the temporal clauses in Herodotus. The numbers are based upon the preceding classified list; bracketed passages are not counted.

ἄχρι οὗ. — Followed by subjunctive aorist, 1.

ἐπεί. — Followed by indicative, historical present, 2; by indicative imperfect, 28; by indicative aorist, 39; by indicative pluperfect, 3; by indicative imperfect and aorist, 1; by indicative imperfect and pluperfect, 1; by indicative aorist and pluperfect, 1; by optative present, 1; by infinitive present, 2; by infinitive aorist, 1; by infinitive perfect, 1.

ἐπεάν. — Followed by subjunctive present, 33; by subjunctive aorist, 118; by subjunctive present and aorist, 3.

ἐπειδάν. — [Followed by subjunctive aorist, 1.] †

ἐπειδή. — Followed by indicative imperfect, 10; by indicative aorist, 19; by indicative perfect, 3; by indicative pluperfect, 2; by indicative present and aorist, 1.

ἐπείτε. — Followed by indicative present, 2; by indicative imperfect, 52; by indicative aorist, 126; by indicative perfect, 1; by indicative pluperfect, 6; by optative present, 1; by subjunctive aorist, 3; by infinitive present, 2; by infinitive aorist, 3; by infinitive present and aorist, 1.

ἐς ὅ. — Followed by indicative, historical present, 2; by indicative present, 2; by indicative imperfect, 6; by indicative aorist, 38; by indicative imperfect and pluperfect, 1; by indicative aorist and pluperfect, 1; by indicative future, 1; by subjunctive present, 2; by subjunctive aorist, 7; by infinitive present, 1; by infinitive aorist, 4.

ἔστε. — "Until," followed by subjunctive aorist, 3; by infinitive present, 1; "as long as," followed by subjunctive present, 5; by subjunctive perfect, 1.

εὖτε. — Followed by indicative imperfect, 2; by subjunctive present, 2; by optative aorist, 1.

ἕως. — "Until," followed by indicative aorist, 2; by infinitive present, 1; "as long as," followed by indicative imperfect, 9; "while," followed by indicative present, 1.

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* Twice; also the present.

† Cf. above, p. 213.

ἦμος. — Followed by indicative present, 1.

μέχρι. — "Until," followed by subjunctive aorist, 1; "as long as," followed by indicative imperfect, 1.

μέχρι οὗ. — Followed by indicative aorist, 5; by optative aorist, 1.

μέχρι ὅσου. — Followed by indicative imperfect, 1.

ὁκότε. — Followed by optative present, 1.

ὅκως. — Followed by indicative imperfect, 1; by indicative aorist, 1; by optative present, 16; by optative aorist, 35; by infinitive present, 1.

Forms of *ὅς* (*ὅστις*). — *ἀπ' ἧς*: followed by indicative aorist, 3; by subjunctive aorist, 1; *ἀπ' οὗ*: followed by indicative present, 1; *ἐξ οὗ*: followed by indicative aorist, 2; by indicative perfect, 1; *ἐν ᾧ*: followed by indicative present, 2; by indicative imperfect, 10; by indicative aorist, 1; by indicative imperfect and aorist, 1; by infinitive present, 1; *τόν*: followed by subjunctive present, 1; *ἐν τῇ*: followed by subjunctive aorist, 1; *ἐξ ὅτευ*: followed by indicative aorist, 1.

Forms of *ὅσος.* — *ἐξ ὅσου*: followed by indicative present, 1; *ὅσον*: followed by indicative imperfect, 9; by subjunctive present, 2; *ἐν ὅσῳ*: followed by indicative imperfect, 1; by indicative imperfect and aorist, 1.

ὑσάκις. — Followed by subjunctive aorist, 1.

ὅταν. — Followed by subjunctive present, 3; by subjunctive aorist, 9.

ὅτε. — Followed by indicative present, 1; by indicative imperfect, 18; by indicative aorist, 14; by indicative pluperfect, 1; by optative aorist, 2.

πρίν. — Followed by infinitive present, 1; by infinitive aorist, 14; by infinitive perfect, 1; by indicative, historical present, 1; by indicative aorist, 4; by subjunctive present, 1; by subjunctive aorist, 13.

πρὶν ἤ. — Followed by infinitive present, 2; by infinitive aorist, 25; by indicative aorist, 3; by subjunctive aorist, 8.

πρότερον ἤ. — Followed by infinitive present, 1; by infinitive aorist, 8; by indicative imperfect, 1; by indicative aorist, 2; by subjunctive aorist, 5.

ὡς. — Followed by indicative imperfect, 121; by indicative aorist, 307; by indicative pluperfect, 35; by indicative imperfect and aorist, 6; by indicative aorist and pluperfect, 1; by indicative imperfect and pluperfect, 8; by subjunctive present, 1; by subjunctive aorist, 1; by optative present, 2; by optative aorist, 2; by infinitive present, 7; by infinitive aorist, 17; by infinitive perfect, 1; by infinitive present and aorist, 1.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF
HARVARD UNIVERSITY.

New Series. — No. XXXI.

I. *DESCRIPTIONS OF SPERMATOPHYTES FROM THE SOUTHWESTERN UNITED STATES, MEXICO, AND CENTRAL AMERICA.*

By J. M. Greenman.

II. *DIAGNOSES AND NOTES RELATING TO AMERICAN EUPATORIEAE.*

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Presented by B. L. Robinson, April 12, 1905. Received June 6, 1905.

I. DESCRIPTIONS OF SPERMATOPHYTES FROM THE SOUTHWESTERN UNITED STATES, MEXICO, AND CENTRAL AMERICA.

By J. M. Greenman.

Tradescantia fluminensis, Vell. Fl. Flum. 140, t. 152 (1790); C. G. Clarke in DC. Monogr. iii. 294. Specimens of this white-flowered South American species were secured by Señor Professor A. Dugès in the vicinity of Guanajuato, Mexico. 1904, no. 3 (hb. Gr.). According to Professor Dugès the plant is not cultivated there, but is found growing wild in moist shady places about the city of Guanajuato, although it is rare. The species does not seem to have been recorded hitherto from Mexico.

Hymenocallis riparia, n. sp. Leaves lorate, 2 to 6 dm. long, 7 to 14 mm. broad, entire, bordered with a narrow whitish subcartilaginous margin, glabrous, their bases expanded, persistent and forming a membranous collar 10 to 12 cm. in length: peduncle 3 to 3.5 dm. long, flattened: spathe-valves 2, lance-attenuate, 3.5 to 6 cm. long, thin and scarious: flowers 3 to 7, sessile: perianth white; tube of the perianth 8 to 12 cm. in length; divisions of the perianth nearly or quite equalling the length of the tube, 3 to 5 mm. in breadth, 5–7-nerved: staminal cup broad-funnelform, 2 to 2.5 cm. high, subentire; free portion of the filaments 5 to 6 cm. long, slightly broadened at the base, gradually tapering to the 1.5 cm. long verticillate anther, greenish: style much elongated, greenish; stigma overtopping the anthers: ovary sessile, bearing 3 to 7 ovules in each cell, these ascending from the lower inner angle of the cell and sometimes subsuperposed. — Mexico. State of Morelos: by streams near Cuernavaca, 11 November, 1895, *C. G. Pringle*, no. 5979 (hb. Gr.), and in the same locality, 27 July, 1896 *C. G. Pringle*, no. 7204 (hb. Gr.); rocky river banks, barranca of Cuernavaca, altitude

1735 m., 22 July, 1904, *C. G. Pringle*, no. 8952 (hb. Gr.). The species is well characterized by its long narrow leaves, white flowers with narrow perianth-divisions, greenish filaments and style. It is apparently most nearly related to *H. adnata*, Herb.

Myrica Pringlei, n. sp. Shrub, 3 to 10 dm. high: stem and branches covered with a reddish or gray bark and dotted with numerous lenticels; ultimate branchlets at first pubescent, later glabrate: leaves petiolate, oblanceolate to oblong, 1.5 to 5 cm. long, 0.5 to 2 cm. broad, obtuse to rounded at the apex and submucronate, sparingly dentate towards the apex, entire in the lower half and abruptly to gradually narrowed into the 1–4 mm. long petiole, dark green above, paler beneath, sparingly pubescent and densely resinous-dotted on both surfaces in the early stages, later glabrate, becoming thick and firm in texture and somewhat verrucose above; midrib and the subhorizontally spreading lateral nerves rather prominent on the under surface: pistillate catkins solitary in the leaf-axils, slender, 5 to 12 mm. in length: flowers scattered, 1 in the axil of each bract; bractlets or scales at the base of the ovary 4, ciliate; ovary densely covered, except at the very top, with wax globules; styles somewhat flattened: staminate catkins solitary in the leaf-axils, 1 to 2 cm. in length; rhachis sparingly pubescent, resinous-dotted: flowers scattered; stamens 4 to 12; anthers often roseate to purplish: mature fruit not seen. — MEXICO. State of Puebla: pine forests about Honey Station, altitude 1765 m., 25 April, 1904, *C. G. Pringle*, no. 10,002 (hb. Gr.).

The species here described is quite distinct from anything in the genus hitherto recorded from Mexico, and of the northern representatives it approaches most nearly *M. carolinensis*, Mill., from which, however, it differs in the longer and looser-flowered staminate and pistillate catkins, and in the somewhat smaller more glabrate leaves, which in age become distinctly verrucose on the upper surface.

Lozanella, n. gen. of *Urticaceae* (*Celtideae*). Flowers dioecious. Pistillate flowers in axillary cymes: perianth 5–6-parted; segments slightly imbricated. Ovary sessile; style divided to the base, branches stigmatose; ovules pendulent, solitary. Drupe small, ovoid, compressed; exocarp succulent; endocarp hard, bony. Embryo curved; cotyledons oblong-rotund. Staminate flowers unknown. Ligneous plants with opposite leaves, axillary cymose flowers, and small greenish drupaceous fruit.

L. trematoides, n. sp. Stem terete or slightly compressed at the nodes, and as well as the branches covered with a reddish-brown bark, dotted with numerous whitish lenticels, at first pubescent later glabrate: leaves opposite, petiolate, ovate-acuminate, 5 to 9 cm. long, 3 to 4.5 cm. broad, evenly serrate-dentate, slightly rugulose and hirsute-hispid above, conspicuously reticulate-veined, hirsute and paler beneath; petioles 1 to 3 cm. long, pubescent; stipules lance-linear, about 4 mm. long, caducous: cymes of pistillate flowers rather slender, 1 to 2.5 cm. in length, 1.5 cm. or less in breadth, pubescent: flowers sessile: perianth 5–6-parted, 1.5 to 2 mm. long; divisions narrowly oblong, about 1 mm. in length, obtuse, ciliate-fringed: drupe sessile, ovoid, compressed or sublenticular, somewhat exceeding the perianth, greenish; styles more or less persistent. — MEXICO. State of Hidalgo: near Trinidad, 1904, *C. G. Pringle*, no. 8983 (hb. Gr.).

Although no staminate flowers of the plant here described are at hand, yet there can be no doubt that its affinity is with the *Celtideae*, and that it represents moreover a genus, hitherto unknown, related on the one hand to *Celtis*, and on the other to *Trema*. From both of these genera it differs in having opposite instead of alternate leaves. Habitally it is more like *Trema*, but from this genus it not only differs in leaf arrangement, but also in having pedunculate more or less open cymes, and broad cotyledons. The genus is named in honor of Señor Filemón L. Lozano, a keen and enthusiastic assistant of Mr. Cyrus G. Pringle.

Ficus subrotundifolia, n. sp. A small tree: stem and branches covered with a grayish or brownish bark; ultimate branches pubescent, especially at the nodes: leaves petiolate, subrotund to ovate-oblong, 4 to 7 cm. long, 3.5 to 6 cm. broad, rounded to obtuse at the apex, subtruncate to obtuse at the base, entire to subundulate-margined, somewhat villous-pubescent on both surfaces in the young stages, later glabrate, white-granulose on the upper surface, rather conspicuously netted-veined beneath; petioles 1 to 2.5 cm. long, more or less densely subsericeous-pubescent: receptacles sessile, axillary, usually in pairs, spherical, 8 to 12 mm. in diameter, greenish, covered with reddish-brown spots, densely short-pubescent to essentially glabrous. — MEXICO. State of Morelos: ravines near Cuernavaca, altitude 1525 m., 9 June, 1904, *C. G. Pringle*, no. 8931 (hb. Gr.). State of Jalisco: Guadalajara, *C. G. Pringle*, no. 11,850 (hb. Gr.).

Oxybaphus linearifolius, Watson. Proc. Am. Acad. xvii. 375 (1882). This species, described from specimens secured by Mr. J. G. Lemmon on the plains near Apache Pass, Arizona, in 1881, has again been found by Dr. A. Davidson along the Metcalf Road, near Metcalf, May 27, 1900, no. 327^a (hb. Gr.). Dr. Davidson's specimens conform in every detail with the original and with Dr. Watson's description; they show, moreover, that the slender upright stems originate from a distinctly perennial base.

Cassia (§ **Chamaechristae**) **leptadenia,** n. sp. Annual: stem simple or branched, erect, 1 to 4 dm. high, pubescent with an upwardly subappressed puberulence commonly intermixed with long horizontally spreading hairs: leaves short-petiolate, 1 to 5 cm. long, 1 to nearly 2 cm. broad; leaflets sessile, usually 24 to 36 (sometimes fewer), linear-oblong, 5 to 10 mm. long, 1 to 2.5 mm. broad, mucronate, rather conspicuously ciliate, glabrous on both surfaces or with a few scattered hairs beneath, somewhat oblique at the base; rhachis pubescent with long spreading hairs, slightly canaliculate above, bearing just below the first pair of leaflets a stipitate tack-shaped gland; petioles 2 to 5 mm. long; stipules oblique-lanceolate, 3 to 6 mm. long, ciliate: flowers 1 to 3, on short axillary or supra-axillary branches, small, about 0.5 cm. long, and less than 1 cm. in diameter; pedicels 3 to 5 mm. long, 1–2-bracteate above the middle, somewhat elongated at maturity; sepals lanceolate, acute, slightly pubescent: petals unequal, pinkish or yellowish-pink; stamens 5 to 8, of different lengths: ovary appressed-pubescent: legume 2 to 4.5 cm. long, 3 to 4 mm. broad, on upwardly arched pedicels, subappressed-pubescent: seeds flat, oblique, about 2.5 mm. long. — *C. nictitans*, Gray, Pl. Wright. 1: 59 (1852), not L.; Torr. Bot. Mex. Bound. 59, not L.; Rothrock in Wheeler's Report, 106, not L. — UNITED STATES. Texas: gravelly plains near Comanche Springs, *Dr. J. M. Bigelow* (hb. Gr.); 48 km. east of El Paso, May to October, 1849, *Chas. Wright*, no. 154 (hb. Gr.). New Mexico: without locality and number, *Chas. Wright*, coll. of 1851 (hb. Gr.); Organ Mts., 1 September, 1897, *E. O. Wooton*, no. 435 (hb. Gr.), distributed as "*Cassia calycoides*, DC."; Florida Mts., 27 August 1895, *A. I. Mulford*, no. 1038^a (hb. Gr.). Arizona: without locality, coll. of 1884, *C. G. Pringle*, no. 92 (hb. Gr.); Blue River, Clifton, 1 September, 1903, *Dr. A. Davidson*, no. 1340 (hb. Gr.); Sonoita Valley, 1874, *Dr. J. T. Rothrock*, no. 629 (hb. Gr.). MEXICO. State of Sonora: Ojo de Gavilan, August, 1851, *Geo. Thurber*, no. 1060 (hb. Gr.). State of Durango: vicinity of the city of Durango, April to November, 1896, *Dr. Ed. Palmer*, no. 654 (hb. Gr.). State of Oaxaca: Valley of Etla, September, 1895, *C. Alvarez*, no. 754 (hb. Gr.).

Var. **mensalis,** n. var. Stem 3 to 5 dm. high: leaves 2.5 to 9 cm. long; leaflets 12 to 50 (6 to 25 pairs), 1 to 1.5 cm. long, 2 to 3 mm. broad; pedicels 2 to 8 mm. long: other characters of the species. — MEXICO. State of Chihuahua: Hacienda San Miguel, near Batopilas, September, 1885, *Dr. Ed. Palmer*, no. 175 (hb. Gr.), a strong vegetative specimen. State of Sonora: vicinity of Guaymas, on mesas among grass, October, 1887, *Dr. Ed. Palmer*, no. 242 (hb. Gr.).

Var. **jaliscense**, n. var. Stem rather freely branching from the base: legume pubescent with spreading hairs: in other characters like the species. — MEXICO. State of Jalisco: Guadalajara, September, 1886, *Dr. Ed. Palmer*, no. 460 (hb. Gr.). State of Guerrero: vicinity of Acapulco, October, 1894, to March, 1895, *Dr. Ed. Palmer*, no. 23 (hb. Gr.).

The species here described has been confused hitherto with *Cassia nictitans*, L.; it differs, however, in having a constantly narrower pod, smaller (except in var. *mensalis*) leaflets which are conspicuously ciliate, and also in having usually long spreading hairs as well as shorter subappressed ones on the stem. *C. nictitans* was originally ascribed to Virginia; and from a large suite of specimens now at hand its range in North America may be given as from Vermont and Massachusetts to Georgia, west to New York, Missouri, Kansas, and eastern Texas. *C. leptadenia* occurs in Texas, as far as yet known, west of the Pecos River only, and extends into New Mexico and Arizona southward through the dry parts of Mexico to Oaxaca. *C. leptadenia* is similar to *C. aspera*, Muhl., and to *C. Simpsoni*, Pollard, but from both of these species it is readily separated by the characters of the foliage and pod.

Argythamnia Pringlei, n. sp. Shrub: stem somewhat angulate, covered with a whitish cortex, glabrous; ultimate branches sericeous-tomentose: leaves petiolate, subrotund to oblong-lanceolate, 1.5 to 7 cm. long, 1.2 to 3 cm. broad, rounded to short-acuminate and acute at the apex, entire or denticulate, rounded to obtuse at the base, pale green and pubescent above, densely white sericeous-tomentose beneath; lateral nerves 4 to 6 on either side of the midrib, divergent: racemes axillary, pedunculate, few-flowered, subsericeous-pubescent; peduncles slender, 1.5 to 2.5 cm. long; bracts narrowly lanceolate, acuminate, persistent: flowers monoecious: staminate flowers 4 to 8; sepals linear-lanceolate, acute, 0.5 cm. long; petals unguiculate, oblong-ovate, about two-thirds as long as the calyx, obtusish, pubescent on the back along the midrib, greenish: pistillate flowers usually 1 at the base of each floral cluster; pedicels about as long as the subtending bract; calyx subangulate-winged in cross-section; sepals ovate, subacuminate, acute, 7 to 8 mm. long: petals minute, spatulate, about 1 mm. long; ovary densely pubescent; styles 3-parted, the divisions again 3-parted and crenately lobed: mature capsule not seen. — MEXICO. State of Morelos: sides of a barranca near Cuernavaca, altitude 1525 m., 17 June, 1904, *C. G. Pringle,* no. 8819 (hb. Gr.).

The species here proposed has its nearest affinity with *A. heterantha,* Muell. Arg., but differs in leaf-outline, the more divergent lateral nerves of the leaves, the ovate petals, and in the densely pubescent ovary.

EUPHORBIA CAMPESTRIS, Cham. & Schlecht., var. **subpuberula,** n. var. Stems 3 to 6 dm. high, glabrous below, puberulent above: leaves lanceolate to lance-ovate, 1 to 3.5 cm. long, 3 to 14 mm. broad; leaves and bracts of the inflorescence ovate and mucronate-acute. — MEXICO. State of Hidalgo: wet banks of river below Trinidad Iron Works, altitude 1740 m., 25 April, 1904, *C. G. Pringle,* no. 8906 (hb. Gr.). Distinguished from the species by the puberulence of the upper part of the stem and floral branches, by the less crowded and broader lanceolate to lance-ovate leaves, and by the more ovate or less rotund leaves and bracts of the inflorescence.

Clethra Alcoceri, n. sp. A small tree; ultimate branchlets terete or subangled, striate, densely covered with a close fine reddish brown stellate tomentum: leaves petiolate, elliptic-lanceolate to oblong-obovate, 7 to 14 cm. long, 3 to 6 cm. broad, obtuse to short-acuminate and submucronate-acute, shallowly sinuate-dentate to merely denticulate, slightly unsymmetrical at the base, dark green above, paler beneath, minutely stellate-puberulent on both surfaces; midrib and lateral nerves prominent beneath; petioles 1 to 1.5 cm. long, rather densely stellate-pubescent: inflorescence terminal, consisting of paniculately disposed racemes, the latter 1 to 2 dm. in length; rhachis striate, densely tawny stellate-tomentose; pedicels 2 to 3 mm. long, about equalling the caducous bracts: flowers 5 to 6 mm. high, equally broad: calyx about two-thirds as long as the corolla; divisions of the calyx ovate, obtuse, closely stellate-pubescent on both surfaces: petals white, obovate or obovate-oblong with thin subfimbriate margins: stamens included; filaments glabrous: ovary densely pubescent with silvery white stellate or branched hairs: mature fruit not seen. — MEXICO. State of Hidalgo: barranca near Trinidad Iron Works, altitude 1600 m., 22 August, 1904, *C. G. Pringle,* no. 8923 (hb. Gr.). The species is dedicated to Señor Dr. Gabriel Alcocer of the National Medical Institute, City of Mexico.

Halenia chlorantha, n. sp. Annual, glabrous throughout: stems erect, 4 dm. or more high, simple below, branched above, narrowly winged: leaves ovate to lanceolate, including the petiole 3 to 6 cm. long, 0.5 to 1.8 cm. broad, acute, entire, somewhat gradually narrowed below into the petiole, 3- (5-) nerved; petioles of the lower leaves about equalling the blade, those of the other leaves gradually shorter upwardly on the stem: inflorescence terminating the stem and branches in several-flowered cymose clusters; pedicels 8 mm. or less in length, erect: calyx deeply 4-parted; divisions of the calyx lanceolate, acute, about 4 mm. long: corolla 8 to 10 mm. long, greenish; tube of the corolla about equalling the ovate-oblong short-acuminate lobes; spurs subperpendicular, slightly incurved at the base: capsule oblong, 12 mm. long, curved above, acuminate, glabrous: seeds subspherical, 1 mm. in diameter, smooth. — MEXICO. State of Hidalgo: wet woods near Trinidad Iron Works, altitude 1735 m., 11 July, 1904, *C. G. Pringle,* no. 8939 (hb. Gr.).

Phacelia Coulteri, n. sp. An erect rather stout herb: stem branched above, reddish brown, pubescent with long spreading villous hairs intermixed with stipitate glands: leaves bi-tripinnatisect, oblong to oblong-ovate in general outline, 2 to 9 cm. long, 1 to 6 cm. broad, hirsute on both surfaces with a few glandular hairs intermixed; segments narrow, obtuse to acute: inflorescence consisting of villous-hirsute and somewhat glandular scorpioid cymes subcorymbosely disposed; pedicels 1 mm. or less in length: calyx deeply 5-parted; divisions of the calyx unequal, linear to linear-oblong or subspatulate, 2.5 to 3 mm. long, obtusish, villous-pubescent intermixed with short-stipitate glandular hairs: corolla narrowly campanulate, 4 to 5 mm. long, light blue to white, glabrous; tube 2.5 to 3 mm. long, bearing within on either side of the filaments a small thin reflexed valve; lobes of the corolla subrotund, 2 mm. broad, subentire or minutely denticulate: stamens exserted: ovary hirsute-pubescent above, 4-ovuled: style short-stipitate, glandular at the base: capsule ovoid, 2.5 to 3 mm. long, pubescent above; seeds 4, from 1.5 to 2 mm. long, somewhat scrobiculate or subfoveolate. *P. glandulosa,* Hemsley, Biol. Cent.-Am. Bot. ii. 359 (1882), not Nutt., as to pl. *Coulter.* — MEXICO. State of Hidalgo: fields about Buena Vista Station, altitude 2590 m., 4 August, 1904, *C. G. Pringle,* no. 8988 (hb. Gr.). State of Vera Cruz: Real del Monte, *Dr. Thos. Coulter,* no. 921 (hb. Gr.).

The latter specimen in the Gray Herbarium bears the following note in the handwriting of Dr. Gray, "Between *P. glandulosa* & *P. tanacetifolia* = latter = *tanacetifolia.*" Dr. Gray also placed a question mark opposite the printed word Mexico on the Coulter label, evidently inferring that the plant came from California instead of Mexico; but as Mr. Pringle's specimen above cited is identical in every regard with the Coulter plant there can be no doubt, as Hemsley indicates, that Coulter's specimen no. 921 was collected in Mexico. With *P. tanacetifolia*, Benth., the species here described has little more than generic affinity. In certain forms of *P. tanacetifolia* the leaves are not unlike those of *P. Coulteri*, but the smaller flowers, shorter calyx, and smaller ovoid capsule of the latter readily separate the two species.

Hemsley, in the Biologia Centrali-Americana, Botany, ii. 359, refers the Coulter plant above cited to *P. glandulosa* Nutt. A careful comparison of *P. Coulteri* with the type and with authentic material of Nuttall's species shows very clearly that we have to deal with two quite distinct plants. Contrasted with *P. glandulosa* the Mexican *P. Coulteri* differs in having more divided leaves, smaller flowers, shorter capsules, and a more villous-hirsute stem. Habitally *P. Coulteri* most closely resembles *P. neo-mexicana*, Thurber (*P. alba*, Rydb.), but here again the somewhat more divided leaves, the subentire corolla-lobes, constantly shorter and more ovoid capsule, and smaller seeds of the former enable one to distinguish it at once from the Thurber species.

Mimophytum, n. gen. of *Boraginaceae*. Calyx 5-parted, persistent but not enlarged in fruit. Corolla tubular-campanulate, with a short tube and a spreading 5-lobed limb, bearing in the throat 5 blunt folds or scales; lobes 5, imbricated. Stamens 5, adnate to the corolla-tube, included; anthers small, obtuse. Ovary 4-lobed; style terete; stigma compressed-capitate; ovules subhorizontal. Nutlets 4, sometimes only 1 or 2 attaining maturity, depressed, obliquely affixed to the gynobase, and when removed revealing an ovate-triangular scar, convex above, flattened beneath, glochidiate over the entire upper surface. Seeds subhorizontal. Herbaceous perennials with trailing habit, alternate leaves, racemose inflorescence, and pale blue or whitish flowers.

M. omphalodoides, n. sp. Stems 6 dm. or more in length, somewhat bifurcately branched, glabrous or sparingly pubescent: leaves petiolate; blade ovate, acuminate, 2 to 7 cm. long, 1 to 4 cm. broad, acute, entire, deeply cordate, dark green and sparingly sub-appressed-hirsute above, becoming somewhat tuberculate in age, paler and strigillose beneath, thin and membranous, 5–7-nerved from the base; petioles slender, 1 to 6 cm. in length, pubescent with spreading hairs: inflorescence terminating the stem and branches in elongated slender subsecund strigillose racemes; pedicels filiform, 2 cm. or less in length, spreading or somewhat recurved: calyx about 4 mm. high, deeply 5-parted; divisions lance-oblong, acute: corolla 5 to 6 mm. long, 5-lobed to near the middle; tube upwardly expanded, bearing in the throat opposite the base of each lobe an oblong blunt fold or scale somewhat pruinose on the inner face; lobes oblong-rotund, spreading: nutlets subcircular, plano-convex, 4 to 5 mm. in diameter, glochidiate-spinose over the entire upper surface, espinose beneath, minutely strigillose on both surfaces. — MEXICO. State of Puebla: under wet cliffs in the barranca below Honey Station, altitude 1525 m., 24 May, 1904, *C. G. Pringle*, no. 8822 (hb. Gr.).

The plant here described but for the character of the fruit might well be placed under the genus *Omphalodes*, being in habit and superficial characters almost the counterpart of *O. acuminata*, Rob. The fruit, however, bears a striking contrast to the saucer-like nutlet of *Omphalodes*. It is on the other hand more like that of *Cynoglossum*, but is more distinctly discoid, and bears the glochidiate spines on the upper side only. The trailing habit, moreover, is quite unlike anything known in *Cynoglossum*. On the whole it seems best to characterize the plant in question as a new genus based on the technical characters of its fruit and its habit, occupying a position in the order somewhat intermediate between *Omphalodes* and *Cynoglossum*.

MYOSOTIS PALUSTRIS, Lam. Fl. Fr. ii. 283 (1778). To this species the following specimens are to be referred: MEXICO. State of Hidalgo: near Trinidad, 25 November, 1903, *C. G. Pringle*, no. 11,882 (hb. Gr.); in shallow water along river ledges below Trinidad Iron Works, altitude 1700 m., 14 June, 1904, no. 8933 (hb. Gr.). The species seems not to have been recorded hitherto from Mexico.

Citharexylon Pringlei, n. sp. Shrub, 3 to 5 m. high, glabrous throughout: stem and branches covered with a grayish bark; branchlets somewhat flattened at the nodes: leaves short-petiolate, elliptic-lanceolate to lanceolate, 2.5 to 8 cm. long, 1 to 2.5 cm. broad. merely obtuse or short-acuminate and obtuse, entire or rarely subdentate towards the apex, narrowed at the base into a 3 to 5 mm. long petiole, dark green and shining above, paler and glandular-punctate beneath with larger circular glands scattered over the entire under surface: inflorescence terminating the stem and branches in elongated arcuate-recurved more or less secund racemes; bracts subulate, nearly or quite equalling the pedicels, 1 to 2 mm. in length: calyx cupulate, about 3 mm. high, 5-angled in cross section, sinuately 5-denticulate, slightly pubescent about the orifice on the inner surface, often purplish: corolla bicolorous, tubular-funnelform, about 4 mm. long, 5-lobed, externally glabrous, densely pubescent in the throat; tube white; lobes subrotund, scarcely more than 1 mm. long and broad, slightly pubescent on the inner or upper surface, deep purple or magenta colored: mature fruit not seen. — MEXICO. State of Hidalgo: barranca below Trinidad Iron Works, altitude 1525 m., 13 July, 1904, *C. G. Pringle,* no. 8932 (hb. Gr.).

The glabrous character of the plant, with the smooth shining upper surface of the leaves, the gracefully recurved racemes and the bicolorous corollas, render this species of easy recognition, and readily separated from all known species of the genus. Its affinity, however, is apparently with the little known *C. lucidum*, Schlecht. & Cham.

Cedronella Wrightii, n. sp. An herbaceous perennial: stem erect, 5 dm. or more high, densely short-pubescent: leaves petiolate, ovate to ovate-lanceolate, 1 to 5 cm. long, 0.5 to 3 cm. broad, obtuse or acute, more or less irregularly crenate-dentate, closely puberulent on both surfaces and somewhat canescent beneath: inflorescence an elongated verticillate spike; the lower verticillasters becoming remote and subtended by foliar bracts, short-pedunculate: flowers small, 6 to 7 mm. long in anthesis: calyx short-tubular, subbilabiate, pubescent especially on the tube; calyx teeth narrowly lanceolate or lance-linear, whitish or tinged with purple, the three upper teeth nearly or quite as long as the tube of the calyx, the two lower teeth about two-thirds as long as the upper: corolla about one-third longer than the calyx, purplish in fresh specimens, becoming more or less faded in the dried state: stamens slightly exserted: nutlets smooth, subtriangular in cross section with convex back, about 1 mm. long. — *C. pallida*, Torr. var. Bot. Mex. Bound. 133 (1859), in part, as to pl. *Wright*, no. 1534. *C. mexicana*, var. *cana*, Gray, forma, Syn. Fl. 1, pt. 2, 377 (1886). *Hyptis spicata*, Torr. l. c. 129. — MEXICO. State of Sonora: mountains near Sta. Cruz, *Wright*, no. 1534 (hb. Gr.); San Bernardino, *Thurber*, no. 780 (hb. Gr.). UNITED STATES. Arizona: Blue River, 8 September, 1902, *Dr. A. Davidson*, no. 840 (hb. Gr.).

The species here proposed has been much confused hitherto, as the literature cited would indicate. Dr. Gray, in his treatment of the genus *Cedronella* for the Synoptical Flora, finally passed Wright's specimen above cited as a small flowered form of *C. mexicana*, var. *cana*, with the comment "fl. not well developed." More material is now at hand, and careful dissections and comparisons of the Wright, Thurber, and Davidson plants show the flowers of each to be perfectly normal, and many, too, are fully developed. These collections evidently represent one and the same species, differing from all other known American species except *C. micrantha* in the size of the flowers and deeply toothed calyx. *C. Wrightii* is distinguished from *C. micrantha* in having a longer more interrupted verticillate spike, longer calyx-teeth, and in the character of the pubescence.

STACHYS BORAGINOIDES, Ch. & Schl. Linnaea. v. 100 (1830); DC. Prodr. xii. 467; Hemsl. Biol. Cent.-Am. Bot. ii. 551. MEXICO. State of Vera Cruz: Jalapa, *Schiede & Deppe;* in moist places near Jalapa, altitude 1200 m., 13 April, 1899, *C. G. Pringle*, no. 7788 (hb. Gr.).

Var. **glandulosa**, n. var. Branches and inflorescence pubescent as in the species, but with long-stipitate glandular hairs intermixed: lower verticillasters sometimes developed into pedunculate racemose-spikes: calyx copiously glandular-pubescent. — MEXICO. State of Hidalgo: barranca below Trinidad, altitude 1675 m., 5 May, 1904, *C. G. Pringle*, no. 8894 (hb. Gr.).

Stachys Pringlei, n. sp. An herbaceous perennial: stems erect or ascending, about 9 cm. in length, green or purplish, villous-hirsute: leaves petiolate, oblong to oblong-ovate, 2 to 5 cm. long, 0.5 to 2.5 cm. broad, rounded to obtuse at the apex, crenate-dentate, cordate, dark green above, paler beneath, hirsute-pubescent on both surfaces; petioles of the lower leaves 3 cm. long, villous-hirsute, gradually diminishing upwardly on the stem: inflorescence elongating in age, becoming 2 dm. or more long: verticillasters usually 6-flowered, subtended by lance-oblong to linear foliaceous acute bracts longer or shorter than the flowers, the lower verticillasters 4.5 cm. apart: flowers short-pedicellate, in anthesis 13 to 15 mm. long: calyx campanulate, short-hirsute, 5 to 6 mm. long; teeth acerose-acuminate, about equalling the tube: corolla conspicuously bilabiate, 13 to 15 mm. long, pale purple fading to nearly white; tube about 8 mm. long, bearing a ring of hairs just above the base, pubescent in the throat; galea erect, externally pubescent; lower lip spreading, about 6 mm. long, maculate, 3-lobed, the median lobe broader than long, emarginate: stamens nearly equalling the galea: filaments pubescent; anthers deep purple: nutlets black, minutely roughened, about 1.5 mm. in diameter. — MEXICO. State of Hidalgo: woodlands near Trinidad Iron Works, altitude 1735 m., 11 July, 1904, *C. G. Pringle*, no. 8941 (hb.Gr.). This species suggests the little known *S. mexicana*, Benth., which, however, is said to be hispid on both surfaces of the leaves and on the calyx, characters that do not apply at all to the plant here described.

Satureia procumbens, n. sp. Stems procumbent or somewhat ascending, slender, 1 to 2.5 dm. in length, rooting at the lower nodes, closely puberulent: leaves petiolate, ovate or broadly spatulate, including the petiole 7 to 22 mm. long, 3 to 12 mm. broad, rounded to subobtuse at the apex, crenate to entire, narrowed below to the slender puberulent petiole, nearly or quite glabrous on both surfaces, paler and glandular-punctate beneath: verticillasters 2–4-flowered; pedicels 3 to 4 mm. long, puberulent: flowers in anthesis 12 to 13 mm. long: calyx bilabiate, 6 to 7 mm. long, 13-nerved, glabrous or with a few scattered hairs on the outer surface, glandular-punctate, pubescent in the throat; anterior lip of 2 rather slender long-ciliated teeth; posterior lip of 3 short obtuse or blunt teeth: corolla 12 mm. long, pale purple, somewhat pubescent in the throat; anterior lip spreading, 3-lobed; posterior lip suberect, emarginate: stamens 4, unequal; the anterior pair much longer than the posterior, and with somewhat larger anthers: nutlets ovoid, about 1 mm. long, minutely scrobiculate. — MEXICO. State of Hidalgo: shaded banks, barranca below Trinidad Iron Works, altitude 1685 m., 5 May, 1904, *C. G. Pringle*, no. 8895 (hb. Gr.). Mr. Pringle's plant here cited was distributed under the name "CALAMINTHA."

Salvia (Heterosphace) Davidsonii, n. sp. An herbaceous perennial: stems slender, ascending or erect, finely pubescent: leaves petiolate, the lower or basal ones lyrately pinnatifid, the upper rhombic-ovate, 1 to 3 cm. long, 0.5 to 2.5 cm. broad, sinuately lobed to subentire, pale green in color at least in the dried state, sparingly pubescent on both surfaces, resiniferous-dotted, commonly 3-nerved from a symmetrical or not infrequently unequal base; petioles narrowly winged, 1 to 6 cm. long: flowers solitary in the axils of the upper leaves or disposed in terminal few-flowered verticillate racemes: pedicels erect or slightly recurved, 5 mm. or less in length: calyx externally short-hirsute-pubescent especially on the rather conspicuous veins, dotted with yellowish glands, sparingly ciliate-hirsute in the throat: corolla about 1 cm. long, red: nutlets smooth. — Arizona: Chiricahui Mts., September, 1881, *Lemmon*, no. 3077 (hb. Gr.); southern Arizona, *Lemmon*, no. 492 (hb. Gr.); Metcalf, October, 1900, *Dr. A. Davidson*, no. 615 (hb. Gr.).

S. Davidsonii is most nearly related to *S. Henryi*, Gray, to which species one of the Lemmon plants above cited was referred by the late Dr. Sereno Watson. The several specimens now at hand, however, representing *S. Davidsonii*, are so uniform in habit, foliar and floral characters, and so distinct in leaf-outline, size of the corolla, etc., from *S. Henryi* and other species of the group, that the writer has no hesitation in regarding them as representing a separate and well defined species.

Bassovia stellata, n. sp. Shrub, 2 m. or more in height; stem covered with a reddish brown cortex; branches, as well as the leaves, pedicels and calyx, tawny stellate-pubescent, somewhat glabrate: leaves usually in pairs and of unequal size, ovate to ovate-lanceolate, 2 to 8 cm. long, 1 to 3 cm. broad, often acuminate, obtuse or acute, dark green above, a little paler beneath, stellate-pubescent on both surfaces, more densely so beneath, somewhat glabrate especially on the upper surface, entire, narrowed below to an obtuse or rounded base; petioles 3 to 10 mm. long, stellate-tomestose: inflorescence axillary, usually several-flowered; pedicels 6 to 12 mm. long, upwardly enlarged: calyx about 2 mm. high, crenately 5-lobed: corolla rotate or rotate-campanulate, about 7 mm. high and 8 mm. in diameter, 5-lobed to a little below the middle, pubescent in the throat; lobes ovate, obtusish, externally densely puberulent: stamens shorter than the corolla; filaments about equalling the anthers, densely pubescent with tawny matted hairs: fruit spherical, about 5 mm. in diameter, black, smooth, and glabrous. — MEXICO. State of Hidalgo: shaded bluffs of the barranca below Trinidad Iron Works, altitude 1525 m., 24 May, 1904, *C. G. Pringle*, no. 8824 (hb. Gr.).

Castilleja glandulosa, n. sp. Annual, or perennial by an indurated base, pubescent throughout with gland-tipped hairs intermixed with longer hirsutish hairs: stems usually simple, erect or nearly so, 1 to 3 dm. high, green or purplish: leaves sessile, a little dilated at the base and semi-amplexicaul, lance-attenuate, 1.5 to 5 cm. long, 1 to 6 mm. broad, acute, entire and often crenate-margined, 3-nerved from the base: inflorescence subspicate, 2 to 18 cm. in length, the lowermost flowers about 1 cm. apart; bracts of the inflorescence lance-attenuate to oblong, somewhat broader than the stem-leaves, the lower bracts entirely foliaceous, the middle and upper ones green below, and as well as the calyx-lobes tipped with color from yellow to scarlet: flowers sessile, in anthesis 2 to 2.8 cm. long: calyx about equally divided anteriorly and posteriorly, 2.5 cm. or less in length; lateral lobes oblong, about 8 mm. long, obtusish to slightly emarginate: corolla 2 to 2.7 cm. long, exceeding the calyx by 3 to 5 mm.: galea erect, 7 to 9 mm. long, with a greenish, glandular-puberulent keel and yellowish white thin margins: stamens 4, didynamous, the posterior pair a little shorter: ovary and style glabrous: capsule ovate-acuminate, 10 to 12 mm. long, smooth: seeds clavate, about 2 mm. long. — MEXICO. State of Mexico: hills near Lecheria Station, altitude 2225 m., 2 July, 1904, *C. G. Pringle*, no. 10,000 (hb. Gr.) *type*. State of San Luis Potosi: Valley of San Luis Potosi, *Schaffner*, no. 741 (hb. Gr.); 22° N. Lat., altitude 1800 to 2400 m., *Parry & Palmer*, no. 691 (hb. Gr.) Federal District: hills above Santa Fé, altitude 2400 m., 25 September, 1899, *C. G. Pringle*, no. 7979 (hb. Gr.). Valley of Mexico, *Schaffner*, no. 322 (hb. Gr.). N. W. Mexico, *Seemann*, in part (hb. Gr. under *C. lithospermoides*, HBK.).

This species is related evidently to *C. scorzonerifolia*, HBK., but differs in being glandular-pubescent throughout with longer hirsutish hairs intermixed, in having shorter narrower leaves, somewhat smaller flowers, and in having the upper floral bracts as well as the calyx-lobes orange- to scarlet-colored. From *C. lithospermoides*, HBK., with which it has been confused also, it differs in having the leaves more dilated at the base, the upper stem-leaves more attenuated, in being less hispid, and finally in having an ovate acuminate capsule.

CASTILLEJA HIRSUTA, Mart. & Gal. Bull. Acad. Brux. xii. pt. 2, 29 (1845); Walp. Rep. vi. 651; Hemsl. Biol. Cent.-Am. Bot. ii. 460. *C. obovata*, Benth. DC. Prodr. x. 528 (1846). To this species are referred specimens collected by Mr. C. G. Pringle in a barranca below Trinidad Iron Works, altitude 1525 m., 13 June, 1904, no. 8935 (hb. Gr.). Mr. Pringle's specimens agree well with the original characterization of the species except there is a tendency for the leaves to become incised-dentate or laciniate.

Lamourouxia brachyantha, n. sp. Stem about 1 m. high, branched above, yellowish green, somewhat hirsute below, densely stipitate-glandular above: leaves sessile, oblong-lanceolate to ovate, 1 to 3 cm. long, 0.5 to 1 cm. broad, acute, sharply and somewhat unequally dentate, half-clasping the stem, glandular-hirtellous on both surfaces: inflorescence racemose-paniculate; bracts of the inflorescence similar to the stem-leaves but smaller: flowers in anthesis 2 to 2.5 cm. long, short-pedicellate: pedicels 3 mm. long: calyx about 12 mm. long, 4-lobed to the middle; lobes ovate, acute, unequally dentate, and as well as the pedicels glandular-hirtellous: corolla 18 to 20 mm. long, bilabiate; posterior lip or galea conspicuously hooded, overarching the stamens, 2-lobed with the lobes soon becoming reflexed; lower or anterior lip recurved, 2-plicate-ventricose in the throat, 3-lobed with oblong rounded lobes: stamens 4, fertile; the posterior pair with smaller anthers: ovary glabrous below, above as well as the persistent style pubescent: capsule ovate, short-acuminate, about 1 cm. long. — MEXICO. State of Mexico: rocky hills, Lecheria, Valley of Mexico, altitude 2285 m., 1 July, 1904, *C. G. Pringle*, no 8815 (hb. Gr.). This species belongs to the § *Hemispadon*, Benth., and according to the revision of the genus by Robinson and Greenman in the American Journal of Science l. 169, it should be placed next to *L. rhinanthifolia*, HBK. The short corolla with its strongly hooded galea combined with the ovate dentate calyx-lobes serve to distinguish *L. brachyantha* from all other known species of the genus.

Hygrophila Pringlei, n. sp. Stems simple or branched, 6 to 10 dm.

high, subterete, green to dark purple, pubescent with spreading or subreflexed short fine hairs: leaves lanceolate, 2.5 to 6.5 cm. long, 0.3 to 1.5 cm. broad, acuminate, acutish, entire, hirtellous-margined, glabrous or with a few scattered hairs on both surfaces, often reflexed: flowers in dense axillary clusters; bracts linear, attenuate: calyx about 1.5 cm. long, equally 5-parted to a little below the middle; tube 5-angled in cross-section, essentially glabrous, minutely lineolate; divisions lance-attenuate, keeled, sparingly pubescent on the keel, ciliate-margined: corolla tubular-funnelform, curved, bilabiate, 2.5 to 3 cm. long, crimson, externally pubescent; tube gradually ampliated above, about 2 cm. long; posterior lip suberect, 2-lobed; anterior lip spreading, 3-lobed: stamens about equalling or sometimes slightly exceeding the upper lip; filaments below minutely pubescent, above glabrous: ovary glabrous; style puberulent: ovules two in each cell: mature fruit not seen. — MEXICO. State of Michoacan: hills near Uruapan, altitude 1675 m., 13 October, 1904, *C. G. Pringle*, no. 8847 (hb. Gr.).

Although mature fruit of this species has not been seen by the writer, yet on account of the convolute corolla-lobes and other technical characters of the flower, as well as the general habit and inflorescence, the plant is confidently placed in the genus *Hygrophila*.

Deppea microphylla, n. sp. A much-branched shrub: stem and branches covered with a grayish bark; wood reddish; the ultimate branchlets puberulent: leaves somewhat elliptic to oblanceolate or subspatulate, including the petiole 6 to 15 mm. long, 2.5 to 5 mm. broad, obtuse, entire, dark green above, paler beneath, glabrous on both surfaces or with a few scattered crisp hairs on the upper surface in the very early stages, narrowed below into a puberulent petiole: inflorescence terminal and axillary; peduncles 1–2-flowered, usually unibracteate, 6 mm. or less in length; pedicels slender, 2 to 5 mm. long: calyx turbinate or obconical, 5-lobed, glabrous; lobes linear-oblong, 1 to 1.5 mm. long, obtusish, about as long as the adnate tube, persistent: corolla rotate, yellow; tube 1.5 mm. long; lobes oblong, 6 mm. long, 2.5 mm. broad, rounded or obtusish at the apex: stamens exserted: fruit an obconical capsule, 2 to 3 mm. long, longitudinally ribbed, loculicidally dehiscent and the two halves later separating along the septa: seeds small, angled. — MEXICO. State of Hidalgo: river bluffs below Trinidad Iron works, altitude 1735 m., 26 April, 1904, *C. G. Pringle*, no. 8907 (hb. Gr.). A very distinct species and readily separated from all the hitherto known members of the genus by the small leaves and few flowers.

Galium pendulum, n. sp. Stems slender, 1 to 2.5 m. in length, hirsute to nearly glabrous, inconspicuously hispidulous along the angles: leaves in fours, elliptic-oblong to subobovate, 0.5 to 1 cm. long, one-half to two-thirds as broad, abruptly short-acuminate, acutish, sparingly hirsute on both surfaces, entire, ciliate, often somewhat revolute-margined, narrowed below to a subpetiolate base, 3-nerved, punctate with oblong pellucid glands: inflorescence an elongated many-headed cymose panicle: lateral branches of the inflorescence dichotomously branched; bracts small; pedicels 1 to 14 mm. long, glabrous: flowers 2 to 3 mm. high: calyx-limb obsolete: corolla rotate-campanulate, about 2 mm long, 4-dentate to the middle; lobes ovate, acute: mature fruit didymous, 1.5 mm. long, 2 mm. broad, pubescent with upwardly incurved or subappressed uncinate-hispid hairs; the two halves of the fruit often separating or diverging at maturity. — MEXICO. State of Hidalgo: hanging from wet cliffs and banks, barranca below Trinidad Iron Works, altitude 1585 m., 6 May, 1904, *C. G. Pringle*, no. 8985 (hb. Gr.). The pendulous habit, quaternate leaves, elongated paniculate cyme with its spreading dichotomous branches, and the upwardly subappressed-hispid fruit abundantly characterize this species, and enable one to readily recognize it among all the other species of the genus. Its affinity is with *G. Pringlei*, Greenm., on the one hand, and *G. uncinulatum*, DC., on the other.

Relbunium mazocarpum, n. sp. Stems prostrate or reclining, slender, 2 to 30 cm. long, glabrous; internodes longer or shorter than the leaves: leaves in fours, ovate-elliptic to elliptic-lanceolate, 1 to 5 mm. long, one-half as broad, terminated by a deciduous setiform mucro, entire, sparingly hirsute above, glabrous beneath: inflorescence axillary: peduncles slender, 4 mm. or less in length, 2-bracteate above; bracts opposite, similar to the leaves but smaller: flowers short-pedicellate: calyx-limb obsolete: corolla rotate, less than 2 mm. in diameter; lobes ovate, acute, externally bearing a few stiff hairs: ovary and mature fruit densely papillose-roughened, in the dried state appearing somewhat scaly. — MEXICO. State of Hidalgo: moist banks, barranca below Trinidad Iron Works, altitude 1675 m., 6 May, 1904, *C. G. Pringle*, no. 8834 (hb. Gr.). Related to *R. sphagnophilum*, Greenm., and *R. humile*, Schum., but differs from the former in being pubescent on the upper leaf-surface and in having a papillose-roughened instead of glabrous and smooth fruit; from the latter species it differs in having a papillose instead of pubescent fruit.

Viburnum caudatum, n. sp. Shrub or small tree, 4 to 6 m. high: stem and branches covered with a reddish brown cortex, subterete;

branchlets somewhat fuscous, glandular-puberulent: leaves ovate to rhombic-ovate, 9 to 13 cm. long, 5 to 9 cm. broad, caudate-acuminate, acute, subentire, 2–4-glandular-toothed at the rounded to cordate base, dark green above, paler beneath, slightly glandular-puberulent on the veins and bearing small patches of white stellate hairs at the junction of midrib and lateral nerves beneath, otherwise glabrous; petioles 8 to 12 mm. long, fuscous, canaliculate above, glandular-puberulent: inflorescence terminating the stem in a glandular-puberulent compound umbel, about 8 cm. in diameter; peduncle about 4 cm. long; rays of the umbel usually 7, from 1.5 to 2.2 cm. long: flowers many: tube of the calyx linear, 3 mm. long, compressed, densely glandular-puberulent; calyx-teeth obtusish, ciliolate: corolla rotate-campanulate, 6 to 7 mm. in diameter, white, 5-lobed to the middle; lobes ovate-rotund, reflexed: mature fruit not seen. — MEXICO. State of Hidalgo: barranca below Trinidad Iron Works, altitude 1525 m., 24 May, 1904, *C. G. Pringle*, no. 8826 (hb. Gr.).

This species is related evidently to *V. sulcatum*, Hemsl., but differs in having essentially terete instead of angulate-sulcate branches, caudate-acuminate larger leaves with a distinctly acute apex, and in not being in the least tomentose on the peduncle or under leaf-surface.

Viburnum ciliatum, n. sp. Shrub: stem and branches subterete, covered with a grayish or somewhat purplish cortex, glabrous; branchlets sulcate-angled, at first pubescent but soon glabrate: leaves broadly ovate, 4 to 7.5 cm. long, 3 to 5 cm. broad, somewhat acuminate, acute, sinuate-dentate, conspicuously ciliate, rounded to shallowly cordate at the base, dark green and except for a few scattered hairs near the margin glabrous above, sparingly hirsute intermixed with a few minute glandular hairs on the veins beneath, sometimes with small patches of stellate tomentum at the junction of midrib and lateral nerves, otherwise glabrous; petioles short, 5 mm. or less in length, pubescent with a few scattered hirsute hairs: inflorescence a terminal slightly glandular-puberulent compound umbel, 3 to 6 cm. in diameter; peduncles 1 to 2.3 cm. long; rays of the umbel usually 5, from 5 to 14 mm. long: flowers numerous: calyx-tube elongate-turbinate, about 2 mm. long, glabrous; calyx-teeth obtuse: corolla rotate-campanulate, 6 to 7 mm. in diameter, white, 5-lobed to the middle; lobes subrotund, spreading: fruit not seen. — MEXICO. State of Hidalgo: wet woodlands near Trinidad Iron Works, altitude 1675 m., 30 April, 1904, *C. G. Pringle*, no. 8881 (hb. Gr.).

The species to which *V. ciliatum* seems most nearly related is *V. mem-*

branaceum, Hemsl., but differs in having larger leaves which are entirely devoid of stellate hairs on the upper surface, in having very sparingly hirsute not tomentose petioles, and finally in having from 6 to 9 lateral veins on either side of the midrib instead of 4 or 5.

Valeriana calcicola, n. sp. An erect herbaceous annual: stem simple to the inflorescence, springing from a subterranean tuberous root, 8 to 9 dm. high, densely puberulent below, glabrous above: radical leaves not seen: stem leaves 1 to 3 pairs, bipinnatisect, oblong to oblong-ovate in general outline, 4 to 9 cm. long, one-half as broad, glabrous or under a lens slightly puberulent beneath; segments linear to linear-oblong, 4 to 15 mm. long, 1 to 3 mm. broad, obtuse; the uppermost pair of leaves much reduced: inflorescence a terminal paniculate cyme, when fully expanded 2 to 2.5 dm. long, 1 to 1.2 dm. broad; bracts short, narrowly ovate, 1 to 3 mm. long, acute, often subconnate at the base: flowers sessile, perfect, proterandrous: corolla subcampanulate, 2.5 to 3 mm. long, 5-lobed to the middle, purple; tube strongly gibbous at the base, densely villous in the throat; lobes at first erect, later reflexed: stamens 3, at the time of dehiscence of the anthers subincluded, later exserted: fruit somewhat flask-shaped, 3.5 mm. long, 2 mm. broad at the base, compressed, plano-convex and usually pubescent on the side of the fertile cell, 3-ribbed and glabrous on the other side with the lateral ribs prominent and rounded on the back; the persistent plumose calyx-teeth as long as or longer than the body of the fruit. — MEXICO. State of Pueblo: on limestone hills near Tehuacan, 22 August, 1901, *C. G. Pringle*, no. 9622 (hb. Gr.).

Valeriana macropoda, n. sp. An herbaceous perennial: stem simple, about 3 dm. high, rising from a tuberous root, glabrous or very sparingly puberulent: leaves pinnatisect with numerous mostly crowded primary divisions and linear obtuse to acute ultimate segments, rather thick and firm in texture; radical leaves including the petiole 10 to 18 cm. long, 3 to 6 cm. broad, rhachis rather broad and continuous in breadth with the petiole, the latter 6 cm. or less in length; stem-leaves 1 to 2 pairs, the lower usually petiolate, the petioles ciliate and frequently connate, the upper pair of leaves sessile and much reduced often to mere bracts: inflorescence cymose, either simple or subcorymbosely disposed; bracts small, lance-linear, 2 mm. or less in length, acutish, green with scarious margins: flowers essentially dioecious, sessile: corolla of the staminate flowers tubular-funnelform, somewhat unequally 5-lobed, white or roseate; tube about 3 mm. long, slightly gibbous at the base, sparingly pubescent within; lobes ovate-oblong, about 1 mm. long, obtuse:

stamens 3, exserted: corolla of the pistillate flowers similar but smaller, 5-lobed to the middle: style exserted: fruit oblong-ovate, 6 to 6.5 mm. long, two-thirds as broad, crowned by the persistent plumose calyx-teeth, strongly compressed, unicostate on one side and 3-ribbed on the other, puberulent to essentially glabrous. — MEXICO. State of Mexico: hills near Lecheria Station, altitude 2225 m., 4 July, 1904, *C. G. Pringle*, no. 8998 (hb. Gr.). This species is closely related to, and possibly may prove eventually to be the same as, the inadequately characterized and obscure *V. Napus*, Lindl., but until more can be learned of the Lindley species and its identity established it seems best to regard Mr. Pringle's plant as representing a distinct species.

Valeriana Nelsonii, n. sp. An erect herbaceous perennial: stems 5 to 7 dm. high, simple or sparingly branched above, striate, glabrous, often purplish at the base: leaves bipinnatisect, oblong to oblong-ovate in general outline, including the petiole 6 to 24 cm. long, one-third as broad, glabrous; segments linear, 0.5 to 6 cm. long, 1 to 3 mm. broad, acute or obtusish; petioles 5 cm. or less in length; stem-leaves 1 to 2 pairs, the upper often reduced to mere attenuated bracts: inflorescence in terminal rather close cymes; bracts minute: flowers sessile, perfect: corolla subcampanulate, 3 mm. long, 5-lobed to or a little below the middle, roseate, proterandrous; tube but slightly gibbous at the base, densely villous in the throat; lobes oblong-ovate, obtuse, spreading: fruit oblong, upwardly a little narrowed, 5 mm. long, 3 mm. broad just above the base, glabrous, compressed, plano-convex, unicostate on the side of the ovuliferous cell, 3-ribbed on the other side with the lateral ribs somewhat more prominent and rounded on the back. — MEXICO. State of Durango: near El Salto, altitude 2400–2650 m., 12 July, 1898, *E. W. Nelson*, no. 4574 (hb. Gr.); El Oro to Guanacevi, 14 to 16 August, 1898, *E. W. Nelson*, no. 4744 (hb. Gr.). Mr. Nelson's specimens have been referred to *V. ceratophylla*, HBK., from which species they differ in having a less cespitose habit, taller stems, more crowded primary leaf-divisions and longer ultimate segments, and finally in the somewhat larger and glabrous fruit. In habit *V. Nelsonii* resembles *V. calcicola*, but it is readily distinguished by the glabrous stem, larger more oblong and glabrous fruit.

ELEPHANTOPUS MICROPAPPUS, Klatt, Jahrb. Hamburg. wissensch. Anstalt. ix. pt. 2, 124 (1892), not Less., is GOMPHRENA PERENNIS, L., as is shown by an examination of Ule's no. 1184, collected in Campo near Laguna, Brazil, March, 1889, now in the Gray Herbarium from the herbarium of Dr. Klatt.

Achaetogeron ascendens, n. sp. An herbaceous perennial: stems several from a common ligneous base, subprostrate or ascending, 1 to 2 dm. in length, subappressed-hispidulous-pubescent: leaves sessile, linear and entire to subpinnatifid with short linear-acute or obtusish divisions, 1 to 2 cm. long, 1 to 5 mm. broad, hirsute-hispid: peduncles 1 to 5 cm. in length, sparingly pubescent with appressed or subappressed stiffish hairs: heads including the fully expanded rays 2 to 2.5 cm. in diameter: involucre campanulate, about 5 mm. high; bracts of the involucre in 2 to 3 series, linear or linear-lanceolate, scarious-tipped and acute, narrowly scarious-margined, hirsute-pubescent, more or less purplish: ray-flowers 2–3-seriate, numerous, fertile; rays including the tubular portion 7 to 9 mm. long, less than 1 mm. broad, white or roseate: disk-flowers many: pappus of both ray- and disk-flowers a laciniated crown of coalescent scales, mostly less than 1 mm. long, nearly or quite equalling the short tube of the corolla: mature achenes about 1 mm. long, laterally compressed, sparingly pubescent. — MEXICO. State of Hidalgo: meadows near Buena Vista Station, altitude 2530 m., 4 August, 1904, *C. G. Pringle*, no. 8851 (hb. Gr.).

Achaetogeron griseus, n. sp. An herbaceous perennial: stem erect, branched, about 3 dm. high, bearing several offsets at the base, green, striate, pubescent with appressed or subappressed grayish hairs: leaves of the offsets and the lower stem-leaves oblanceolate to spatulate, 3 to 8 cm. long, 0.5 to 1.5 cm. broad, obtuse or submucronate-acute, entire to more or less pinnatifid with oblong obtuse lobes, narrowed below to a subpetiolate base, subappressed-pubescent on both surfaces; upper stem- and rameal-leaves narrowly oblanceolate to lance-linear, entire: heads on rather long very pubescent peduncles, including the rays 2.5 to 3 cm. in diameter: involucre campanulate; bracts of the involucre lance-attenuate, narrowly scarious-margined and purplish, pubescent with subappressed hairs: ray-flowers numerous; rays including the tube about 1.5 cm. long, 1 mm. broad, white: disk-flowers many: pappus an inconspicuous fringed crown: achenes of both ray- and disk-flowers laterally compressed, slightly pubescent. — MEXICO. State of Durango: vicinity of the city of Durango, April to November, 1896, *Dr. Edward Palmer*, no. 821 (hb. Gr., and hb. U. S. Nat. Mus.). The affinity of this species is with *A. Wislizeni*, Gray, but the leaves are more or less pinnatifid, the pubescence in general is less spreading, the involucral bracts are purplish, and the pappus is much more reduced. Intermediate forms between *A. Wislizeni* and *A. griseus* may be found, but until such are known to occur it seems best to regard them as distinct species.

Achaetogeron purpurascens, Greenm. Proc. Am. Acad. xxxix. 94 (1903). By typographical error the generic name was here published as "*Archaetogeron*." It may be noted here, that specimens of this species were also secured from its original station, near Colonia Garcia in the Sierra Madre, State of Chihuahua, by *E. W. Nelson*, 1 to 20 August, 1899, no. 6213 (hb. Gr., and hb. U. S. Nat. Mus.).

Achaetogeron versicolor, n. sp. An herbaceous annual or sometimes indurated at the base and thus becoming perennial, 1.5 to 3 dm. high: stems much-branched from the base, erect or ascending, pubescent with spreading hirsute hairs: leaves alternate, sessile, linear-lanceolate to oblanceolate-spatulate, 8 to 18 mm. long, 1 to 6 mm. broad, acute, entire to 3–5-toothed, hirsute-pubescent: peduncles slender, 6 cm. or less in length, upwardly enlarged just beneath the head, pubescent with horizontally spreading hairs: heads including the fully expanded rays about 12 mm. in diameter: involucre campanulate, about 4 mm. high; bracts of the involucre biseriate, lanceolate to lance-ovate, acute, scarious-tipped and with thin margins, hirsute-pubescent, purplish: ray-flowers in about 3 rows, numerous, fertile; rays 4 to 5 mm. long, half a millimetre wide, white or whitish at the base, tipped with color varying from magenta to dark purple: disk-flowers many: pappus of both ray- and disk-flowers a minute inconspicuously fringed crown: mature achenes laterally compressed, about 1 mm. long, sparingly pubescent to essentially glabrous. — MEXICO. State of Hidalgo: meadows near Buena Vista Station, altitude 2530 m., 7 August, 1904, *C. G. Pringle*, no. 8849 (hb. Gr.).

Erigeron alcicornutus, n. sp. Annual: stem erect, simple and purplish below, branched and green above, striate, pubescent with long spreading or slightly reflexed jointed hairs: leaves sessile, semiamplexicaul, auriculate, oblong-lanceolate, 2 to 6.5 cm. long, 0.5 to 3 cm. broad, deeply laciniate-toothed in the terminal half with diverging teeth or divisions, hirsute on both surfaces: heads subcorymbosely disposed on naked or bracteate peduncles, including the rays 2.5 to 3 cm. in diameter: involucre campanulate; bracts of the involucre narrowly lance-attenuate, in 2 (3) series, hirsute-villous with some glandular hairs intermixed: rays white, many (180–200) nearly 1 cm. long, less than 1 mm. wide: disk-flowers many: pappus of both ray- and disk-flowers in a single series consisting of few slender setae, 1 to 2 mm. in length, very caducous: mature achenes about 1 mm. long, glabrous. — MEXICO. State of Chihuahua: on alluviums of Arroyo Auco, Sierra Madre, 15 October, 1887, *C. G. Pringle*, no. 1273 (hb. Gr.).

Mr. Pringle's plant was originally referred to *Achaetogeron Seemannii*, Gray (*Erigeron Seemannii*, Greene), from which, however, it differs in the character of the foliage, the number of ray-flowers, and in other technical characters of the head. *E. alcicornutus* has the leaves relatively broad at the base, and this breadth is retained up to the point of laciniation, not gradually narrowed to a subamplexicaul base. Moreover, the rays are more numerous, longer and narrower than in the Seemann plant, and the achenes again are smaller and quite glabrous. On account of the simple setose pappus with comparatively few setae Mr. Pringle's plant above cited seems to the writer best referred to *Erigeron*, related to *E. neo-mexicanus*, Gray, and *E. delphinifolius*, Willd.

Erigeron calcicola, n. sp. Suffruticose: stems several from a stout ligneous base, erect or nearly so, 1.5 to 3 dm. high, usually branched, pubescent with short spreading or slightly reflexed somewhat crisp hairs: leaves spatulate to linear, 0.5 to 2.5 cm. long, 1 to 5 mm. broad, acute or obtusish, entire or the lower ones few-toothed, pubescent on both surfaces: peduncles 10 cm. or less in length, hirtellous-hirsute with short spreading or somewhat reflexed hairs: heads 6 to 9 mm. high, including the rays 1.5 to 2 cm. in diameter: involucre campanulate, a little shorter than the flowers of the disk: bracts of the involucre lanceolate, acute, more or less scarious-margined, hirtellous-hirsute: ray-flowers numerous, subbiseriate; rays narrow, white or faintly tinged with purple: disk-flowers many: pappus double, consisting of long slender setae and of short laciniate squamellae, similar in both disk- and ray-flowers but somewhat shorter in the latter: mature achenes 1 to 1.5 mm. long, pubescent under a lens. — MEXICO. State of Hidalgo: calcareous bluffs near Tula, altitude 2070 m., 11 September, 1899, *C. G. Pringle*, no. 8223 (hb. Gr.); chalk bluffs, altitude 2070 m., 16 September, 1902, *C. G. Pringle*, no. 9899 (hb. Gr.). State of San Luis Potosi: 22° N. Lat., altitude 1800–2400 m., *Parry & Palmer*, no. 391 (hb. Gr.). State of Coahuila: Saltillo, 15–30 April, 1898, *Dr. Edward Palmer*, no. 20 (hb. Gr.).

E. calcicola resembles *E. pubescens*, HBK. (*E. Ervendbergii*, Gray), but differs in having a stouter more distinctly ligneous base, and a spreading or somewhat reflexed pubescence on the stem and peduncles not upwardly appressed or strigillose.

Erigeron morelensis, n. sp. Perennial: stem scapiform, slender, more or less flexuous, 1.5 to 2.5 dm. in length, bearing 3 to 5 linear acute bracts between the leafy base and the solitary terminal head, striate, green or slightly purplish, sparingly pubescent: leaves at the base of the stem crowning the oblique rootstock, petiolate, lanceolate to somewhat obovate-lanceolate, including the petiole 3 to 10 cm. in length, 0.5 to 2 cm. broad, acuminate, acute, remotely and rather coarsely mucronate-dentate, gradually narrowed at the base into the slender petiole, dark green and pubescent above with scattered hirsute hairs, paler and glabrous beneath; petioles reddish-purple, usually as long as or sometimes exceeding the blade: heads 6 to 8 mm. high, including the rays 1.5 to 2 cm. in diameter: involucre campanulate, about as high as the flowers of the disk; bracts of the involucre 2–3-seriate, subequal, lanceolate, acute, pubescent with a few jointed hairs: ray-flowers 60 to 70, in 2 series; rays white or tinged with magenta, 2–3-dentate: pappus of both ray- and disk-flowers alike, consisting of a single series of numerous slender setae: mature achenes about 1 mm. long, nearly or quite glabrous. — MEXICO. State of Morelos: wet cliffs, mountain cañon near Cuernavaca, altitude 1980 m., 15 May, 1898, *C. G. Pringle*, no. 7668 (hb. Gr.).

Erigeron oaxacanus, n. sp. A low herbaceous perennial: stems several from a common base, ascending or suberect, simple or branched, about 1 dm. high, hirsute-pubescent: leaves oblanceolate to lance-linear, 0.5 to 1.5 cm. long, 1 to 4 mm. broad, incised-serrate with 3 to 7 teeth to entire, subappressed-hirsute-pubescent on both surfaces; the lowermost leaves narrowed to a subpetiolate base, the upper sessile: peduncles terminating the stems and branches, 4 cm. or less in length, slender, pubescent: heads excluding the rays 6 to 8 mm. in diameter: involucre campanulate; bracts of the involucre narrowly lance-attenuate, hirsute-pubescent: ray-flowers 2–3-seriate, 80 to 85; rays white, twice as long as the involucre: disk-flowers numerous: pappus of both ray- and disk-flowers biseriate; the outer series of fimbriated scales, the inner of numerous slender setae: achenes pubescent. — MEXICO. State of Oaxaca: near San Mateo del Mar, 16 May, 1895, *E. W. Nelson*, no. 2606 (hb. Gr., and hb. U. S. Nat. Mus.).

Erigeron oreophilus, n. sp. Perennial: stems erect from a ligneous base, 3 to 8 dm. high, branched above, green or purplish, striate, glandular-puberulent and sparingly hirsute with scattered horizontally spreading hairs: leaves mostly pinnatifid, 1.5 to 8 cm. long, 4 cm. or less broad, glandular-puberulent and sparingly hirsute; divisions linear or linear-oblong, acute or obtuse, entire; the broad petiole-like base of the leaves usually rather conspicuously ciliate; the uppermost leaves undivided, linear-lanceolate and entire: inflorescence a terminal corymbose cyme: heads including the rays 2.5 to 3 cm. in diameter: involucre campanulate, about 6 mm. high; bracts of the involucre linear, attenuate, somewhat scarious-tipped and more or less purplish, glandular-puberulent or somewhat atomiferous-glandular with a few scattered spreading stiffish hairs intermixed: ray-flowers numerous, 2- (3-) seriate; rays white: disk-flowers many: pappus of both ray- and disk-flowers alike, double; the inner of slender deciduous setae, the outer of short laciniate squamellae: mature achenes about 1 mm. long, laterally compressed, pubescent. — MEXICO. State of Chihuahua: southwestern part of the state, August to November, 1885, *Dr. Edward Palmer*, no. 419 (hb. Gr.); dry ledges of La Bufa Mt. above Cusihuiriachic, altitude 2375 m., 31 August, 1887, *C. G. Pringle*, no. 1271 (hb. Gr.); cool slopes, foothills of the Sierra Madre, 11 October, 1887, *C. G. Pringle*, no. 1272 (hb. Gr.); near Colonia Garcia in the Sierra Madre, 1–20 August, 1899, *E. W. Nelson*, no. 6218 (hb. Gr., and hb. U. S. Nat. Mus.); between Colonia Garcia and Pratt's Ranch below Pacheco, 20–24 August, 1899, *E. W. Nelson*, no. 6265 (hb. Gr., and hb. U. S. Nat. Mus.); near Colonia Garcia in the Sierra Madre, altitude 2255 m., 4 September, 1899, *Townsend & Barber*, nos. 305, 306, (hb. Gr.). — UNITED STATES. Arizona: Mt. Graham, altitude 2740 m., September, 1874, *Dr. J. T. Rothrock*, no. 731 (hb. Gr.); Ft. Whipple, 1865, *Coues & Palmer*, no. 421 (hb. Gr.).

Forma **tenuilobus**, n. form. Stems and leaves very sparingly pubescent; division of the leaf linear, elongated even 3 cm. in length, 2 mm. or less in breadth. — MEXICO. State of Sinaloa: Sierra de Choix, 15 October, 1898, *E. A. Goldman*, no. 255 (hb. Gr., and hb. U. S. Nat. Mus.).

Forma **latilobus**, n. form. Stem and leaves copiously glandular-puberulent; divisions of the leaf linear-oblong, obtusish, 3 cm. or less in length, 2 to 5 mm. in breadth. — MEXICO. State of Chihuahua: base of Mt. Mohinora, 12.8 km. from Guadalupe y Calvo, altitude 2130 to 2285 m., 23–31 August, 1898, *E. W. Nelson*, no. 4861 (hb. Gr., and hb. U. S. Nat. Mus.).

Var. **dactyloides**, n. var. Leaves lance-oblong, 2 to 7.5 cm. long, 0.3 to 2.5 cm. broad, laciniate-lobed in the upper portion with 3 to 7 slightly divaricating lance-linear acute lobes 12 mm. or less in length, the lower two-thirds of the leaf-blade of nearly uniform breadth, 5 to 12 mm. broad, entire: rays white or roseate-tinged in the young stage. — MEXICO. State of Chihuahua: near Colonia Garcia in the Sierra Madre, 1–20 August, 1899, *E. W. Nelson*, no. 6220 (hb. Gr., and hb. U. S. Nat. Mus.). The variety connects directly with typical forms of the species through Townsend & Barber's no. 305, and Pringle's no. 1272.

E. oreophilus is closely related to *E. delphinifolius*, Willd., and to *E. neo-mexicanus*, Gray, and has been confused with both of these species.

From the former it differs in having horizontally spreading hirsute hairs on the stems, and in being glandular-puberulent; from the latter it differs in being glandular-puberulent or at least atomiferous-glandular, and in having heads averaging fully one-third larger not only in the length of the ray-flowers but in the longer involucral bracts, etc.

ERIGERON REPENS, Gray, var. **psammophilus**, n. var. Stem slender, somewhat flexuous, sparingly pubescent: leaves glabrous or inconspicuously pubescent: other characters of the species. — MEXICO. State of Vera Cruz: Lizardo, *Dr. Wawra*, no. 276 (hb. Berlin, and hb. Gr.). Mr. C. G. Pringle's no. 9673 collected on sand dunes near the City of Vera Cruz, and noted in the Proceedings of the American Academy xxxix, 95, 1903, forms a good intermediate between the type of the above species and the variety here described.

Baccharis Palmeri, n. sp. Suffruticose: stems one to several from a ligneous base, erect, 2 to 3 dm. high, striate-angled or furrowed, glabrous, glandular-viscid: leaves sessile, lanceolate to oblanceolate, 7 to 28 mm. long, 1 to 8 mm. broad, acute, entire, glabrous, punctate and glandular-viscid, uninerviate or with 2 lateral inconspicuous nerves: inflorescence corymbose or corymbose-paniculate: heads 3 to 5 at the ends of the branches on rather short peduncles: heads of the staminate flowers 8 to 10 mm. high, 11–13-flowered; involucral bracts in 3 to 4 series, lanceolate, 3 to 7 mm. long, acute, the outer greenish glandular and shorter than the inner almost colorless ones: heads of the pistillate flowers 1 to 1.5 cm. high, 15–18-flowered; bracts of the narrowly campanulate involucre 3–4-seriate, elliptic-lanceolate to lanceolate, 2 to 6 mm. long, the outer greenish, glandular-viscid, obtuse and shorter than the pale almost colorless or subhyaline inner acute ones: mature achenes 2 mm. long, striate, glabrous, bearing the rather persistent pappus, the latter 10 to 12 mm. in length. — MEXICO. State of Durango: vicinity of the city of Durango, April to November, 1896, *Dr. Edward Palmer*, no. 801 (hb. Gr., and hb. Berlin) staminate plant, and no. 802 (hb. Gr.) pistillate plant. The species is related to *B. Seemanni*, Gray, which it superficially resembles, but differs in having shorter peduncles, more numerous heads, and a fewer-seriated involucre.

Baccharis Pringlei, n. sp. Stem striate, purplish, pubescent with crisp hairs; internodes 5 to 6 cm. in length: leaves alternate, sessile, ovate, acuminate, 4 to 12 cm. long, 2 to 5.5 cm. broad, abruptly narrowed below the middle to about 1.5 cm. in breadth, amplexicaul, mucronate-acute, subsinuate-dentate and the teeth terminated by a mucro, ciliate, hirsute-pubescent above, paler and pubescent on the midrib and lateral veins beneath: inflorescence a terminal leafy hirsute-pubescent many-headed panicle; peduncles 6 mm. or less in length, slender; bracts of the ultimate inflorescences lanceolate to subsetaceous: heads of the staminate flowers about 5 mm. high and broad, about 25-flowered: involucre campanulate, 3-seriate; bracts of the involucre lanceolate, purplish, the outer acute, the inner longer, obtuse, and more or less scarious on the margins and at the tip: pistillate flowers unknown. — MEXICO. State of Oaxaca: in wet ravines, Sierra de San Felipe, altitude 2285 m., 11 December, 1895, *C. G. Pringle*, no. 7014 (hb. Gr.).

In foliar characters *B. Pringlei* suggests *B. oaxacana*, Greenm., from which, however, it is easily distinguished by its larger acuminate leaves, many-headed paniculate inflorescence, smaller heads, and finally by the absence of glandular hairs on the stem and in the inflorescence.

Melampodium Nelsonii, n. sp. Perennial: stems ascending or erect, much-branched, hirsute-pubescent: leaves sessile, linear-lanceolate, 1.5 to 5 cm. long, 1 to 4 mm. broad, entire and undivided or pinnately 3-cleft into linear divaricately spreading divisions, pubescent above, densely sericeous-villous beneath, revolute-margined: peduncles rather slender, 1 to 8 cm. in length, pubescent with spreading hairs: heads including the conspicuous orange-yellow rays 1 to 1.5 cm. in diameter: outer involucral bracts herbaceous, ovate-deltoid, gradually narrowed to an acute apex, densely hirsute-pubescent: ray-flowers about 12; rays oblong, 5 to 6 mm. long, 3 mm. broad: fruit somewhat quadrangular in cross-section, ribbed on the sides, and more or less tuberculate; the hood conspicuously developed and usually prolonged into a spirally coiled puberulent appendage. — MEXICO. State of Michoacan: Volcano of Jorullo, 28 March, 1903, *E. W. Nelson*, no. 6939 (hb. Gr., and hb. U. S. Nat. Mus.).

The fruit characters place *M. Nelsonii* in the § *Eumelampodium* near *M. longipes*, Rob., from which species, however, Mr. Nelson's plant is readily separated by the distinctly ligneous stem, more numerous ray-flowers, longer rays, and a ribbed fruit. In habit *M. Nelsonii* assimilates *M. heterophyllum*, Lag., but differs in having a well-developed hood with an attenuated coiled appendage, narrower leaves, and a more densely pubescent upper leaf-surface.

SANVITALIOPSIS, Schz. Bip. Among Liebmann's plants of Mexico is one which was determined by Schultz Bipontinus as "*Sanvitaliopsis Liebmanii*." The earliest reference to *Sanvitaliopsis* appears in Bentham and Hooker's Genera Plantarum, where it is mentioned under *Zinnia* with the following brief characterization: "*Sanvitaliopsis*, Schz. Bip. in Pl. Liebm. est species habitu *Heliopsidis*, acheniis tamen 2-aristatis *Zinniae*." The first adequate description and publication of the name was in 1887, when Klatt published a careful diagnosis of the Liebmann plant under *Zinnia Liebmannii*, Benth. & Hook. f., citing *Sanvitaliopsis Liebmannii*, Schz. Bip., as a synonym. On this description of Klatt the genus *Sanvitaliopsis* must rest.

The essential characters of the plant in question are such that it does not seem to the writer to have been correctly referred to *Zinnia* where Bentham and Hooker, f., placed it. The narrow attenuated bracts of the involucre, the uncinate-tipped pales, as well as the habit are all greatly at variance with *Zinnia ;* indeed these differences are so well marked that our only course is to accept *Sanvitaliopsis* to generic rank, as a member of the subtribe *Zinnieae*. The recently described *Grypocarpha*, Greenm., is unquestionably congeneric with *Sanvitaliopsis*, but represents a second species of the Schultz Bipontinus genus. The complete synonymy with citation of specimens may be given as follows: —

SANVITALIOPSIS, Schz. Bip. in Benth. & Hook. f. Gen. Pl. ii. 357 (1873), and in Leopoldina xxiii. 89 (1887), reprint p. 2 in synonymy. *Grypocarpha*, Greenm. Trees & Shrubs, i. 145, t. 73 (1903).

S. LIEBMANNII, Schz. Bip. in Leopoldina, l. c. *Zinnia Liebmannii*, Benth. & Hook. f. Leopoldina, l. c.; Rob. & Greenm. Proc. Am. Acad. xxxii. 19 (1896). — MEXICO. Rio Taba, *Liebmann*, no. 552 (hb. Copenhagen, and hb. Gr.).

S. Nelsonii, n. comb. *Grypocarpha Nelsonii*, Greenm. Trees & Shrubs, l. c. — MEXICO. State of Chiapas: top of ridge back of Tonala, *Nelson*, no. 2892 (hb. Gr., and hb. U. S. Nat. Mus.).

Rumfordia floribunda, DC. Prodr. v. 550 (1836); Deless. Ic. Sel. 4. t 30 (involucre poorly represented); Hemsley, Biol. Cent.-Am. Bot. ii. 157. Typical forms of this species are now represented in the Gray Herbarium by the following collections: MEXICO. State of Oaxaca: without further locality, *Ghiesbreght*. State of Morelos: on mountainside about Parque Station, 2130 m., 26 November, 1902, *Pringle*, no. 9955. State of Michoacan: mountains near Patzcuaro, 26 November, 1891, *Pringle*, no. 3940; north slope of Mt. Patambau, altitude 3150 m., 28–31 January, 1903, *Nelson*, no. 6570; north slope of Mt. Taucilaro, altitude 2590 to 3200 m., 24 February, 1903. *Nelson*, no. 6889. State of Jalisco: mountains near Talpa, altitude 1340 to 1525 m., 7 March, 1897, *Nelson*, no. 4024; near Compostela, altitude 915 to 1830 m., 7–8 April, 1897, *Nelson*, no. 4172.

Forma **pubescens**, n. form. Similar to the species in every way except that the leaves are tawny pubescent on the under surface. —

MEXICO. State of Oaxaca: Cerro de San Felipe, altitude 2440 m., 29 November, 1895, *C. Conzatti*, no. 30 (hb. Gr.).

Sabazia (?) anomala, n. sp. Annual, 3 to 4.5 dm. high: stem dichotomously branched from near the base, pale green or purplish, hirsute-pubescent with glandular hairs intermixed: leaves opposite, sessile, ovate-lanceolate, 2 to 5 cm. long, 7 to 22 mm. broad, acute, remotely and coarsely sinuate-dentate, cuneate to slightly rounded at the base, pale green and hirsute-pubescent on both surfaces, 3-nerved from near the base: peduncles terminating the stem and branches, 7 cm. or less in length, hirsute-pubescent with glandular hairs intermixed: heads including the fully expanded rays about 1.5 cm. in diameter: involucre campanulate, 1–2-seriate; bracts of the involucre ovate to lance-ovate, 4 to 5 mm. long, acute or obtuse, herbaceous or somewhat tinged with purple: receptacle conical, paleaceous; pales narrow, trifid: ray-flowers 8, fertile; rays white or roseate, obtusely 3-dentate: disk-flowers numerous; corollas yellow: pappus of both ray- and disk-flowers of 10 obovate or narrowly oblong ciliate-fringed persistent scales, half as long as the achenes: mature achenes subterete, 1.5 mm. long, striate, rather sparingly pubescent. — MEXICO. State of Hidalgo: rocky mesas near Buena Vista Station, altitude 2530 m., 7 August, 1904, *C. G. Pringle*, no. 8853 (hb. Gr.).

This species is placed doubtfully in the genus *Sabazia*, because of the presence of a scaly pappus. In every other regard the plant is truly congeneric, having a habit closely assimilating *S. humilis*, Cass., from which it differs not only in the presence of a pappus but in the pubescent shorter achenes. There is a superficial resemblance between *Sabazia anomala* and *Tridax trilobata*, Hemsl.; it is, however, a similarity in habit and foliage rather than in technical or essential characters.

PERYMENIUM BUPHTHALMOIDES,* DC. Prodr. v. 609 (1836); Hemsl. Biol. Cent.-Am. Bot. ii. 181; Rob. & Greenm. Proc. Am. Acad. xxxiv. 523, in part. *P. Cervantesii*, Hemsl. l. c., at least as to *Bourgeau*, no. 378 (hb. Gr.); Rob. & Greenm. l. c.; not DC. — MEXICO. *Alaman* (hb. DC., tracing of type in hb. Gr.). Federal District: hills above

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* Since the revision of this genus by Robinson and Greenman (Proc. Am. Acad. xxxiv. 521–529, 1899) types of several of the earlier species have been examined in the De Candolle herbarium by Dr. Robinson, and additional information was obtained concerning their identity, especially that of *P. buphthalmoides*, DC., which has been much confused with *P. Cervantesii*, DC. The above synonymy and the specimens cited it is hoped may help to clear up the confusion of these two species.

Santa Fé, altitude 3000 m., *Pringle*, no. 9322 (hb. Gr.); *Bourgeau*, no. 378 (hb. Gr.). State of Mexico: Flor de Maria, *Pringle*, no. 3169 (hb. Gr.). State of Jalisco: granitic hills near Guadalajara, *Pringle*, no. 8624 (hb. Gr.).

P. CERVANTESII, DC. l. c. — MEXICO. *Alaman* (hb. DC., photograph of type in hb. Gr.). State of Mexico: rocky hills, Lecheria, Valley of Mexico, 21 km. north of the City of Mexico, altitude 2285 m., 4 July, 1904, *C. G. Pringle*, no. 8816 (hb. Gr.). Mr. Pringle states that this species grows from 9 to 12 dm. in height.

P. flexuosum, n. sp. An herbaceous perennial: stems decumbent or ascending from a ligneous base, slender, flexuous, 1.5 to 6 dm. in length, green or purplish, striate, pubescent with appressed or somewhat spreading stiffish hairs: leaves opposite, sessile or nearly so, ovate to lanceolate, 1.5 to 4.5 cm. long, 0.5 to 2.2 cm. broad, acute, dentate, usually abruptly contracted at the base sometimes to a very short petiole, hirsute-hispid on both sides especially on the midrib and nerves beneath, intermixed on the surface with shorter finer strigillose hairs, 3–7-nerved from or just above the base: heads solitary, including the rays 2 to 3 cm. in diameter, borne on long naked peduncles terminating the stem: peduncles 4 to 8 cm. in length, densely subappressed-canous-pubescent towards the head: involucre subuniseriate; bracts of the involucre lanceolate to lance-oblong, obtuse or obtusish, hirsute with spreading hairs: ray-flowers about 8; rays oblong, including the short tubular portion about 1 cm. long, 4 to 5 mm. broad, yellow: pales of the receptacle often purplish-tipped: disk-flowers about equalling or slightly exceeding the involucre: pappus of both ray- and disk-flowers much reduced, consisting of a few very minute setae and a single long awn or seta borne on the inner or posterior angle of the achene: achenes slightly pubescent above. — *P. buphthalmoides*, Rob. & Greenm. Proc. Am. Acad. xxxiv. 523 (1899), not DC. — MEXICO. Federal District: Serrania de Ajusco, altitude 3000 m., 9 July, 1898, *C. G. Pringle*, no. 7636 (hb. Gr.).

This species differs from *P. buphthalmoides*, DC., in having lanceolate to ovate rather than eliptic-lanceolate leaves, and the same are as a rule more numerously dentate and less conspicuously nerved; the pubescence on the involucre is of spreading not appressed hairs; and the pappus is much more reduced.

Bidens Pringlei, n. sp. Perennial: stems ascending from a ligneous base, 3 to 4.5 dm. in length, green or slightly purplish, nearly or quite glabrous below, pubescent above with short canous subappressed or somewhat spreading hairs: leaves, except the uppermost, petiolate, bi-tripinnatisect, 2 to 6 cm. long, 1 to 4 cm. broad, glabrous; divisions linear, acute: peduncles long, slender, 1.5 dm. or less in length, naked, subappressed-canescent-pubescent: heads few, including the fully expanded rays 2.5 to 3 cm. in diameter: involucre campanulate, 5 to 6 mm. high; outer bracts of the involucre about 15, linear, acute, equalling or slightly exceeding the inner ones, canous-pubescent, the inner bracts oblong-lanceolate, obtuse, pubescent along the median line, scarious-margined: ray-flowers 8 to 10, neutral; rays whitish or roseate: disk-flowers yellow, numerous, 45 to 50; mature achenes linear, about 1 cm. long, hirsute-hispid above, bearing 2 or 3 rather short retrorsely barbed awns 2 mm. or less in length. — MEXICO. State of Michoacan: fields near Uruapan, altitude 1525 m., 8 October, 1904, *C. G. Pringle*, no. 8814 (hb. Gr.).

BIDENS ROSEA, Schz. Bip. in Seem. Bot. Herald, 308 (1852–57). *Cosmos pilosus*, HBK. Nov. Gen. & Sp. iv. 241 (1824); Hemsl. Biol. Cent.-Am. Bot. ii. 200 (1884). Although the Humboldt and Bonpland specimen on which this species was based has not been examined by the writer, yet there are in the Gray Herbarium several specimens which correspond fairly well with the original description. The mature achenes in the material at hand are elongated, as in many species of *Bidens*, but not attenuated above into a beak as in *Cosmos*, hence the disposition of the plant as made by Schultz Bipontinus is here maintained. The following collections are taken to represent the above species: MEXICO. State of San Luis Potosi: Alvarez, 28 September to 3 October, 1902, *Palmer*, no. 192 (hb Gr.). State of Michoacan: Los Reyes, 8 to 12 February, 1903, *E. W. Nelson*, no. 6868 (hb. Gr., and hb. U. S. Nat. Mus.). GUATEMALA, Department of Santa Rosa: Cuijiniquilapa, altitude 800 m., *Heyde & Lux*, no. 6172 (hb. Gr.).

Var. **calcicola**, n. var. Leaves more divided; divisions lanceolate: involucre rather densely canous-pubescent. — MEXICO. State of Morelos: on limestone hills, Yautepec, near Cuernavaca, altitude 1220 m., 21 October, 1902, *C. G. Pringle*, no. 11,340 (hb. Gr.).

Bidens sarmentosa, n. sp. A low almost prostrate herbaceous perennial: stems freely branched, 1.5 dm. or less in length, glabrous or bearing a few scattered hairs: leaves petiolate, bipinnate, 1 to 1.5 cm. long, two-thirds as broad, glabrous on both surfaces; segments obtuse or obtusish: heads small, 5 to 7 mm. high, about 1 cm. in diameter including the rays, borne on glabrous or essentially glabrous peduncles 4.5 cm. or less in length: involucre double, the outer bracts usually 5, linear or linear-spatulate, obtusish, glabrous, sometimes ciliate-margined, spreading or subreflexed. inner involucral bracts lanceolate to lance-oblong, obtuse: ray-flowers 5 to 6, sterile; rays short-oblong. 4 to 5 mm. long, nearly as broad, orange-yellow: disk-flowers about 20: mature achenes linear, 2 to 6 mm. long, glabrous or sparingly hispidulous, awnless or with reduced awns. — MEXICO. Federal District: Serrania de Ajusco, Ajusco Station. altitude 2895 m., 9 November, 1903, *C. G. Pringle*, no. 11,486 (hb. Gr.) This plant was distributed as "*Bidens ferulaefolia*, DC.," but the prostrate habit, the small leaves and heads, and the short rays amply distinguish it from that species.

BIDENS SEEMANNII, Schz. Bip. in Seem. Bot. Herald, 307 (1852–57); Hemsl. Biol. Cent.-Am. Bot. ii. 203, as to plant of Seemann, not of Parry and Palmer. *B. Seemanni*, Gray, Proc. Am. Acad. xix. 16. *Cosmos Seemanni*, Gray, l. c. This species has been referred both to *Bidens* and to *Cosmos*. The mature achenes are linear, elongated, 13 mm. or less in length and nearly of the same diameter throughout, but not attenuated above into a beak as in true *Cosmos*. Certainly if the two genera are to be kept separate the above species must be retained in the genus *Bidens*, where it was originally placed by Schultz *Bipontinus*. Besides the original specimen collected by Seemann, fragments of which are in the Gray Herbarium, and the Ghiesbreght plant cited by Dr. Gray in the paper referred to above, there are now at hand excellent specimens collected by Mr. C. G. Pringle in fields, near Uruapan, State of Michoacan, altitude 1525 m., 8 October, 1904, no. 8845 (hb. Gr.). The finely divided pinnatisect leaves, the deep purple rays, and the long linear smooth-bodied usually 6-awned achenes render this species one of the most striking and easily recognized of the entire genus.

Cosmos ocellatus, n. sp. Annual: stem erect or ascending, simple or branched, 3 to 5 dm. high, more or less pubescent with crisp jointed hairs: leaves opposite, petiolate, 2 to 8 cm. long including the petiole, 1.5 to 4 cm. broad, bi-tripinnately parted, dark green above, pale beneath, subappressed-pubescent on both surfaces; segments lanceolate or lance-linear, acute; petioles 0.5 to 4 cm. long, pubescent: heads 1.2 to 2 cm. high, including the rays 3 to 4 cm. in diameter, borne on long naked striate somewhat pubescent peduncles 1 to 2 dm. in length: involucre double, the outer bracts subfoliaceous, linear or lance-linear, callous-tipped, 8 to 10 mm. long, densely pubescent with long jointed somewhat interwoven hairs; inner involucral-bracts narrowly elliptic-ovate, acutish, yellowish-brown in color, scarious-margined: ray-flowers about 8, sterile; rays obovate-oblong, 1.5 cm. long, two-thirds as broad, notched with a broad truncated notch, orange-yellow, usually with a reddish-brown or chocolate-colored eye near the base: pales of the receptacle tipped with black: disk-flowers numerous; mature achenes about 1.5 cm. in length, attenuated above into a long slender beak, glabrous towards the base, pubescent above. — MEXICO. State of Guerrero: thin soil of the knobs of the Sierra de Tepoxtlan, altitude 2285 m., 4 October, 1900, *C. G. Pringle,* no. 8386 (hb. Gr.), distributed under the name "*Bidens daucifolia*, DC."

A portion of the type-number of De Candolle's species is in the Gray Herbarium, and a careful comparison of Mr. Pringle's plant with it shows the two to be rather remotely related. The larger heads, the broader deeply notched bicolorous rays readily separate the Pringle plant from *Bidens daucifolia*, DC.; and moreover its distinctly long-attenuated or beaked achenes place it in the genus *Cosmos*.

Eutetras Pringlei, n. sp. An herbaceous perennial, more or less glandular pubescent throughout: stems erect or ascending, 1.5 to 3 dm. or more in length, often with tufts of white hairs in the leaf-axils: leaves opposite, petiolate, deltoid-ovate, 1 to 4.5 cm. long, 1 to 4 cm. broad, acute, irregularly and rather coarsely sinuate-dentate, dark green, 3-nerved from the cordate base, the lateral nerves again branching; petioles 0.5 to 3 cm. in length, canaliculate above: inflorescence a few-headed corymbose cyme; peduncles 2 to 5 cm. long, upwardly enlarged just below the involucre: heads about 12 mm. high, including the fully expanded rays 2.5 to 3 cm. in diameter: involucre broadly campanulate, a little shorter than the flowers of the disk; bracts of the involucre uniseriate, linear-lanceolate, acuminate, about 1 cm. long, acute: ray-flowers 12 to 15, fertile; corolla-tube 3.5 mm. long, stipitate-glandular; rays oblong, 10 to 12 mm. long, 4 to 5 mm. broad, 3-dentate, white: disk-flowers numerous; corollas tubular, 7 mm. long, 4-dentate: anthers conspicuously appendaged with oblong appendages: pappus of both ray- and disk-flowers of 4 short lacerated scales alternating with an equal number of slender setae, persistent: mature achenes 3.5 mm. long, pubescent along the angles, nearly or quite glabrous on the four faces. — MEXICO. State of Guanajuato: about basaltic cliffs, near Acambaro, 6 October 1904, *C. G. Pringle*, no. 8813 (hb. Gr.).

This genus has been known hitherto only through *E. Palmeri*, published by Dr. Gray in 1880. The discovery of a second species is noteworthy, especially as the plant was found in a region quite remote from Dr. Palmer's locality. *E. Pringlei* exhibits perfectly the general characters ascribed to the genus by Dr. Gray, and it is readily distinguished from *E. Palmeri* by being larger throughout, and of a less caespitose habit.

SENECIO ALBONERVIUS, Greenm. Shrub, 2 to 3 m. high: stem in the younger parts white-tomentose, later becoming glabrous and covered with a grayish bark: leaves petiolate, broadly ovate, cordate, 3 to 12 cm. long, nearly or quite as broad, palmately nerved, 5–9-angulately lobed, at first tomentulose on both surfaces, later more or less glabrate and discolorous, persistently white-tomentulose on the nerves especially of the upper surface, short mucronate-denticulate on the margins; lobes short-mucronate-acute: inflorescence a terminal many-headed corymbose panicle: heads 10 to 12 mm. high, radiate; involucre calyculate with short triangular-lanceolate acute bracteoles; bracts of the involucre usually 8, lance-linear to oblong, obtuse, about 6 mm. long, thickened along the median line, glabrous or slightly tomentulose-puberulent, the inner with broad scarious margins: ray-flowers commonly 5; pappus surpassing the tube; rays 4-nerved: disk-flowers 8 to 10: achenes glabrous. — Monogr. Senecio, 26 (1901) & Engl. Bot. Jahrb. xxxii. 22 (1902). — MEXICO. State of Mexico: Tamascaltepec, *Schiede*, coll. of 1831 (hb. Gr., and hb. Mus. Bot. Berol.); Mt. Ixtaccihuatl, altitude 2440–3350 m., 1903, *C. A. Purpus*, no. 201 (hb. Gr., hb. T. S. Brandegee). State of Michoacan: north slope of Mt. Tancilaro, altitude 2285–3200 m., 24 February, 1903. *E. W. Nelson*, no. 6904 (hb. U. S. Nat. Mus., and hb. Gr.). State of Morelos: Sierra de Tres Marias, altitude 3050 m., 15 April, 1904, *C. G. Pringle*, no. 8903 (hb. Gr.). State of Vera Cruz: Mineral del Monte, *Ehrenberg*, no. 324 (hb. Gr., and hb. Mus. Bot. Berol.).

SENECIO BRACTEATUS, Klatt. Leopoldina, xxiv. 125 (1888), reprint p. 7. *S. leucanthus*, Greenm. Monogr. Senecio, 25 (1901) & Engl. Bot. Jahrb. xxxii (1902). *Cacalia amplexicaulis*, Schz. Bip., acc. to Klatt, l. c., reprint p. 8. I am indebted to Professor Eug. Warming of Copenhagen for a critical comparison of Mr. Pringle's no. 5713 with Liebmann's no. 192, the original of *S. bracteatus*, Klatt (*Cacalia amplexicaulis*, Schz. Bip.). There can be no doubt of the identity of the two plants, although the Liebmann specimen is said to be rather incomplete. — MEXICO. Between St. Andres and St. Miguel, *Liebmann*, no. 192 (hb. Copenhagen, fragment and drawing ex herb. Klatt in hb. Gr.). State of Oaxaca: Sierra de San Felipe, altitude 3050 m., 19 September 1894, *Pringle*, no. 5713 (hb. Gr.).

Cirsium pinetorum, n. sp. Stem 1 to 2 m. high with a few widely spreading branches above, striate, tawny arachnoid-tomentose: leaves pinnatifid, lanceolate-oblong in general outline, acuminate, the lower 3 to 4 dm. long, 1 to 1.5 dm. broad, the upper gradually smaller and sessile but not decurrent on the stem, spinose-margined, hirsute-hispid above, densely white-tomentose beneath; the terminal and lateral segments prolonged into a stoutish straw-colored spine: heads solitary, terminating the stem and branches, large: involucre 5 to 6 cm. high; bracts of the involucre disposed in many series, essentially uniform, narrowly lance-attenuate, 2.5 to 5 cm. in length, spiny tipped, entire or sparingly spinose-margined, somewhat glabrate on the back, deep purple in color, the outer successively shorter: flowers apparently white or slightly purplish: mature achenes oblong, 5 to 6 mm. long, glabrous. — MEXICO. State of Puebla: in pine forests, Honey Station, altitude 1765 m., 15 September, 1904, *C. G. Pringle*, no. 8884 (hb. Gr.).

This species is one of the most showy of the genus and is easily recognized among all known American species on account of the very large heads, the narrow lance-attenuated deep purple essentially uniform bracts of the involucre.

Onoseris conspicua, n. comb. *Rhodoseris conspicua*, Turcz. Bull. Soc. Nat. Mosc. xxiv. pt. 2, 95, t. 2 (1851); Hemsl. Biol. Cent.-Am. Bot. ii. 254 in synonymy.— MEXICO. Sierra San Pedro, Nolasco, *Jurgensen*. State of Oaxaca: near Plumia, altitude 1000–1500 m., 17 March, 1895, *E. W. Nelson*, no. 2480 (hb. Gr.). The few-flowered heads and the long involucre are important diagnostic characters of this species.

Onoseris rupestris, n. comb. *Caloseris rupestris*, Benth. Pl. Hartw. 88 (1841). *Pereziopsis Donnell-Smithii*, Coulter. Bot. Gaz. xx. 53, t. 6 (1895). Mr. W. Botting Hemsley of the Royal Gardens at Kew has kindly compared for me *Heyde & Lux* no. 4527, exsiccatae John Donnell Smith, with the original of *Caloseris rupestris*, Benth., and states that the two plants are conspecific. The species is characterized especially by the long involucre, the purplish involucral bracts, and by the mingling of hirtellous hairs with the arachnoid tomentum of the peduncles. *O. rupestris* is easily separated from *O. Isotypus*, Benth. & Hook. f., with which it has been confused by having a much longer involucre.

Perezia Lozani, n. sp. Stem striate, purplish, hirtellous-puberulent: leaves sessile, amplexicaul, ovate-oblong, in specimens at hand 2.5 to 7 cm. long, 1 to 4 cm. broad, acute, unequally and subspinosely dentate, slightly hirtellous above, glandular-hirtellous and rather prominently reticulate-veined beneath: inflorescence a terminal corymb or corymbose panicle, leafy: heads mostly short-pedunculate, 12 to 16 mm. high, about 25-flowered: involucre narrowly campanulate; bracts of the involucre imbricated in 5 to 6 series, linear-oblong to lanceolate, 4 to 8 mm. long, the outer bracts successively shorter, squarrose, mostly herbaceous, slightly expanded at the tips and mucronate, hirtellous-puberulent, the inner lanceolate and obtuse or acute, greenish or purplish: mature achenes 5 mm. long, glandular-hirtellous. — MEXICO. State of Hidalgo: under dry cliffs between Metepec and Zontecomate Stations, altitude 2590 m., 19 September, 1904, *C. G. Pringle*, no. 8871 (hb. Gr.).

The rather unusual involucre with its squarrose broad-tipped more or less herbaceous bracts readily characterizes this species and renders it easily distinguishable among all other species of the genus. It is named in honor of Sr. Filemón Lozano, worthy assistant of Mr. C. G. Pringle.

Perezia megacephala, n. sp. An herbaceous perennial, glabrous throughout: stem simple, erect, about 6 dm. high, springing from an enlarged ferrugineous-tomentose base, striate: leaves obovate-oblong to oblanceolate, 2.5 to 11 cm. long, 1 to 5 cm. broad, obtuse or acutish, serrate-dentate to entire, thick and firm in texture, conspicuously reticulate-veined on both surfaces, strongly ascending and subimbricated on the stem; the lower leaves largest, semiamplexicaul and serrate-dentate, the upper smaller, narrowed to a petiole-like base and entire: heads large, 3.5 to 4 cm. high and about as broad, solitary, terminating the stem: involucre broadly campanulate; bracts of the involucre 6–7-seriate, those of the outer series oblong and mucronate-acute, the innermost lanceolate, acute and purplish-tipped: flowers numerous, 2 to 2.5 cm. long: corollas purplish: achenes about 7 mm. long, glabrous. — *P. Wislizeni*, var. *megacephala*, Gray, Proc. Am. Acad. xxii. 433 (1887). — MEXICO. State of Jalisco: Rio Blanco, October, 1886, *Dr. Edward Palmer*, no. 655 (hb. Gr.). Dr. Palmer's plant was regarded by Dr. Gray as a variety of *P. Wislizeni*. The latter is now well represented in the Gray Herbarium through the collections of Wislizenus, Pringle, Palmer, and Nelson; it is a species showing little tendency to extreme variation, and is characterized especially by the glaucous nature of stem and foliage, the upper portion of the stem being essentially naked, and by the comparatively few broad outer involucral bracts. In *P. megacephala*, on the other hand, the bloom is absent, the stem is leafy to the single terminal head, and the outer involucral bracts are more numerous, longer, and broader. In view of these apparently constant differences the writer has no hesitation in raising Dr. Gray's variety to specific rank.

PEREZIA MONTANA, Rose. Contr. U. S. Nat. Herb. i. 105, t. 8 (1891). Specimens of this very distinct species have been secured by Dr. Edward Palmer at Santiago Papasquiaro, State of Durango, April and August, 1896, no. 59 (hb. Gr., and hb. U. S. Nat. Mus.). The achenes, although originally described as glabrous, are not infrequently somewhat pubescent. The Durango collection adds another station towards determining the geographical range of the species.

Trixis megalophylla, n. sp. Shrub: stem covered with a grayish brown bark, at first pubescent, later glabrate, somewhat winged from the decurrent foliage: leaves short-petiolate or the uppermost subsessile, oblong-lanceolate, 5 to 17 cm. long, 1.5 to 6 cm. broad, acute, entire or denticulate, revolute-margined, mostly contracted at the base into a short narrowly winged petiole, hirtellous-puberulent above, tomentulose beneath; midrib and lateral veins prominent on the under surface: inflorescence a terminal leafy glandular-hirtellous paniculate cyme; bracts of the inflorescence ovate to ovate-lanceolate, foliaceous: heads 15 to 18 mm. high, about 12-flowered: outer or accessory bracts of the involucre lanceolate or slightly oblanceolate, nearly equalling or a little exceeding the linear-oblong about 12 mm. long short-acuminate inner true involucral-bracts; mature achenes about 7 mm. long, hirtellous-pubescent with a few glandular hairs intermixed. — MEXICO. State of Guerrero: between Sochi and Tlalkinsala, altitude 1100 to 1465 m., 29 November, 1894, *E. W. Nelson*, no. 2038 (hb. Gr., and hb. U. S. Nat. Mus.). On account of the large leaves and the glandular inflorescence the species is very characteristic, and readily recognized among all the other species of the genus.

Trixis Nelsonii, n. sp. Shrub: stem covered with a brownish bark, wingless; ultimate branches tawny-pubescent: leaves petiolate, elliptic-ovate to oblong-lanceolate, 3 to 7 cm. long, 1 to 3 cm. broad, abruptly acuminate-apiculate, entire or denticulate, narrowed at the base into a short petiole, rugulose and somewhat hirtellous on the upper surface, densely and permanently tomentose beneath, revolute-margined; petioles 5 to 8 mm. long: heads about 1.5 cm. high, rather crowded at the ends of the branches: bracts of the outer involucre mostly oblanceolate, 5 to 8 mm. long, acute; inner bracts of the true involucre 8, linear-oblong, about 12 mm. long, short-acuminate, acutish, and as well as the outer bracts and peduncles tawny-pubescent intermixed with glandular hairs or glandular papillae: flowers about 16. — MEXICO. State of Chiapas: between San Cristobal and Teopisca, altitude 2040 to 2590 m., 4 December, 1895, *E. W. Nelson*, no. 3459 (hb. Gr., and hb. U. S. Nat. Mus.). Of the known species of this genus *T. Nelsonii* is nearest *T. rugulosa*, Rob. & Greenm., but differs in having broader leaves, which are abruptly contracted at the apex not gradually attenuated, and in being densely and permanently tomentose beneath; moreover, the outer involucral bracts in *T. Nelsonii* are usually broader than in *T. rugulosa*.

II. DIAGNOSES AND NOTES RELATING TO AMERICAN EUPATORIEAE.

By B. L. Robinson.

Ophryosporus venosissimus, n. comb. *Eupatorium venosissimum* Rusby, Mem. Torr. Bot. Club, vi. 57 (1896). The anthers of this species, even when examined with a compound microscope, show no vestige of an apical appendage. In other respects, such as the few subequal scales of the involucre, the species is in agreement with *Ophryosporus*. In habit and foliage it somewhat resembles *O. Cumingii*, Benth.

The genus Ageratella seems never to have received a formal or detailed characterization. It was originally founded (without generic description) upon two plants which Dr. Gray regarded as varieties of a single species. Hoffmann, in Engl. & Prantl, Nat. Pflanzenf. iv. Abt. 5, p. 137 (1890), also regarded the genus as monotypic and gave only a comparative, 2-line description of it. With the more abundant material now at hand it is clear that there are two distinct species. The generic characters and specific distinctions may be recorded as follows: —

Ageratella, Gray. Capitula homogama, cylindrica vel anguste campanulata, ca. 15-flora. Involucri squamae valde inaequales, in seriebus 5 verticalibus imbricatae, exterioribus brevibus ovatis, interioribus longioribus lanceolato-oblongis, intimis linearibus angustissimis deciduis paleiformibus. Receptaculum hemisphaericum, nudum, parvum. Corollae anguste tubulosae sine ullis faucibus distinctis summa parte plus minusve contractae, externe sparse glandulosae, dentibus limbi 5 anguste oblongis brevissimis erectis. Antherae oblongae, basi obtusae, connectivo apice incrassato truncato vel in appendicem brevem obtusam producto. Achaenia 5-gona, prismatica, deorsum leviter angustata, in costis sursum hispidula. Pappi paleae ovatae vel lanceolatae, translucidae, erosae, costa unica hispidula in aristam longam purpurascentem producta. Styli rami elongati, leviter clavellati. — Gray ex Wats. Proc. Am. Acad. xxii. 419, 420 (1887); Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv. Abt. 5, p. 137 (1890). — Frutices vel suffrutices graciles, humiles, ramosi. Folia parva, opposita vel alterna, ovata vel oblonga vel linearia. Capitula numerosa, mediocra, racemosa vel paniculata.

A. microphylla, Gray, l. c. Fruticosa copiose paniculatim ramosa; ramis gracillimis subteretibus decumbentibus puberulis apice nudiusculis

in pedunculos gracillimos ca. 6 cm. longos laxe spicato-pluricapitatos productis; foliis subsessilibus oppositis late ovatis vel ovato-rhomboideis crenato-serratis obtusiusculis 6–10 mm. longis 4–7 mm. latis utrinque puberulis et glanduloso-atomiferis; pedicellis ad 3 mm. longis; involucri anguste campanulati squamis viridibus vel brunnescentibus dorso puberulis, exterioribus acutis, interioribus obtusiusculis vel apice rotundatis; capitulis ca. 17-floris. — *Ageratum microphyllum*, Sch. Bip. in Seemann, Bot. Herald, 298 (1856). *Decachaeta Seemanni*, Benth. & Hook. f. Gen. ii. 239 (1873). *D. Seemannii*, Hemsl. Biol. Cent.-Am. Bot. ii. 78, t. 42 (1880). *Ageratella microphylla*, var. *Seemanni*, Gray, l. c. — Northwestern Mexico, *Seemann*, no. 2043 (co-type, in hb. Gr.).

A. Palmeri, n. sp. Suffruticosa erecta; caulibus saepius plurimis 5–6 dm. altis teretibus pilis brevissimis crispis griseo-puberulis foliosis; foliis alternis lanceolatis vel lineari-oblongis obtusiusculis vel apice rotundatis inciso-serratis vel integris 1–3-nerviis 12–20 mm. longis griseo-puberulis et obscure atomiferis; inflorescentia elongata angusta folioso-bracteata, ramulis gracilibus ad 4 (rarius 10) cm. longis adscendentibus 1–6 (rarius 15)-capitatis; squamis involucri subcylindrici vel anguste campanulati viridibus striatis, exterioribus brevibus plus minusve griseo-puberulis capitulis ca. 13-floris. — *A. microphylla*, var. *Palmeri*, Gray, l. c. — Rio Blanco, Jalisco, Mexico, *Palmer*, no. 537 (type, in hb. Gr.); on dry porphyritic hills near Guadalajara, *Pringle*, no. 1815; Sierra de San Esteban, *Pringle*, nos. 9840, 11,480. This species is readily distinguished from the other by its narrow alternate leaves and different inflorescence.

Oxylobus adscendens, Robinson & Greenman, n. comb. *Ageratum adscendens*, Sch. Bip. ex Benth. & Hook. f. Gen. ii. 242 (1873), nomen subnudum; Hemsl. Biol. Cent.-Am. Bot. ii. 80 (1881), where first described; Klatt, Leopoldina, xx. 75 (1884). This species, both from habit and technical characters, should be placed in *Oxylobus*, Moc., a marked group of three or four ageratoid *Eupatorieae* regarded as a genus by Mociño, as a subgenus of *Phania* by De Candolle, Prod. v. 115 (1836), as aberrant species of *Ageratum* by Bentham and Hooker f., l. c., but restored to generic rank by Dr. Gray, Proc. Am. Acad. xv. 25–26 (1879), who, however, failed to include the species here mentioned. Dr. Gray's two binomials, *O. arbutifolius* and *O. glanduliferus*, appear to have been entirely overlooked by the compilers of the Index Kewensis. In this connection it may be worth while to call attention to a probable error by which Mr. Hemsley (Biol. Cent.-Am. Bot. ii. 82) extends the range of the latter of these two species to "Venezuela, &c., *Fendler*, 1155." It seems more than likely that the specimen referred to was Linden's 1155 from Southern Mexico, which is, in fact, *O. glanduliferus*, while Fendler's 1155 is, at least in the Gray Herbarium, a piperaceous plant.

Fleischmannia Langlassei, n. sp. Suffruticosa decumbens plus minusve ramosa 2 dm. vel ultra alta; caulibus teretibus griseo-brunneis, ramulis leviter angulatis infra glabratis supra glanduloso-pilosis foliosis; foliis alternis graciliter petiolatis ovato-lanceolatis inciso-serratis vel etiam profunde trilobatis tenuibus 1–2.5 cm. longis 5–12 mm. latis utrinque viridibus subconcoloribus glabratis, petiolo 5–14 mm. longo juventate glanduloso-pilosiusculo maturitate omnino glabrato; capitibus multifloris longissime pedunculatis, pedunculis 1–4 in apicibus ramorum dispositis gracilibus plus minusve flexuosis obscure glanduloso-pilosiusculis vel in maturitate glabris; involucro turbinato-campanulato 8–14 mm. diametro squamis lanceolato-linearibus acutissimis striatis valde inaequalibus multiseriatim imbricatis glaberrimis purpurascentibus; receptaculo leviter convexo nudo; corollis albidis anguste tubulosis glabris 4.5 mm. longis sub limbo brevissimo erecto 5-dentato saepe leviter constricto; styli ramis longe exsertis conspicuis purpureo-roseis; achaeniis linearibus in angulis 5 sursum hispidis 1.7 mm. longis basi callosis, setis pappi 5–6 purpureis capillaribus sed firmiusculis 4 mm. longis. — Moist rocks, Arroyo de Barabas, Michoacan or Guerrero, Mexico, alt. 1000 m., 11 March, 1898, *E. Langlassé*, no. 27 (type, in hb. Gr.). Vernacular name, *clavellilo*. In inflorescence, flowers, and achenes this species is much like *F. Schaffneri*, Gray, but it differs conspicuously in its chiefly undivided very thin and completely glabrate leaves, as well as in its glabrate petioles and peduncles. In *F. Schaffneri* the leaves, which are of firm texture and somewhat grayish green with a fine permanent glandular puberulence, are regularly cleft nearly to the base into three linear lobes.

Piptothrix aegiroides, n. sp. Erecta; caule 6–12 dm. alto subtereti purpureofolioso simplici glaberrimo; foliis oppositis graciliter petiolatis ovato-deltoideis acuminatis serrato-dentatis vel crenatis firmiusculis utrinque subreticulatis supra viridibus glabris subtus pallidioribus in nerviis 3 pubescentibus 6 cm. longis 5 cm. latis, petiolo purpureo supra subsulcato 1.5–3 cm. longo; capitulis ca. 20-floris numerosissimis in corymbo planiusculo dispositis, pedicellis 5 mm. longis bracteolatis glabris; involucri campanulati squamis ca. 12 subaequalibus sub-biseriatim imbricatis oblongis viridibus vel apice obtuso eroso purpurascentibus 3 mm. longis; receptaculo paleaceo, paleis linearibus obtusis; corollis albis tubulosis glabris tubo proprio gracili saepe curvato ca. 1.3 mm.

longo in fauces longos subcylindricos leviter gradatimque ampliato, dentibus limbi ovato-oblongis obtusis erectis vel plus minusve patentibus; antheris apice longe appendiculatis basi rotundatis; filamentis glabris; achaeniis atrofuscis prismaticis valde 5-costatis glabris saepius leviter curvatis basi albocallosis 1.7 mm. longis; pappi setis paucis inaequalibus sursum scabridis purpurascentibus ca. 2 mm. longis caducissimis. — Under oaks, cool slopes of mountains above Etzatlan, Jalisco, Mexico, 24 October, 1904, *C. G. Pringle*, no. 8859 (type, in hb. Gr.)

Eupatorium chrysostylum, n. sp. Herbaceum e radice lignosa suberectum 2–3 dm. altum, caulibus compluribus flexuosis subsimplicibus mollissime albo-lanatis, pilis perlongis tenuibus patentissimis plus minusve viscosis; foliis oppositis petiolatis deltoideo-ovatis acutis grosse et argute dentatis ca. 3 cm. longis 2.5 cm. latis tenuibus concoloribus utrinque viridibus sed a grisea pubescentia tectis; petiolo 1–1.5 cm. longo patente tomentoso; paniculis rotundatis compactis multicapitulatis terminalibus vel subterminalibus, pedicellis 3–7 mm. longis viscoso-tomentosis; capitulis ca. 30-floris 12 mm. longis, involucro turbinato-cylindrico, squamis lanceolato-linearibus viridibus striato-nerviis valde inaequalibus multiseriatim imbricatis adpressis acutissimis dorso margineque griseo-pubescentibus; corollis glabris flavescentibus vel virescentibus longe tubulosis sursum sub limbo brevi 5-dentato erecto vel modice patente leviter angustatis ca. 5 mm. longis; antheris linearibus apice appendice ovato-lanceolata munitis; stylis clavatis valde exsertis aureis conspicuis; achaeniis gracilibus 4 mm. longis nigris pubescentibus deorsum leviter angustatis basi callosis; pappi setis ca. 25 vix barbellatis valde inaequalibus albis. — Dry ledges, rocky hills near Chihuahua, Mexico, 17 April, 1885, *C. G. Pringle*, no. 135 (type, in hb. Gr.) This plant, although distributed as *E. Parryi*, Gray, differs from that species conspicuously in its far more copious and less glandular pubescence, its more sharply toothed leaves, which are truncate rather than cordate at the base, and in its dense inflorescence, the pedicels in *E. Parryi* being two to four times as long and the heads relatively few.

Eupatorium leucoderme, n. sp. Fruticosum; ramis teretibus flexuosis albidis laevibus, ramulis fulvo-tomentellis; foliis oppositis oblongo-lanceolatis tenuibus utrinque viridibus 3-nerviis serratis vel subintegris 5–7 cm. longis 2–2.5 cm. latis utrinque praecipue in nerviis fulvo-puberulis subtus crebre nigro-punctatis apice attenuato-acuminatis basi obtusis vel rotundatis; nerviis a venulis transversariis quam eae Miconiae connexis; petiolo ca. 6 mm. longo gracili flexuoso fulvo-tomentello; paniculis lateralibus oppositirameis dense corymboso-thyrsoideis 6–9 cm. diametro; capitulis ca. 6-floris sessilibus in glomerulis rotundatis congestis; squamis involucri lanceolato-linearibus acuminatis valde inaequalibus subuniseriatis dorso fulvo-puberulis; corollis albis anguste tubulosis 4 mm. longis, limbo resinoso-punctato leviter ampliato brevissime 5-dentato; achaeniis prismaticis vix 3 mm. longis; setis pappi ca. 40 barbellatis 3.5 mm. longis sordido-albis; antheris breviter sed distincte in apice appendiculatis. — Sandy soil, Chuta, Michoacan or Guerrero, Mexico, alt. 25 m., 20 June, 1898, *E. Langlassé*, no. 183 (type, in hb. Gr.). In habit this species much resembles *E. albicaule*, Sch. Bip., but differs in its thin punctate leaves, acuminate involucral scales, etc.

Eupatorium Lozanoanum, n. sp. Fruticosum oppositirameum glaberrimum; ramis striatulis teretibus fuscescentibus, ramulis teretibus viridibus foliatis, internodiis 2–9 cm. longis; foliis oppositis petiolatis ovato-lanceolatis falcatim caudato-acuminatis cuspidato-serratis basi inaequali obtusis vel rotundatis supra basin trinerviis sed etiam prope basin nerviis duabus parvis brevibus marginalibus plus minusve obscuris munitis utrinque laete viridibus sublucidis pulcherrime pellucide punctatis reticulatisque 8–10 cm. longis 2–3 cm. latis; paniculis terminalibus laxis trichotomis 5–10 cm. diametro pyramidalibus 60–80-capitatis, ramulis gracilibus saepe divaricatis plus minusve puberulis capitula subsessilia ad apicem gerentibus; capitulis cylindricis 1 cm. longis 3 mm. diametro 4-floris; involucri squamis paucis (ca. 8) valde inaequalibus viridibus striatulis convexis apice obtusiusculis, exterioribus minimis ovatis, interioribus oblongis; corollis viridi-albis tubulosis 5 mm. longis glabris, tubo proprio faucibus vix ampliatis subaequanti, dentibus limbi 5 brevibus triangularibus patentibus; antheris apice longe appendiculatis; achaeniis prismaticis deorsum angustatis 3.7 mm. longis in costis 5 hispidulis inter costis obscure puberulis; pappi setis ca. 40 albis sursum-barbellatis, plurimis sublongioribus corollae superantibus prope apicem subdilatatis; styli ramis rectiusculis longissimis vix clavatis flavido-brunneis. — In the deep barranca below Trinidad Iron Works, Hidalgo, Mexico, 12 July, 1904, *C. G. Pringle*, no. 8942 (type, in hb. Gr.). Dedicated to Mr. Filemón L. Lozano, the faithful and efficient assistant of Mr. Pringle in his field work.

Eupatorium petraeum, n. sp. Frutex 1.5 m. altus, caule solitario tereti purpureomaculato minute puberulo; foliis alternis late ovatis vel suborbicularibus acutiusculis angulatis vel sublobatis undulato-dentatis petiolatis supra basin 3–(5)-nerviis utrinque reticulatis in nerviis puberulis 10 cm. longis 9 cm. latis, petiolo puberulo subtereti 2–2.6 cm. longo; panicula ramosissima ampla pyramidata obscure pubescenti, bracteis minimis angustis, ramulis prope apicem solum capituliferis, pedicellis filiformibus divergentibus 3 mm. longis; capitulis numerosissimis parvis 4 mm. longis ca. 14-floris; involucri campanulati squamis valde inaequalibus sub-3-seriatim imbricatis, extimis brevissimis parvis paucis lanceolatis subacutis puberulis, interioribus elliptico-oblongis obtusis vel rotundatis supra mediam partem pubescentibus 2.2 mm. longis, receptaculo leviter convexo paleaceo, paleis caducis linearibus obtusiusculis apice puberulis; corollis albis minute sparseque glandulosis 2 mm. longis, faucibus gradatim ampliatis tubo proprio vix longioribus, dentibus limbi ovato-deltoideis acutiusculis brevissimis recurvatis; antheris subtruncatis sed inter apicibus thecarum connectivo valde expanso evasculoso; achaeniis brevibus deorsum angustatis in costis 5 sursum hispidulis, setis pappi ca. 15 albis sursum barbellatis 1.7 mm. longis. — In granitic soil, Sierra Madre in the State of Guerrero, Mexico, altitude 1200 m., 4 November, 1898, *E. Langlassé*, no. 565 (type, in hb. Gr.). This species shows in its anthers a transition to *Ophryosporus*. However, the modification of the upper part of the connective into an expanded transparent spongy cushion devoid of vascular tissue is clearly a rudimentary form of the terminal appendage. In the true species of *Ophryosporus*, on the other hand, no such rudiment is visible even under the compound microscope. Furthermore, the true *Ophryospori* have an involucre of nearly equal subuniseriate scales and a naked disk.

Eupatorium Michelianum, n. sp. Fruticosum 1.25 m. altum; ramulis teretibus gracilibus rectis foliosis fulvo-glanduloso-hirsutulis; foliis oppositis subsessilibus basi rotundatis vel cordatis subamplexicaulibus elongato-oblongis attenuatis argute serrulatis firmiusculis ad 13 cm. longis 3 cm. latis glabriusculis penninerviis utrinque reticulato-venulosis subtus pallidioribus aureo-atomiferis; paniculis terminalibus oppositirameis bracteatis rotundatis ca. 1 dm. diametro ca. 90-capitatis, pedicellis teretibus ca. 5 mm. longis; capitulis ca. 22 floris ovoideo-cylindricis 1 cm. longis; squamis involucri arcte adpressis multiseriatim imbricatis apice rotundatis erosis discoloribus, extimis brevissimis suborbicularibus, intimis anguste linearibus; corollis albis anguste tubulosis sursum vix ampliatis, limbo patente breviter 5-dentato; antheris apice longe appendiculatis; achaeniis prismatis 5-costatis deorsum leviter angustatis 3.3 mm. longis in costis hispidulis. — In granitic soil of the Sierra Madre, Guerrero, Mexico, altitude 1200 m., 7 December, 1890, *E. Langlassé*, no. 589 (type, in hb. Gr.). This species is allied to *E. glaberrimum*, DC., but is glandular-hirsute. It is also near *E. Oerstedianum*. Benth., from which it differs in its hispidulous achenes and narrower involucre, the lowest scales being suborbicular and not at all herbaceous. It is dedicated to

the memory of the late Marc Micheli, at one time editor of the interesting exsiccatae of which it forms a part.

EUPATORIASTRUM NELSONII, Greenman, var. **cardiophyllum**, ROBINSON & GREENMAN, n. var. Habitu et floribus formae typicae; foliis suborbicularibus duplicato-dentatis basi valde cordatis. — Los Piños, Chiapas, Mexico, 2 June, 1904, *E. A. Goldman*, no. 1052 (type, in hb. U. S. Nat. Mus., tracing and fragments in hb. Gr.).

KANIMIA NITIDA, Bak. in Mart. Fl. Bras. vi. pt. 2, 370 (1876). In the Index Kewensis, ii. 3 (1895), Hooker, f. and Jackson reduce this species to *K.* "*erythralina*," crediting the latter name to Bentham and Hooker, f. Gen. ii. 247 (1873). If the combination had been correctly made in the Genera Plantarum it would obviously antedate Baker's binomial and stand according to the rule of priority under the genus, but this is not technically the case. At the place indicated, Bentham and Hooker, f. merely refer to *Kanimia*, the plant which they call "*Mikania erythralina*, DC." The combination *Kanimia erythralina* is not made, and indeed there is no assurance that the authors of the Genera Plantarum regarded the transferred plant as a distinct species or that in naming it under *Kanimia* they would have adopted the combination *K. erythralina*. All that their statement conveys is the fact that the generic affinities of De Candolle's plant are with *Kanimia* rather than with *Mikania*. There is, however, an added reason why the transfer in the Genera Plantarum should not in this instance be regarded as equivalent to the creation of the new binomial *K. erythralina*, for De Candolle's specific name was *erithalina*, given from a fancied resemblance of the plant to *Erithalis*. *Kanimia nitida*, Baker, is not only the first correct combination accompanied by accurate synonymy, but happily it is also in accordance with the most rigid priority of the specific name, for it is founded on *Eupatorium nitidum*, DC. Prod. v. 180 (1836), which has priority of position over *Mikania erithalina*, DC.

CARPHEPHORUS REVOLUTIFOLIUS, DC. Prod. v. 133 (1836). This species, described by the eldest De Candolle, was based upon a specimen sent to him by Sternberg from the herbarium of Haenke and supposed to have been collected in Mexico. Concerning the plant Bentham and Hooker, f. (Gen. ii. 249) say " *C. revolutifolius*, DC. l. c. 133, e Mexico, et *C. cordifolius*, DC. Prod. vii. 267, e Brasilia, nobis ignoti, certe e descr. e genere expellendi sunt." By Hemsley (Biol. Cent.-Am. Bot. ii. 108), *C. revolutifolius*, DC., is still included in the Mexican flora, but with the comment that both this and *C. ? triangularis*, Gray, are doubtful species. At the request of the writer, Mr. Casimir De Candolle has been so kind as to lend for examination a bit of the original plant of Haenke from the Prodromus Herbarium at Geneva, calling attention to the fact that Schultz Bipontinus had once seen the specimen, and recorded on the sheet his opinion that it was not Mexican, but was the Chilean *Pleocarphus revolutus*, Don. It appears that this supposition has never been put on record in print. It is, however, fully confirmed by an examination of the fragment sent by Mr. De Candolle. The plant is certainly not a *Carphephorus*, nor does it belong to the *Eupatorieae*. The corollas are distinctly bilabiate, and the achenes are not at all angled. It agrees very closely with Don's description of *Pleocarphus revolutus* and with Gay's admirable colored plate of the Chilean plant (Fl. Chil. t. 43). In the Gray Herbarium there is furthermore a specimen of *Pleocarphus revolutus* collected by Gay, and with this also the plant of Haenke is in close agreement, the only differences noted being a slightly greater pubescence on the pedicels and a tendency for the bracts of the involucre to be a little narrower, differences of degree only, and so slight that they may be confidently attributed to individual variation. The genus *Pleocarphus* has no floral distincions from *Jungia*, with which it has been united by nearly all writers, who have had occasion to mention it in recent years. In accordance with this view, the plant in question should be called :

Jungia revoluta, n. comb. *Pleocarphus revolutus*, D. Don. Trans. Linn. Soc. xvi. 228 (1830); Remy in Gay, Fl. Chil. iii. 427, t. 43. *Carphephorus revolutifolius*, DC. Prod. v. 133 (1836). Of the same habitally divergent section of *Jungia* is also

J. dentata, n. comb., *Pleocarphus dentatus*, Phil. Linnaea, xxxiii. 51 (1864).

It is well known that confusion has existed in the collections of Haenke and Née, whose plants came partly from western South America and partly from Mexico. It is, therefore, a matter of no surprise that the problematic *Carphephorus revolutifolius*, which many acute and diligent collectors in Mexico have never succeeded in rediscovering, should be found identical with a plant from Chili, whence without doubt the plant of Haenke originally came. The species should, therefore, be eliminated both from the Mexican flora and from the genus *Carphephorus*.

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THE RELATIVE POSITIONS OF THE ARC AND SPARK LINES OF THE SPECTRA OF TITANIUM, ZINC, AND IRON.

By Norton A. Kent.

With a Plate.

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By Norton A. Kent.

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Historical Survey and Statement of the Object of the Investigation.

In 1901 Haschek[*] stated that the lines of the spark spectrum of titanium exhibit considerable shift as compared with the same lines in the arc, a shift in one instance as great as 0.13 t.-m.

In 1903 the writer of the present paper made a careful experimental investigation of the subject.[†] No such large shifts were discovered: but it was shown that in all probability the wave-lengths of certain lines of the titanium spark spectrum are sensitive to various conditions of the electric circuit, such, for instance, as the amount of capacity, self-induction, and ohmic resistance employed, and the presence or absence of an auxiliary spark-gap in series with that under investigation.

The instrument used was a ten-foot Rowland grating. It seemed advisable to use greater dispersion. Hence the present work was undertaken with a twenty-one-foot grating. During its progress, Eder and Valenta[‡] (using a fifteen-foot instrument) have published a paper in which they state that neither in the case of pure zinc nor brass wire do the lines studied by Haschek show any shift. They attribute the results of these investigators to a photographic displacement of the centre of gravity of the lines due to over-exposure — the lines in question being those which exhibit very strongly the phenomenon of unsymmetrical broadening; and they suggest also the possibility of a pseudo-shift due to poor focus. The conclusion drawn by Eder and Valenta is that, at minimum photographic exposure, the coincidence of arc and spark line is perfect. Moreover, Middlekauff[§] has lately studied the iron spectrum and finds no shifts in the region investigated. The focal length of his grating was twenty-one feet, six inches.

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[*] Astrophysical Journal, **14**, 181 (1901).

[†] Ibid., **17**, 286 (1903).

[‡] Ibid., **19**, 251 (1904).

[§] Ibid., **21**, 116 (1905).

Such an inconsistency in the results of different investigators emphasizes the importance of taking extreme care in all the details of the experiment, guarding against any possible false shifts and stating with exactness all the conditions under which the investigation is made.

The objects of the present series of experiments are, then, to attempt to settle definitely the question of the coincidence or non-coincidence of arc and spark lines, to discover the determining conditions in case shifts do exist, and to harmonize, if possible, the results obtained by previous investigators.

EXPERIMENTAL CONDITIONS; CONSTRUCTION AND ADJUSTMENT OF THE GRATING-MOUNT.

The conditions under which the present investigation has been carried out are as follows: The science hall of the college is a substantial building located not less than seventy-five yards from a roadway upon which there is heavy traffic; the foundations of the building are laid solidly upon glacial drift; and the room in which the spectroscope is mounted is situated in the cellar. The grating is a twenty-one-foot Rowland concave, of 15,000 lines to the inch, having a ruled surface of six by two and one-half inches. The grating holder is of brass, and is substantially built. The entire mount is of steel or iron. Seven-inch channel beams form two sides of the triangle, the third is a beam of five inches. The tracks are of two and three-fourths by one-half inch rolled steel. The carriages are massive; the truss-rod connecting the two is of 2¼-inch piping; and three draw-bars conduce to great rigidity. The camera box is of iron and brass, designed by the author, and is especially massive. The photographic plate is held in its place by brass screws. The slit is connected rigidly to the channel beams, and the whole mount is supported by rubber disks on three substantial brick piers. After adjusting the instrument for any definite region of the spectrum, the bases of the grating-holder and camera box were always clamped to the steel tracks by nine iron clamps of various sizes.

The camera shutter, being supported from the floor, is entirely free from the camera box, and is so arranged that there is absolutely no chance of mechanical jar from the operation of changing the shutter between exposures. The slit mechanism is surrounded by cardboard cylinders which are held free from the slit by supports connected rigidly to the walls of the room. Around these cylinders is tied the black cloth which separates the dark spectrometer room from the light room in which are placed the spark and arc. The entire mount, including channel beams, steel tracks, trucks, truss-rod, draw-bars, and all possible parts of grating and camera box, is carefully wrapped in several layers of paper or cloth to facilitate obtaining constant temperature conditions. The steam pipes in the room are protected thoroughly by asbestos covering or several layers of paper.

In adjusting the instrument the slit and grating rulings were made parallel by the use of a plumb bob. The coincidence of the normals to the grating and to the plane of the camera box was determined by using an illuminated slit 1 mm. wide and adjusting until the image was horizontally coincident with the object. The ordinary method of superposing the image of a candle flame upon the flame itself was not regarded as sufficiently delicate.

Duration and Method of Exposure.

In general the exposures were short, ranging from three to ten minutes, and all the quantitative work of the investigation was carried on when temperature conditions were most favorable, the change in temperature as given by one or more thermometers whose bulbs were placed underneath the cover of the mount usually being absolutely unreadable, and never more than a half of a tenth of a degree Centigrade. Moreover, the method of expose was such that pseudo-shifts, due either to mechanical jar or temperature change, must have been eliminated. The camera shutter was arranged as shown in the following cuts. Usually the arc was exposed first, its spectrum being placed along the centre of the photographic plate — this by use of the slit as given in Figure A; then the spark image was thrown upon the slit and its

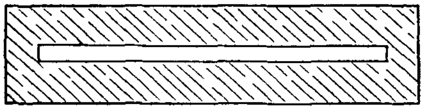

Figure A.

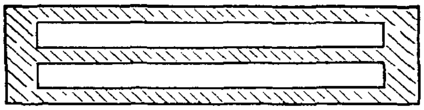

Figure B.

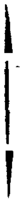

Figure C.

spectrum placed on the edges of the plate — this by the use of the slits shown in Figure B; then the first operation was repeated, and the arc spectrum again thrown upon the centre. But in most cases, always during the latter part of the investigation, the shutter was so arranged that the vertical superposition of the first and third exposures was not perfect, so that the central portion of the plate containing the arc spectrum presents some such appearance as that shown in Figure C. A mechanical shift may thus be easily detected.

Production of the Sources of Light.

The spark was produced in the following manner: The city current (110 volts, 133 cycles per second) was led into a 5 kilowatt transformer (110 to 15,000, 30,000 or 60,000 volts) manufactured by the Central Laboratory Supply Co., of Lafayette, Indiana, in series with which was a home-made water rheostat. The secondary voltage ranged from approximately 1600 to 14,000 volts, as the ratio of transformation which was employed was 30,000 to 110. The high potential current was led into glass-paraffin-tin condensers giving a range of capacity varying from about 0.008 to 0.08 microfarads (see Table I). Both the charging and discharging high potential wires were very heavy (about No. 8 standard wire gauge).

TABLE I.

Capacity of Condensers.

No. of Condenser.	Approximate Area of one Element in cm^2.	Mean Thickness of Glass in cm.	No. of Elements.	Total Capacity in Microfarads.
1	900	0.245	4	0.0076
2	900	0.249	12	0.0226
3	900	0.241	10	0.0195
4	730	0.246	20	0.0315
5	730	0.470	8	0.0067

The self-induction coil employed consisted of three concentric cylindrical coils of dimensions given below (see Table II). Generally, when photographing the spark, there was placed in the rear of it, at a distance of 80 cm., a 12-inch Westinghouse fan, the strong air current from which was directed upon the spark-gap by a megaphone 70 cm. long, the smaller opening (4 cm. in diameter) being about 2 cm. distant from the spark. When a secondary spark was used in series with that under investigation, the side draft from the fan was allowed to play upon it also. The fan undoubtedly produces greater disruptiveness and increases the potential of the discharge; but its steadying influence is very desirable, especially when self-induction is used in the discharge circuit. The luminous image was formed on the slit by a glass lens (aperture 3 inches, focal length 25 inches, made by Bausch & Lomb) situated about 110 cm. from the source of light and 152 cm. from the slit. This lens was held in a metal casing clamped to a marble slab which was set on an iron pipe imbedded in a cement foundation.

TABLE II.

DIMENSIONS OF SELF-INDUCTION COIL IN CM. GAUGE NUMBER OF WIRE = 14 S.W.G. EXTERNAL DIAMETER = 0.54. CM.

Coil.	Length.	External Diameter.	Total No. of Turns.
Inner	66.0	5.22	124
Central . . .	65.3	8.59	122
Outer	64.7	9.95	122

In all cases the spark which was being photographed was placed in a horizontal position in view of the possibility that different regions of the spark might show different spectra.

In the primary circuit of the transformer were placed a Thomson indicating watt-meter (maximum voltage 150 volts, maximum current 50 amperes, graduated in hectowatts) and a Thomson alternating ammeter (maximum current 100 amperes). The primary voltage was read by a Weston direct reading, alternating and direct current, voltmeter (two scale instrument, 15 and 7½ volts, 1 to 10 multiplier).

The arc was developed by a 133 cycle alternating circuit, the current used being about 12 amperes, measured by a 50-ampere Aryton and Perry spring ammeter. The discharge took place between one-half inch carbons, the lower one of which was of the cored type and was hollowed out a trifle to receive the small pieces of metal used. The distance between the terminals was always in the neighborhood of 3 mm. That the energy for the spark and arc discharges could be obtained from the city circuit was a distinct advantage, for during the exposures there were no mechanical vibrations due to heavy machinery, which would not have been the case if the current had been generated in the building.

Reduction of Astigmatism, Method of Measuring Plates, etc.

The investigation was confined to the second spectrum, the dispersion being one Ångström unit to 0.75 mm. The length of the rulings of the grating surface was diminished by reducing the vertical aperture to about 2 cm. (using black paper for the purpose); this process, together with stopping down the length of the slit to 5 or 6 mm., so much reduced the astigmatism that the tips of the spectral lines could be seen in the field of the micrometer used in measurement. We thus avoid a shift due to over-exposure, for, by setting the cross-hair upon the tip of the spark image, where it is just visible to the eye, we are certainly making use of minimum exposure conditions.

The filar micrometer (manufactured by Bausch & Lomb) was reinforced by adjustable trusses to produce greater rigidity.

A test of the accuracy of the instrument showed that the errors of measurement were greater than those of the screw. The magnification employed was about seven diameters, and a single cross-hair was found to give the best results. The plates were placed violet left, and four settings were made on the external or spark spectrum, then two upon the internal or arc spectrum; then the same set of measurements was repeated with the violet to the right. The plates used were the regular seed "Gilt Edge" 27, 5 × 7, cut into strips 1¼ × 7 inches.

The developer generally used was that described by Wallace[*] as giving with the seed 27 plate a definite and regular silver grain deposit.

Several plates were developed with Jewell's solution,[†] to which was

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[*] Astrophysical Journal, **20**, 118 (1904). The formula was kindly sent the author by Mr. Wallace, and is as follows:

A. Hydrochinon	1 gm.	B. KBr	1 gm.
Metol	1 gm.	NaOH	12 gms.
Adurol	2 gms.	H_2O	175 gms.
Na_2SO_3 (crystals)	24 gms.		
H_2O	175 gms.		

Take equal parts of A and B.

[†]

A. Hydrochinon	30 gms.	B. Potas. ferrocyanide	30 gms.	C. K_2CO_3	30 gms.
Na_2SO_3 (crystals)	150 gms.	H_2O	180 gms.	H_2O	180 gms.
H_2O	750 gms.				
Alcohol	10 c.c.				

Take one part of B, one of C, and six of A. Normal amount of bromide, 15 drops (10 per cent solution) to 100 c.c. of developer.

added an excess of bromide (roughly 0.5 gram of potassium bromide to a 3 oz. solution).

RESULTS.

The results are contained in the following tables, concerning the data given in which a few general remarks may be made: —

The time of the two arc exposures varied from 10 to 60 seconds, the short exposures predominating.

The length of the slit, unless otherwise stated, was either 5 or 6 mm., its width between 0.025 and 0.050 mm.,

The arc spectrum was generally thrown on the centre of the plate.

The values of current, power, and voltage given in the tables are the means of from two to six (average three) readings taken at fairly regular intervals during the exposures. The conditions of the circuit were quite stable, the three elements seldom varying more than 1 ampere, 20 watts, and 0.3 volts respectively.

The metals used were: Titanium carbide or "cast titanium" (made by Eimer & Amend of New York), 85 per cent titanium, 15 per cent carbon; a titanium-iron alloy, 20 per cent titanium, 80 per cent iron (Eimer & Amend); zinc battery rods; brass wire, 30.2 per cent zinc, 69.8 per cent copper; ordinary iron rod.

Unless otherwise stated the part of the spark image used was that near one terminal and the fan was employed to steady the discharge.

The secondary spark took place between brass rods 4 mm. in diameter.

The focus, to an accuracy of 1 mm., was determined repeatedly from week to week as the temperature of the room changed. This was a precautionary measure: no change in focus was noticed.

The given values of the shifts represent generally but one set of measurements on each plate, never more than two. This course was pursued because the different plates agreed so well that numerous measurements upon the same plate were deemed unnecessary. The average deviation from the mean of the measurements on the different plates in all the sets was about 0.0027 tenth-meters for titanium and iron, and 0.0070 for zinc. The average deviation from the mean of different measurements of a line on any one plate was about 0.0025 tenth-meters for titanium, 0.0018 for iron, and 0.0060 for zinc.

To gain in another manner an idea of the degree of accuracy of measurement, four settings, two red-right and two red-left, were made *

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* A suggestion due to Professor Frost.

on each of three lines: Ti, λλ 3898.645, 3901.114, and 3904.926 (Rowland's table of solar wave-lengths), as they appeared on a focus plate. The first line only was reversed: all were somewhat too dense for very accurate settings. Assuming the values of the wave-lengths of the first and third lines as given, that of the second was calculated. The first determination gave 3901.112, the second 3901.114; the mean 3901.113, but 0.001 tenth-meter different from the wave-length as given above.

Unless otherwise stated the developer used was the Wallace solution.

The spark lines which were studied most thoroughly, namely Ti, λλ 3900 and 3913; Zn, λλ 4680, 4722, and 4810, generally appear enhanced, diffuse, and unsymmetrically broadened. Many iron lines also show unsymmetrical broadening to some extent.

Figures 1, 2, and 3 of the accompanying plate are illustrative of the lines studied. The scratches, appearing at the tips of the spark and between the arc and spark segments, were drawn by pricking two holes in the film of a positive of the original negative at points which seemed to lie in the "centre of gravity" of the very tips of the spark lines. These two points were then joined by a ruler and the film carefully scratched away along its edge in the four regions above mentioned. A negative of the positive was then made. From the three negatives prepared in this manner the positive reproductions were developed.

The arc line lies considerably to the left, or violet, in all cases but that of Figure 2, which is a "self-induction line." It will be noticed that the double arc exposure as sketched in Figure C appears only in Figure 3 of the plate. As stated previously, this non-superposition method was not used at the very first of the investigation.

The scale of the enlargements is about five to one.

Let us discuss first the results as given by the titanium plates in Table III: —

1. The members of the set of six plates, set 1 (see next to last column of the table), agree in showing that with large capacity shifts exist (about 0.023 t.-m.), even in the absence of a secondary gap.

2. Set 2 (5 plates) shows that, with a secondary gap of 4 mm. and lessened capacity, considerable shifts exist (about 0.037 t.-m.). The effect of a secondary gap is to increase the potential of the discharge, and this gives the same result as an increase of capacity.

3. A condition (set 3) given by an increase of secondary gap and further reduction of capacity still presents a shift (about 0.033 t.-m.). It may also be seen that plates Nos. 8, 16, and 25 (sets 1, 2, and 3), on which the arc spectrum was photographed outside of the spark spectrum,

show no marked or consistent difference from those obtained in the ordinary manner. Care was taken that the spark exposure be very short.

4. Introducing the self-induction coil into the discharge circuit of the condenser (set 4) unquestionably conduces to coincidence of arc and spark line (mean value of shift 0.005 t.-m.).

5. Sets 5 and 6 show that the use of the alloy, 20 per cent titanium, 80 per cent iron, give much less displacement (0.002 t.-m.) than that given by titanium carbide, 85 per cent titanium, 15 per cent carbon (about 0.02 t.-m.).

6. The part of the spark image used is extremely important; set 7, given by that region of the image near the terminal, shows a shift of 0.018 t.-m., while the central portion of the spark (set 8) 4.5 mm. from the terminal shows but about 0.006 t.-m.

7. Sets 9 and 10 show that ohmic resistance (2.7 ohms) in the discharge circuit of the condenser produces an effect exactly similar to that of self-induction (see result No. 4), reducing the shift from about 0.017 t.-m. to practically zero.

8. Sets 7 to 10, in which no fan was used, show that the same is not a *sine qua non* in the matter of shift.

9. Further, the developer used appears to have no effect upon the shift. Therefore in sets 5, 7, 8, and 10 the measurements made on plates developed with the Jewell solution are averaged in with the others. Indeed it is difficult to see how there could be a difference, provided the tip of the line be used in measurement. If, however, the setting be made upon the body of the line a difference might very easily be introduced (see below, result No. 16), a "softer" developer, such as Wallace's, tending to displace the apparent centre of gravity of the line toward the side of greater broadening.

10. Sets 11 and 12, also 11 and 13, Table IV, show that the alloy and self-induction effects of sets 6 and 7, also 3 and 4, Table III, respectively, hold good with lines of the spectrum other than λλ 3900 and 3913.

11. Set 14 shows that large shifts exist in lines in other regions, e. g., λ 4827.74, 0.024 t.-m.; λ 4836.25, 0.036 t.-m.

It is also noticeable that whenever both spark and arc lines are reversed the central portions of the reversal are coincident.

Passing to zinc, Table V, we have:

12. Sets 15, 16, and 17 give results comparable with those of 3, 1, and 2 for titanium, and sets 18 and 19 confirm the alloy effect of 6 and 7 and also 11 and 12.

TABLE III.

SHIFT OF SPARK LINE TOWARD RED IN

No. of Plate.	Date.	Length of Spark Exposure in Min.	CONSTANTS OF ELECTRIC CIRCUIT.							Character of Lines. S = Sharp. D = Diffuse.
			Primary.			Secondary.				
			Amps.	Watts.	Volts.	Capacity in Micro-farads.	Lgth. of Gap in mm.		Part of Image used.	
							Prim'y.	Secondary.		
6	Dec. 15	15	40.6	193	6.8	0.0812	1.0	0	End	D
7	" "	15	41.1	180	7.0	0.0812				
8	" 16	15	40.4	188	6.5	0.0812				
9	" "	22	40.9	172	5.8	0.0812				
10	" 17	26	39.7	188	6.3	0.0812				
28	" "	15	40.0	160	7.2	0.0803				
12	" "	5	39.0	390	12.0	0.0195	1.5	4.0	End	D
13			37.8	390	12.5		2.0			
14			38.3	400	13.0		1.7			
15			38.5	380	11.3		2.1			
16			38.5	380	12.2		2.5			
22	Jan. 14	6	35.5	1010	46.0	0.0067	2.0	31	End	D
23	" 17	5	34.1	910	37.0		1.0	27		
24	" "	6	34.4	910	36.0		2.0	27		
25	" "	5	34.0	883	38.3		2.0	27		
26	" "	5	35.0	717	35.0	0.0067	2.0	27	End	S
27	" 21	7	33.6	775	41.0			25		
70	April 15	5	41.5	415	11.6	0.0226	2.0	5	End	D
72	" "	3	42.4	400	11 3					
93	May 5	8	41.4	395	10.9					
99	" "	11	41.2	400	11.2					
77	April 15	5	41.7	397	11.1	0.0226	2.0	5	End	S
78	" "	9	41.2	420	11.3					
73	" "	4	41.5	440	12.0	0.0226	9.0	0	End	D
74	" "	3	41.1	410	11.5					
97	May 6	9	39.8	417	11.2					
101	" "	10	41.2	410	11.5					
102	" "	10	41.0	427	11.5					
75	April 15	4	41.6	425	11.7	0.0226	9.0	0	Centre	D
76	" "	3	41.5	435	11.5					
96	May 5	6	41.5	425	11.4					
98	" 6	9	41.0	425	11.1					
80	April 29	10	41.3	297	9.1	0.0226	4.0	0	End	S
81	" "	15	41.9	313	9.0					
82	" "	15	41.9	313	9.0					
83	" "	5	42.1	320	8.9	0.0226	4.0	0	End	D
84	" "	4	41.5	315	9.0					
87	May 2	8	41.8	300	9.2					
88	" "	7	42.0	300	9.1					
103	" 10	10	40.7	307	8.7					
104	" "	10	40.2	300	8.5					
105	" "	8	40.9	315	8.6					
106	" "	8	41.3	310	8.4					

TITANIUM.

TENTH-METERS: POSITIVE UNLESS OTHERWISE STATED.

Metal used.	Fan.	Shift.				No. of Set.	Remarks.
		λ 3900.68.		λ 3913.58.			
		Separate Measurements.	Mean.	Separate Measurements.	Mean.		
85 % Ti 15 % C	Yes	0.036* 25 21 18* 21 18	*0.023*	0.031 24 20 22 24 22	*0.024*	1	Plate No. 8, spark inside; arc outside.
85 % Ti 15 % C	Yes	0.042 35 38 37 39	*0.038*	0.048 30 39 31 34	*0.036*	2	Plate No. 16, spark inside; arc outside.
85 % Ti 15 % C	Yes	0.045 36 26 32	*0.035*	0.041 25 28 30	*0.031*	3	Plate No. 25, spark inside; arc outside.
85 % Ti 15 % C	Yes	0.002 8	*0.005*	0.004 7	*0.005*	4	Triple coil of self-induction used.
85 % Ti 15 % C	Yes	0.027* 15* 21 17	*0.020*	0.026* 23 16* 24	*0.022*	5	Developed with Jewell solution.
20 % Ti 80 % Fe	Yes	0.001 4	*0.002*	. . 0.003	*0.003*	6	
85 % Ti 15 % C	No	0.020 14* 22 21 17	*0.019*	0.017* 17* 19 23 15	*0.018*	7	Developed with Jewell solution.
85 % Ti 15 % C	No	0.006 6 6 7	*0.006*	0.003 6 5 7	*0.005*	8	Developed with Jewell solution.
85 % Ti 15 % C	No	−0.001 −7 10	*0.001*	−0.001 −6 3	*−0.001*	9	Two meters of fine wire resistance 2.7 ohms. in discharge circuit of condenser.
85 % Ti 15 % C	No	0.020* 18* 16* 14* 13 20 18 14	*0.017*	0.023* 18* 12* 15* 15 20 17 17	*0.017*	10	Developed with Jewell solution.

* Mean of two measurements.

TABLE IV.

Shift of Spark Line toward Red in Tenth-
End of Image near Terminal Set upon Slit;

Number of Plate.	Date.	Length of Spark Exposure in Minutes.	Constants of Electric Circuit.						Shift and Character of b = bright line. r = reversed line.					
			Primary.			Secondary.			λ 4443.97		λ 4468.65		λ 4489.24	
			Amps.	Watts.	Volts.	Capacity in mf.	Gap-Length in mm.		A S b b	Mean.	A S b b	Mean.	A S b b	Mean.
							Pri.	Sec.						
57	Mar. 18	5	40.4	433	11.9	0.0226	2	5	—0.005*	*—0.002*	0.009	*0.008*	0.008	*0.008*
60	"	5	40.8	393	11.1				2		8*		8	
63	Apr. 8	10	41.9	403	11.3	"	"	"	0.003	*0.003*	—0.001	*0.002*		
64	"	12	41.8	373	11.1				. .		5			
59	Mar. 18	5	41.2	357	12.1	"	"	"	*—0.001**		*0.003*		*—0.001*	
									λ 3223.1 †		λ 3228.0 †		λ 3235.0 †	
									A	S	A	S	A	S
									r	r	b	b	r	r
49	Mar. 13	12	40.6	450	12.0	0.0226	2	5	*—0.001*		*0.002*		*0.001*	

* Mean of two measurements.

† Third order spectrum. Wave-lengths given approximately.

TITANIUM.

METRES; POSITIVE UNLESS OTHERWISE STATED.

METAL 85% Ti, 15% C, UNLESS OTHERWISE STATED.

ARC AND SPARK LINES.
A = arc line. S = spark line.

λ 4533.42		λ 4534.15		λ 4549.79		λ 4563.94		λ 4572 15			No. of Set.	Remarks.
A S r b	Mean.	A S b b	Mean.	A S b b	Mean.	A S b b	Mean.	A S b b	Mean.			
0.013 . .	*0.013*	0.010 4	*0.007*	0.022 15	*0.020*	0 003 4	*0 003*	0.016 25	*0.020*		11	
		—0.007 . .	—0.007	0.006 5	*0 005*			0.001 2	*0.001*		12	20 % Ti, 80 % Fe.
—0.006 ‡		*0.001*		*—0.001*		*0.001*		*—0.003*			13	Triple coil of self-induction used.

λ 3237.5 †		λ 4827.74		λ 4836.25		λ 4844 13		λ 4848.62		λ 4856.18		No. of Set.
A	S	A	S	A	S	A	S	A	S	A	S	
r	r	b	b	b	b	b	b	b	b	b	b	
0.000		*0.024*		*0.036*		*0.015*		*0.015*		*—0.005*		14

‡ Spark line on this plate is reversed.

TABLE V.

SHIFT OF SPARK LINE

No. of Plate.	Date.	Length of Spark Exposure in Minutes.	CONSTANTS OF ELECTRIC CIRCUIT.								Metal Used.
			Primary.			Secondary.					
			Amps.	Watts.	Volts.	Capacity in Micro-farads.	Length of Gap in mm.		Part of Image used.	Shape of Terminals.	
							Pri.	Sec.			
31	Jan 27	10	33.4	955	36.0	0.0067	?	25	End	Flat 3 × 6 mm.	Zn battery rods.
33	Jan. 28	10	32.5	946	38.8		2	24			
34	Jan. 28	15	32.8	965	41.0		2	21			
36	Feb. 3	15	34.8	983	41.0		2	21			
37	Feb. 3	15	33.2	993	42.5		2	21			
38	Feb. 3	30	41.1	193	7.4	0.0736	1	0	End	"	"
41	Mar. 4	4	41.0	440	12.8	0.0195	1.5	3	End	Round 3 mm. diameter.	"
42	Mar. 4	6	40.8	445	13.2						
45	Mar. 11	20	41.2	343	9.6	0.0226	2	2.5	End	" Flat 1 × 3 mm.	30.2 % Zn 69.8 % Cu
46	Mar. 11	18	41.7	350	9.7						
47	Mar. 11	10	40.7	490	14.2	0.0226	2	2	End	"	Zn battery rods.
48	Mar. 11	10	40.8	463	13.8			2			
43	Mar. 4	5	42.0	385	11.1			2.5			

ZINC.

TOWARD RED IN TENTH-METERS.

Fan.	Shift.						No. of Set.	Remarks.
	λ 4680.38.		λ 4722.26.		λ 4810.71.			
	Separate Measurements.	Mean.	Separate Measurements.	Mean.	Separate Measurements	Mean.		
	. .		0.066		0.048			
	0.019		43		30			
Yes	37	*0.028*	40	*0.042*	30	*0.039*	15	
	24		36		48			
	32		26		. .			
"		*0.025*		*0.047*		*0.039*	16	
"	0.034	*0.033*	0.047	*0.047*	0.071	*0.065*	17	
	33		48		59			
"	0.003	*0.003*	0.000	*0.003*	0.017	*0.012*	18	Filings from zinc battery rod used in arc as in other cases. Lines sharp.
	. .		6		8			
	0.029*		0.059*		0.084*			
"	. .	*0.032*	63	*0.065*	125	*0.102*	19	Plate No. 48 developed with Jewell solution and normal amount of bromide.
	35		74		97			

* Mean of two measurements.

13. In iron, Table VI, we find that, among the lines investigated, the shifts are in general very small, but are nevertheless present.

Further facts are as follows:

14. A plate on which three exposures were made in the usual manner, the first and third being superposed, and the source of light being the titanium arc, shows that no shift is introduced by throwing a diffuse image on the slit, destroying the focus by moving the arc two inches nearer the lens. This proves that even if either arc or spark were out of focus to a small extent the shift would not be altered.

TABLE VI. IRON.[*]

SHIFT OF SPARK LINE TOWARD RED IN TENTH-METERS.

Wave-length.	Shift.	Wave-length.	Shift.
3688.58	0.004	3788.01	0.001
3705.10	2	95.13	3
9.37	3	99.68	3
20.07	3	3815.97	10
27.78	3	27.96	6
48.39	5	34.37	7
58.36	5	40.58	2
63.93	6	41.19	8
67.34	5		

15. A plate exposed to the carbon and graphite arcs shows that no shift occurs between lines which are due to the same impurities in both. The currents used were 33 and 75 amperes respectively. The graphite arc is produced with considerable difficulty, a large current being required,

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[*] The conditions under which the iron plates were obtained are as follows: Six plates taken at various times; average spark exposure, 12 minutes; average current, power, and voltage, 41.3 amperes, 420 watts, 11.5 volts; capacity, 0.0226 microfarads; length of primary gap, 9 mm.; no secondary gap and no fan; region of spark image used, that near terminal. The above values of the shift are means of measurements made on from two to six plates.

and the discharge is quite explosive. It was thought that this explosive effect might produce a small shift toward the red.* Evidently the disruptive effect is of too great a period and not sufficiently violent.

16. A plate was taken under conditions similar to those of Nos. 83 and 84, except that the slit-length was extended to about 1 cm., the whole vertical aperture of the grating was used, and the exposure lengthened to ten minutes. Under these conditions there results a greatly over-exposed spark spectrum. Upon careful measurement, setting on what appears to be the maximum of intensity of the body of the line, shifts as great as 0.031 and 0.025 t.-m. appear; whereas set 10 gives values of 0.017 and 0.017. It is evident, then, as Eder and Valenta suggest, that by over-exposure a false displacement may be introduced in a line marked by unsymmetrical broadening.

TABLE VII. TITANIUM.

DATA TRANSFERRED FROM TABLE III.

Mean Current in Amperes.	Mean Power in Watts.	Mean Potential Difference in Volts.	Mean Shifts for $\lambda\lambda$ 3900, 3913.	Set.	Effect of
40.5	180	6.6	0.023	1	
38.4	388	12.5	37	2	
34.5	928	39.3	33	3	self-induction.
31.3	746	38.0	5	4	
41.6	402	11.3	21	5	alloy.
41.5	409	11.2	2	6	
40.9	421	11.5	18	7	part of spark image.
41.4	428	11.4	5	8	
41.7	308	9.0	0	9	ohmic resistance.
41.3	308	8.8	17	10	

17. By referring to Table VII, the data given in which are taken from Table III, it will be seen that the energy involved is not the only

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* The suggestion that a comparison of these two spectra be made is due to Professor Crew.

element in the shift, for sets 3 and 4, in which nearly the same power (928 and 746 watts) was used show a large difference in shift. This is due to self-induction, which causes less disruptiveness in the discharge.

Ohmic resistance (sets 9 and 10) gives the same effect.

The influence of a low percentage alloy appears in sets 5 and 6, and the difference between the centre and end regions of the spark is shown in sets 7 and 8.

It may be remarked that set 3 (power 928 watts) shows less shift, 0.033, than set 2 (power 388 watts), 0.037; but the difference is very small, and moreover the disruptiveness in the latter case may have been greater.

Summation of Results.

In conclusion it may be said that the writer's former results have been qualitatively and, to a certain degree, quantitatively confirmed. It is difficult to see how we can avoid the following deduction: Under certain conditions, namely, when self-induction and ohmic resistance are absent, large capacity or long secondary gap are used, and the terminals are of a high percentage alloy, the part of the spark which lies near the terminal gives a spectrum, the wave-lengths of the lines of which are greater than those of the arc spectrum. The shifts occur when the energy involved in each discharge is great, and the discharge is disruptive in nature. These conditions mean the creation of a large amount of metallic vapor in a short time — a highly explosive phenomenon. That an increase of wave-length should result is rather to be expected. This may be rendered more evident from the following discussion:

In addition to those theoretical considerations (mentioned in the writer's former paper) * which would lead us to expect an increase in wave-length, namely those dealing with the pressure, "p," in a gaseous medium as a function of the number of particles per c.c., "n," the mass of each, "m," and the mean squared velocity, "u^2," postulating the truth of the equation $p = \frac{1}{3} m n u^2$, it may be stated that such an increase in pressure may come from the fact that, at the beginning of the discharge, the air acts as an incompressible medium (compare the downward explosive effect of dynamite placed on the surface of the earth). The development of the metallic vapor takes place so quickly that the air has no time to move. The lessened shift in the centre of the spark can then be easily explained, for it is known that some time

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* Astrophysical Journal, **17**, 296 (1903).

elapses before the metallic particles, advancing from the terminal, reach the central region of the spark. When they have finally reached this region the air has yielded and the pressure fallen. An unusual amount of vapor is disengaged in the discharge between electrodes of titanium and zinc in water (not so prominently the case with iron). That the pressure developed is also great was noticed by the writer in his work with Professor Hale on the spark under water.* The terminals were badly shattered; indeed there is good reason to expect that the lines of the spectrum given by the discharge in air pass through a series of changes which resembles that in water. Certain titanium lines appear reversed on some of the present series of plates and not so on others. The reversal comes up unsymmetrically on the violet edge of the centre of gravity of the line, a common occurrence in the spectra of the spark under water. The question of the exact parallelism of the two phenomena must be reserved for future investigation.

POSSIBLE EXPLANATION OF THE INCONSISTENT RESULTS OF HASCHEK, EDER AND VALENTA, AND MIDDLEKAUFF.

Haschek undoubtedly dealt with over-exposed lines and increased the true shift by a photographic displacement of the region of maximum intensity.

Eder and Valenta probably used a vertical spark and set the central region of the same on the slit of the spectrometer (the lines studied by the writer, Zn λλ 4680, 4722, and 4810, are among those investigated by Eder and Valenta). We are assuming that the same effect obtains in the case of zinc as with titanium (sets 7 and 8, deduction No. 6 above) with reference to the central and end portions of the spark. That no shift exists under the conditions employed between the absorption regions of reversed arc and spark lines, as stated by Eder and Valenta for zinc, was found to be true in the case of titanium.

Lastly, that Middlekauff found no shift with iron may easily be explained by the fact that few lines show a shift, and these generally to but a small degree, under highly disruptive conditions and near the region of the terminal. Middlekauff probably used a comparatively weak spark, set its image vertically on the slit, and integrated with his spectrometer the spectra of the central regions.† Moreover Middlekauff and

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* Astrophysical Journal, **17**, 154 (1903).

† The statement is made that the spark was eight or nine mm. in length and its image on the slit about 6 mm.

Eder and Valenta may have used small wires in the discharge circuit of the condenser and so have introduced ohmic resistance.

Influence of Results upon the Mapping of Spark Spectra and upon Stellar Line of Sight Work.

The bearing of the above results upon radial velocity investigations in which a titanium spark is employed to give the comparison spectrum is such as to emphasize the fact that low-energy and non-disruptive conditions should be employed if the lines of the resultant spectra are assumed to be severally of the same wave-length as those of the arc or the absorption and bright lines of the stars. Furthermore, if a highly disruptive spark be used, the same circuit conditions should be employed at all times in any one observatory, to make the plates there taken mutually comparable; or, finally, the same conditions should prevail at all observatories if mutually consistent results are desired.

The general bearing upon the mapping of spark spectra may be seen in the fact that an agreement between the wave-lengths of spark lines as measured by various investigators is impossible unless the conditions of the electric circuit be the same in all cases.

In conclusion, I wish to acknowledge my indebtedness to the Rumford Committee for the two grants which made this work possible, and to Professors Hale and Frost of the Yerkes Observatory for the loan of the grating used. I am also under obligation to Dr. George S. Isham for the gift of the filar micrometer, and to Mr. Cornelius Vanderbilt for generous financial aid. My assistants, Messrs. Hartley and Miller, have proved invaluable at various times; and to Mr. Boord I am indebted for the analysis of the brass wire.

Wabash College, Crawfordsville, Indiana,
June, 1905.

EXPLANATION OF PLATE.

FIGURE 1.

Positive of Ti λ 3913.58 as apparent on plate No. 22.
Mean shift as given by plates 22 to 25, 0.031 t.-m.

FIGURE 2.

Positive of Ti λ 3913.58 as apparent on plate No. 27.
Influence of self-induction shown.
Mean shift as given by plates 26 and 27, 0.005 t.-m.

FIGURE 3.

Positive of Zn λ 4722.26 as apparent on plate No. 47.
Mean shift as given by plates 47, 48, and 43, 0.065 t.-m.

CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF
HARVARD UNIVERSITY. — LXII.

PRELIMINARY DIAGNOSES OF NEW SPECIES OF LABOULBENIACEAE. — VI.

By Roland Thaxter.

Received June 18, 1905.

With the present contribution, which comprises such new forms of Laboulbeniaceae as have accumulated during the past two years. the writer proposes to close the series of preliminary diagnoses which he has issued from time to time since 1899. With these additions the total number described becomes approximately five hundred, including about forty-eight genera; and plates illustrating all the forms not included in the writer's "Monograph," comprising not far from one thousand figures, are now completed. It is a matter of regret, however, that the publication of these illustrations must necessarily be still longer deferred, owing to the fact that the writer proposes to be absent during the coming year, and will be unable to give this matter his attention until after his return.

Dimeromyces Labiae nov. sp.

Male individual. Receptacle consisting normally of three superposed cells, the basal projecting laterally to form the base of the lowest antheridium, bearing distally a two-celled appendage distinguished by a basal blackish septum; the two upper cells of the receptacle producing two, rarely three, antheridia: the latter short and stout, the efferent region containing the several tubes rather abruptly distinguished from the venter, as well as from the common efferent tube, which curves outward somewhat abruptly. Total length to tip of appendage, including foot, 50 μ. Appendage 25 μ. Antheridium 16–18 × 6–7 μ.

Female individual. Subhyaline, or straw-colored, consisting of an irregular axis of three or four superposed cells, terminated by a simple, usually two-celled, appendage distinguished by a blackish septum: the subbasal cell bearing a broad blunt bicellular upcurved appendage or protrusion, which subtends either the single perithecium, or a several-celled appendage similar to the primary one. Perithecium solitary, colorless, somewhat elongate, tapering but slightly to the more or less abruptly distinguished rather broad somewhat flattened tip. Total length to tip of perithecium 65–75 μ. Perithecium 50–60 × 12 μ. Spores about 18 × 12 μ.

On the elytra of *Labia minor* Burm., Cambridge, Mass.

Dimeromyces minutissimus nov. sp.

Male individual. Receptacle consisting of three cells, the basal externally suffused with blackish-brown, and projecting beneath the base of the antheridium; the subbasal giving rise to the solitary antheridium; while the distal is separated by a blackened septum from a two-celled erect rigid distally spinose appendage. Venter of antheridium small, tapering gradually to the stout outcurved neck. Total length, including foot, 32 μ. Antheridium 16 × 5 μ.

Female individual. Receptacle very irregularly formed, externally blackened above the large foot; consisting of several irregularly superposed cells, the subbasal giving rise to a two-celled prominence subtending a blackish tapering appendage, which projects at right angles to the colorless primary appendage; the latter two-celled, distinguished by a blackish septum, and often subtended by a similar appendage from the same (distal) cell of the receptacle. The single perithecium rather short and broad, tapering slightly to the more or less distinctly differentiated somewhat truncate tip. Length to tip of perithecium 50 μ. Longest appendage 40–50 μ. Perithecium 34 × 10 μ.

On the inferior surface of the rhabdites of *Labia minor* Burm., Cambridge, Mass.

These two species may prove merely variations of the same form due to the differences in their position of growth, and are only provisionally separated.

Monoicomyces Leptochiri nov. sp.

Receptacle consisting of a main axis of superposed cells, usually long and slender, normally simple, sometimes once to several times branched, the whole structure apparently a branch from the subbasal cell of a small primary receptacle: the axis somewhat similar to that of Rhachomyces, consisting of a series of superposed cells, variable in number (about twelve to twenty-five), each of which bears on one side, so as to form a unilateral series, a pair of simple cylindrical few-celled appendages, which become better developed from below upward; arising side by side, closely associated and appressed, characteristically blackened at the base, which is usually indistinguishable from the small blackened cell from which they arise, and which is obliquely separated just below each septum of the axis: this opaque region in the lower cells of the axis usually projecting characteristically like a notch or tooth: the cells of the axis near its base deeply suffused or opaque; the suffusion becoming, from below upward, gradually associated with the septum only: the axis in some specimens nearly hyaline throughout, with blackish-brown suffusions at or about the septa. The paired appendages replaced somewhat irregularly below the perithecium by antheridia of small size, resembling the appendages in general appearance when viewed laterally, and usually terminated by two hyaline simple appendages of no great length. Perithecium solitary, terminating the main axis, or its secondary axes; faintly yellowish, straight, nearly symmetrical, rather abruptly narrower at the tip, the apex rather small, symmetrical, truncate. Total length to tip of perithecium 250–800 μ, the axis about 18–20 μ in diameter. Perithecium 70–90 × 18–25 μ. Longer appendages sometimes 50 μ.

On all parts of several species of *Leptochirus*, Java.

Monoicomyces similis nov. sp.

Pale straw-colored. Receptacle consisting of two superposed cells, terminated by a straight tapering simple several-celled appendage, sometimes reaching to the tips of the perithecia, and distinguished by a black basal septum. The subbasal cell of the receptacle also giving rise to a pair of normally symmetrical branches, consisting of a single basal cell, from which arises the stalk-cell of the perithecium on the inner, and that of the antheridium on the outer side. Stalk-cell of the perithecium narrowed at the base, the narrow portion red-brown, expanding above and hyaline. Perithecium somewhat asymmetrical or bent, deeper straw-colored, tapering gradually to the blunt tip, which is hardly distinguished. Antheridium of the normal type, the distal appendages usually two, one sometimes exceeding the tip of the perithecium. Total length to tip of perithecium 150–200 μ. Perithecium 90–110 × 30 μ, the stalk-cell 35 × 46 μ. Spores about 35 × 4 μ.

On the abdomen of *Homalota* sp., infesting species of *Lactarius*, Kittery Point, Maine.

Eucantharomyces Madagascarensis nov. sp.

Rather pale straw-colored. Perithecium large, subcylindrical, tapering very slightly distally and basally, the rounded tip rather abruptly distinguished, and subtended by more or less distinct elevations; the inner lip-cell short, prominently rounded, but not extending beyond the other lip-cells and rendering the apex subsymmetrical; the base not differentiated from the body of the receptacle, the cells relatively short, not extending down beside the stalk-cell, from which they are rather clearly distinguished: stalk-cell elongate, tapering below. Basal and subbasal cells of the receptacle of about equal length, the former about half as broad as the latter, and extending to the base of the stalk-cell of the perithecium. Basal cell of the appendage somewhat larger than the subbasal, slightly overlapping the subbasal cell of the receptacle, the marginal cell not prominent below the discharge tube which is curved outward, sometimes horizontally; the antheridial cells in three rows of seven, seven, and five cells respectively. Spores about 60 × 4 μ. Perithecium, including basal cells, 385 × 60–70 μ, the stalk-cell 275–290 × 40–55 μ. Receptacle 165 × 35–45 μ. Antheridium, total length 130–150 × 30–35 μ, above basal cell 75 μ.

On elytra of *Callida* sp., Madagascar; Hope Collection, No. 273.

Chitonomyces dentiferus nov. sp.

Pale straw-colored. Receptacle nearly straight, the foot large, the basal cell about twice as long as broad, but slightly broader distally, the subbasal cell broader than long, distinguished above and below by a more or less distinct constriction, the lower marginal cell longer than the next above it, the terminal cell subcylindrical with broadly rounded apex bent slightly inward. Perithecium distally curved inward above a suberect, tooth-like, unicellular appendage, which is curved slightly outward, and originates above a rounded more deeply colored prominence, which subtends it and lies nearly opposite the extremity of the distal appendiculate cell of the receptacle, which it about equals in length. The tip of the perithecium tapering (somewhat as in *C. appendiculatus*), the lip-cells slender and somewhat elongate, with rounded extremities. Total length to tip of perithecium 90–110 × 18–20 μ. Perithecium about 65 × 18 μ. Basal portion of receptacle about 35 μ. Spores about 28 × 3 μ.

On the margin of the left elytron of *Laccophilus* sp., near the base, Lake Eustis, Eustis, Florida.

Chitonomyces Javanicus nov. sp.

Foot relatively small. Basal cell of the receptacle small and narrow, its basal half or more suffused with dirty blackish-brown, above hyaline; subbasal cell longer than the basal, and of nearly the same diameter throughout; the three cells above it subtriangular, about equal in size, darker straw-colored, or pale amber-brown: the lower marginal cell of the distal portion of the receptacle narrow, clean cut, darker amber-colored; the lower appendiculate cell above it hyaline, small: the two remaining cells above and external to it, forming an opaque appendage shaped like a knife-blade, its tip rounded outward from the inner side to an external blunt apiculus. Perithecium suffused below with pale amber-brown, long and slender, narrower opposite the insertion of the lower appendage, expanding slightly above it, the small hyaline apex curved inward and subtended externally by an erect, clear, dark amber-brown projection, hardly or not at all exceeding it; and on the inner side by a larger, broader, less deeply colored, spreading cell, which is curved toward the receptacle, the blackened portion of which projects about half its length above it. Total length to tip of perithecium 165–185 μ, to the tip of the black prolongation of the receptacle 180–200 μ, greatest width 22–25 μ. Perithecium 90–110 × 16 μ. Basal portion of the receptacle 75 μ, including the slender foot.

On the margin of the left elytron of *Laccophilus* sp., Java.

Chitonomyces spinosus nov. sp.

Basal and subbasal cells of the receptacle subequal, the latter somewhat broader; the lower marginal cell relatively short and broad; the lower appendiculate cell relatively large; the upper marginal cell bearing distally a long unicellular amber-brown outgrowth, about half the total length of the individual, projecting upward and outward at an angle of about 45°, with a slight terminal twist, and subtended by a prominent bulge of the cell which bears it: the distal appendiculate cell relatively large and broad. Perithecium rather stout, the transverse limits of the wall-cells indicated externally by more or less pronounced elevations, the blunt tip somewhat abruptly distinguished above the nearly straight inner margin. Total length to tip of perithecium, including foot, 130 μ, greatest width 33 μ. Marginal appendage 45–60 μ. Spores about 28 × 2.5 μ. Perithecium 75×20 μ.

On the left posterior leg of *Laccophilus* sp., Java.

Distichomyces nov. gen.

Receptacle consisting of a basal and subbasal cell surmounted by two parallel series of cells of indefinite number, any of which may bear either a sterile appendage or an antheridium externally, one of the series ending in a perithecium, the other terminated by the primary appendage. Appendages of the same type as those of Rickia and Peyritschiella. Antheridia at maturity terminal on a unicellular branch, becoming quite free in a compact group.

A genus very closely resembling Rickia, but differing in its biseriate receptacle, and its free simple antheridia, the terminal cell which in other genera of the Peyritschiellae persists as a common delivery tube, becoming apparently completely disorganized, although the early stages of development have not been seen. The primary appendage is borne on a bicellular base, the two series of cells forming the main axis of the receptacle resulting from the activities of two cells immediately below this base; one representing an original branch and finally producing the perithecium which terminates this "perithecial axis"; the other being an intercalary cell, so that the development of this half of the axis, which may be called the "primary axis," is in a sense trichothallic.

Distichomyces Leptochiri nov. sp.

Slender and considerably elongated, or sometimes short and stout: the primary axis consisting of from twelve to thirty cells, more or less, above the subbasal cell of the receptacle, somewhat proliferous distally beyond the base of the primary appendage, the small cells thus formed, one or two of them appendiculate, extending to the free tip of the perithecium; the perithecial axis consisting of from six to twenty-six cells, more or less; the perithecium slightly broader, but not distinguished from the axis, wholly united on its inner side to the primary axis, more or less strongly curved outward to its blunt subtruncate free tip. Appendages as in Peyritschiella, hyaline, rather slender, very variably developed, borne on a small triangular cell, separated distally and externally from the cells of the axis; sometimes numerous, often few and scattered, or almost wanting except distally, where all the cells of the primary axis are usually appendiculate. Antheridial branches borne like the sterile branches, irregular in position and number, sometimes even absent, usually confined to the lower half of the axis, more often near the base, consisting of a short stalk-cell bearing terminally a compact erect group of flask-shaped antheridia, usually four or five in number, their short broad necks at first closely approximated or coherent, divergent only after full maturity. Perithecium 50–55 × 15–20 μ. Longer sterile appendages 75 × 6 μ. Antheridial branch, including antheridia, 18 μ. Total length, longer 475 × 10 μ, shorter and stouter 125 × 36 μ. Secondary perithecia may develop from cells of either axis if the primary perithecium does not develop or is destroyed.

On the head, antennae and under surface of *Leptochirus* sp., Java; Rouyer, No. 1403.

Herpomyces Anaplectae nov. sp.

Male individual consisting of four superposed cells, the two or three upper ones becoming variably branched, and bearing a variable number of antheridia on the branchlets; the subbasal cell apparently in some cases producing one or two small secondary receptacles, each of which bears an irregular tuft of antheridia. Total length, including antheridia, 35–50 μ. Antheridia 24 × 2 μ.

Female individual hyaline. Primary receptacle mostly 4-celled, the distal cell at first long, conical, mucronate, becoming more or less collapsed and irregular at maturity; the subbasal giving rise to a branch from which are formed a pair of small secondary receptacles, each producing a single perithecium; the two symmetrically placed, and symmetrically curved away from the axis of the primary receptacle. Perithecium long, slender, tapering from below the middle to its attenuated apex; the tranverse boundaries of the wall-cells often indicated in the middle region by more or less distinct elevations, the apex unmodified, blunt. Primary receptacle 25 × 3 μ. Secondary receptacle 16 × 13 μ. Perithecium 80–90 × 16 μ. Spores about 18 × 2 μ.

On the antennae of a species of *Anaplecta* allied to *A. Dorhniana* Sauss.-Zehntn., collected under stones along the margin of a stream near Caracas, Venezuela, by Dr. A. F. Blakeslee.

Herpomyces Nyctoborae nov. sp.

Male individual. Axis consisting of five or six cells, the basal somewhat elongate, the rest short and somewhat rounded, the terminal one abruptly apiculate; the distal cells producing branches which in turn branch three to four times, the ultimate branchlets bearing the antheridia terminally in groups, the whole forming a dense tuft. Axis about 54 × 10 μ, the total length to the tips of the antheridia about 100 μ. Antheridia 20 × 3 μ.

Female individual. Colorless, the primary receptacle consisting of four or five cells, the terminal one short and apiculate; the subbasal (perhaps others also?) giving rise to branches which produce a variable number of secondary receptacles arranged with the younger behind the older as in *H. Paranensis.* The secondary receptacles much as in *H. Paranensis*, the first formed broadest. Perithecium long and slender, tapering, especially above the middle; the stalk-cells flattened, so that the perithecium is almost sessile on the receptacle; the ascigerous portion often abruptly somewhat narrower than the base, the fourth lateral external wall-cell bearing a long curved attenuated wholly free spine-like cell, which extends just beyond a subterminal short distally incurved sharp spinous process, which subtends, and is nearly twice as long as, the free curved tip of the perithecium: the fourth anterior wall-cell also producing a long curved spinous process, slightly shorter than the lateral one, or more divergent. Secondary receptacles 65 × 45–50 μ. Base and stalk-cells of perithecium 30–35 × 15–18 μ, the ascigerous part to tip of longest spine 150–165 μ: width near base 20–25 μ, below lower spine 10–12 μ; external spine 50 μ, terminal spine 12 μ.

On antennae of *Nyctobora latipennis*, Texas; Scudder Collection, No. 1383.

Herpomyces Phyllodromiae nov. sp.

Male individual consisting of four superposed cells: producing distally one or several antheridia directly. Total length about 30 μ.

Female individual. Colorless, the primary receptacle minute, consisting of four subequal cells: the subbasal sending down a branch which furcates to produce a pair of small secondary receptacles, consisting of only two or three vertically elongated cells and each bearing a solitary perithecium. Perithecia more or less symmetrically placed, the ascigerous portion slightly inflated, tapering above to the somewhat slender distal portion, which bends abruptly outward from the substratum, diverging but slightly; the tip subtended by a blunt pointed spine-like projection, directed nearly at right angles to the axis of the free tip. Primary receptacle about 25 μ long. Secondary receptacle 20 × 10 μ. Perithecium, to tip of process, about 80–100 × 15–20 μ.

On antennae of *Phyllodromia* sp., Abyssinia; Scudder Collection, No. 1381.

Herpomyces Platyzosteriae nov. sp.

Male individual consisting of four superposed cells bearing distally one or several antheridia. Total length to tip of antheridia about 40 μ.

Female individual, colorless. Primary receptacle consisting of four superposed cells, the terminal one acuminate; the subbasal giving rise to a branch which produces a single secondary receptacle, made up of a vertical series of cells, obliquely superposed, and variable in number, giving rise above to a solitary perithecium. Perithecium erect, bent inward toward the seta on which the individual grows, the ascigerous portion hardly, or not at all, inflated; the distal portion curved outward, tapering to the pointed apex, which is subtended on the inner side by a stout, bluntly rounded projection about as long as the free apex of the perithecium which extends outward at right angles from its base. Total length to tip of subterminal projection 160–225 μ. Perithecium 150 × 33 μ, its terminal process 10 × 3.5 μ.

On the antennal setae of *Platyzosteria ingens* Scud., Mexico; Scudder Collection, No. 1384.

Acompsomyces brunneolus nov. sp.

Colorless, the tip of the perithecium and its posterior margin suffused with brown, the suffusion extending to the base of and including the appendage. Receptacle short, the basal cell hyaline, relatively large, slightly inflated above, tapering to the small foot; the subbasal cell very small, twice as broad as long; the basal cell of the appendage deeply suffused with blackish brown, especially externally; relatively small and short, the rest of the appendage much as in the other species, somewhat narrow and suffused with brown. Stalk-cell of the perithecium short, slightly longer than broad, the perithecium relatively large and stout, nearly straight, the inner margin nearly straight, and suffused with brown, the outer bent abruptly inward below the tip, which is brown, short, stout, well differentiated, broadly truncate; the lip-cells rising in a truncate conical apiculus subtended by the four clearly distinguished broad subterminal wall-cells. Total length to tip of perithecium 125 μ. Perithecium 80 × 20 μ, the stalk-cell 12 × 10 μ. Receptacle 28 × 12 μ. Appendage to tip of spine 45 × 10 μ, the basal cell 10 μ long.

Near the base of the right elytron of *Corticaria* sp., Kittery Point, Maine.

Stigmatomyces Elachipterae nov. sp.

Receptacle hyaline, the basal and subbasal cells of nearly equal length, the former more or less distinctly inflated distally in the region of a partial septum, above which it may be somewhat faintly marked by transverse lines; the lower portion having its walls thickened from above downward in a characteristically laminate fashion. Stalk-cell of the appendage abruptly distinguished from its much narrower basal cell, which becomes considerably broader distally; a rather large cell being separated from it terminally, bearing three antheridia: above it the appendage consists of three cells, the upper very small, bearing in all normally five antheridia, including the spinose terminal one. Venter of the perithecium pale amber-brown; the wall-cells bulging conspicuously throughout their length, and rather abruptly distinguished above the neck, usually stout and somewhat shorter than the venter; the tip asymmetrical, and slightly oblique, with projecting bluntly pointed apex. Perithecium, venter 55–65 × 30 μ, neck 50 × 16–20 μ. Appendage 80–90 μ, the stalk-cell 15–20 μ. Receptacle about 65 × 18 μ. Spores 40 × 3.5 μ.

On *Elachiptera longula* Loew., Intervale, New Hampshire.

Stigmatomyces micrandrus nov. sp.

Venter of the perithecium, together with its basal cells and the stalk-cells of the appendage, dark amber-brown, symmetrically oval, or typically subconical; at maturity roughened by numerous transverse, not very prominent, ridges; the neck usually longer than the venter, slender, nearly cylindrical above its slightly spreading base, and usually more or less distinctly curved; the tip slightly narrower, and subtended by slight elevations, distally asymmetrical, somewhat oblique, the lip-cells prominent, somewhat pointed and divergent, subtending a pointed prominent apex. Basal cells of the perithecium small, subequal, and characteristically prominent. Receptacle typically rather short and stout, the two cells subequal, or the subbasal cell more or less elongate, finally more or less distinctly marked by transverse striations. Stalk-cell of the appendage rather short, more or less prominent: the basal cell abruptly narrower, squarish, brown, bearing a single antheridium; the rest of the appendage short and stout, hyaline, consisting of two superposed cells, each bearing two antheridia, surmounted by the single terminal spinose antheridium. Spores 45 × 4 μ. Perithecium, venter 72–80 × 50–60 μ, the neck 140–150 × 16–18 μ. Appendage above stalk-cell 35–40 μ; the stalk-cell 20 μ. Receptacle 90–110 × 30 μ. Total length to tip of perithecium 275–325 μ.

On the superior surface of the abdomen of a small blackish fly, Ralum, New Pomerania; Berlin Museum, No. 1284.

Stigmatomyces pauperculus nov. sp.

Short and stout, dark amber-brown, except the hyaline receptacle, and the antheridial portion of the appendage. Venter of perithecium short, stout, inflated; more or less distinctly granular, abruptly distinguished from the neck, which is slightly inflated above the middle, distally curved, and slightly tapering; the apex somewhat asymmetrical and blunt, the lip-cells forming inconspicuous papillae. Receptacle hyaline, rather short, the cells subequal, abruptly distinguished from the parts above. Stalk-cell of the appendage short and prominent, separated from the darker basal cell by an abrupt constriction; the rest of the appendage very broad, short, hyaline, curved inward; consisting of three superposed cells and the terminal antheridium, the total number of antheridia eight (possibly nine), scarcely more than their small necks free. Total length to tip of perithecium 165–175 μ. Venter of perithecium 40–45 × 35–38 μ, the neck 50 × 16 μ. Appendage, including basal cell (16–20 μ), 58–65 × 15 μ. Spores 32 × 3 μ.

On legs of a small dark fly with white inferior abdomen, Ralum, New Pomerania; Berlin Museum, No. 1291.

Stigmatomyces Sarcophagae nov. sp.

Normally long and slender. Color nearly uniform dull amber-brown, except the hyaline receptacle and the appendage above its basal cell. Venter of perithecium minutely transversely granular-striate, rather narrow, straight, the wall-cells prominent and slightly spiral; the neck rather abruptly distinguished, the cells more or less spirally disposed, a rather distinct subterminal enlargement, above which the subconical tip ends in a blunt slightly oblique apex, the paired posterior lip-cells slightly prominent; the basal cells small, somewhat prominent; the stalk-cell relatively large. The receptacle normally very elongate, hyaline, nearly isodiametric; the subbasal cell much longer than the basal. The appendage long, slender, recurved; its stalk-cell about two or three times as long as broad; the basal cell more than twice as long as broad, the portion above it consisting of five superposed cells, and a terminal antheridium; the total number of antheridia normally nine. Spores about 35 × 4 μ. Venter of perithecium 75–90 × 35–42 μ, the neck 150 × 18–22 μ. Appendage, to edge of curvature, 110 μ, the stalk-cell about 35 μ, the basal cell 18–20 μ. Receptacle 200–325 × 30 μ. Total length of larger individuals 600 μ.

Growing in tufts on the abdomen of *Sarcophaga* sp., Island of Margarita, Venezuela; Dr. A. F. Blakeslee.

Stigmatomyces Venezuelae nov. sp.

Perithecium slender, curved throughout, the venter pale amber-brown, hyaline externally, not distinguished from the neck on the inner side; the outer wall-cells bulging more or less conspicuously; the neck tapering slightly, the tip blunt, and unmodified; the basal and stalk-cells hyaline, distinguished from the venter by an abrupt external constriction. Cells of the receptacle hyaline, subequal, the basal bulging distally, below the subbasal, to which it is thus asymmetrically adjusted. Stalk-cell of the appendage tinged with amber-brown, more than twice as long as broad, abruptly distinguished from the lower part of the basal cell, which is small and brown: the appendage proper small, hyaline, withering at maturity, except the lower part of the basal cell; consisting apparently of about three cells producing about four nearly free antheridia. Perithecia 95 × 30 μ. Receptacle 90 × 14 μ. Appendage 45 μ, exclusive of stalk-cell, which is 22 μ long.

Growing at tip of abdomen of *Limosina* sp., Island of Margarita, Venezuela; Dr. A. F. Blakeslee.

Rhachomyces Javanicus nov. sp.

Perithecium straight, symmetrical, tapering slightly distally, translucent, brown, the tip opaque blackish brown, the apex hyaline and blunt, the whole surrounded and more or less hidden up to the tip by the appendages. Receptacle rather long and slender, the main axis normally consisting of more than twenty cells which are clearly distinguishable, more or less suffused with blackish brown, less so toward the base, more or less hyaline at and obliquely below the septa; the appendiculate cells giving rise to a dense series of similar black subequal appendages, their inner edges somewhat hyaline, with slightly upturned and enlarged bluntly pointed hyaline tips, more numerous distally and forming a dense tuft below and around the perithecium which they conceal, except for its projecting tip. Perithecium 95–110 × 35–40 μ. Receptacle 275–350 μ by about 18 μ. Total length 350–460 μ. Appendages longer, about 90–100 μ.

On legs of a small Harpaloid (?) beetle brought from Buitenzorg, Java, by Dr. H. M. Richards.

Rhachomyces Aphanopsis nov. sp.

Perithecium but slightly inflated, hyaline, straight, the tip blunt symmetrical. Receptacle variable, hyaline; the main axis consisting of about ten to fifteen hyaline cells, the two lowest larger and suffused with brown, the rest broader than long, and separated by somewhat oblique septa: the hyaline appendiculate cells relatively large, and not concealed by the appendages, which are rich black brown, the lower ones, when unbroken, extending upward far beyond the tip of the perithecium and curved toward it: those arising about its base, and even below it from the inner cells of the appendiculate series, unlike the rest, erect, straight, consisting of a paler basal part, five to six septate, with constrictions at the septa. and a distal unicellular more or less elongate portion. Perithecium (not fully matured) about 85 × 25 μ. Receptacle 75–165 × 15–18 μ. The longest appendages about 300–350 μ.

On the elytra of *Aphanops cerberus*, nearly always immature, Ariège, France; Sharp Collection, Type, No. 1142; Berlin Museum, No. 877; Paris Museum, No. 193.

Laboulbenia bilabiata nov. sp.

Form stout, the basal cell of the receptacle nearly hyaline, the subbasal becoming amber-yellow, the parts above it clear amber-brown. Perithecium stout, nearly symmetrical, tapering to the broad tip, the anterior lip-cells modified subterminally to form a closely approximated pair of broadly rounded projections, which are subtended on the inner side by the slightly prominent rounded apex, which is in turn subtended by a blackish suffusion of no great size. Receptacle normal, rather short and stout, the basal and subbasal cells slightly prominent anteriorly, of nearly equal length, the walls very thick and striate. Appendages normal, the outer consisting of a black-brown basal cell, twice as long as broad, bearing distally the two divergent branches, which are mostly once branched (antero-posteriorly), all the lower cells suffused with brown, becoming hyaline toward the tips: the inner appendage consisting of a basal cell, slightly shorter than that of the outer, paler brown, bearing the normal branch on either side; the branchlets more or less elongate, or bearing loose tufts of antheridia. Perithecium 240 × 75 μ, including the terminal projections, which measure 25 × 22 μ. Receptacle 225–250 μ. Total length 450 × 115 μ. Appendage 200–300 μ.

On *Brachinus armiger* Dej., Cape of Good Hope; Berlin Museum, No. 982.

Laboulbenia olivacea nov. sp.

Perithecium nearly free, broader distally, olivaceous brown, paler and subhyaline distally on the inner side; the inner margin bulging characteristically above and curved abruptly outward to the tip, which forms a flat blunt hyaline papilla, turned outward and hardly distinguished from the body of the perithecium, and wholly subtended by a wide blackish suffusion, the outer margin forming a slight curvature continuous with that of the receptacle. Receptacle relatively small, the basal cell small, narrow, and quite hyaline, except distally, where it is partly involved in the dark olive-brown suffusion of the rest of the receptacle, which is coarsely punctate in the region of cells II and III, becoming wholly opaque and indistinguishable from the base of the perithecium, with the exception of cell V and a corresponding cell of about equal size which lies external to it, being cut off distally from cell IV, the receptacle being abnormal in this respect. These two cells hyaline or translucent, the whole anterior margin, from the base of cell II to the tip of the perithecium, forming a nearly symmetrical continuous curvature. Outer appendage consisting of a small slightly prominent basal cell, slightly constricted above, where it is separated by a blackish septum from a hyaline erect slender nearly straight distal portion or branch: the basal cell of the inner appendage somewhat smaller, giving rise to several erect branches, like the outer, which reach about to the tip of the perithecium, or may be longer. Perithecia 80–100 × 32–40 μ. Receptacle 90 μ. Appendages 75–110 μ. Total length to tip of perithecium 185–220 × 40–48 μ.

On legs and on the inferior surface of the abdomen of *Lebia* sp., Java; Rouyer, No. 1396. Closely resembling *L. curtipes* in general form.

Laboulbenia pusilla nov. sp.

Short, stout, pale yellowish. Perithecium free, straight, very slightly narrower distally, the distal portion, except externally, about the hyaline pore, deep black, the suffusion extending less than half-way down externally, and about a third on the inner side: the distal end broad, flat, projecting on the inner side to form a rounded prominence; the pore lateral and external, hyaline, with adjacent paler shades; the black, slightly projecting area below it forming a more or less clean-cut line of demarcation. The receptacle short and stout; cells I and II subequal, as are also cells III and IV. Appendages as in *L. orientalis,* hyaline, the two basal cells partly overlapping and bearing each four to five branches four to five times branched, the cells distinguished by constrictions and blackish septa. Spores 45 × 5 μ. Perithecia 90 × 36–40 μ. Receptacle 100–110 μ. Total length to tip of perithecium 200 × 210 μ. Appendages 100–110 μ.

On inferior surface of *Brachinus scotomedes*, Japan; Berlin Museum, No. 994.

Laboulbenia rotundata nov. sp.

Perithecium deep brown, except below the tip; broadly rounded, the inner margin of the upper half free above the insertion-cells, and formed by a single long cell extending to the tip, somewhat as in *L. fallax*, and perhaps representing a marginal extension of the receptacle, as in that species. The tip blunt, with two minute inner tooth-like prominences of unequal size, the larger subtended by a narrow black suffusion, the whole tip more deeply suffused externally. Receptacle straight, elongate, tapering throughout to the base, the basal cell more deeply suffused distally, and protruding somewhat anteriorly below the subbasal cell, which is pale dirty yellowish-brown, inconspicuously granular-punctate: the distal portion dark dirty brown, the cell-boundaries hardly visible. The insertion cells small, giving rise to numerous (a dozen or more, for the most part broken in the type) primary slender clavate cells, from which closely septate short branchlets arise externally. Perithecium 185 × 65 μ, not including the marginal cell, which is 18 μ broad. Receptacle 360 μ. Appendages (broken) about 50 μ. Total length to tip of perithecium 450 μ.

On *Dineutes spinosus*, Java; Sharp Collection, No. 1086.

Laboulbenia Chaetophora nov. sp.

Perithecium dirty olive brown, straight, subconical; the upper two-thirds free, tapering from a broad base, the wall-cells with a distinct spiral twist, very slightly enlarged below the tip, which is slightly, but rather abruptly, distinguished, symmetrical, black, except distally; the apex rounded, and bearing an erect shorter, and an externally divergent, much longer, straight, black-brown, spine-like process. Receptacle broad, distally tapering to a slender base; the basal and subbasal cells about equal in length, dirty yellowish; cells III and VI about equal in length, much darker; the cells above them almost opaque, bulging externally below the insertion cells, which bear numerous short, rounded appendiculate cells, one or two of them larger and opaque; the branches numerous, the branchlets similar to those which in general characterize species of this section of the genus (mostly broken in the type). Perithecia 380 × 110 μ, the longer spine 40 μ. Receptacle 525 μ, or over. Total length to tip of perithecium 800 × 185 μ.

On the tip of abdomen of *Dineutes solitarius*, Madagascar (?); Sharp Collection, No. 1075.

Laboulbenia pallescens nov. nom.

Laboulbenia pallida Thaxter. Proc. Am. Acad. of Arts and Sci. xxxviii, p. 49.

By an oversight the preoccupied name *pallida* was used for this species, which occurs on species of *Clivina* in the Central American region.

Coreomyces curvatus nov. sp.

Nearly hyaline, or but faintly tinged with brownish. Receptacle becoming narrow toward the small basal cell, the two cells above the latter subequal in length, the appendiculate cells usually four, broader than long, subequal or the lower shorter. Stalk-cell of the perithecium relatively long, slightly or not at all inflated; the base of the perithecium often slightly inflated, the upper third or more bent abruptly outward, tapering to the blunt curved apex, the concave side of which is edged with amber-brown, the convex marked by slight indentations due to the prominence of the persistent wall-cells in this region. Appendages as in *L. Corisae*, for the most part distinctly shorter. Spores 55 × 4 μ. Perithecium 130–140 × 36 μ, its stalk-cell 70 × 25 μ. Total length, average, about 350–360 μ. Appendages about 125 μ.

On under side of margin of left elytron of two species of *Corisa*. Near Fresh Pond, Cambridge.

Ceratomyces falcifera nov. sp.

Allied to *C. furcatus*, hyaline, the inner margin of the perithecium and the base of the appendage suffused with amber-brown, becoming blackish and opaque below the appendage. External margin of the perithecium nearly straight, the inner strongly convex; the pointed tip curved to the base of the large, long sickle-shaped, external, subterminal, perithecial appendage, which consists of about twenty-five to thirty superposed cells, and is curved outward. Receptacle consisting of three superposed cells, surmounted by two cells from which the perithecium and antheridial appendage arise; the latter consisting of six or more cells, bearing branches like those of the other allied species on *Berosus*. Spores 55 × 4 μ. Perithecium 125–150 × 55–65 μ, its appendage 435–470 × 22 μ. Receptacle 90–110 × 35 μ. Appendage, without branches, 110–125 μ. Total length to tip of perithecium 250 μ.

On left inferior margin of abdomen of *Berosus* sp., Java; Rouyer, No. 1393.

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REPORT OF AN EXPERIMENT TO DETECT THE FITZGERALD-LORENTZ EFFECT.

By Edward W. Morley and Dayton C. Miller.

With Two Plates.

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REPORT OF AN EXPERIMENT TO DETECT THE FITZGERALD-LORENTZ EFFECT.

By Edward W. Morley and Dayton C. Miller.

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A null result was obtained in 1887,* in an experiment to detect, if possible, a difference of velocity of light in different directions owing to the motion of the apparatus towards or away from waves of light in the stationary ether. FitzGerald and Lorentz then suggested that the dimensions of the apparatus might be modified by its motion through the ether. If this modification depend on the resilience or other physical properties of the materials, it may perhaps be detected by experiment.

We have constructed two apparatus with which to examine this question. In the first, we replaced the sandstone used in 1887 by a structure of white pine. A strong cross was built up of planks, fourteen inches wide and two inches thick, and fourteen feet long. One was laid east and west, then one across it north and south, and so on. They were slightly notched where they crossed. On their intersection was secured a cast-iron bedplate for certain optical parts of the apparatus. At the ends, after filling the spaces between the planks, were bolted iron supports for our mirrors. The whole was placed on a round float, which in turn rested in a basin of mercury.

Our sixteen mirrors were each four inches in diameter. The mirrors rested each on the points of three adjusting screws, against which they were held by springs. On the bedplate at the intersection of the arms of the cross were placed a plane half-silvered mirror and a compensating plate; these had been, as is usual, cut from the same plane-parallel disk.

Figure 1 is a diagram, not to scale, of the optical arrangements. Light from a source S reaches the mirror D. Part is transmitted, reaching the mirror II. It is successively reflected to 2, 3, 4, 5, 6, 7, and 8. From 8 it returns by the same path to D, where part is reflected to the observer at T. Another part of the incident ray is reflected along the other arm of the cross, is similarly passed to and fro, returned, and at last transmitted to the observer. In the apparatus actually used, mirror 5 lay above 3, rather than to one side of it; Figure 2 shows this arrangement. The whole path of the light along these mirrors was enclosed and covered, to lessen the effect of air currents and other local disturbances. An acetylene flame was carried as a source of light. A telescope magnifying thirty-five diameters gave distinct vision of mirror 8, at whose surface the interference fringes are apparently localized.

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* On the Relative Motion of the Earth and the Luminiferous Ether. A. A. Michelson and E. W. Morley. Am. Jour. Sci., **34**, 333.

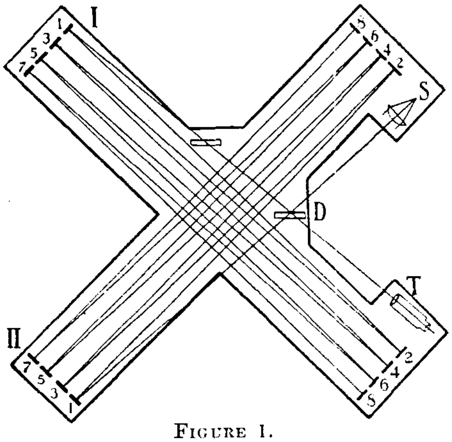

FIGURE 1.

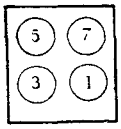

FIGURE 2.

The mirrors, being silvered and polished, were put in place, and the lengths of the two paths were measured with a split rod and then made nearly equal. Establishing interferences in sodium light, we found the central part of a series of some seven hundred interferences which are brighter than the adjoining three hundred. With no long search, we could see interferences in white light, although we had provided no screw for moving a mirror with its surface always parallel to a given surface. This we had avoided, in order to have everything about the two arms as symmetrical as possible.

We now computed the direction and velocity of the motion of the centre of the apparatus by compounding the annual motion in the orbit of the earth with the motion of the solar system towards a certain point in the heavens. During part of August, the whole of September, and nearly all of October, this motion never coincides with the plane of our apparatus. For other dates, there are two hours in each day when the motion is in the desired plane, except for two days when the two hours coalesce into one. At the beginning of June the two hours are about 11 h. 20 m. A.M., mean solar time, and 9 h. 50 m. P.M. At the time of our last set of observations, July 5th to July 9th, the hours were 11 h. 40 m. A.M. and 8 h. 20 m. P.M., local mean time.

After many trials, with filar micrometer, and with scale on mirror 8, we found it advisable to accumulate a great number of observations made as rapidly as might be. What we had to do, in presence of all the local disturbances of density of the air which sometimes made observation impossible and always made it difficult, was as if we were trying to measure the solar atmospheric tide. If we could vary the period of this tide at will by controlling the revolutions of the earth, we should doubtless get a result sooner by accelerating the latter and making a great number of observations in a given time, rather than by retarding the period in order to measure with very great precision the hourly height of a barometer. We therefore proceeded as follows. One observer walked around with the moving apparatus, his eye at the telescope, while he maintained the rotation by an occasional gentle pull on a cord so fixed as not to bring any strain to bear on the cross arms of the apparatus. The room was darkened. The other observer also went around with the apparatus; as an index showed the azimuth of the apparatus to be that indicated by one of sixteen equidistant marks, he called out the number or some other signal. The first observer replied with the reading for the given azimuth, which the second observer recorded. The next azimuth was called at the proper instant, the reading given, and so on. Half the time, perhaps, the observations were interrupted before they became numerous enough to be useful, being stopped by excessive displacement of fringes owing to temperature changes and the like. But patience is a possession without which no one is likely to begin observations of this kind. Runs of twenty and thirty turns, involving 320 or 480 readings, were not uncommon. A run of thirty turns meant that the observer, who could sometimes make a turn of sixteen readings in sixty-five or seventy-five seconds, walked half a mile while making the severe effort involved in keeping his eye at the moving eyepiece without the least interruption for half an hour. The work is, of course, somewhat exhausting.

Observation with this apparatus could not begin till the month of August, and we had to stop without having accomplished as much as was desirable. During the busy season of the school year, observation is impossible. We had therefore expected to resume our work in June. But we then found that our pine apparatus had so much suffered from the dryness of the building that we could not maintain the adjustment of our fringes. We could not, in the time, build another apparatus of timber which had not been dried all winter, nor was it thought well to construct another apparatus closely resembling the first. While planning a new apparatus, we made a couple of experiments to show, what was well enough known, that difference of magnetic attraction on the iron parts of our apparatus could not disturb our observations. We suspended two massive pieces of iron at the ends of one arm, so that one should be in the lines of magnetic force of the earth's field, and the other transverse to them, these relations being reversed on reversing the position of the apparatus. But observations with this load of iron gave the same result as before. Next we placed an analytical balance on one arm, with which to weigh a bar of iron at the extremity of that arm. It was so placed that at one azimuth the bar was nearly in the lines of force, and at another was transverse to them. If there were a difference of half a milligram in twelve hundred grams, it would have been detected, but no such difference existed. We found by trial how much a weight of a hundred grams displaced our fringes, and so learned, as was known before, that the influence of the earth's magnetism could not be a disturbing factor.

The Rumford Committee of the American Academy of Arts and Sciences having made a grant in aid of this experiment, we carried out our original plan of making a steel structure so rigid as to permit easy and satisfactory observation. In this new apparatus all the optical parts are carried by a steel frame built of plate and angle-iron, somewhat like a bridge girder. A cubical steel box, fourteen inches on each edge, constitutes the centre of the structure, which is in the form of a cross. To each of the four sides of this cube are firmly attached arms, each about six feet and a half in length. Each arm is made of steel plates, three eighths of an inch thick, eighteen inches wide at one end, and six inches wide at the other, standing on edge, and kept fourteen inches apart by suitable plates, angle-irons, and other braces; thus are formed hollow beams of great rigidity, especially in a vertical direction. This framework is shown in Plate 1, from which it is seen that the structure is in effect two rigid beams, each fourteen feet long, crossing at right angles, and symmetrical as regards strength and rigidity.

On two ends of the cross, S and T, Figure 1, are two upright cast-iron frames, fastened by bolts, each of which carries four mirrors, marked 2, 4, 6, and 8. Against the corners of each of these frames rest four pine rods, three quarters of an inch in diameter and fourteen feet long. Each rod is supported throughout its length by a brass tube an inch in diameter; each pair of tubes is joined together in a vertical truss, as shown in Plate 1. Against the farther end of these rods there rest the frames which hold the two sets of mirrors, I and II, Figure 1. Each of the latter frames is freely suspended by two thin steel ribbons and is held firmly against the pine rods, and through these against one of the two fixed mirror holders; the pressure is applied by means of adjustable spiral springs. Thus the distance between the opposite systems of mirrors depends upon the pine rods only. This construction permits the convenient substitution of distance rods of other material, so that experiments might be easily made to test the theory that the dimensions of different materials are differently affected by motion of translation through the ether. The diagonal mirrors are carried by adjustable supports bolted to the steel frame near its centre.

The observing telescope of an inch and a half aperture with a magnifying power of thirty-five is attached to a support bolted to the steel frame. The acetylene lamp and the four-inch condensing lens stand on a wooden shelf as far as convenient from the mirrors, which are protected by asbestos screens with air spaces. The whole path of light through the apparatus is enclosed by a wooden cover made of pine seven eighths of an inch thick, having doors and glass windows where these are required. The observer's eyes are protected from extraneous light by a dark cloth.

The entire apparatus, weighing about nineteen hundred pounds, rests upon a circular wooden platform about five feet in diameter. An annular projection on the under side of the platform is immersed in mercury of such depth as to float the platform and the apparatus. The mercury is contained in an annular cast-iron trough of such dimensions as to leave a clearance of about half an inch between the iron and the wooden float. A small pin at the centre of the iron trough enters a socket in the wooden float, so as to keep the float from touching the sides of the trough.

Plate 1 shows the steel framework and float, together with the trusses which are to support the distance pieces. The mirror frames and the telescope are in position, but lamp and lens are not in position. Plate 2 shows the apparatus as it appeared at the time of the observations.

With this apparatus, fringes adjusted on a certain Monday remained in adjustment throughout the whole of the week during which our observations continued. Observations were made in precisely the same manner as with the previous apparatus.

We obtained 260 complete observations, consisting each of readings at sixteen azimuths around a circumference. At the date of the observations, the annual motion of the earth, together with the motion of the solar system, may be taken as 33.5 kilometers a second. It is assumed that the solar system is moving towards a point whose right ascension is 277.5°, and whose north declination is 35°, with a velocity of eleven miles a second. The velocity of light being 300,000 kilometers a second, the ratio of the squares of the velocities is 0.72×10^8. The length of path of a ray in our apparatus was 3224 centimeters, in which distance there are contained 5.5×10^7 wave lengths of sodium light. The expected effect being doubled by rotation through 90, the displacement of fringes expected on the simple kinematic theory will be $11 \times 10^7 \div 0.72 \times 10^8$. This is 1.5 wave length.

As was indicated, there were two times in the day when observation was advisable. The direction of the motion with reference to a fixed line on the floor of the room being computed for the two hours, we were able to superimpose those observations which coincided with the line of drift for the two hours of observation. Doing this, and subtracting a constant so as to make the algebraic sum of the observations equal to zero, we get a certain result. Then adding the first term to the ninth, and so on, since the effect repeats itself in a circumference, we get our final result, as follows: —

Result of observations at various azimuths.

Azimuths	8	7	6	5	4	3	2	1
Wave lengths	+0.0075	+0.0088	+0.0113	−0.0102	−0.0123	+0.0027	−0.0021	−0.0062

Azimuth mark 1 denotes that the telescope of the apparatus was directed N. 29° E.; 3, N. 16° W.; 5, N. 61° W., &c.

These numbers may be confidently pronounced to be due to errors of observation. We computed from them several curves of the theoretical form, having their origin at sixteen equidistant points in the half circumference; this was done by the method of least squares. The most probable of these curves had an amplitude of 0.0073 wave lengths, and its zero was half-way between the azimuths marked 4 and 5. The average of the given observations is 0.0076 wave lengths; after subtracting the ordinates of the computed curve, the mean residual was 0.0066 wave lengths. The sum of the squares of the residuals before was 565×10^{-4}; afterwards, it was 329×10^{-4}.

We may therefore declare that the experiment shows that if there is any effect of the nature expected, it is not more than the hundredth part of the computed value. If pine is affected at all, as has been suggested, it is affected to the same amount as is sandstone. If the ether near the apparatus did not move with it, the difference in velocity was less than 3.5 kilometers a second, unless the effect on the materials annulled the effect sought.

Some have thought that the former experiment only proved that the ether *in a certain basement room* was carried along with it. We desire to place the apparatus on a hill, covered only with a transparent covering, to see if any effect could be there detected. As the Rumford Committee have allowed us thus to utilize an unexpended balance, we hope to make the experiment in this form, should it be possible to make observations in trying conditions.

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THE SPERMATOGENESIS OF THE MYRIAPODS.

IV. — ON THE KARYOSPHERE AND NUCLEOLUS IN THE SPERMATOCYTES OF SCOLOPENDRA SUBSPINIPES.

By Maulsby W. Blackman.

With a Plate.

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IV. — ON THE KARYOSPHERE AND NUCLEOLUS IN THE SPERMATOCYTES OF SCOLOPENDRA SUBSPINIPES.*

By Maulsby W. Blackman.

Presented by E. L. Mark. Received June 14, 1905.

In making a comparative study of the spermatogenesis of different species of chilopods the phenomena observed in Scolopendra subspinipes are in some respects so markedly different from those already reported as characteristic of S. heros (Blackman, :02 and :05) as to warrant their treatment in a separate paper. The most striking differences are dependent upon the fact that a large plasmosome, or true nucleolus, is present within the nucleus of the spermatocytes of S. subspinipes during stages of mitotic inactivity, while in the cells of S. heros no such structure is to be found. The presence of this body in S. subspinipes seems to necessitate a corresponding variation in the behavior of the chromatin. Although these differences are often quite marked, the process is essentially similar in the two species.

The material upon which this paper is based was collected by the author near Flatts, Bermuda, during July, 1903. I am glad of this opportunity of thanking Dr. E. L. Mark, director of the Bermuda Biological Station, for numerous courtesies extended to me while working in the laboratory.

The testes obtained from Scolopendra subspinipes are not so far advanced in development as those of S. heros, used in observations recorded in my former papers (Blackman, :03 and :05). Very few spermatozoa, spermatids, and secondary spermatocytes are to be found, nearly the entire bulk of the testis consisting of growing spermatocytes and cells in what I have elsewhere (:05) designated as the vesicle stage. The proportion of cells of the smaller type does not seem to be as great as that found in S. heros (Blackman, :01, :05) or in S. morsitans (P. Bouin, :03). This may be, and probably is, due to the age of the individuals; for I have concluded from a study of the cells of the two types in S. heros that the difference in size is due to differences in nutrition.

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* Contributions from the Bermuda Biological Station for Research, No. 8.

The testes were fixed in Gilson's fluid, embedded in paraffine, and reduced to sections 3 to 5 micra thick. These were stained in Heidenhain's iron-haematoxylin (some were counterstained with Congo red), and studied under a magnification of from 1200 to 1800 diameters.

The spermatogonia of S. subspinipes in the resting stage are similar in all respects to those of S. heros. They are small cells containing an oval nucleus, in which no indication of chromatin can be observed except the large, deeply staining nucleolus-like body, the karyosphere. During the prophase small granular aggregations of chromatin appear (Figure 1), and at the same time the karyosphere becomes so altered as to show conclusively that, as in the sister species, the chromosomes are derived from its substance.

After the formation of the chromosomes from the karyosphere in S. heros nothing of this body remains except the accessory chromosome. In the present species, however, the residue is different. It consists of two parts, a small, deeply staining chromatic body, the accessory chromosome, and a larger, less dense part, the plasmosome (Figure 1). The latter part does not retain the haemotoxylin, but is nearly colorless, or assumes a reddish-brown tint, when the haematoxylin is followed by Congo red. During the later part of the prophase the plasmosome is dissolved, and in the metaphase and later stages of mitosis no indications of such material are to be found (Figure 2).

The phenomena occurring during the later stages of the last spermatogonium seem to be identical in all essential respects to those already described as existing in S. heros. At the end of the anaphase the chromosomes are massed together so closely at opposite poles of the elongated cell as to make their outlines indistinguishable (Figure 2). Later, during the telophase, these chromosomes still aggregated into a dense mass, lengthen out into slender granular segments (Figure 3). A single element, however, the accessory chromosome, does not assume this granular condition, but retains its homogeneous appearance and is on this account very conspicuous (Figure 3).

Upon the reconstruction of the nuclear membrane the segments of chromatin become distributed throughout the nuclear space (Figures 4, 5), causing the cells at this stage to bear a close resemblance to the sperm cells of insects in the spireme condition. Here, however, the chromatin is not a continuous thread, as it is said to be in the cells of many animals, but is in the form of a number of granular segments, one half as numerous as the ordinary chromosomes of the spermatogonia.

These chromosomes are produced by the union in pairs of the chromosomes of the division period. That this is an end to end union is very strongly indicated, although not definitely proved, by the appearance of the chromatin segments at this stage. In Figures 4 and 5 the nuclear vesicle is occupied by a number of slender granular chromatin segments, each of which is bent near its middle, forming a V-shaped figure. At the angle of the V, which doubtless represents the point of union of the two component elements, the chromatin is often interrupted in such a manner that the halves of the segment seem to be connected by only a strand of linin. These conditions in Scolopendra are quite similar to those described by Montgomery (:00) in Peripatus, and later (:01) in Hemiptera, and they lead to the same conclusion as did the more convincing observations made by Sutton (:02) on Brachystola. The latter author has shown conclusively that the synapsis of the chromosomes occurs during the telophase of the last spermatogonium, and that it is brought about by an end to end union of entire spermatogonial chromosomes.

Thus in S. subspinipes the pseudo-reduction of the chromosomes is accomplished in exactly the same manner as in S. heros. Up to this stage (Figure 4) the appearance of the accessory chromosome has been entirely similar to that in S. heros, but from now on the behavior of all of the chromatin is quite noticeably different. This is evidently due to the introduction within the nucleus of a new structural element, — the true nucleolus, or plasmosome.

The first indication of the nucleolus in the spermatocyte is seen at the stage when the nuclear membrane has just been re-formed (Figure 5). At this time a transparent, non-stainable structure appears upon one side of the accessory chromosome. This fundament of the nucleolus at first has very much the appearance of a vacuole occupying one side of the accessory chromosome (Figure 5).

In the telophase of the last spermatogonial division the accessory chromosome, when it first becomes distinguishable from the other chromosomes, is of an oval shape (Figures 3, 4), but after the origin of the plasmosome the compound element thus arising is at first spherical. The chromatin representing the accessory chromosome assumes a crescent shape and nearly surrounds the unstained part — the plamosome. In this manner the accessory chromosome forms one half or more of the outline of the sphere (Figures 5, 8 *b*).

With the reappearance of the nuclear membrane the cell enters upon a period of phenomenal growth and the nucleolus increases in size very rapidly, while the accessory chromosome remains of nearly the same size. Stages in this growth of the nucleolus are shown in Figures 6, 7, and 8, *b*, *c*, *d*, *e*. As the nucleolus grows it also changes in its reaction to stains, and assumes the diffuse brownish tint characteristic of the plamosome when stained with iron-hamotoxylin (Figure 8 *c*). Its shape also is so altered that from now on it is an approximate sphere, upon one side of which lies the deeply stained accessory chomosome. This body, which has again assumed an oval shape, is usually not embedded in the plasmosome, but is closely apposed to one side of it (Figures 8 *f*, 9). Quite often there is a cup-like depression in the plasmosome at the place where the accessory chromosome comes into contact with it (Figure 10).

It is during the period represented by Figures 4 to 19 that the most striking and interesting differences between the germ cells of Scolopendra subspinipes and S. heros occur. These variations, as I have already intimated, primarily concern the arrangement and behavior of the chromatic structures.

The conditions in S. heros are briefly as follows: Upon the reconstruction of the nuclear membrane after the last spermatogonial division, the cells, now containing the number of chromatin segments characteristic of spermatocytes, "enter upon a period remarkable for the extraordinary changes which take place in their structure." Of these changes the most remarkable are the enormous increase in the size of the cells and the peculiar and characteristic arrangement of the chromatin.

Concerning the growth of the cells in S. heros it is sufficient to say that the diameter of the large spermatocytes is about ten times that of the spermatogonia from which they arise. As regards the chromatin structures, I may quote from one of my papers cited (:03): "As the cell continues in its growth the chromatin segements become larger and more diffuse. They no longer retain the stain with the persistency which has characterized them heretofore. Gradually they break down and their substance is accumulated about the accessory chromosome, thus seemingly increasing this element greatly. This process continues until all of the chromatin of the cell is aggregated in one large, intensely staining body situated peripherally in close contact with the nuclear membrane." This body, the karyosphere, is not a homogeneous mass of chromatin, but, "on the contrary, is a rather complex structure, consisting of chromatin, linin, and karyolymph." The chromatin composing the karyosphere is in the condition of a very fine spireme, which is so densely massed as to present the appearance under moderate magnification of a homogeneous sphere of chromatin. It is only in thin sections and with high magnification (1200 to 1800 diameters) that the spireme character is shown. Later, during the prophase, the chromosomes arise from this mass by a mere unwinding of the chromatin threads. When this process is completed nothing remains of the karyosphere but the accessory chromosome. There is apparently no nucleolar material contained in it, as no residue can be found within the nucleus and no such structure can be seen at any previous stage.

I shall now describe the changes during the same period in S. subspinipes and compare the processes in the two species. During the telophase of the last spermatogonium the appearance of the cells of the two species is identical, but at the time of the formation of the nuclear membrane the true nucleolus arises in S. subspinipes in close contact with the accessory chromosome and grows rapidly as the cell increases in size. Thus the conditions in the two species are slightly different, and it is to be expected that the subsequent behavior of the cells would also vary.

In S. heros it was impossible to follow in detail the process of the massing of the chromatin into the karyosphere. In S. subspinipes, however, this process, for reasons which I shall soon make plain, can be followed much more satisfactorily. In this species the chromatin, instead of becoming closely aggregated about the accessory chromosome, is deposited in a rather thin layer upon the periphery of the large nucleolus, or plasmosome. This structure, staining so differently from the chromatin, furnishes an excellent background, by means of which the various stages in the formation of the karyosphere may be studied with ease.

In S. subspinipes, as in S. heros, the formation of the nuclear membrane around the chromosomes may be taken as marking the transition from spermatogonium to spermatocyte. In one species, precisely as in the other, this stage ushers in a period remarkable for the concentration of the chromatin. As the cells grow the chromosomes gradually become much more diffuse and therefore stain less deeply. During this time the nucleolus, still in close apposition to the accessory chromosome (Figure 8 *b*), increases in size so rapidly that its volume is soon several times that of the accessory chromosome (Figure 8 *c*, *d*).

Shortly after the stage shown in Figure 8 *d*, the nucleolus seems also to change somewhat in its staining reaction. Irregular areas upon its periphery now show a marked affinity for chromatin stains (Figures 6, 7, 8 *e*), causing it under moderate magnification to appear of a uniform gray color.

With higher amplification, however, it is readily seen that this change is due to the deposition of some of the chromatic threads upon the surface of the nucleolus. These deposits are in the form of very diffuse spiremes, which are similar to the segments which are still free in the nuclear space (Figure 6). The nucleolus as seen in profile no longer presents an even contour, but at the places where the dark areas are found the diameter of the mass is increased (Figure 8 *e*). It is thus evident that loose threads of chromatin are being merely applied to the outside of the nucleolus, not embedded in its substance. The accessory chromosome still preserves the same relation to the nucleolus as before, being closely apposed to one side of this structure (Figure 8 *d*, *e*).

As the cell continues to grow the free chromatin within the nucleus becomes more and more scanty (Figures 6, 7), until soon only a few chromatin segments are to be seen where formerly there were many (Figure 7). Finally, in the "vesicle" stage (Figures 9–13), no chromatin is to be found in the nucleus except that which is massed about the nucleolus to form the karyosphere.

During the growth period of the spermatacyte the nucleolus has increased very markedly in size (Figure 8 *e*, *f*). This is due both to the increase of nucleolar material and to the deposition of chromatin upon the periphery of the nucleolus. At the time of the vesicle stage the appearance of the karyosphere* varies considerably in different cells, this variation being due to the different manner in which the chromatin is arranged with regard to the nucleolus. Different appearances presented by the karyosphere at this stage are shown in Figures 8 *f*, and 9–13. Usually the chromatin is distributed more or less unevenly over the surface of the nucleolus (Figures 8 *f*, 9) but often it forms a nearly complete (Figures 11, 13) or complete (Figures 10, 12) layer about it, thus causing the karyosphere to stain very intensely. In preparations stained with haemotoxylin and Congo red the karyosphere appears black when the surface is in focus, but when the centre is in focus the middle portion of the body shows a reddish-brown coloration, thus disclosing the presence of nucleolar material. Likewise, when a portion of one side of the karyosphere has been removed in the process of sectioning, the true structure of the body is shown. Figure 10, representing such a condition, shows the outer layer of intensely staining chromatin and the inner nucleolar material, containing several vacuoles.

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* In my former papers I have described the karyosphere as a highly organized structure found within the nucleus and containing "chromatin (in a granular, reticular, or spireme form), karyoplasm,— i. e., linin,— and karyolymph. It is in fact a miniature nucleus." It is now seen that it may be still more complex, since it may also contain nucleolar material.

More commonly, however, the karyosphere exhibits the reticular appearance represented in Figures 8*f*, 9. Here the chromatin is very plainly of the nature of a superficial spireme or a reticulum, which does not completely invest the nucleolus. The appearance of the karyospheres of this nature is strikingly similar to that of a small nucleus in which the chromatin is spread out over the periphery of the nuclear area in flaky masses (Figure 9).

Meanwhile the accessory chromosome has undergone no apparent change, and throughout the growth period has preserved the same relation to the nucleolus which it had during the early spermatocyte stages (Figures 5–11). Occasionally an accessory chromosome is not to be seen, owing to the fact, doubtless, that the section is cut in such a plane that the chromosome is concealed behind the karyosphere or does not lie in the same section with it (Figures 11, 12).

In S. subspinipes, just as in S. heros, a number of small granular masses of a deeply staining substance are scattered throughout the nucleus. These stain similarly to chromatin; inasmuch, however, as such masses are also present at various places in the cytoplasm outside the nucleus, it seems probable that they are not chromatin, but metaplasm — i. e., that they are either food material or by-products of the cell's activity.

In S. heros the first changes in the active prophase of the division of the first spermatocyte manifest themselves in the cytoplasmic structures; they consist in the dissolution of the archoplasm and the migration of the centrosomes. In S. subspinipes this is not true. Here the first marked change affects the chromatin. The chromosomes do not arise as distinct entities from the karyosphere by a simple disentanglement of the spireme, but the chromatic portion of the karyosphere is merely detached in the form of a varying number of flaky reticular masses, which thus come to lie free in the nuclear vesicle. These spongy masses of chromatin do not represent single chromosomes, for they later break up into several smaller masses, which soon take on the form characteristic of tetrads in the prophase. Figure 14 represents a spermatocyte in the very early prophase, in which the chromatin has just left the karyosphere. There are several chromatin masses; of these the larger one contains nearly all of the chromatin. Figures 15–18 represent slightly different stages, a little older than that shown in Figure 14. In Figure 15 the chromatin is in several masses, of which the larger ones doubtlessly represent several chromosomes. In Figures 17 and 18 the chromatin has separated into distinct chromosomes, and some of these already show plainly their tetrad nature. In Figures 15 and 16 not all of the chromatin has as yet left the nucleolus, several diffuse flaky masses being still spread out over its surface.

After leaving the karyosphere, the chromatin soon breaks up into a number of diffusely granular segments, each representing a chromosome. These segments are not at first very similar to those found in S. heros, as they are not long slender chromatic filaments,but much shorter and thicker. They, however, undergo the same changes (Figure 19); i. e., they are divided first longitudinally and later crosswise. The latter division takes place at the point where in the preceding telophase the spermatogonial elements united. These soon come to resemble exactly the tetrads found in the sister species. Many variations and distortions of the tetrad are to be found, just as in S. heros, but, as in that species, they all may be referred to the typical cruciform or double-V figures described by Paulinier ('99) and McClung (:00).

These quadripartite bodies arise in the following manner. In the early prophase the chromatin of the cell is in the form of a number of granular segments, each one of which represents one of the bivalent chromosomes obtained by the end to end union in pairs of the univalent chromosomes of the spermatogonium. Each of these segments splits longitudinally throughout its entire length, thus forming a double thread of chromatin. Soon after this cleavage is effected, each half of the chromosome divides near the middle of its length,— i. e., at the point at which the component elements united to form the bivalent chromosome,— and the adjacent ends produced by this cross division become drawn out in the same direction and perpendicular to the axis of the original thread, but the adjacent ends resulting from the cross division of one half of the thread move in a direction opposite to that of the other half, thus forming the cross-shaped figures so common in the spermatocyte prophase (Figure 19).

In later stages the chromosomes come together more closely, thus causing the chromosome to become more compact and to lose its granular appearance. By this condensation the planes of cleavage separating the four component chromatids, which are quite pronounced in earlier stages (Figure 19), become so masked that they can no longer be distinguished. The chromosomes of this stage — the late prophase — have typically the four-lobed appearance represented in Figure 20; but some of them may be so distorted as to show little evidence of relationship with the tetrads of an earlier stage.

During the early prophase, up to a time when the tetrads have begun to be formed, the accessory chromosome remains apparently unchanged. It is still a small, deeply staining, homogeneous body closely apposed to one side of the plasmosome. Later, when the other chromosomes have already assumed the tetrad shape and have begun the process of condensation, it leaves this position, and thereafter may lie in any part of the nucleus. Owing to the ordinary chromosomes being of irregular form during the late prophase (Figure 20), and to their being approximately equal in size to the accessory chromosome, the latter is in many cells not distinguishable from them. Consequently it has been impossible to trace the later history of the accessory chromosome in S. subspinipes with as much certainty and in such detail as has been done in S. heros (Blackman, :05). However, I can say that the accessory chromosome, unlike the other chromatic elements, undergoes but one — a longitudinal — division.

Thus it is certain that the behavior of the modified chromosome in the two species of Scolopendra which I have studied is similar, at least in all essential particulars. In both species the accessory chromosome is produced not by the union of two spermatogonial chromosomes during synapsis, as the other chromosomes are, but is derived from one of the spermatogonial elements directly, being in no way altered by synapsis. Since the object of the reduction division is the separation of the chromosomal elements which conjugated during the telophase of the last spermatogonium, it naturally follows that the accessory chromosome, not having taken any part in this conjugation, does not undergo a reduction division. In Scolopendra heros (Blackman, :03, :05), as well as in many insects, Pyrrhocoris (Henking, '91), Anasa (Paulmier, '99), Orthoptera (McClung, :02, Sinéty, :01, Sutton, :03), the absence of this element in one half of the spermatids has been demonstrated. This is also true for S. subspinipes.

At the end of the growth period, during the vesicle stage, the inner, lightly staining portion of the karyosphere, i. e., the nucleolus, often contains several small clear spaces. As these show no reaction whatever to the stains, and as they are bounded by a definite spherical outline, it is probable that they are vacuoles filled with a fluid or gaseous substance. Before the beginning of the prophase the conditions are such that these structures are not often visible. It is only in such cases as that represented in Figure 10, where a part of the karyosphere has been removed in sectioning, that they may be observed. Later, however, in the prophase, when the chromatic portion of the karyosphere has become detached, leaving the nucleolus plainly visible, the conditions for observing them are favorable. At this time there are usually three or four of these vacuoles, their diameters being about one sixth to one tenth that of the plasmosome (Figures 14–18). They are clear, transparent, unstained spherules bounded by a definite outline, but enclosed in no distinguishable membrane. Later, as the prophase advances and the tetrads take on their characteristic form (Figures 19, 20), these vacuoles increase both in size and number, until, just before the nuclear membrane breaks down, they are often so numerous that the nucleolus is a spongy mass bearing little resemblance to its previous appearance. During this time the plasmosome has not appreciably altered in size; however, in the very late prophase, only a little of the original material remains.

This, I believe, might be explained on the supposition that the vacuoles represent material — probably hyaloplasm — obtained from without, while the nucleolus at the same time gives off material to the nucleus. The material thus gained by the nucleus may represent merely a by-product of cell metabolism, but the fact that the nucleolus degenerates immediately preceding cell division might be taken to indicate a more important function. Three explanations concerning the purpose and fate of the nucleolus suggest themselves: (1) the material may be used for the nutrition of the cell in general, (2) it may be used directly in the formation of the spindle fibres, or (3) it may contribute to the achromatic portion of the chromosomes. I believe that the first of these alternatives is realized in the case of S. subspinipes. In S. heros, where the archoplasmic structures are very well developed, no nucleolus is to be found at any stage of the spermatocyte. It further seems very often to be true that where the proportion of chromatin in the cell is especially large, as in the spermatocytes of insects and amphibians, no nucleolus whatever is present, thus pointing to the independence of the chromosomes. On the other hand, in cells in which there is a great preponderance of cytoplasm, as in egg cells, nucleolar structures are of very general occurrence.

The behavior of the chromosomes during the two following mitoses is so similar to that already described in S. heros that it hardly requires separate description. The result of the two spermatocyte divisions is four spermatids from each primary spermatocyte. Of these four, two contain one more chromosome each than do the other two, just as is the case in S. heros. The accessory chromosome is not divided in the first mitosis, but goes entire to one cell. In the second division, when the other chromosomes are undergoing a reduction division, this divides longitudinally. In the spermatid it is for a time distinguishable from the other chromosomes on account of its homogeneous condition.

No traces of a nucleolus are to be seen at any stage after the late prophase or early metaphase of the first spermatocyte. No such structure reappears during the short prophase of the second spermatocyte, nor have I observed it in the few spermatids to be found in my material. It would seem that, in S. subspinipes at least, the nucleolus is present only in stages of protracted mitotic inactivity.

The principal facts of interest presented in this paper are due to the presence of a true nucleolus or plasmosome in the nucleus of the sperm cells during the so-called resting stages of mitosis. The presence of this body seems to result in a considerable difference in the behavior of the chromatin and in the structure of the karyosphere. Carnoy ('85) in his "Cytodiérèse chez les arthropodes" has divided nucleolar bodies into four classes, his conclusions being based in part upon observations on the spermatogenesis of various Chilopods. These classes are as follows: "(*a*) les nucléoles nucléiniens; sperules de nucléine amorphe, ou remassée en peleton serré; (*b*) les nucléoles plasmatiques, masses albuminoides renfermant de la plastine; (*c*) les nucléoles mixtes, qui sont constitués par la réunion des deux espèces précédentes en un corps unique, où chacune se maintient cependant sous une forme figurée; (*d*) les nucléoles noyaux, ou noyaux en miniature, renfermant par conséquent tous les éléments d'un noyau véritable; membrane, portion plasmatique et portion nucléiniene.

In Lithobius, Scutigera, and Geophilus nucleoli of the fourth type, nucléoles noyaux were found by Carnoy. These are similar in general characteristics to the karyosphere in Scolopendra heros, although differing in many details. In Scolopendra dalmatica, however, according to Carnoy ('85), "L'élément nucléin d'ailleurs volumineux, est généralement irrégulier, bosseli, parfois moniliforme," and the nucleolus "n'est pas un nucléole-noyau mais un nucléole plasmatique." The structure of the nucleus "est donc celle d'un noyau ordinaire." The nucleolus does not stain with the chromatin stains and takes no part in the formation of the chromosomes, which arise directly from the chromatic network of the nucleus.

In attempting to correlate these results of Carnoy with my own observations upon American species of the same genus, one of two conclusions is necessary. Either Carnoy mistook the early prophase of the first spermatocyte for the "resting stage," or the condition of the chromatin in S. dalmatica presents yet another variation in its behavior from that found in S. heros. In the light of these later observations upon S. subspinipes, the latter view seems probable.

If Carnoy's observations be accepted as correct, we have three variations in the behavior of the chromatin during the growth period; and these variations form a graded series. In Scolopendra dalmatica the chromatin, as in the majority of metazoa, is in the form of an apparent reticulum or spireme distributed irregularly throughout the nucleus; in S. subspinipes the chromatin threads are aggregated in a rather loose manner and are deposited upon the surface of the large nucleolus, and in S. heros the chromatin filaments are so densely massed in an approximately spherical body — containing no nucleolar material — that it is only in the most favorable cases that its true spireme structure can be observed.

ZOÖLOGICAL LABORATORY, HARVARD UNIVERSITY,
June 14, 1905.

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EXPLANATION OF PLATE.

All drawings are of *Scolopendra subspinipes* and were made by the author from camera lucida outlines at a magnification of 1800 diameters. They were reduced one third in reproduction, making a final magnification of 1200 diameters.

FIGURE 1. Prophase of the last spermatogonial division, showing the plasmosome with the accessory chromosome closely apposed to one side, and the other chromosomes lying free in the nucleus.

FIGURES 2, 3. Early and late telophase of the last spermatogonial division. In Figure 3 the accessory chromosome can be readily distinguished from the other chromosomes, which are granular filaments.

FIGURES 4, 5. Early spermatocytes. The nuclear membrane is reappearing and the chromatin segments of the reduced number are arranged irregularly throughout the nuclear area. Evidences of the origin of the elements by an end to end union of spermatogonial chromosomes is seen in the V-shaped character of most of them. In Figure 5 the plasmosome is arising in close relation with the accessory chromosome.

FIGURE 6. A pair of spermatocytes during the growth period. The plasmosome has increased in size and some of the chromosomes have become arranged upon its periphery. Others are still free in the nucleus.

FIGURE 7. Nucleus at a slightly later stage.

FIGURE 8. Series of figures (*a-f*) showing the evolution of the plasmosome and origin of the karyosphere.

FIGURES 9–12. Various appearances presented by the karyosphere during the "vesicle" stage.

FIGURE 13. Large spermatocyte in the vesicle stage.

FIGURE 14. Early prophase of a large spermatocyte. The chromatin has become detached from the plasmosme in the form of several granular masses.

FIGURES 15–18. Nuclei in the early prophase.

FIGURE 19. Mid prophase of a large spermatocyte, showing the character of the tetrads in Scolopendra subspinipes The nucleolus is becoming more vacuolated.

FIGURE 20. Later prophase. The tetrads have become condensed into homogeneous chromosomes, which are typically four-lobed.

FIGURE 21. Late prophase in a small spermatocyte.

FIGURE 22. Prophase of secondary spermatocyte.

FIGURE 23. Metaphase of a small second spermatocyte.

FIGURE 24. Spermatid of Scolopendra subspinipes.

Proceedings of the American Academy of Arts and Sciences.

Vol. XLI. No. 14. — December, 1905.

ON THE FIRST OBSERVATIONS WITH REGISTRATION BALLOONS IN AMERICA.

By A. Lawrence Rotch.

ON THE FIRST OBSERVATIONS WITH REGISTRATION BALLOONS IN AMERICA.

By A. Lawrence Rotch.

Presented March 8, 1905. Received July 22, 1905.

In a communication to the Academy in 1897,* the author described the so-called "ballons-sondes," which had been first employed in Europe a few years before to ascertain the meteorological conditions of the high atmosphere. He also gave an account of the more recent use of kites at his own Observatory on Blue Hill to investigate the conditions prevailing in the free air at lesser heights. In concluding he expressed the wish, as the American member of the International Committee for Scientific Aeronautics, that similar explorations of the high atmosphere with the "ballons-sondes" might be attempted in America.

Although the successful experiments at Blue Hill led to the extensive use of kites for meteorological observations abroad, the "ballons-sondes" were not tried in America until 1904, when the author was enabled to make a series of ascensions at St. Louis through the co-operation of the management of the Lousiana Purchase Exposition. A grant of $2,500 was authorized, but only about half this sum was spent for apparatus, travelling and living expenses.

In Europe two types of these balloons have been employed recently; viz., large balloons of silk or paper and small balloons of india-rubber, all being filled with hydrogen gas. The economy and ease of inflation, with the advantage of a quick ascension to the culminating point and restricted drift, led to the adoption in the St. Louis experiments of the latter form, devised by Dr. Assmann, director of the Royal Prussian Aeronautical Observatory, and made by the Continental Caoutchouc and Gutta Percha Company of Hanover, Germany. The balloons used had an initial diameter of approximately six feet when inflated with about 100 feet cubic feet of hydrogen gas, and carrying parachute and instrument exerted a net lift of nearly two pounds, which is sufficient. Being closed at the mouth they rise at an almost uniform rate of speed, expanding meanwhile until they burst at a height which is dependent upon their initial distension. A parachute, covering the top of the balloon, moderates the fall and it reaches the ground comparatively near the point of departure. The times at which the balloon left the ground and returned to it being automatically recorded and these places being known, the average direction and velocity of the balloon can be calculated, although its velocity is usually greater over the upper portion of its course. In some cases two balloons, unequally filled and coupled tandem, are employed, and, as only one balloon bursts, the other is borne slowly to the ground, where it continues to float and serves to attract persons to the spot. If the balloons are not badly torn they may be mended and used again, but, since the rubber has been permanently stretched, its elasticity is impaired and consequently the balloons will not rise as high as before. The quality of the rubber also deteriorates rapidly, especially when exposed to light and heat. The recording instruments, which were furnished by M. Teisserenc de Bort of Paris, consist of a metallic thermometer and a Bourdon barometric tube which record upon a smoked cylinder turned by clockwork. The smoked sheet of aluminum is afterwards removed from the cylinder and the record made permanent by flowing shellac over it. The temperature and barometric pressure are recorded simultaneously, and from them the corresponding curve of height may be computed with tolerable accuracy by Laplace's formula. Although the instruments had been calibrated in Paris, they were again tested before each ascension for barometric pressure under the receiver of an air-pump, in which the natural rate of change of pressure was imitated as nearly as possible, and for temperature in a bath of alcohol cooled with solidified carbon dioxide. Each instrument is contained in a mica case and further protected against rough usage by a wicker basket suitably screened against the sun's rays, the whole weighing about a pound and a half. It is suspended from the parachute at a considerable distance below the balloon and directions printed on a water-proof envelope request the finder to pack the instrument carefully in a box and return it either to St. Louis or to Blue Hill, a reward being promised for the service. A card contained in the envelope, after being filled out with the finder's name and address and the time of finding, is to be mailed immediately to the sender. The fact that all but one of 23 balloons and instruments despatched were found and returned shows the efficiency of these arrangements as well as the excellent topographical situation of St. Louis for this work.

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* These Proceedings, **32**, No. 13.

Owing to difficulties in obtaining hydrogen gas on the Exposition Grounds, the experiments were not begun until the middle of Septem-

OBSERVATIONS WITH REGISTRATION BALLOONS DESPATCHED FROM ST. LOUIS, U. S. A.

Date.	Time of Start.	Minimum Temp. (Deg. F.)	Corresponding Height. (Feet.)	Direction of Flight. (From.)	Distance travel'd. (Miles.)	Minutes in Air.	Velocity. (Miles per Hour.)
1904.							
Sept. 15	4.33 P.M.	–62.5	55,925	WNW.	60	114	26.3
Sept. 23	5.39 P.M.	–64.7	51,120	W.	57	. . .	. . .
Sept. 24	5.35 P.M.	–19.7	26,800	W.	60	130	23.1
Sept. 26	4.20 P.M.	† . . .	. . .	WNW.	52	. . .	. . .
Nov. 22	5.18 P.M.	–11.6	21,035	NNW.	65	156	25.0
Nov. 23	5.45 P.M.	46.4	7,650	NW.	55	65	50.8
Nov. 24	6 25 P.M.	39.4	8,200	NW.	145	. . .	. . .
Nov. 25	6.00 P.M.	–63.9	37,730	W.	280	167	100.7
Nov. 26	5.27 P.M.	–76.2	35,400	NNW.	235	140	100.6
Nov. 29	10.56 A.M.	–20.9	21,330	W.	140	142	59.1
Nov. 29	5.53 P.M.	. . .	36,075*	WSW.	260	. . .	. . .
Nov. 30	6.47 P.M.	. . .	2,800*	S.	16	28	34.2
Dec. 1	7.25 P.M.	–1.8	21,735	SW.	92	132	41.9
Dec. 2	12.19 P.M.	–72.4	51,870	SW.	105	112	56.2
1905.							
Jan. 22	5.20 P.M.	–70.8	41,570	W.	162	. . .	. . .
Jan. 23	5.20 P.M.	† . . .	. . .	WNW.	77	. . .	. . .
Jan. 25	5.25 P.M.	–111.0	48,680	NNW.	285	. . .	. . .
Jan. 26	5.20 P.M.	–85.2	28,950	NW.	140	. . .	. . .
Jan. 27	5.15 P.M.	. . .	55,090*	NW.	108	. . .	. . .
		–4.7	17,160	. . .	. . .	. . .	. . .
Jan. 28	5.36 P.M.	–6.7	14,510	WNW.	30	. . .	. . .
Feb. 10	. . .	. . .	. . .	W.	250	. . .	. . .
March 2	. . .	. . .	. . .	NNW.	68	. . .	. . .

* These are the maximum heights, no temperatures being obtained.

† The records of these ascensions were obliterated by the finders.

ber, 1904. During that month four ascensions were made, all the balloons falling about 50 miles east of St. Louis within a distance of 15 miles of one another, and at the maximum height of ten and a half miles a temperature of —62° F. was recorded. Another series of 10 ascensions was made during the closing days of the Exposition in the latter part of November and the first days of December, mostly after sunset to avoid the possible effects of insolation. Owing to the stronger air currents at this season, two of these balloons travelled with a mean velocity of more than one hundred miles an hour and all but one fell within the eastern half of a circle having St. Louis as its centre and a radius of 280 miles. The exception was a balloon which did not rise above the surface wind. In general the drift was away from the areas of low barometric pressure at the ground, thus confirming the theory of the outflow of air above them. An extreme height of almost ten miles was attained, and a temperature of —76° F. was found once somewhat below seven miles.

In order to continue these observations during the winter, the author executed at his own expense another series of nine ascensions during the latter part of January and on the days of the international ascensions abroad in February and March, respectively. The gas required was generated in barrels by the acid process. One of these balloons was lost, but the others fell nearly within the semi-circle described above. Several of the records are incomplete, owing to stoppage of the clock movements, and another was obliterated by the finder, but a height exceeding ten miles was indicated, and the extraordinarily low temperature of —111° F. was registered at the height of about nine miles during the prevalence of a high barometric pressure at the ground. Most of the ascensions were conducted by Messrs. Clayton and Fergusson of the Blue Hill Observatory staff, and it is due to their skill and perseverance in overcoming difficulties, consequent on undertaking a new work away from home, that such good results were obtained. The records have been partially reduced by Mr. Clayton and a summary of them is given in the accompanying table, but the temperatures and heights must still be regarded as more or less approximate. A complete publication will be made with the investigations of the Blue Hill Observatory in the "Annals of the Astronomical Observatory of Harvard College."

The annual variation of the temperature at great heights in the free air above the American continent is not yet known, and to ascertain this the writer will undertake a fourth series of observations with registration balloons this summer at St. Louis, the expense being met by a grant of $1,000 from the Hodgkins Fund held by the Smithsonian Institution to aid research relating to the atmosphere.

Proceedings of the American Academy of Arts and Sciences.

Vol. XLI. No. 15. — December, 1905.

ON A POSTULATE RESPECTING A CERTAIN FORM OF DEVIATION FROM THE STRAIGHT LINE IN A PLANE.

By G. Hay.

ON A POSTULATE RESPECTING A CERTAIN FORM OF DEVIATION FROM THE STRAIGHT LINE IN A PLANE.

BY G. HAY.

Presented Oct. 11, 1905. Received Oct. 16, 1905.

IN studying the relation of the axiom of the plane to the supposition of the sum of the angles of the rectilinear triangle being less than 180°, it becomes important to decide whether a certain line which presents itself in our supposed plane is to be considered a straight line or not.

"Der Begriff der Geraden ist vermöge seiner Einfachheit nicht definirbar, Richtung ohne die Gerade unverständlich." Baltzer, Elemente der Mathematik, zweiter Baud, Leipzig, 1874. Legendre alludes to the difficulty of defining the straight line. See Erdmann, Axiome der Geometrie, Leipzig, 1877, p. 19.

Suppose in a plane a given straight line; and in the same plane a line defined by its relations to the given straight line. It may be important to decide whether the line so defined is to be considered straight or not.

In the absence of a satisfactory definition of the straight line it is proposed to exclude directly from the category of straight lines by a postulate certain lines in a plane, as being, according to the postulate, forms of deviation from the ideal straight line.

A necessary property of the latter shall be the ideal property of it, that the straight line of any two of its points coincides with that of any other two. This property however leaves the straight line undefined, for a similar relation of any two of the points to their line would hold of the points of a given circular arc, with fixed centre, in a plane.

In the Figure which represents straight lines in a plane, LP, M_2P_2 and L_1P_3 are each a straight line perpendicular to straight line PP_3.

$$LP > M_2 P_2$$

$$L_1 P_3 > M_2 P_2$$

$$QP = M_2 P_2 = HP_3$$

$$\alpha = \text{ang. } M_2\ Q\ P$$

$$\beta = \text{ang. } Q\ M_2\ P_2$$

$$\gamma = \text{ang. } M_2\ L\ Q$$

Supposing we are dealing with a plane, and, the sum of the angles of the rectilinear triangle not being greater than 180°, we have

$$\alpha = \beta$$

$$\gamma < 90^\circ$$

Thus in the Figure, the straight lines $L\ M_2$ and $P\ P_3$ are subject to the fifth postulate, as the Figure represents straight lines in a plane. As this postulate concerns straight lines, it cannot, if true, be inconsistent with any other true proposition about such lines.

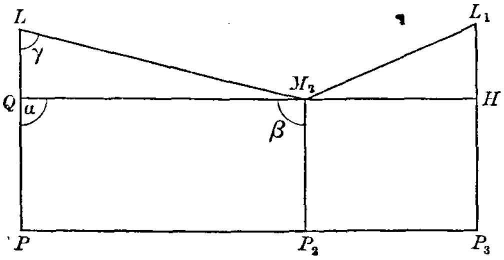

FIGURE.

As to the question of line $L\ M_2\ L_1$ in the Figure being straight or not, besides what may be deduced from the fifth postulate, use may also be made of the property mentioned above. In accordance with this, the straight line of $L\ M_2$ should coincide with that of $M_2\ L_1$, if $L\ M_2\ L_1$ is straight.

But in the Figure, straight-line portion $L\ M_2$ may be considered as converging towards $P P_3$, and straight-line portion $M_2\ L_1$ as diverging from $P\ P_3$, the two lines having point M_2 in common, and being on different sides of the perpendicular $M_2\ P_2$, and on the same side of $P\ P_3$.

Now the straight-line portions $L\ M_3$ and $M_2\ L_1$ being conditioned as above described, the postulate may be stated: Straight line passing through points L and M_2 cannot coincide with straight line passing through points M_2 and L_1. The condition stated by the postulate excludes the line $L\ M_2\ L_1$ from being straight, and the postulate may be designated as postulate for a certain form of deviation from the straight line in a plane, or briefly as deviation-postulate.

The ground of this postulate may be shown by means of the following subsidiary postulate. The plane is here regarded as a perfectly flat surface. Consider the straight line in which the limited straight line $L\ M_2$ lies; and in this straight line, outside the portion $L\ M_2$ and adjoining M_2 an additional small portion of it, small enough to lie on the same side of $P\ P_3$ as M_2 does. Now the subsidiary postulate: The additional small straight-line portion commencing at M_2 will converge towards $P\ P_3$ as $L\ M_2$ so converges. Consequently $M_2\ L_1$, diverging from $P\ P_3$, must be in a different straight line from that in which $L\ M_2$ lies, as they both pass through point M_2, and in the respective portions near to M_2 the one converging to, and the other diverging from, $P\ P_3$, and therefore by the ideal property mentioned above, line $L\ M_2\ L_1$ is not straight.

It may be noted that the convergence of $L\ M_2$ towards $P\ P_3$ depends on the given conditions of the construction of the diagram; and in using this convergence in the reasoning it is not necessary to consider whether the straight line of $L\ M_2$, when prolonged, meets $P\ P_3$ or not. That is, the reasoning of the subsidiary postulate does not make use of the relation of the fifth postulate which holds valid between the lines $L\ M_2$ and $P\ P_3$.

COMPARISON OF THE DEVIATION-POSTULATE WITH THE FIFTH POSTULATE AS THEY ARE INDICATED ON THE FIGURE.

In using the former to show that $L\ M_2\ L_1$ is not straight, it is not necessary to consider whether the course of the straight line $L\ M_2$ meets $P\ P_3$ or not; but only that the two limited straight lines $L\ M_2$ and $M_2\ L_1$ being differently inclined to $P\ P_3$, the one converging towards and the other diverging from it, then, according to the postulate, the straight lines of $L\ M_2$ and $M_2\ L_1$ cannot coincide. This reasoning, it is submitted, does not make use of the fifth postulate.

That $L\ M_2\ L_1$ is not straight may also be considered as a consequence of the fifth postulate, because by that postulate the straight line $L\ M_2$ would have to meet $P\ P_3$; but the line $L\ M_2\ L_1$ not doing so, cannot be straight. Here we take into account whether the line $L\ M_2\ L_1$ meets $P\ P_3$ or not, which was not the case in the preceding reasoning.

That the two postulates which we are comparing are not identical will appear from the following:

The fifth postulate excludes $L\ M_2$ from taking the course $M_2\ H$, which, as $M_2\ P_2$ equals $H\ P_3$, can never meet $P\ P_3$; but the other postulate considers only those continuations of $L\ M_2$ which, like $M_2\ L_1$, diverge from $P\ P_3$, and disregards course of $M_2\ H$.

It is one proposition to say, as the deviation-postulate does, that straight line $L M_2$ cannot take the course $L M_2 L_1$ as described, thereby excluding $M_2 L_1$ from being the continuation of $L M_2$, and another to say, with the fifth postulate, that the same line must meet $P P_3$, for in this case both $M_2 L_1$ and $M_2 H$ are excluded from being the continuation of $L M_2$.

The two postulates exclude the line $L M_2 L_1$ (see Figure) from being straight, but respectively on different grounds.

The deviation-postulate, considering also the ideal property referred to above as belonging to a straight line, excludes $L M_2 L_1$ from being straight on the ground of its being a course inconsistent with a generic property of the straight line.

The fifth postulate makes the exclusion, because $L M_2 L_1$ being so conditioned at its intersection with $L P$ that $L M_2$ and $P P_3$ are subject to the fifth postulate, does not take the course which, according to this postulate, it should take, or because it does not take the course of one of two straight lines related to each other in a certain way.

Now the line $L M_2 L_1$, considered as a part of diagram (see Figure), embodies both the two sets of conditions which the two postulates respectively present for the line not being straight. It is submitted that either of the two sets may be used for arguing that the line is not straight.

The deviation-postulate may be stated as follows:

Given in a plane two straight-line segments, having a common point outside of a given third straight line in the same plane, and situated on different sides of the perpendicular from the common point to the third straight line; let each of the two straight-line segments, considered as proceeding each from the common point, diverge from the third straight line, then postulate: The two straight lines of the segments do not coincide.

The above postulate is in one respect interesting in comparison with the fifth postulate, in that the former concerns special conditions which prevent a *single* line in a plane from being straight; whereas the latter tells us that *two straight lines* in a plane, and related to each other in a certain way, intersect.

AN ADDITIONAL DIFFERENCE BETWEEN THE TWO POSTULATES.

The deviation-postulate is a postulate about two straight lines, and states a condition which prevents a certain line from being straight (see the Figure). Given two adjoining straight-line segments $L M_2$ and $M_2 L_1$, having a given point in common and related in a certain way, as indicated in description of the Figure, to a given third straight line $P P_3$, which the segments do not meet. The point in common excludes parallels.

With the given relation to each other that the two segments have a given point in common, a possible case for the two straight lines of these segments would be that they coincide. But, according to the postulate, the relations of the two straight lines to the given third straight line are such that they do not coincide. Then the condition of these two lines is that they intersect at one point. The part of this condition which the postulate of the two lines states, is

The exclusion of coinciding.

Fifth postulate: With the relation to each other that the two straight lines of the postulate do not coincide, and are related in a certain way to a third straight line which they both intersect; then postulate. The two straight lines of the postulate intersect. This, the intersection, is equivalent to

Excluding parallels.

Therefore, the postulate in this case virtually states the exclusion of parallels, whereas, in the case of the deviation-postulate, it states exclusion of coinciding.

Two uses may be made of the above deviation-postulate: one, to show that the sum of the angles of the rectilinear triangle is equal to 180°; and the other, to show that if that sum is supposed less than 180° then the axiom of the plane does not hold good.

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SURFACE TENSION AT THE INTERFACE BETWEEN CERTAIN LIQUIDS AND VAPORS.

By Arthur L. Clark.

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SURFACE TENSION AT THE INTERFACE BETWEEN CERTAIN LIQUIDS AND VAPORS.

By Arthur L. Clark.

Presented by A. G. Webster, May 10, 1905. Received Oct. 11, 1905.

All determinations of the capillary constants of liquids are made necessarily with the liquid used in contact with its saturated vapor, a gas, or another liquid. The constant for a free liquid surface, i. e., one not in contact with any other substance, cannot be determined, although the value for a liquid near its freezing-point and in contact with its own vapor cannot be far from the true value. It is true, however, that almost all methods for the determination of the constant measure the tension at the line of contact with a solid and not on the undisturbed surface.

The values of the constant for various liquids have been determined by a large number of observers.* Quincke † has determined the constant for the interface of two different liquids, and Kundt ‡ for liquids in contact with gases under pressure. Both of these observers worked at room temperatures. Quincke found that $\alpha_1 < \alpha_2 + \alpha_{12}$, because of solution of the two liquids in each other. Kundt found that the surface tension between a liquid and a gas decreases rapidly with increasing pressure. Wroblewski § showed that this effect was due to solution of the gas in the liquid. Since the surface tension of mixtures obeys approximately the centre of mass law, we can readily see that we must have a very great lowering of the capillary constant, when one liquid is replaced by a gas whose surface tension is practically zero.

A very large number of experimenters ‖ have investigated the effect of raising the temperature of a substance upon its capillary constant, and have found that, in every case, a rise in temperature is accompanied by a decrease in the constant. Kasterine,* and Ramsay and Shields † have pushed the observations to the critical point and have shown that the constant becomes zero at this point. This had been studied qualitatively many times, but the above-mentioned observers have carried out quantitative investigations.

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* See Winkelmann, Handbuch der Physik, Vol. 1, page 497, for a very complete bibliography of the subject.

† Quincke, Pogg. Ann., **139**, 1 (1870).

‡ Kundt, Wied. Ann., **12**, 538 (1881).

§ Wroblewski, Compt. Rend., **95**, 284 (1882).

‖ See Winkelmann, Handbuch der Physik, loc. cit.

The vanishing of the capillary constant of the interface noticed by these observers is to be attributed not to the vanishing of the constant of the liquid, but to the fact that the values for the liquid and vapor have approached equality, causing the boundary surface to disappear.

The capillary constant of the boundary between two liquids at high temperatures and pressures, and between a liquid and the saturated vapor of another liquid has not been investigated. The following research is an attempt to supply to some slight extent these missing data. It will be readily seen that the choice of liquids is limited, if the experiments are to be pushed to temperatures much above the room temperatures. The substances should be as nearly insoluble as possible in each other, should not act upon each other chemically; the vapor density of one, at least, should be very small in experiments on vapors. As yet little more than a beginning has been made, as the larger part of the available time has been spent in devising and testing the apparatus and methods.

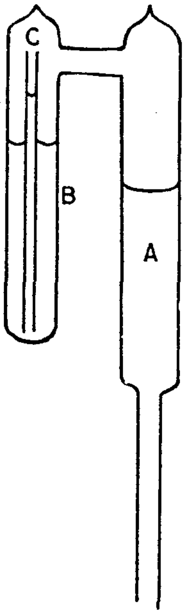

FIGURE 1.

THEORY OF EXPERIMENTS.

The method of experimenting on the surface between a liquid and the saturated vapor of another liquid will be described first. The substances were placed in a glass tube of the general appearance shown in Figure 1.

The part *A* contains the liquid to be vaporized, the part *B*, the liquid whose surface is to be studied. *C* is a small capillary tube suspended inside *B*. The surface tension is found by measuring the height of the meniscus in the capillary above that in the large tube, the diameters of the two tubes, and the angle of contact. Then

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* Kasterine, Journ. de Phys. (3), **2**, 529 (1893).

† Ramsay and Shields, Phil. Trans., **184**, 647 (1893).

$$a = \frac{g\,H\,(d - d')}{2\cos\omega\left(\frac{1}{r_1} - \frac{1}{r_3 - r_2}\right)}\ *$$

where a is the surface tension, H the height of the meniscus, d and d' the densities of the liquid and vapor respectively, ω the contact angle, and r_1, r_2, and r_3, the inner and outer radii of the capillary tube and the inner radius of the containing tube respectively. The angle of contact was found by measuring the depth of the meniscus and the radius of the tube. Then considering the meniscus a portion of a sphere,

$$\omega = \frac{\pi}{2} - 2\tan^{-1}\frac{h'}{r_1},$$

h' being the depth of the meniscus. This method of determining ω is not satisfactory, for it assumes a spherical meniscus, which shape is of course realized less and less as we near the wall of the tube. It also assumes a perfectly circular cross-section. A better method has been projected, but not yet tested.

The capillary tube method of measurement of surface tension leaves much to be desired when applied under such severe conditions as in the present work.

The tube used in the liquid experiments was not of the complicated form used in the vapor tests, but both liquids were placed in the same tube. The measurements for the determination of a were made exactly as just described.

These tubes were placed in an electric heater controlled by an automatic thermostat, and the constant observed for different temperatures. The lower part of the experimental tube was connected with a manometer. In this way the capillary constant and pressure could be deter-

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* The system being in equilibrium, the pressures inside and outside the capillary tube at the level of the meniscus outside are equal.

So

$$\frac{2\pi r_1 T\cos\omega - \pi r_1^2 g\,d_1}{\pi r_1^2} = \frac{2\pi\,(r_3 + r_2)\,T\cos\omega - \pi\,(r_3^2 - r_2^2)\,g\,H\,d_2}{\pi\,(r_3^2 - r_2^2)}$$

or

$$2\,T\cos\omega\left(\frac{1}{r_1} - \frac{1}{r_3 - r_2}\right) = g\,H\,(d_1 - d_2)$$

whence

$$T = \frac{g\,H\,(d_1 - d_2)}{2\cos\omega\left(\frac{1}{r_1} - \frac{1}{r_3 - r_2}\right)}$$

mined for any temperature at will. The manometer also served as a criterion of constancy of temperature.

DESCRIPTION OF APPARATUS.

Heating Apparatus and Thermostat.

The heating is done electrically and the temperature is controlled by a self-regulating thermostat. As a large amount of work has been planned requiring an automatically controlled thermostat working over a wide range of temperature, it was decided to build it once for all; and, though for a large amount of work a simpler controller might be used, the ease of manipulation and certainty of operation repaid for the extra trouble in building.

The thermostat is a modification of the one used by Griffiths * in his work on "The Value of the Mechanical Equivalent of Heat."

It consists of a cast-iron box *A* (Figure 2), the walls of which are hollow and filled with mercury. The expansion of the mercury closes the relay circuit, which controls the heating current. The construction of this box was attended with serious difficulty because, on account of the openings for fan shafts, windows, and experimental tubes, it was deemed best to make the box in one casting. The walls are hollow except where the openings occur, where they are solid for a space of about 1 cm. about the openings. I am indebted to Mr. L. S. Burbank for the method of making the patterns for this casting. The hollow walls and the interior of the box itself were cored, the pattern being one solid piece.

The top of the box was planed smooth, and then ground by rubbing on a large sheet of plate glass covered with powdered emery and oil. The cover of the mercury chamber was also planed and ground, and screwed on tightly with just enough silicate of soda to ensure perfect tightness. Finally the removable cover was fitted on. Then the holes for the windows and fan shafts in the sides and for the experimental tubes in the bottom were drilled. These holes were of course bored through the solid places in the walls. The dimensions of the box are 15 × 15 × 15 cm. outside. The walls are 6 cm. thick, the hollow space being the same size. Both the cover to the mercury chamber and the removable cover are .6 cm. thick. The mercury chamber contains 420 c.c. There are two openings in the cover to this chamber, one for the overflow to the regulator and the other for convenience in filling, which later is closed with a steel plug. The chamber was filled by heating to about 250° and pumping out the air at one opening and allowing clean dry mercury to run in at the other. In this way all air and moisture could be removed from the mercury and the chamber. When full the box was allowed to cool while still connected with the mercury supply.

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* Griffiths, Phil. Trans., **184**, 361 (1893).

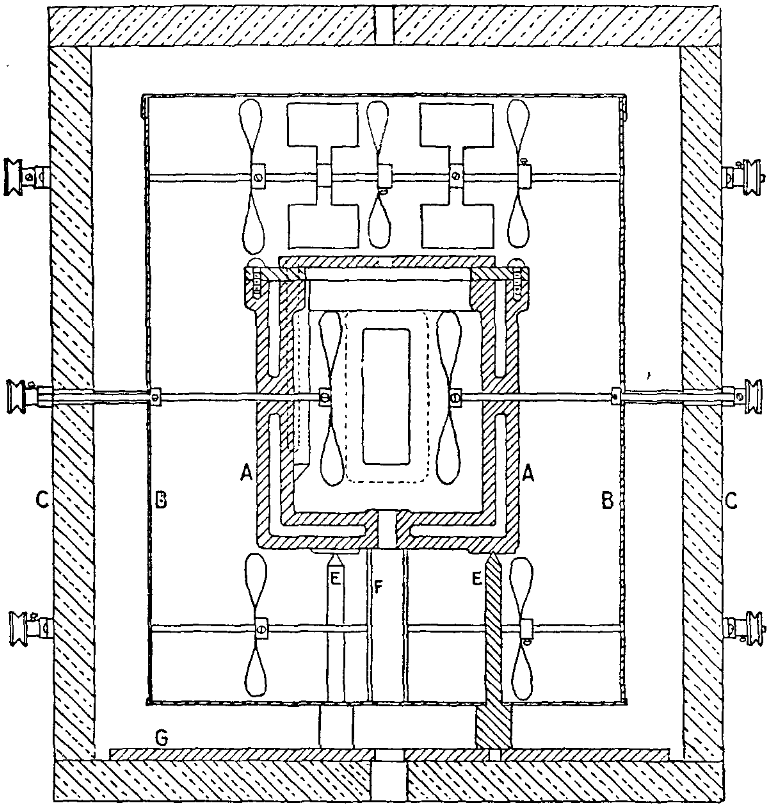

Figure 2.

Then the second hole in the cover was closed by a steel plug securely screwed in.

The windows are of glass held in place by springs of heavy steel wire wound horizontally around the box. On the bottom of the box are three bosses provided with conical hole, groove, and plane respectively for the support of the box which rests on three pillars to be described.

At two opposite points on the inside wall of the box are projections into which vertical holes are bored for the insertion of thermometers. These holes are 8 cm. deep and .75 cm. in diameter, and are filled with mercury which thus surrounds the thermometers. In the covers are holes exactly coinciding with the thermometer holes.

As already stated, the heating is done electrically. This is accomplished by means of coils of German silver wire, .035 cm. in diameter, which surround the iron box on five sides. These coils are suspended on supports made of glass tubing fastened to the sides of the enclosing box.

This enclosing box *B* (Figure 2) is made of sheet copper, the dimensions of which are 23 × 23 × 32 cm. There are holes for windows, thermometers, shafts, and tubes corresponding to the holes in the iron box.

The copper box is in turn enclosed in a wooden box or case, the space between the copper and wood being packed with magnesia.

The external dimensions of the wooden case are 35 × 35 × 43 cm. Where the fan shafts pass through the magnesia, metal tubes were inserted and the magnesia was moistened with water so that when hard the tubes were held in position. These tubes serve to prevent the magnesia from falling upon the shafts.

It will readily be seen that, on account of the numerous holes in the iron box as well as in the enclosing box, a liquid bath for heating is out of the question. There is no doubt that a liquid may be stirred more easily than air; but after all efficient stirring means the rapid and constant moving of the heated substance from the source to the body under experiment. In the heater used, this is accomplished by a system of sixteen fans of sheet iron, carried on six shafts which are run at high speed by an electric motor. These fans are about 7.5 cm. in diameter and create a strong circulation in the heated space around the iron box. There are two fans inside the iron box to ensure uniformity of temperature. This is necessary on account of the breaks in the continuity of the iron box. The shafts for the fans are made of steel rod about .3 cm. in diameter and run in bearings screwed to the outside of the wooden case. The shafts for the inner fans have their inner bearings in the iron box itself, and these bearings are oiled by means of oil tubes which project through the wooden case.

In the bottom of the case was screwed a heavy iron plate *G*, in which were fastened the steel pillars *E*, which support the iron and copper boxes. These pillars are 16 cm. long and are .6 cm. in diameter for a distance of 7 cm. from the conical end. Then they are 1.2 cm. in diameter for the remainder of the length. The copper box rests on the shoulders thus formed, while the iron box rests on the points. Thus, whenever it is necessary to take down the apparatus, it may be put together again in perfect alignment with very little trouble. The whole system, which weighs about 40 kg., is slung from the ceiling by rope and pulleys. The case is supported by heavy iron straps and is also provided with large screw-eyes which slide on two upright steel rods, securely fastened to the work-table. This allows vertical adjustment and prevents swaying and tipping. *F* is a porcelain tube inserted in the heated space to protect the experimental tube from becoming heated too rapidly, since otherwise the lower part of the latter, being filled with mercury, would be unduly heated in the warming-up stages and cause distillation of the liquid in the tube above.

The resistance of the heater is 10 ohms, and is used on the 110-volt alternating lighting circuit. This, though unsteady through a small range of voltage, answers very well, since the changes of temperature in the heater take place slowly.

It remains now to describe the automatic part of the regulator. Since the thermostat was designed for temperatures between room temperature and 300° C., an overflow chamber of adjustable capacity is necessary. This is shown in Figure 3.

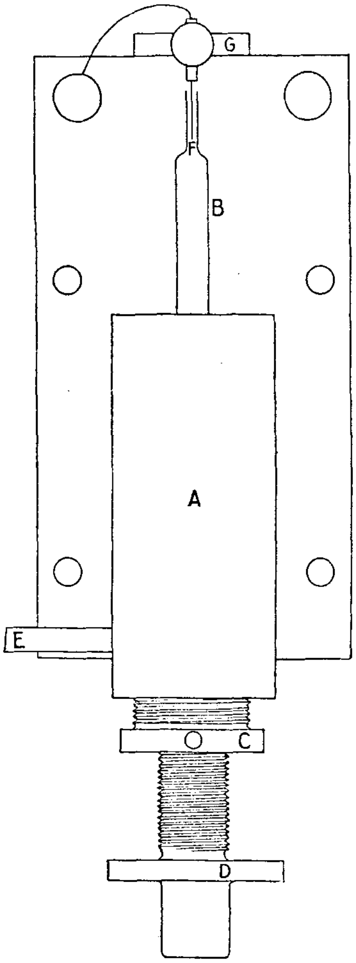

FIGURE 3.

A cast-iron cylinder *A*, with an opening for a glass tube *B* in the upper end, and a stuffing-box *C* with screw *D* in the lower end, is screwed firmly to the front of the wooden case. The side tube at *E* communicates with the mercury chamber of the iron box of the heater. The glass tube at *B* is .2 cm. in diameter at the upper part, the lower part being much larger.

A platinum wire *F* is held in place by a screw in the hard rubber block *G*. The mercury rising by the expansion completes the relay circuit at *F*, and the heating current is either cut off or reduced until the apparatus cools off again. The screw in the lower part of the chamber enables the capacity to be suited to the temperature desired. The total capacity of this chamber is about 20 c.c., the expansion of the mercury at 300° being about 18 c.c. The screw is three-fourths of an inch in diameter and the pitch is sixteen threads to the inch. The glass tube is cemented in with silicate of soda and the screw is packed with rubber washers held between leather ones. The tube communicating with the iron box is of steel about .5 cm. in diameter. A rise of temperature of one degree causes an ascension in the glass tube of about 2 cm. With the regulator cutting off the entire current, the regulation is within

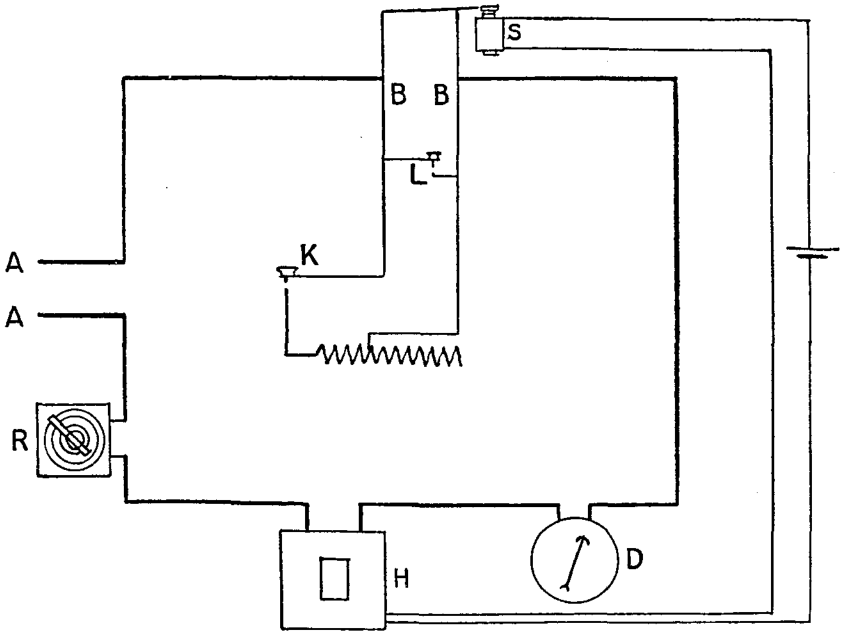

FIGURE 4.

$\frac{1}{10}$° C. at a temperature as high as 200°. By cutting off only a small portion of the current the regulation to within $\frac{1}{100}$° is very simple.

The arrangement of the circuit is shown in Figure 4. *A A* are the mains, *R* a large resistance adjustable through a wide range, *H* is the heater, *D* the ammeter. The gap *BB* is bridged over either by the relay *S* or the key *L*. When the relay is operated by the thermostat, the circuit is broken and the current sent through the variable resistance provided the key *K* is closed, otherwise the circuit remains open.

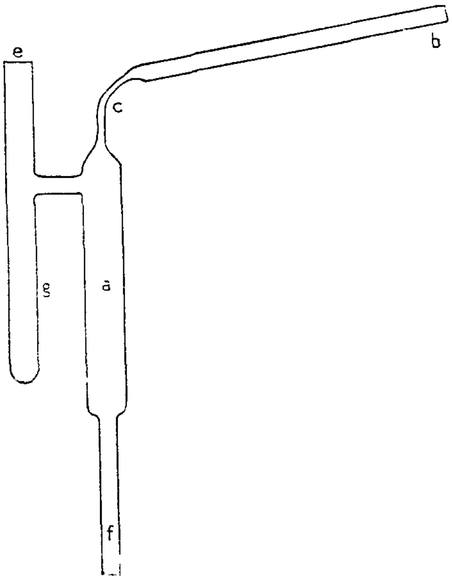

FIGURE 5.

EXPERIMENTAL TUBES.

Considerable difficulty was experienced in making a style of tube which would do the work required of it, be easy to insert in the apparatus and easy to fill. After many months of trial and failure and almost innumerable accidents, the form shown in Figure 1 was adopted. Since the pressures are considerable, the glass tubing must be selected with great care. The tubing finally adopted was obtained from the Bausch & Lomb Optical Company, and with its purchase ended a great many difficulties.

The tubing was carefully washed with aqua regia and caustic potash and finally with distilled water, and dried with a current of air.

The tubes are made in the form shown in Figure 5, the lower end being as in Figure 8.

The part *a* is about 6 cm. long, *e* is left open for the insertion of the capillary. This capillary is held in place by a platinum clip shown developed in Figure 7. The end *b* is left open for the attachment of the distilling apparatus; *c* is drawn out, being left large enough for safe handling. After the insertion of the capillary, care being taken to centre it accurately in the tube *g*, the part *d* is drawn down as shown in

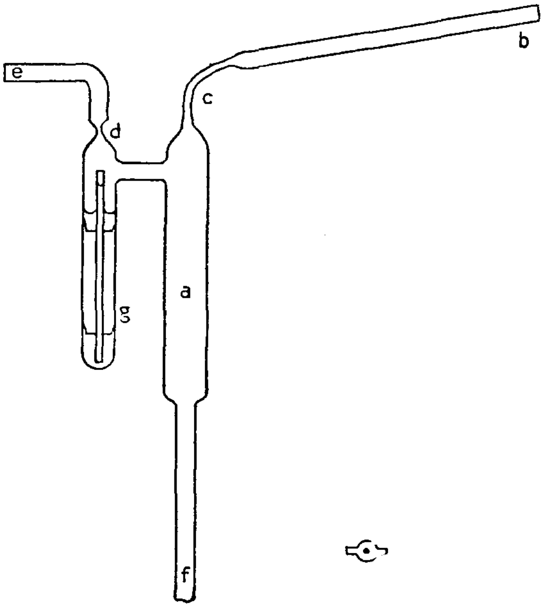

FIGURE 6.

FIGURE 7.

Figure 6. The heavy liquid is now placed in the tube *g* and the end *e* bent over for attachment to the pump. The lower end of the tube as well as the method of holding it in the steel chamber is shown in Figure 8.

L is a large steel chamber (capacity 20 cm.) containing pure mercury; *K* is a stuffing-box screwed down on the rubber packing *g*. The glass tube *f* is inserted in the heater down through the hole in the bottom. The piece *J* is slipped on. then the steel washer *e*, and finally a rubber stopper *c*. By slightly moistening the tube with glycerine, the stopper may be worked over the enlargement *M*. Then *J* is screwed down and the stopper entering the narrow place at *d*, effectually closes the opening. The expanded part *M* prevents the tube from being pressed up through the stopper. This form of joint is perfectly tight up to 100 atmospheres and probably would be at much higher pressures. The joint is very flexible and the tube may be rotated very easily. After the insertion of the tube, the heater is lowered as far as possible, the tube *b* is sealed on to the retort containing the liquid to be distilled in. There is a stop-cock in this tube. The tube *e* (Figure 6) is attached to the air-pump also through a cock. Next the narrow parts in the two tubes are drawn out very small and when cool the air-pump is started and the lower part of the tube is heated. The mercury in the steel chamber is allowed to rise very slowly, being kept very hot to drive out air and moisture. This is continued until the part *a* is partly filled. The tube *a* is now allowed to cool. The air-pump still operating, *b* is gradually heated with a coil of wire to ensure the driving out of any air that may have entered during the filling. Now *a* is kept cool with ice, and *b* hot with the electric current, and the liquid distilled over from the retort. The first distillate is boiled away to carry off any air still remaining. Then the liquid is distilled over and when the proper amount is obtained the cock to the still is closed, the narrow part sealed off, then the other tube sealed off, and the tube is complete. The tube is now rotated until in the best position for observation, the heater is raised to the proper height and the cover put on, thermometers inserted, and the experiment begun.

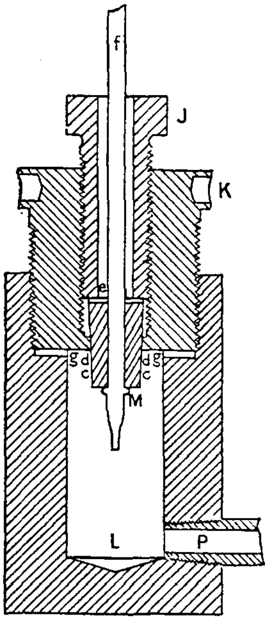

FIGURE 8.

MANOMETER.

The manometer is patterned after Amagat's and was made from drawings loaned by Professor Barus of Brown University, the sizes of the pistons being changed to suit the work in hand. The large piston is 5 cm. in diameter and the small one about 1 cm., the reduction factor being 24.42. A complete description of this manometer is given in

Amagat's paper,* and in the translation by Barus in the book edited by him entitled "The Laws of Gases," p. 55. A few modifications deserve mention, — the change of piston areas, and the substitution of light engine oil in place of castor oil, and of a heavy cylinder oil in place of molasses, this substitution being made according to a suggestion of Professor Barus'. Figure 9 shows the manometer used in this work.

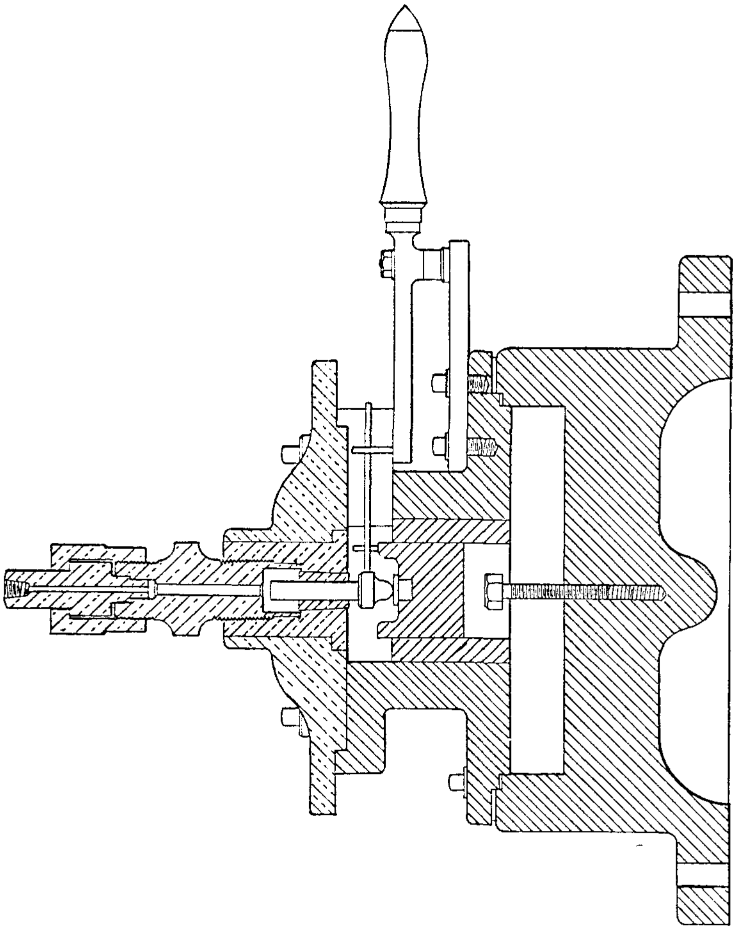

Figure 9.

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* Amagat. Annales de Chimie et de Physique, **6**, 19 (1893).

The pistons were made of tool steel hardened and ground as were the cylinders in which they move. The fit is exceedingly good, there being very little leakage of oil and scarcely any friction. It was feared that the manometer would not give accurate results for pressures below ten atmospheres, but the results are surprisingly good. It was compared with an open mercury manometer and the error for readings as low as one atmosphere was less than 1 per cent.

The glass tube in which the pressures are read is 4 mm. in diameter internally, and the scale is made of ordinary laboratory wooden meter sticks. A displacement of one millimeter represents about 2.5 cm. on an open manometer. The pump of the manometer is smaller than that described by Amagat, and is used merely to control the position of the pistons. The vertical motion of the pistons is aided by a twist given to them by means of a lever, as described by Amagat. The manometer has given the best of satisfaction and has proved itself a very valuable adjunct to the laboratory. I am indebted to Professor Barus for the drawings, the castings, and for many valuable suggestions.

Since there are unavoidable minute leaks, and since when the manometer operates there is an appreciable motion of the pistons and necessarily an increase of volume in the experimental tubes, it is necessary to provide some means of compensation. This is accomplished by a screw forced into a steel oil chamber which is connected with the mercury chamber under the heater (Figure 8) and with the manometer. This oil chamber is simply a piece of steel shafting bored out to a diameter of seven-eighths of an inch, and a steel screw three-fourths of an inch, forced in by a hand wheel ten inches in diameter. The pitch of the screw is sixteen, and the thread is packed with leather washers which had been previously boiled in marine glue. Professor Barus has shown the efficacy of marine glue for such purposes. The packing thus formed is screwed down with the gland of the stuffing-box.

Between the compressor just described and the mercury chamber of Figure 8 is another reservoir for mercury. This is simply a chamber filled with mercury and oil, connecting at the bottom with the mercury reservoir of Figure 8 and at the top with the compressor. This obviates soiling the mercury which comes in contact with the glass experimental tube, and the mercury chamber, too, is kept clean.

Between the two mercury chambers and also between the compressor and the manometer are steel cocks which deserve mention. These are made as shown in Figure 10. The especial feature is the hardened steel conical stem which rotates on the stem proper. This cone is ground to fit the seat, which is relieved at the tip. These cocks always close tightly and do not break down at critical times as do such cocks when seated with some soft metal like tin or lead.

All joints, whether containing mercury or oil, are heated, smeared with marine glue, and screwed together hot, and never leak.

MICROSCOPE.

The observing is done with a micrometer microscope whose working focal length is 10 cm. It is carried on a micrometer slide so that the stationary wire of the eyepiece may be kept on the surface of the liquid in the outer part of the experimental tube, one observation only being required to measure the capillary height. One division of the eyepiece micrometer corresponds to .005 mm. The whole system is supported on a rigid iron base, whose height is adjustable. The thermometers used were made by Baudin, readings being taken to $\frac{1}{10}$°.

METHOD OF EXPERIMENTATION.

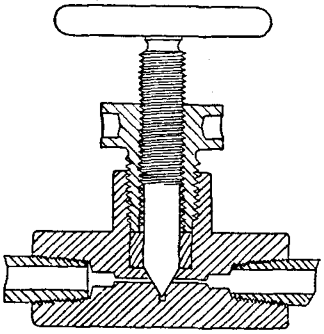

FIGURE 10.

The microscope is focused on the tube and the stationary wire set on the base level, the movable one on the meniscus, and several readings made on each of the instruments. The current is turned on and the regulator set for the next higher temperature. As the temperature rises, there is always a slight displacement of the liquid in the experimental tube which is easily compensated by the compressor. It is desirable that the volume should be kept as nearly constant as possible, since any considerable change may produce decided changes in temperature, especially at high temperatures. After the temperature is steady, as shown by the thermometer, readings were taken on the manometer to ensure constancy of temperature.

The entire run with one tube occupies from ten to fifteen hours when there are no accidents.

The density of the heavier liquids was determined with a pyknometer, being treated in the heater at various temperatures and then weighed with the contents.

The following figures show the results for glycerine and olive oil:

Glycerine.		Olive Oil.	
Temp.	Density.	Temp.	Density.
31.3°	1.255	21°	.9127
100°	1.207	43°	.8986
150°	1.167	90.4°	.8654
190°	1.134	134.7°	.8372
235°	1.089	178.2°	.8081
		225.5°	.7790

Curves shown in Figure 11 were plotted and the densities used in calculation taken from these.

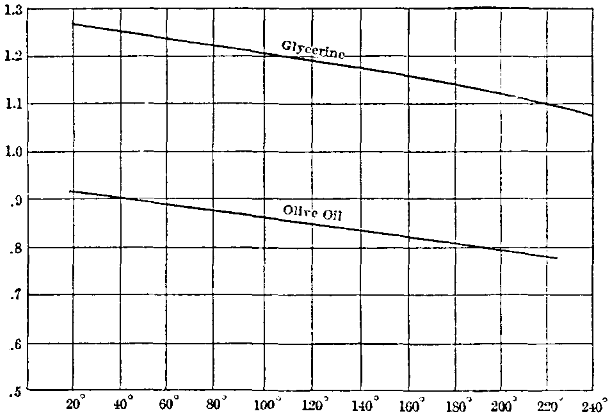

Figure 11.

The values of vapor densities of lighter liquids were taken from results obtained by other observers, and the curves of Figure 12 plotted. These particular figures are from Ramsay and Young.[*]

The diameters of the capillary tubes were determined by breaking and measuring with microscope.

Thus far the liquids observed divide themselves into two groups: 1st, those in which the vapor of the vaporized liquid condenses on the other liquid at a temperature higher than the temperature of the saturated vapor, and 2d, those in which this condensation temperature is lower than the saturation temperature. Cantor[†] has shown that whether or not a vapor spreads out on another liquid depends on the values of the surface tension of the liquid, of the condensed vapor, and of the interface; i. e., when the liquid from the condensed vapor wets or spreads out over the second liquid, the vapor will be condensed even when the temperature is above the saturation temperature. In order to keep such a surface dry, the temperature must be maintained several degrees higher than the saturation temperature. This makes observation with such a pair of substances very difficult. In the earlier parts of the experiment several months were spent determining whether this condensation was due to the above-mentioned phenomenon or to unequal heating, bringing about a distillation. The appearance of the condensation was always noticed by the change in shape of the meniscus. Ethyl ether and glycerine, while serving admirably in every other way for experiment, belong to the second class of substances. The condensation was very rapid as the critical point of ether was approached, and the following very curious effect was noticed. After the critical point had been passed by several degrees, the liquid ether had entirely disappeared in the ether part of the tube, while in the glycerine part of the tube liquid ether still persisted and appeared to be permanent, though no trial was made for a long period of time. The temperature was pushed several degrees higher and all liquid ether disappeared: then the temperature was lowered gradually and liquid ether appeared first in the capillary above the glycerine, then in the outer tube above the glycerine, and finally in the ether tube.

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[*] Ramsay and Young, Phil. Trans., **178**, 57 (1887).

[†] Cantor, Wied. Ann., **56**, 493 (1895).

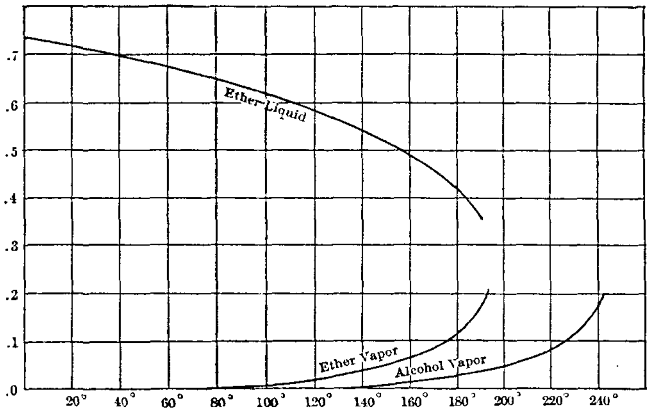

FIGURE 12.

This was interpreted as follows. The effect of Cantor comes first, forming a film of ether on the glycerine surface. Owing to the concavity of this surface, the ether vapor is not in equilibrium with an ether surface at this temperature and is consequently condensed. The presence of a small quantity of glycerine vapor undoubtedly facilitates this condensation. It will be noticed that the capillary tube contains the hottest ether vapor while the whole is cooling off, and here the vapor should condense last. This excludes the possibility of unequal heating being the reason for this effect.

In order to measure the surface tension between liquid glycerine and the vapor of ethyl ether, a special method was employed to keep the temperature of the glycerine above that of the ether so as to avoid the above-mentioned condensation. The glycerine tube was surrounded by a small coil of German silver wire and a separate heating current sent through this. The method of procedure was as follows. When the desired temperature of the heater was obtained, a measured current was sent through this auxiliary coil for a measured time, and then when the condensed liquid had disappeared the current was shut off and measurements taken of the height of the meniscus until the shape of the meniscus changed due to the redeposit of ether. After the completion of the experiments, enough of the tops of the two tubes was removed to insert the two junctions of an iron and German silver thermo-couple. Then the same current that was used previously was sent through the auxiliary coil for the same time and the change in temperature inside the capillary tube measured. Of course the thermo-couple was calibrated by a separate experiment. In this way a rough determination of the rise in temperature was made. It is in error, for the specific heat of ether under such circumstances is unknown. I have used the value for ordinary temperatures.

This temperature of equilibrium between a vapor of one liquid and another liquid has apparently not been studied except by Cantor at room temperature. This will be taken up again by a new method, and instructive results are expected.

Ethyl alcohol and olive oil belong to the first class of liquids, and no precautions were necessary to prevent distillation. Here, however, there is considerable absorption of the vapor by the oil especially at high temperatures. The mixture becomes homogeneous at about 240°.

The alcohol was the purest obtainable and distilled into the tube as described, and the oil was freed from volatile matter by boiling at high temperature.

The following table gives the results of experiments with these substances.

T is the temperature, *P* the pressure in atmospheres, *H* the height of the meniscus in cm. (corrected for concavity inside and outside the capillary *), ω the contact angle, $d - d'$ the difference between the densities of olive oil and alcohol vapor, and α the surface tension in dynes per cm.

T	P	H	ω	$d - d'$	α	
°						
12		.594	0	.92	23.2	$r_3 =$ .279 cm.
35		.579	0	.91	22.4	
73	1.18	.519	0	.88	19.4	$r_2 =$ .088 cm.
128	5.48	.367	0	.86	13.3	$r_1 =$ .060 cm.
151	9.88	.308	0	.81	10.4	
176.5	17.7	.234	0	.77	7.5	
201	29.2	.127	0	.75	4.03	
219	41.0	.070	0	.71	2.11	
232	51.4	.040	0	.68	1.16	
241.2	60.3	?	?	.62	?	

Above 232° the meniscus flattened and absorption took place so rapidly that observation was impossible.

LIQUID ALCOHOL AND OLIVE OIL.

An experiment over any considerable range in temperature was impossible with the capillary used, as above 40° the meniscus sunk so low as to be invisible.

At 30.2° the value of α was 14.2 dynes per cm.,† ω being 0 at this temperature although it did not remain so at higher temperatures. r_1, r_2, and r_3 were .348 cm., .102 cm., and .068 cm. respectively.

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* This correction was made by adding $\frac{1}{3} r_1$ and subtracting .11 $(r_3 - r_2)$.

† This is the result of a single experiment.

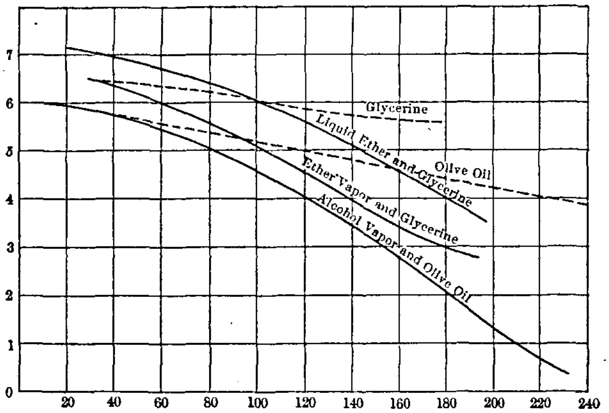

FIGURE 13.

ETHYL ETHER VAPOR AND GLYCERINE.

T	P	H	ω	$d - d'$	a	
°						
31	.86	.645	0	1.25	28 8	$r_3 = .337$
110	7.44	.487	0	1.17	19.7	
120	9.40	.456	0	1.15	18.1	$r_2 = .076$
140.1	14.87	.392	0	1.13	15.3	$r_1 = .056$
150	17.65	.362	0	1.11	13.9	
160.5	21.12	.341	0	1.08	12.7	
170.3	25.17	.315	0	1.05	11.5	
175	27.21	.308	0	1.03	11.0	
185.3	32.94	.300	0	.99	10.3	
193.4	37.19	.288	0	.90	8 7	

T here is the temperature of the heater. $d - d'$ is calculated for the liquid at the temperature of equilibrium, and for the vapor at saturation temperature.

LIQUID ETHYL ETHER AND GLYCERINE.

T °	H	ω	$d - d'$	α	
18.8	.714	17° 25′	.55	15.4	
35.8	.690	no sure change	.55	14.9	$r_3 = .356$
73.2	.617		.56	14.3	$r_2 = .080$
105.8	.586		.59	13.5	$r_1 = .061$
132.3	.526		.63	12.9	
152.3	.470		.65	12.0	
178.0	.418		.71	11.7	
195.6	.350		.80	11.0	
198.8	.345		.82	11.3	
200	.343		.86	11.6	

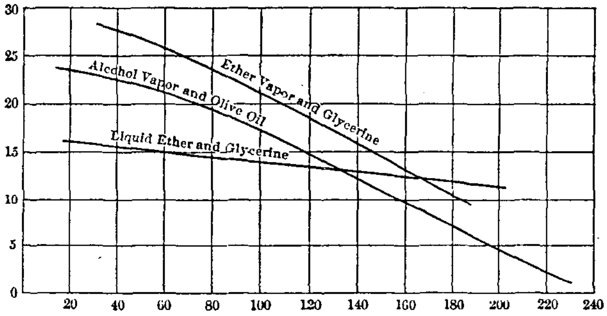

FIGURE 14.

The last two tables show a discrepancy in the value of α not wholly explained. It is probably due to two very large sources of error: 1st, to the impossibility of determining $d - d'$ at temperatures near the normal critical temperature for the liquid-liquid case, and 2d, to the uncertainty concerning the same quantities in the liquid-vapor case, as well as to the fact that α for the latter case is not the value for a true saturated vapor but rather for a superheated vapor.

Figure 13 shows the capillary height plotted as a function of the temperature, the dotted curves showing the height in millimeters when the liquid glycerine and olive oil are in contact with their own vapors, the figures for these results being taken from Weinstein's tables.*

Figure 14 shows the values of a in degrees per cm.

Figure 15 shows the results of the work with the thermopile on the temperature of equilibrium for the case where the vapor condenses on the liquid. Experiments were conducted at temperatures between 132° and 188°. It will be noticed that the equilibrium temperature corresponding to the normal critical temperature is 200°, the temperature at which the meniscus disappeared in the liquid case.

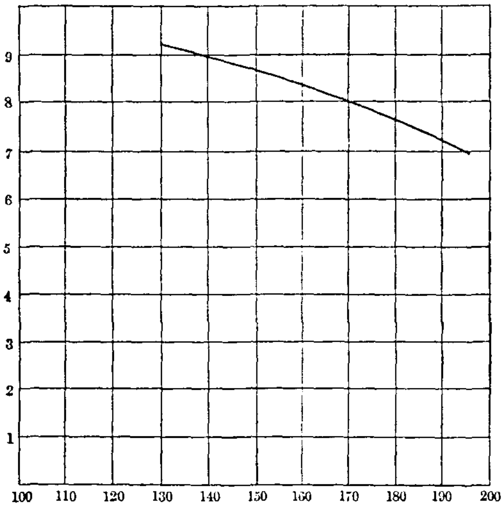

Figure 15.

The curves resemble very closely the curves given by Ramsay and Shields,† showing that the surface tension of a liquid in contact with its own vapor obeys approximately the same laws as for a liquid in contact with the vapor of another liquid.

In closing I wish to extend my heartfelt thanks to the American Academy of Arts and Sciences for their generous appropriation for the apparatus used in the work, to the authorities of Clark University for the opportunity of carrying on the work, and to Professor A. G. Webster for his unfailing kindness and interest.

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* Weinstein, Thermodymik und Kinetik der Körper, Vol. II, p. 49.

† Ramsay and Shields, loc. cit.

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TITLES OF ARTICLES ON THE HALL EFFECT, WITH THE TITLES OF SOME ARTICLES ON THE RELATED PHENOMENA SINCE DISCOVERED BY OTHER INVESTIGATORS.

By T. C. McKay.

TITLES OF ARTICLES ON THE HALL EFFECT, WITH THE TITLES OF SOME ARTICLES ON THE RELATED PHENOMENA SINCE DISCOVERED BY OTHER INVESTIGATORS.

By T. C. McKay.

Presented by E. H. Hall,* October 11, 1905. Received November 15, 1905.

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[*] As Dr. McKay is no longer in Cambridge, the proof-reading of this paper has fallen to me, and I must take a good deal of the responsibility for any errors which may be found in it. I have verified many but not all of the references. — E. H. H.

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GALVANIC POLARIZATION ON A MERCURY CATHODE.

By Gilbert Newton Lewis and Richard Fay Jackson.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

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Our knowledge concerning the real mechanism of chemical reactions is very meagre, even in the case of those occurring in homogeneous systems. Still less do we know of what the processes really are that occur in or near the surface of discontinuity in the case of a heterogeneous system. To the latter class belong all electrolytic reactions, and here the phenomenon of galvanic polarization offers an especially promising field for investigation. Unfortunately, most studies in polarization have been made under conditions too complicated to permit a ready interpretation of their results. Even in cases where the polarization of a single electrode has been studied alone, the essential phenomenon has been frequently obscured by the presence of substances produced at the other electrode and the existence of secondary reactions due to other impurities, notably atmospheric oxygen. In cases also where slow diffusion processes occur, — such, for example, as a diffusion of gases into platinum,* — the previous history of the cell is an important and usually an unknown factor.

It has been our purpose in the present work to choose a single definite electrolytic reaction of a simple character, and, eliminating with the greatest care secondary reactions of all kinds, to study the polarization accompanying this reaction under varying conditions. The reaction chosen was the deposition of hydrogen from normal sulphuric acid upon a mercury cathode. In other words, we have studied the phenomenon known as the over-voltage (Ueberspannung) of hydrogen. Mercury was chosen for the cathode metal because of the high over-voltage upon it, because of its small capacity for absorbing hydrogen, because it is the only metal which is not liable to accidental variations in surface, and finally because we believed that with this metal, if with any, we would find evidence of the break in continuity (Knickpunkt) of the polarization curve, which has been described by numerous observers, but which has been attributed by others to erroneous observations or to incorrect interpretation of results.

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* See Cottrell, Zeit. phys. Chem., **42**, 385 (1903).

For measuring polarization two general methods have been employed. One consists in measuring directly the potential of the electrode in question against a standard electrode, either while the main current is passing or immediately after it is broken. The other method consists in measuring the total electromotive force between the two electrodes during the flow of a given current. The difference between this electromotive force and that which exists when an infinitely small current passes, may be called the total polarization, and is the sum of three terms: the polarization potentials at the two electrodes, and the fall of potential through the electrolyte. By diminishing the resistance in the electrolyte the latter term can be made negligibly small (except in case a resisting film forms at one of the electrodes). If, then, we use at one terminal an unpolarizable electrode, the total polarization is a measure of the polarization at the other electrode. This was the method adopted.

In order to exclude all side reactions, the liquid about the cathode must be carefully protected from any contamination by substances produced at the anode. This might be accomplished by separating the two electrodes so far as to prevent diffusion from one to the other during the time of the experiment. This would not only have necessitated a very complicated apparatus, but, by introducing a large resistance, would have prevented the simple calculation of the polarization at the cathode from the total polarization of the cell. It seemed better, therefore, to look for a suitable anode which could be used directly in the same liquid with the cathode without introducing any foreign substances into the electrolyte. The choice apparently lay between a lead peroxide electrode and an electrode of hydrogen upon platinized platinum. Since, as will be shown presently, the merest traces of impurity in the electrolyte may vitiate the results, it was feared that even the minute quantities of lead that would go into solution from the former electrode might be dangerous. We finally adopted, therefore, the hydrogen electrode in the usual form of a spiral of platinized platinum foil, half immersed in the electrolyte and bubbled over with hydrogen. It was not certain in advance that such an electrode could be regarded for our purpose as unpolarizable, but preliminary experiments indicated this to be the case for such small currents as were to be used. This conclusion was completely verified by our later experiments, as will be shown.

The electrolytic cell and the measuring apparatus are sketched in Figure 1. The cell consists of a glass bottle, which is immersed in a thermostat (the latter not shown in the figure). B represents the tube by which the hydrogen enters the cell; C is the platinum electrode, made of a strip of platinum foil of about thirty square centimeters, platinized with pure platinum chloride, carefully cleaned and treated for some time with boiling water, to remove any loose particles that might otherwise be disengaged by the hydrogen bubbles and carried to the cathode; E is the exit tube for the hydrogen; D is a tube two millimeters in internal diameter, containing mercury, constituting the cathode. In order to determine whether diffusion processes in the electrolyte play an important part in the phenomenon cited, it was our original purpose to vary the height of the mercury in D, and find the effect upon the magnitude of the current, at potentials insufficient to produce a bubble of hydrogen in the tube.* As a matter of fact, however, we found that after complete saturation of the electrolyte with hydrogen no potential

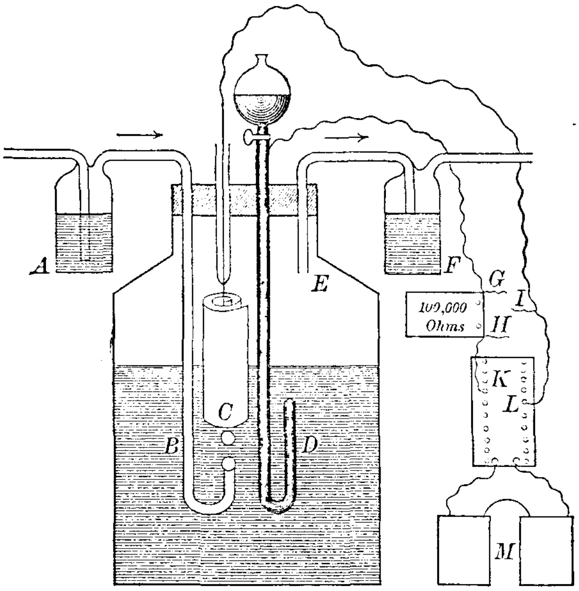

Figure 1.

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* Cf. Caspari, Zeit. phys. Chem., **30**, 89 (1899).

was too small to produce a bubble, which indeed appeared to form quantitatively according to Faraday's law. A current as low as 10^{-8} amperes, resulting from a potential of approximately .2 volt, gave a bubble visible to the naked eye within a minute, although the mass of this bubble must have been less than 10^{-11} grams. The apparatus, therefore, as a current detector, is almost as sensitive as a good galvanometer. On account of the formation of these bubbles it was found impracticable to experiment with the mercury below the end of the tube D, and in all our experiments the top of the mercury meniscus was kept at a level with the end of the tube.

The various tubes leading into the cell passed tightly through a rubber stopper. The use of rubber, although possibly objectionable, was decided upon in order to facilitate such alterations in the apparatus as the course of the experiments might make desirable. All possible precautions were taken to prevent the rubber from contaminating in any way the contents of the cell.

For preparing pure hydrogen a large generator with electrodes of zinc amalgam and copper, of the type first used by Cooke and Richards,* was employed. It was so arranged that the liquid could be withdrawn and new zinc and acid added without the entrance of a trace of air. The new acid, before it was introduced, was always treated for several days beforehand with a stream of hydrogen.

Before entering the cell the hydrogen passed through a wash-bottle A, containing sulphuric acid of the same strength as that in the cell, and on emerging it passed through a wash-bottle of mercury F, so that the pressure in the apparatus was always several centimeters of mercury above that of the atmosphere, thus preventing the leaking in of air.

The figure shows also the essential electrical connections. M represents a storage battery whose current passes through a resistance box of the form invented by Ostwald for measuring potentials. By moving the pegs L and K of this box a current of any desired strength could be tapped off to the cell, in circuit with which a resistance of 100,000 ohms was placed.

The two quantities to be measured were the potential between the electrodes and the current. The latter was obtained by measuring the potential fall in the 100,000 ohms' resistance. In other words the measurements consisted in finding the potentials between G and I, and between H and G. These were obtained by means of a potentiometer

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* Amer. Chem. Jour., **10**, 81 (1888).

(not shown in the figure) based on the Poggendorff method. In order to avoid changes in the polarization the potentials were first compensated as nearly as possible with a Lippmann electrometer, and then more exactly with a delicate galvanometer.

Since the internal resistance of the cell was low and the currents small, the fall of potential in the cell never amounted to more than a small fraction of a millivolt and could therefore be disregarded. Having then a cell with an unpolarizable hydrogen anode and zero fall of potential in the liquid, the total potential between the electrodes is a direct measure of the polarization of the cathode, for the whole process of the cell consists in hydrogen dissolving at the anode and forming again at the cathode, and the whole counter electromotive force of the cell is therefore merely a measure of the work lost in that irreversible process at the cathode which is responsible for the phenomenon of over-voltage.

The importance of using pure materials in this work is illustrated by the experiments of Walker and Paterson*. According to these authors, in the amalgam soda process where a solution of sodium chloride is electrolyzed with a mercury cathode, the presence of minute quantities of metallic impurities sometimes causes an enormous difference in the proportion between the amounts of sodium and hydrogen deposited. Thus, for example, the yield in sodium was lowered from 100 per cent to less than 70 per cent by one part in 10,000 of iron, cobalt, or nickel. In other words, the impurity in some way † assists enormously the formation of hydrogen from hydrogen-ion at the electrode, the very reaction with whose velocity we are concerned.

The sulphuric acid used in our experiments was prepared by the distillation of the best acid obtainable for analytic purposes. For preparing the mercury a new method was employed, which seems more trustworthy than any of those in common use. This method consists in heating mercuric oxide of high purity obtained in the wet way, and condensing the mercury set free. The oxides of such metals as zinc and cadmium are not decomposed by heat, while metals like platinum and silver will be set free but not volatilized. Mercury is the only readily volatile metal whose oxide is decomposed at red heat. For preparing small quantities of very pure mercury this method may, therefore, be recommended. In order to accelerate the decomposition and also to prevent the reunion of mercury and oxygen, the process is best carried on in vacuo.

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* Trans. Amer. Elec. Chem. Soc., **3**, 185 (1903).

† Mr. R. A. Hubbard has been kind enough to make a few experiments to verify the work of Walker and Paterson. He corroborates fully their main results and finds, as they did, that the formation of hydrogen at the cathode is associated with the formation of a scum on the mercury surface. In order to obtain some idea of its nature, the experiments were repeated in a weak acid solution. In this case the scum did not appear, nor was the yield of sodium appreciably affected by the impurities. It is probable, therefore, that the scum is due to the hydroxide of the metallic impurity and that it favors the formation of hydrogen in the same way that platinum favors its formation from zinc in the presence of acid, by lowering the over-voltage. Notwithstanding this evidence of the comparative harmlessness of metallic impurities in an acid solution, we have deemed it advisable to take all possible precautions to avoid impurities of every kind.

After the apparatus was set up and hydrogen had passed through the cell for several days to drive out the enclosed air, measurements were begun, but at first no constant results could be obtained. The current was subject not only to gradual changes but to sudden and capricious fluctuations, which rendered the experiments valueless. This unsatisfactory behavior, which may have been due to traces of oxygen clinging obstinately to the cell, or perhaps to incomplete saturation of the platinum electrode with hydrogen, continued for a surprisingly long time, and only after several weeks could definite and constant readings be obtained. Indeed, most of the results noted below were obtained after nearly three months, during which time hydrogen had been passing continuously through the cell at the rate of one cubic centimeter or more per hour. During this time also a current of about five-millionths of an ampere was sent through the cell continuously in order to precipitate any mercury which might have found its way into the solution and especially to help establish constant conditions at the cathode, partly by reducing the last traces of oxygen and partly by the mechanical stirring caused by the bubbles of hydrogen.

At first it seemed that these bubbles, which, as we have stated, were invariably formed by the current, might cause considerable inconvenience and uncertainty in the measurements. The bubble always started at the line of contact between the glass and mercury, but on becoming large enough to be easily visible to the eye it would mount suddenly to the top of the meniscus and there continue to grow, clinging to the mercury with great tenacity. The only easy way of removing the bubble was to tip the cell so as to bring the meniscus for the moment above the surface of the acid. Fortunately this treatment proved to have no effect whatever on the polarization, so that with a given potential the bubble could, as a rule, be allowed to form and be removed repeatedly without changing the current. Occasionally, however, this was not the case, and therefore, in most series of experiments the current at every potential was read both before and after removing the bubble.

As a test of the serviceableness of our apparatus, it was important to determine how quickly a constant current would be established after increasing the external electromotive force. The results not only were eminently satisfactory for our purpose, but gave evidence of a remarkable and unique phenomenon. In cases where polarization depends upon processes of diffusion either in the electrolyte or in the interior of the electrode, experiments have shown that the current falls gradually after any increase in the electromotive force and that it takes hours, days, or even weeks for the establishment of a constant condition. In such cases comparable results can be obtained only by reading the current always at a definite, arbitrarily chosen time after the change in electromotive force. In our present case we found that when the electromotive force was suddenly increased the current became constant in less than a minute, and that before reaching constancy the current was *rising* instead of falling. Thus, in a typical case the current at the end of one-fifth of a second was a few per cent below the final value, after a quarter of a minute only about half of one per cent below.

This unusual phenomenon could hardly be caused by the heating effect of the extremely small currents, and must be due either to the breaking down of some surface resistance or to the formation by the current of some substance which accelerates the electrolytic reaction, that is, to a kind of autocatalysis. We will presently describe another phenomenon closely connected with this one.

The constancy of the current with a given potential and the immediate attainment of this constancy, together with the fact that neither tipping the cell so as to bring a new portion of the hydrogen electrode under the electrolyte, nor suddenly bubbling hydrogen over the electrode, produced any change in the current, we regarded as convincing proofs that the hydrogen electrode was perfectly unpolarizable for the currents used.

The method of finding the relation between potential and current strength was to begin at a low potential and at the end of regular intervals of time to raise the potential step by step to the highest value desired, and then by similar stages to bring it back to a low potential. To illustrate, let us consider a particular case, in which the external electromotive force was changed every ten minutes, the current being observed at the end of this period both before and after removing the hydrogen bubble. The following table gives the potential in volts and the corresponding currents in millionths of a milliampere. When the potential of .652 volts was reached, the current began to fluctuate, and therefore, without taking any readings, the potential was immediately lowered and the series continued. The whole experiment was at 21.9° C.

E	I	E	I	E	I
0.314	69	0.582	3720	0.544	2250
0.436	444	0.597	4815	0.524	1690
0.477	815	0.652		0.494	1220
0.498	1130	0.608	6290	0.474	910
0.522	1580	0.593	4920	0.424	490
0.546	2140	0.578	3700		
0.564	2880	0.561	2950		

It has been shown by Jahn and Shoenrock,* by Haber † and others, both theoretically and experimentally, that in many cases the potential is a linear function of the logarithm of the polarizing current. In Figure 2, therefore, we have plotted from the values given in the table the potential E in one direction, in the other the logarithms of the current I. The various values obtained from the table are represented by the points of the arrows whose direction upward or downward indicates whether the values were taken on the way up to the high potential or on the way back to the low. We see that the points obtained in the first part of the experiment fall very exactly upon a straight line whose equation is

$$E = A \log I + B, \tag{1}$$

where A and B are constants. The points on the way back from the high potential lie in every case above this line.

The latter behavior is entirely unexpected. Many cases have been studied in which the current is diminished by previous high polarization of the electrode, and this is obviously due to the slow disappearance of the polarization. But we see that in our case the current at a low potential is increased by the previous passage of a larger current at a higher potential. If this phenomenon had been observed in a single instance only, we should have attributed it to chance errors, but it occurred regularly in all our experiments. It can only be explained by the assumption that the current itself produces, either some substance that catalyzes the electrolytic reaction, or some condition of the cathode surface which aids the reaction, and that this effect is increased by increasing the current and persists for some time when the current is diminished. This is the very conclusion to which we have already been led by the other phenomenon mentioned above. Whatever the true cause of these two phenomena may be, it is in all probability the same for both. According to the observations of Russ,* who investigated the reducing power of the electric current when a number of different solid metals were used as cathodes, the cathode occasionally became "active," that is to say, permitted with the same potential a much greater current to flow. This "activity" appeared to be produced by the current, and more rapidly the larger the current. While this is similar to the phenomenon we have noted, we might expect a solid metal to suffer occasional changes of surface, but it is hard to see how any peculiar condition could establish itself on a mercury surface and persist after the removal of its cause

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* Zeit. phys. Chem., **16**, 45 (1895).

† Ibid., **32**, 193 (1900).

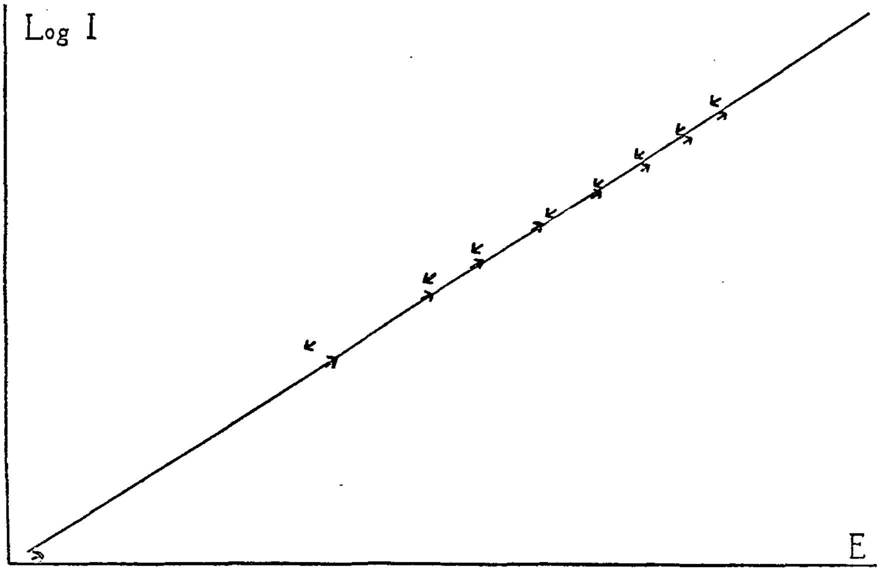

FIGURE 2.

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* Zeit. phys. Chem., **44**, 641 (1903).

and after standing and shaking, unless indeed it be due to the formation there of some new substance.

When we compare series of experiments conducted on different days and at different temperatures, ranging from 20° to 40°, we find that all give curves similar in every respect to the one shown in Figure 2. In every case the points obtained on the way up to the maximum potential lie on a straight line, while those obtained on the way down form an irregular curve lying above the line. Let us confine our attention to the former points and consider the theoretic grounds upon which the logarithmic formula is based. They are briefly as follows:

We assume that all true polarization (excepting the apparent polarization caused by an electric resistance at the electrode) is due to a counter-electromotive force caused either by the exhaustion of the substances used in the electrolytic reaction faster than they can be replaced, or by the accumulation of the products of this reaction faster than they can be removed. The degree of polarization is therefore a measure of the slowness of some irreversible process or processes, these being either processes of diffusion to or from the field of action (situated at or near the electrode surface), or chemical processes which supply the factors or destroy the products of the electrolytic action. From this point of view, the potential of the electrode, when the current is passing, differs from that of the unpolarized electrode by the potential of a concentration cell. From the Nernst equation, therefore, we are able to write for the potential of an electrode through which a current is passing the equation,

$$E = \frac{R\,T}{F} \ln \frac{c_1^{\frac{1}{n_1}} c_2^{\frac{1}{n_2}} \cdots}{c_1'^{\frac{1}{n'_1}} c_2'^{\frac{1}{n_2'}} \cdots} + \text{constant}, \qquad (2)$$

Where c_1', c_2', etc., represent the concentrations of the factors, c_1, etc., those of the products of the electrolytic reaction, n_1', etc., and n_1, etc., are whole numbers, each of which represents the number of gram-equivalents in a gram-molecule of the substance in question.

Let us assume as a special case that the polarization is due solely to the accumulation of the products of the reaction. Let us further assume as a more special case that the polarization is due to the accumulation of only one of the products, and that the process by which this product is removed is a chemical one, obeying the law of Guldberg and Waage. Accordingly, equation (1) becomes

$$E = \frac{R\,T}{F} \ln c_1^{\frac{1}{n_1}}, \qquad (3)$$

while the velocity V of the reaction is given by the equation, $V = k\, c_1{}^{m_1}$ where m_1 is the "order" of the reaction. The current can flow only as the reaction proceeds, and is therefore proportional to V. We may write therefore, $I = k'\, c_1{}^{m_1}$ whence $\ln\, c_1 = \ln I^{\frac{1}{m_1}} + \text{constant}$. Substituting in equation (3) we have,

$$E = \frac{R\,T}{m_1\, n_1\, F} \ln I + \text{constant},$$

or

$$E = \frac{.0002\ T}{m_1\, n_1} \log_{10} I + B. \tag{4}$$

Attempting to obtain a similar equation for the more general case, that two or more of the products are accumulated, we find that this is only possible when certain simple relations exist between the several values of *m* and *n*.

One might consider from the development of equation (4) given by Haber (l. c.) that for all cases of polarization our accepted theories, without the aid of further hypotheses, would lead directly to an equation of the same logarithmic form. We see, however, that this is not the case. If the polarization be due to the exhaustion of one or more of the substances used up in the electrolytic reaction, the equation will take an entirely different form. We shall return to this point presently. But even assuming, as we did above, that the polarization is due to the accumulation of a single one of the products, we must introduce an important assumption in order to obtain equation (4), namely, that the substance is removed according to the simple law of Guldberg and Waage. This assumption includes two others, namely, that the process in question is a homogeneous chemical reaction, that is, that it includes no processes such as those of diffusion; and secondly, that the reverse reaction takes place to a negligible extent. The latter assumption is undoubtedly justified in most cases of the sort that we are considering, but whether the former assumption is justified can only be decided by experiment.

Equation (4) is more special and more definite than equation (1), for the former attempts to tell us more about the quantity A than that it is a constant at a given temperature. Now, as a matter of fact, several investigators have shown that in a number of cases equation (1) holds with considerable accuracy, but that on the other hand, the values of A not only cannot be calculated from equation (4), but are altogether inconsistent with it, A proving in most cases to be greater than .0002 T, while the equation demands that it be either equal to this number or some simple fraction thereof, since n and m are always positive whole numbers by definition.

We can obtain the values of A for the experiment whose results are given in Figure 2 by finding the slope of the straight line which best satisfies the individual points. This method permits a possible error of perhaps one per cent. The value so obtained is 0.159, if E is expressed in volts. Similarly, from another experiment at the same temperature of 22° we obtain a value for A of 0.157. The maximum value consistent with equation (4) is 0.058, when m and n are both equal to unity.

Between 35.5° and 36.5° four experiments were made. The values of A obtained from these were 0.166, 0.157, 0.162, 0.166, with an average 0.163. The difference of over 3 per cent between this value and the average value, 0.158, found at 22°, is probably considerably greater than the error in determining these values, and shows that A increases with the temperature; but whether proportionally to the temperature, as equation (4) demands, cannot be determined accurately from our experiments.

Since we find that our experiments are inconsistent with equation (4), we must conclude that at least one of the assumptions upon which that equation was based is false. The assumption which seems most likely to be erroneous is that the process determining polarization is a chemical reaction in a homogeneous phase. Even without the evidence of our experiments, is not this assumption improbable?* Is it not more probable that the processes, whatever they are, take place in that surface film which marks the transition from metal to electrolyte, and that processes of diffusion to, from, or through this film play an important rôle?

Two important diffusion processes suggest themselves at once as possible causes of the polarization, — the diffusion of gaseous hydrogen from, and the diffusion of hydrogen-ion to, the electrode. But these can both be excluded by the aid of certain general criteria.

If any polarization is due to the accumulation of the *final* products of electrolysis, there will be, as a rule, especially if this product is a gas, some potential at which its concentration becomes so great that a new phase will suddenly form and the current will begin to rise more rapidly. There will be, in other words, a break in continuity in the polarization curve. As a matter of fact, we observed no discontinuity, in the region of our experiments, that is up to over 0.7 volt, although at this potential, if the polarization were due to the accumulation of gaseous hydrogen, the hydrogen would be at a pressure of over 10^{24} atmospheres, — an inconceivable state of affairs. If further evidence were needed, the fact that no change in the current is observed, when the hydrogen bubble appears at the electrode, would show that the gaseous hydrogen plays no essential part in the phenomenon.

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* See Nernst, Zeit. phys. Chem., **47**, 52 (1904).

We can also show the unimportance of the diffusion of hydrogen-ion. Assume that the concentration of hydrogen-ion at a given distance from the electrode is kept constant by stirring or by convection, then, if the current depend on the diffusion of the hydrogen-ion, it would be proportional to the difference between that constant concentration and the concentration at the electrode. With the decrease of the latter this difference would approach constancy, and therefore our logarithmic formula would no longer hold. The current, instead of increasing more and more rapidly with increase of potential, would tend to assume a constant value. Brunner,* although recognizing this principle, holds to the view that the depletion of hydrogen-ion in the neighborhood of the electrode is responsible for the polarization, and he cites experiments to justify this view; but besides the evidence against this idea that we have presented, other entirely decisive arguments could be adduced; thus, for example, when hydrogen is deposited on certain metals, like platinum and iron, the polarization is not destroyed by thoroughly washing the electrode, and only disappears after the lapse of hours or days. It is obvious that the diffusion of hydrogen-ion can have nothing to do with this phenomenon.

One might ask with some reason whether it is possible that the process consists wholly or in part in a process of diffusion, since we have seen that a stationary condition is established almost instantly, when the external electromotive force is applied, while diffusion processes ordinarily reach a stationary condition only after considerable time. The answer must be that, if any part of the polarization is due to a process of diffusion, this diffusion must occur through a film of extremely small thickness. We may add that, since the diffusion must be slow in order to account for the polarization, either the film must present a very high resistance to the diffusion, or the diffusing substance must be at an extremely low concentration.

We have, then, reason to believe that the process responsible for polarization is not a homogeneous reaction, nor a process of diffusion through any considerable distance.

The theory of over-voltage, with which the authors began these experiments, was as follows: The reaction, $2\,\overset{+}{H} + 2\,\Theta = H_2$, does not occur in a single stage, but in the two following: $\overset{+}{H} + \Theta = H$, and $2\,H = H_2$. The reaction whose slowness causes the polarization is the second of these. In other words, the potential of a hydrogen electrode depends on the concentration of monatomic hydrogen, and this concentration increases rapidly during cathodic polarization on account of the slowness of the reaction by which it is removed. For example, at a polarization potential of 0.7 volt the concentration of this substance at the electrode would be 10^{12} times as great as it would be when in equilibrium with ordinary hydrogen. Nevertheless, both of these concentrations might be absolutely very small, and probably are. The difference in polarization with different cathodes would be explained by the different catalytic action of the material at the electrode, the polarization being less the greater the catalysis. This theory, although it has been suggested by Tafel,[*] has otherwise had no place in the numerous discussions which the phenomenon of over-voltage has occasioned. It possesses, nevertheless, a good deal of plausibility. We may in fact cause the electrolytic reaction to proceed in the very two stages which we have written above. When hydrogen is deposited on a palladium cathode, the hydrogen, forming without marked polarization, is absorbed by the metal, where it has been shown by two independent methods[†] to exist in the monatomic condition. If the hydrogen is then withdrawn from the palladium, it appears in the form of ordinary hydrogen, H_2.

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* Zeit. phys. Chem., **47**, 56 (1904).

But we have more striking arguments in favor of this theory. Those metals, notably platinum and palladium, in whose presence the electrolytic deposition of hydrogen and the reverse reaction, the electrolytic solution of hydrogen, progress most readily, are the very ones which we have every reason to believe catalyze the reaction $2H \rightleftarrows H_2$ (of course, if in one direction, in both). This reaction is doubtless a very slow one under most conditions. Hydrogen at ordinary temperatures is a pretty inert substance, but in the presence of palladium or platinum black it readily reduces a large number of substances. So also these metals aid the union of hydrogen with other elements, such as oxygen and the halogens. Furthermore, in every known case where hydrogen is produced by a reaction, the reaction is catalyzed by these metals. We may mention the action of metals on acids, the reduction of water by chromous salts, the decomposition of a solution of sodium in liquid ammonia.* It is possible that each of these reactions is catalyzed in a specific way; but it is much easier to believe that in every case the metal simply catalyzes the slow reaction, $2H \leftrightarrows H_2$, and that this is also the explanation of the electrolytic reaction that we are considering. It is to be noted that metals like mercury and lead, on which the over-voltage is highest, — that is, which are the poorest catalyzers of the electrolytic reaction, — have little influence on any of the reactions mentioned above.

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[*] Zeit. phys. Chem., **34**, 200 (1900).

[†] Hoitsema, Zeit. phys. Chem., **17**, 1 (1895). Winkelmann, Ann. der Phys., (4), **6**, 104 (1901).

Although we have laid stress upon the *a priori* plausibility of this explanation, we are nevertheless forced to admit that, while our experiments are in no way opposed to the theory, they fail at the same time to give any evidence in its favor, except in as far as they enable us to exclude certain other explanations and thus limit the field of possibilities. If the main reaction at the basis of polarization is the formation of ordinary hydrogen from monatomic, then our experiments only show that this reaction does not occur in a single homogeneous phase. Since in all probability the monatomic hydrogen is present in extremely small concentration, this would account for the slowness of any diffusion process in which it enters, even if the diffusion were through a thin film.

Let us return to our experimental results. We have seen that they may be represented by a number of almost parallel straight lines, whose equations are of the type $E = A \log I + B$. Let us now consider the quantity B.

While the cell gave consistent and constant results during the time required for a single series of experiments, there was some variation from day to day, so that the results of two different days gave, when plotted, two parallel but not always coincident lines. That is to say, the value of B was not found to be constant, but for some unknown reason varied irregularly with the time.†

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* The interesting fact that this latter reaction is greatly accelerated by platinum was brought to our notice by Dr. C. A. Kraus.

† It is not impossible that this variation might have been due to changes in the barometric pressure, for, while small changes of pressure have no appreciable effect upon the potential of the reversible hydrogen electrode, they may have a considerable effect upon the velocity of the irreversible process, which is responsible for polarization. In fact, this is rendered highly probable by the experiments of Tammann and Nernst (Zeit. phys. Chem., **9**, 1, (1892)), who showed the great retarding influence of pressure on the reactions between metals and acids. Since in these reactions the only common feature is the formation of gaseous hydrogen from hydrogen-ion, it is probable that this is the reaction that the pressure retards. It is our intention in the near future to study especially the influence of pressure on the polarization of hydrogen.

Regarding the various straight lines as practically parallel within the range of our experiments, and giving to A its average value, we may express all our results approximately by the equation $E = .0160 \log_{10} I + B$, where B varies between the extreme values 0.014 and 0.069, E being given in volts and I in millionths of a milliampere.

On account of the temporal changes in B it was difficult to find how this quantity changed with the temperature, for the values obtained on different days at different temperatures differed hardly more from one another than those obtained on different days at the same temperature. Attempts to solve this problem by suddenly changing the temperature back and forth failed because we found that after a change of temperature several hours at least were necessary for the establishment of constant conditions. The effect of temperature on the polarization cannot be large, and on the whole our experiments indicate, if anything, that at a given potential the current is smaller the higher the temperature.

This result appears at first sight opposed to the general law that the velocity of physico-chemical reactions increases rapidly with the temperature, but it must be borne in mind that this law holds only for given concentrations of the reacting substances, while, when the potential is kept constant and the temperature raised, the concentrations of the reacting substances may diminish very considerably.

We have not yet mentioned an interesting phenomenon which appeared when the potential was carried much higher than 0.7 volt. At these high potentials the current would frequently begin to fluctuate, and at the same type the hydrogen, instead of making a single bubble at the top of the mercury meniscus, would form a number of small bubbles which appeared not only at the surface of the mercury but also between the mercury and the tube, sometimes as much as a centimeter below the surface. This is not such a break in the polarization curve as we have mentioned above, which would indicate the sudden formation of some new product of the electrolysis, for it is not associated with a sudden rise in the current, but on the contrary usually brought the current down to an abnormally low value.

Probably this phenomenon is due to changes in the surface tension of the mercury produced by the high polarization, but it might be explained by the formation of an unstable amalgam of hydrogen, and this hypothesis should not be discarded until further experiments have been made.

It will be seen that the experiments which we have described in this paper have presented a number of new problems rather than solved the old, and they must therefore be regarded as preliminary to a more thorough investigation. Since, however, circumstances prevent a continuation of our work at this time, we publish the results so far obtained.

SUMMARY.

The polarization observed when hydrogen is deposited electrolytically upon a mercury cathode is studied, the apparatus being designed to exclude as fully as possible all impurities and all side-reactions.

The polarization curve up to 0.7 volt is regular and shows no break in continuity. The relation between potential E and current I is given very exactly by the equation $E = A \log I \times B$ where A and B are constants.

The theory of this logarithmic equation is discussed.

It is shown that the polarization cannot be due to the slowness of diffusion either of hydrogen-ion to the electrode or of the hydrogen away from the electrode. but must be due to some intermediate reaction.

A theory is proposed concerning the nature of this reaction.

The effect of temperature on the polarization is considered.

Certain unexpected phenomena accompanying the polarization are discussed.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE ELECTRICAL CONDUCTIVITY OF SOLUTIONS IN LIQUID IODINE.

By Gilbert Newton Lewis and Plumer Wheeler.

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THE ELECTRICAL CONDUCTIVITY OF SOLUTIONS IN LIQUID IODINE.

BY GILBERT NEWTON LEWIS AND PLUMER WHEELER.

Presented by T. W. Richards. Received September 2, 1905.

As the result of certain speculations concerning metallic conductivity and the possibility of a gradual transition [*] from metallic to electrolytic conductivity, one of the authors was led several years ago to investigate the conductivity of liquid iodine [†] and of solutions in that solvent. The results of the preliminary experiments, although promising no definite answer to the questions which originally suggested the experiments, were considered of sufficient interest to warrant the present more extended research.

In the last decade our knowledge of conductivity in non-aqueous solutions has grown rapidly, and many new solvents, chiefly oxygen and nitrogen compounds, have been found to give conducting solutions. All of the inorganic solvents which have been studied, if we except sulphur dioxide and certain less important compounds of sulphur and of phosphorus, are salts or substances of similar dualistic type.

At the time when our work was begun no element had been found to act as a dissociating agent, but recently a paper by Plotnikow [‡] on conductivity in bromine solutions has appeared. The extraordinarily interesting and puzzling phenomena that he has studied appear to have little in common with those we are about to discuss, notwithstanding the close relationship between bromine and iodine.

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[*] Cf. Quincke, Zeit. anorg. Chem., **24**, 220 (1900).

[†] It may seem to the chemist a little odd to regard iodine as in any way allied to the metals, but in the periodic classification it stands among the elements which are on the border line between the metals and the non-metals, and which frequently partake of the character of both; thus if we consider the elements of the three adjoining groups, P, As, Sb; S, Se, Te; Cl, Br, I, we observe that in the first group arsenic and antimony both have marked metallic characteristics, in the second group tellurium is usually regarded as a metal, while one of the modifications of selenium is metallic, and so we might expect that in the last group iodine would show some metallic properties. It does in fact possess metallic lustre, that is, it behaves like a metal towards electric waves of high frequency.

[‡] Zeit. phys. Chem., **48**, 220 (1904).

Iodine bears no resemblance to any of the other dissociating solvents, except bromine, which have been studied. One property, however, it possesses in common with them all, the tendency to form complexes.* Thus Walden and Centnerszwer have shown liquid sulphur dioxide to be an excellent dissociating agent,† and have also pointed out the large number of complexes into which this substance enters.‡ Ammonia, which forms such a bewildering number of complex compounds, has been shown by Franklin and Kraus § to be one of the very best dissociating solvents. The case of water is too familiar to dwell upon.

The complexes in which iodine occurs have, with the exception of the polyiodides, been little studied, but doubtless a very large number could be isolated. Almost all salts increase the solubility of iodine in water, indicating the formation of complexes. The various solvents too, in which iodine dissolves with a brown color, are known to enter into combination with it.‖

The dielectric constant of iodine has not been determined, as far as we know.

In undertaking the investigation of iodine solutions we had the option of making a cursory study of a large number of substances or a more thorough study of some one substance. Choosing the latter course we have determined with a fair degree of accuracy the conductivity of solutions of potassium iodide in liquid iodine at temperatures from 120 degrees to 160 degrees, and in concentrations ranging from a few thousandths normal up to 10 times normal, — a range in which the specific conductivity increases 10,000 times.

Experimental Method.

The conductivity cells were of the form shown in Figure 1. They were made of two tubes 1½ cm. in diameter and 15 cm. long, joined below by a smaller tube 8 or 10 cm. long and 3 mm. in diameter. The ends of the cell were closed by tight-fitting glass stoppers. The electrodes (4 sq. cm.) of polished platinum foil were, in the early experiments, attached to platinum wires leading through the sides of the tubes. Later, in order to prevent any possible conduction through the thermostat liquid, these wires were carried down inside the tubes from the top and fastened rigidly below by small rods of glass. This arrangement was fairly satisfactory for dilute solutions, which attack platinum only slightly and superficially, but with solutions stronger in potassium iodide the platinum suffered seriously, the electrode being sometimes nearly eaten away in the course of a few days. For this reason it was desirable to use electrodes which could be removed except during measurements. Electrodes attached to the stoppers, as shown in the figure, proved to be satisfactory in every respect and were used in all the later experiments. Between measurements these stoppers with their electrodes were replaced by plain glass stoppers of similar form. On account of the density of iodine vapor it was found that this could be done without appreciable loss of iodine. The small changes of position in the electrodes that were possible were shown to have no noticeable effect on the resistance of the cell. These movable electrodes had the further advantage that they could be cleaned before each measurement.

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* See Abegg, Zeit. elek. Chem., **5**, 353 (1899).

† Zeit. phys. Chem., **39**, 513 (1901).

‡ Zeit. phys. Chem., **42**, 432 (1903).

§ Amer. Chem. Jour., **23**, 277 (1900).

‖ Strömholm, Zeit. phys. Chem., **44**, 721 (1903).

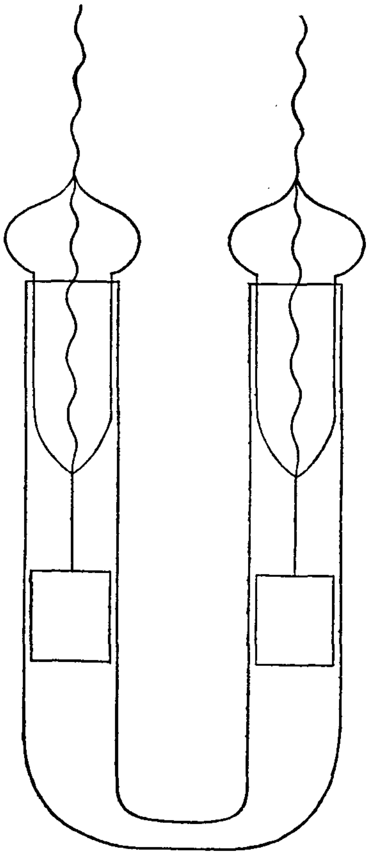

FIGURE 1.

Even with this arrangement, since it was sometimes necessary to leave the electrodes in the solution for an hour or two, it was feared that the platinum might be sufficiently attacked to influence the results. For this reason we used for all concentrated solutions electrodes of platinum-iridium foil and wire, containing 15 per cent iridium. These when first used were slightly etched on the surface, but afterwards they appeared to be unattacked even after long standing in concentrated solutions.

The cells were immersed nearly to the top in the thermostat, which was of the ordinary type and so arranged that it could be readily set at any one of three temperatures, 120°, 140°, and 160°. At first cotton-seed oil was used for the bath, but was later discarded, partly on account of the vapors which, rising from it and condensing on the sides of the cell, endangered the purity of the contents, and partly on account of the gradual hardening which it underwent. After some experimenting a suitable substance was found in a commercial substitute for lard known as cottolene.

The method of carrying out a series of experiments was as follows: Enough iodine (60–80 gm.) was weighed out to stand when melted 2 or 3 cm. above the electrodes. The glass stoppers were inserted at once to prevent loss of iodine, the cell placed in the thermostat, and its resistance determined. Then the stoppers were removed, after gently warming them with a flame to melt down the small quantity of iodine which sublimed into the space between the stopper and the tube. A known weight of potassium iodide, pulverized and pressed into a small pellet, was introduced, and the liquid was stirred by a simple apparatus which drew it back and forth from one arm of the cell to the other. The stoppers were replaced and the resistance again determined. In the same way more iodide was added and the process repeated. At each concentration the resistance could be determined for a number of temperatures.

The measurements of resistance were made by means of a Wheatstone's bridge, both with direct current and galvanometer, and with alternating current and telephone. The direct current was very convenient and accurate, and with dilute solutions gave excellent results. The two electrodes showed no potential difference either before or during the passage of the current. *

In order to calculate the specific resistance of the solution from the observed resistance, the constant of each cell was obtained by means of a standard aqueous solution of potassium chloride or potassium iodide.

The only chemicals used in the research were potassium iodide and iodine. The former was purified by several recrystallizations. Two kinds of iodine were used; one was the pure resublimed iodine prepared

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* This lack of polarization is no evidence that the conductivity is metallic in character. The same phenomenon is observed with an aqueous solution of potassium iodide and iodine.

by Kahlbaum, the other was made according to the method of Andrews,* by heating together potassium bichromate and potassium iodide. Both varieties were further purified by resublimation and gave identical results. For determining the conductivity of pure iodine a still purer variety was desired. Its preparation will be described later.

EXPERIMENTAL RESULTS.

In the first experiments measurements were made at the three temperatures, 120°, 140°, and 160°. Later, on account of certain irregularities at the lowest temperature, measurements were confined to the two higher at which uniformity in the composition of the solution could be established with less difficulty.

A large number of experiments made before the technical difficulties were entirely overcome need not be considered in detail, as they were followed by more accurate experiments covering the whole field. The example given in Table I (Preliminary Series) will suffice to show the general character of the results. L represents the specific conductivity. C expresses the strength of the solution in grams of potassium iodide to 100 grams of iodine. For the sake of convenience we will refer to this as the concentration. We have not yet made exact enough determinations of the specific gravities of the different solutions to enable us to give the true concentrations in gram-molecules per liter, but for dilute solutions this molecular concentration may be considered as proportional to C.

TABLE I.

PRELIMINARY SERIES.

C.	L.		
	120°	140°	160°
0.	.0000644	.0000613	.0000563
.0315	.000228	.000214	.000195
.0561	.000402	.000362	.000336
.108	.000678	.000648	.000592
.162	.00110	.00102	.000932
.222	.00174	.00165	.00135

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* Amer. Chem. Jour. **30**, 428 (1903).

We note at once the marked decrease of conductivity with increase of temperature. This negative temperature coefficient, although it suggests the behavior of metals, in no way proves that we are dealing here with a case of metallic conductivity. Many electrolytic conductors are known which show the same behavior; usually, it is true, in the case of solvents not far removed from their critical temperatures, but even in water a few substances, such as phosphoric acid, have negative coefficients. We shall see presently how the temperature coefficient of the iodine solutions changes with the concentration.

It is also evident from the table that the conductivity increases rapidly with the concentration and in fact more rapidly than the concentration itself. This fact can be better shown if we pass at once to the consideration of the following tables, containing Series 1 and 2. In these tables M represents the quotient $\frac{L}{C}$, and is therefore for these dilute solutions proportional to the molecular conductivity. Before taking this quotient we have subtracted from the observed values of L the specific conductivity of the iodine, thus correcting for the original conductivity of the solvent, as is customary in aqueous solutions.

TABLE II.

SERIES 1. 140°.

C.	L.	M.
0.	.0000216	
.0135	.0000970	.00555
.0246	.000141	.00484
.0471	.000240	.00464
.0688	.000340	.00464
.0901	.000455	.00480
.206	.00133	.00638
.325	.00271	.00828
.495	.00587	.0118
.671	.0107	.0159
.899	.0185	.0207
1.13	.0277	.0244

TABLE III.

Series 2.

C.	L.		M.	
	140°	160°	140°	160°
0.	.000068	.000066 (?)		
.00991	.000114	.000107	.0045	.0041
.0237	.000172	.000162	.00436	.00405
.0512	.000298	.000272	.00450	.00402
.0729	.000399	.000366	.00454	.00413
.0962	.000538	.000494	.00489	.00444
.123	.000706	.000656	.00518	.00479
.183	.00119	.00110	.00611	.00562
.236	.00178	.00163	.00725	.00662
.348	.00342	.00315	.00961	.00889
.461	.00541	.00515	.0116	.0110
.681	.0114	.0108	.0166	.0158
.900	.0194	.0185	.0213	.0204
1.12	.0275	.0269	.0243	.0239

The results contained in Series 2 are expressed graphically in Figure 2, the lower points representing the values at 160°, the upper those at 140°. It is evident that the values of M increase rapidly with the concentration. This phenomenon, although it has never been observed in aqueous solutions, is not rare in the case of other solvents. The figure shows further that between $C = 0.1$ and $C = 1.0$, M increases *linearly* with the concentration. The form of that portion of the curve lying below $C = 0.1$ deserves little attention, as the values here are much affected by the correction made for the original conductivity of the iodine, and are thus liable to large errors, even assuming that our method of making the correction is entirely correct in theory.

Above $C = 1.0$ the line shows the beginning of a curvature which becomes much more pronounced at slightly higher concentration.

The results obtained at 140° over a wide range of concentration are given in Series 3, 4, 5, 6, 7, and 8. All the values in these series, together with all but the first three points of Series 2 (140°) are plotted in Figure 3 and give a view of the values of M over a wide range of concentration. The points in the right hand portion of the curve, although few in number, were determined with more accuracy than any of the others. It is to be noted that for concentrations above 13, the specific conductivity is nearly constant.

TABLE IV.

C.	L.	M.
	SERIES 3.	
1.53	.0420	.0274
2.28	.0730	.0318
3.09	.113	.0367
3.64	.141	.0388
4.45	.177	.0398
13.2	.384	.0290
	SERIES 4.	
1.49	.0370	.0247
2.03	.0625	.0307
2.56	.0895	.0350
3.13	.119	.0380
3.82	.153	.0400
4.39	.179	.0409
5.62	.233	.0415
8.06	.312	.0390
10.52	.355	.0338
14.25	.394	.0276
	SERIES 5.	
1.47	.0350	.0237
2.12	.0690	.0326
2.77	.0980	.0356
3.56	.140	.0394
4.25	.174	.0410
6.89	.273	.0398
8.99	.321	.0359
	SERIES 6.	
1.34	.0368	.0274
20.1	.383	.0190
	SERIES 7.	
23.2	.390	.0168
	SERIES 8.	
28.1	.396	.0141

The temperature coefficient of the conductivity, which in the case of dilute solutions we have shown to be negative, becomes numerically smaller with increasing concentration, is zero when C is a little less than 3.0 and for stronger solutions becomes positive. The conductivity curve for any other neighboring temperature is therefore very similar to the one for 140°, but crosses that curve at a concentration of about 3. The change in sign of the temperature coefficient might perhaps have been predicted, for with increasing concentration the substance approaches more and more nearly the condition of a pure molten salt and all salts which have been hitherto studied have large positive temperature coefficients.

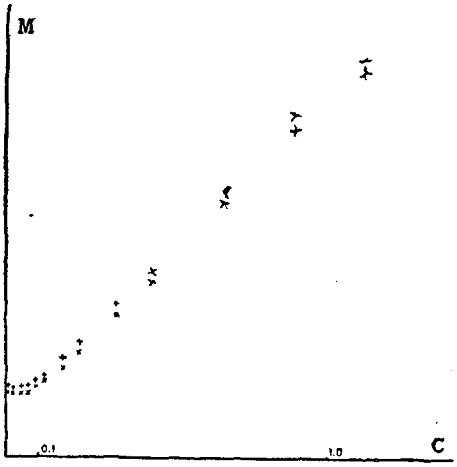

Figure 2.

Table V gives the percentage change in the conductivity, per degree, at several concentrations.

TABLE V.

C.	Temperature coefficient.
	per cent.
0.1	—0.46
0.4	—0.38
2.8	±0.
8.3	+0.30
20.0	+0.55
28.1	+0.67

Discussion of the Conductivity Curve.

Assuming that the whole of the conductivity is electrolytic in character, let us consider from this point of view, the curves of Figures 2 and 3, especially the former, which embraces the region of dilute solutions. Here we may regard M as proportional to the molecular conductivity and, assuming the mobility of the ions to be constant, to the degree of dissociation. The latter quantity, instead of rising with increasing dilution and approaching a constant maximum as in the case of water solution, has a maximum in the neighborhood of C = 5, and from that point decreases rapidly with increasing dilution.

This behavior has been previously noticed by a number of observers in the case of certain organic solvents and other solvents of *small dissociating power.* It is our belief that this is an extreme manifestation of a phenomenon which is common to all solutions and which has been frequently observed in aqueous solutions.

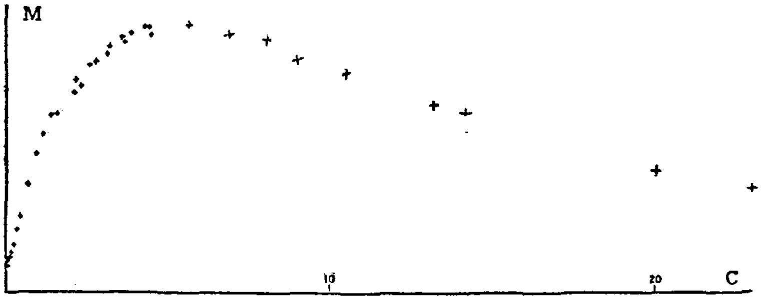

Figure 3.

The mass law applied to the dissociation of a binary electrolyte gives the equation $K C_1 = C_2^2$ where C_1 is the concentration of the undissociated substance and C_2 that of one of the ions. It is well known, however, that this equation is not satisfied in the case of all water solutions. Bancroft * has shown that the empirical equation $K C_1 = C_2^n$, where n is a specific constant depending on the nature of the electrolyte, expresses satisfactorily the behavior of any aqueous solution. This quantity n varies from 2 in the case of weak acids and bases down to 1.36 in the case of potassium chloride. If now we could find some still "stronger" electrolyte than potassium chloride, the value of n would doubtless be still smaller. If a substance could be found with a value of n less than 1 the degree of dissociation and consequently the molecular conductivity would increase with the concentration as they do in the case we have been studying.

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* Zeit. phys. Chem., **31**, 188 (1899).

From this point of view we see that the increase of molecular conductivity with the concentration, which we have found in the case of potassium iodide in iodine and which others have found in the case of salt solutions in many organic solvents, is a deviation from the mass law in greater degree, but of the same kind and in the same direction, as that which is observed in the case of aqueous solutions of strong electrolytes.

What is the cause of this general deviation from the mass law, and why should it be greater in the case of weak dissociating agents?

Before attempting to answer this question let us give definition to an idea which we shall find useful. If we examine a number of liquids chosen at random, we find that as a rule those liquids which, when pure, are the best conductors of electricity, are, as solvents, the best dissociating agents, and when dissolved in other solvents are themselves the most dissociated. In other words the property of dissociating other substances and the property of being dissociated when pure and when dissolved in other solvents go usually hand in hand. Thus pure nitric acid is a conductor, it dissociates other substances dissolved in it, and is dissociated in other solvents. Benzol has none of these three properties. Let us call a substance possessing these properties to a marked degree an *electrophile*. The best electrophiles are substances which consist of two parts, one having a strong positive, and the other a strong negative electro-affinity, as this term is used by Abegg and Bodländer.* In this category are included most inorganic salts. Ether and the hydrocarbons are types of the poorest electrophiles.

Water occupies an intermediate position. That it stands first or nearly first among known dissociating solvents is due doubtless to the fact that none of the best electrophiles are liquid at ordinary temperatures. The only liquids which are good conductors in a pure state are melted salts.† If these salts could be supercooled to ordinary temperatures their conductivity would still be far greater than that of water, and doubtless their dissociating power too.

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* Zeit. anorg. Chem., **20**, 453 (1899).

† Whether the good conductivity of melted salts is due to a high degree of dissociation or to great mobility of the ions has not hitherto been demonstrated. One of the authors together with Mr. B. S. Lacy has attempted to answer this important question by measuring directly the ionic velocities in melted salts. The experiments, although not yet concluded, have progressed far enough to show that the mobilities of the ions are probably even less than in water. The degree of dissociation must therefore be enormously greater than in the case of any other pure liquids.

Let us see how this idea bears upon the deviations from the mass law, $K\,C_1 = C_2{}^2$. Here K is a quantity which does not vary with the concentration in a given medium. In different media however, K has different values, as shown by experiment, and as we should also expect from theory, whether the mass law be derived from the principles of probability or from thermodynamics. If therefore with changing concentration there is an essential change in the character of the medium, K no longer remains a constant but must change as the concentration changes.

In order to give a complete qualitative explanation of the phenomena under consideration we have only to make the assumption that the addition to any solvent of a better or a worse electrophile than itself respectively increases or decreases the electrophilic character of the solvent.

Thus for example, water and acetic acid are both medium good electrophiles. If small quantities of acetic acid are added to water the electrophilic character of the medium is not materially changed and therefore for dilute solutions of acetic acid the mass law holds.

If on the other hand we add to water even a small quantity of a very strong electrophile like potassium chloride the dissociating power of the medium is increased, K increases with the concentration of potassium chloride and we have the familiar deviation from Ostwald's dilution law.

An extraordinarily good confirmation of the view that such deviations are due to a change in the dissociation power of the medium is given by Arrhenius,* who showed that the dissociation of weak acids in water is considerably increased by the addition of small quantities of inorganic salts. In other words a tenth or even hundredth normal salt solution is an appreciably better dissociating solvent than pure water.†

If finally we add to water a substance that is a much poorer electrophile than water itself we should expect a deviation from the dilution law in the opposite direction. That is, the dissociation constant should decrease with the concentration. Such a phenomenon has never been observed, nor, perhaps, is it likely to be until our experimental methods are improved, for to discover this phenomenon it would be necessary to investigate accurately the dissociation of extremely poor electrolytes.

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* Zeit. phys. Chem., **31**, 197 (1899).

† It is likewise probable that the dissociation of water itself is considerably increased in strong salt solution. An experimental proof of this would be interesting and important.

However, fully as convincing evidence is offered by the fact demonstrated by Arrhenius * that the dissociation of an electrolyte in water is always diminished by the addition of a non-electrolyte (in other words a very poor electrophile) such as sugar or glycerine.

When now we consider, instead of water, other solvents which are much poorer electrophiles, we should expect results of the same general character, except that the change in dissociating power should be much more pronounced when a strong electrophile is added. So we may explain in these cases deviations from the mass law so great that the degree of dissociation increases with the concentration, as in the case of potassium iodide in iodine.

THE CONDUCTIVITY OF PURE IODINE.

Does absolutely pure iodine conduct the current? If so, is this conductivity metallic or electrolytic? If the latter, what are the ions? These are not the least interesting questions raised by our work.

The preparation and study of pure iodine present some difficulties. It seems that iodine acts to some degree upon glass and platinum, and that it therefore absorbs some impurity when it is sublimed and collected upon glass and also when it stands in the cell. For this reason we resublimed some of our best iodine in a simple apparatus made entirely of quartz, and from this transferred it directly to a conductivity cell made of a quartz tube 6–8 mm. in diameter and about 15 cm. long, bent in the form of a U. This was placed in the thermostat, and electrodes of platinum-iridium were introduced just before measuring the conductivity.

The specific conductivity of the iodine obtained in this way was not less than that of the iodine previously used, in fact never less than 3×10^{-5}. Similar results were obtained with iodine twice sublimed in quartz.

We do not feel entirely safe, however, in drawing conclusions from these results until we explain a very perplexing *decrease* in conductivity which was observed when the iodine stood for any length of time in the thermostat. It is hard to account for this unusual behavior. It may be due to some gradual molecular change in the iodine, or to the escape of water or other impurity taken from the air during sublimation, or even to an absorption of impurities while standing, although no case is known where the conductivity of a pure *electrolytic* conductor is lowered by impurities. One experiment was made with iodine sublimed in quartz in a vacuum, but this iodine had to be exposed to the air while being introduced into the cell. It gave a conductivity which was of the same order of magnitude as previously observed.

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* Zeit. phys. Chem., 9, 489 (1892).

These experiments unfortunately were interrupted before completion. We will therefore postpone a discussion of them until they can be repeated with more care.

It is our intention to return to this subject as soon as circumstances permit, and also to study the solubility and conductivity of other salts in iodine, to determine the molecular weight in these solutions, and finally, if any suitable method can be found for attacking so difficult a problem, to determine from the changes at the electrodes during the flow of the current whether or not the conductivity is altogether electrolytic in character.

SUMMARY.

Potassium iodide is readily soluble in liquid iodine, forming solutions which when concentrated conduct the electric current as well as the best aqueous solutions.

The dependence of the conductivity upon the concentration is determined over a wide range.

In dilute solutions the molecular conductivity increases linearly with the concentration, rising to a maximum and then falling again.

The probable connection is shown between this anomalous behavior and the deviations from Ostwald's dilution law in water solutions. A hypothetical explanation of both these phenomena is offered.

The temperature coefficient of conductivity is negative for dilute solutions, but with increasing concentration passes through zero and becomes positive.

Pure iodine in all probability possesses a conductivity of its own, but this can be definitely determined only by further experiments.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE TRANSITION TEMPERATURE OF SODIC BROMIDE: A NEW FIXED POINT IN THERMOMETRY.

By Theodore W. Richards and Roger C. Wells.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

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THE TRANSITION TEMPERATURE OF SODIC BROMIDE: A NEW FIXED POINT IN THERMOMETRY.

By Theodore W. Richards and Roger C. Wells.

Presented December 13, 1905. Received November 29, 1905.

Introduction.

Recent work upon the transition temperatures of crystallized salts has shown that many of these transition temperatures are capable of determination with accuracy enough to serve as a means of standardizing thermometers.[1] A number of such points have been found approximately by Richards and Churchill, and one of these points, namely, the transition of sodic sulphate at 32.383°, has been fixed with great precision in reference to the International Scale by Richards and Wells.

It is highly desirable that as many such points as possible should be determined with great precision, because they furnish means of determining temperature intervals dependent only upon the procuring of pure materials, and independent of the possession of a standard thermometer. Among the many transition temperatures to be investigated, two especially promised great usefulness, — one, that of sodic bromide, 50.7°, lying about half-way between the freezing and boiling points of water, and therefore capable of testing the middle of the thermometric scale; and the other, that of sodic chromate, lying at about the ordinary temperature of a laboratory. Of these two points, the former is discussed in the present paper, and the latter will soon be discussed in considerable detail in another paper. Both were determined approximately by Richards and Churchill in the paper already cited, but these determinations were only of a crude and preliminary nature, and required careful verification.

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[1] Richards, Am. Jour. of Science, **6**, 201 (1898); Richards and Churchill, These Proceedings, **34**, 277 (1899); Richards and Wells, ibid., **38**, 431 (1902); also Zeit. f. phys. Chem., **26**, 690 (1898).

It is well known that the presence of impurities affects greatly the transition temperature of a crystallized salt, just as it affects the melting point of a pure substance. Obviously, great purity is demanded if an accurate temperature is to be attained; hence, from the practical point of view, those salts will be most desirable whose purity is most easily attained, other conditions being equal. Thus the turning point of a research upon this subject is the purification of the materials, just as it is in any other accurate chemical investigation. It is therefore necessary that such a research be carried on in a chemical, and not in a physical, laboratory.

As has been said, the ease of purification of the salt chosen is one of the important criteria determining the choice, but this is by no means the only criterion. It is important also that the salt should have as large a heat of transition as possible in order that the accidental addition of outside heat or impurity shall affect the transition temperature as little as possible. Moreover, it is desirable that the salt should not possess the property of forming many hydrates, because there is in this case danger of confusion, and of consequent inexact results. It is also important, although less essential, that the change of volume during the transition should be slight, in order that change of barometric pressure should cause but little difference in the transition temperature. As a rule, however, this last condition is not a very rigorous one, because, even in a case where the change of volume is as much as in melting ice, the changes ordinarily occurring in barometric pressure are barely perceptible on the most accurate thermometers.

From the point of view of the phase rule the transition temperature of a crystallized salt is a quadruple point, involving the coexistence of four phases. As a matter of convenience, however, it is more satisfactory to determine the temperature of another point, almost identical, whose fixity is determined, in the presence of the two components anhydrous salt and water, by the coexistence of three phases, — hydrated salt, dehydrated salt, and solution, under a definite pressure, — that of the atmosphere plus that of the small layer of liquid above the thermometer bulb. These four conditions (three phases and the pressure) are in this case the $n + 2$ conditions which determine invariance in the system composed of the $n = 2$ components. Theoretically, as long as a trace of each of these phases is present the temperature will be constant, but practically it is well to have a fair proportion of each in order to establish the equilibrium as promptly as possible; supercooling or superheating are thus most satisfactorily avoided.

In this case the reaction is a transition from the dihydrate ($NaBr_22H_2O$) to the anhydrous salt ($NaBr$) and its saturated solution; the result is to be represented approximately by the following statement: —

$$103NaBr2H_2O = 61NaBr + 42NaBr, 206H_2O.$$

This statement is based upon the known fact that a saturated solution of sodic bromide at the transition temperature contains about 116.8 parts of anhydrous salt to 100 of water; in other words, almost exactly 42 grams of anhydrous salt dissolve in the two gram-molecules (36 grams) of water set free from one gram-molecule of salt. The transition is represented upon the accompanying diagram of solubility curves.

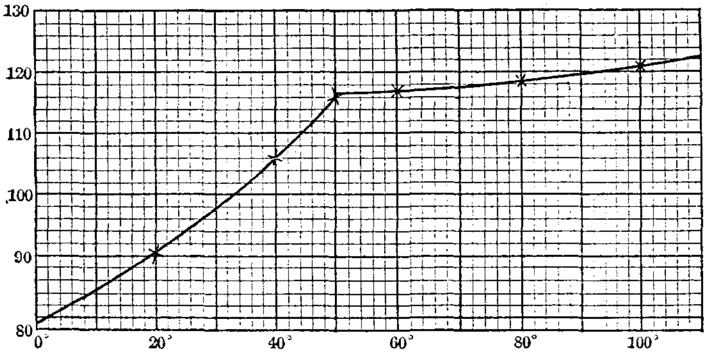

Figure 1. Solubility of Sodic Bromide in Water.

The heat of this reaction may easily be computed approximately, although we have not the data for exact computation. Obviously, except for the effect of the possible change of heat capacity of the reacting system during the reaction, the method of calculation is simply to subtract from one another the heats of solution of the initial and final systems in a large excess of water. This would give the hypothetical heat of reaction at 20°, to which a correction would have to be added in order to compute its value at 50°. Now, Thomsen has found that the heat of solution of a gram-molecule of crystallized sodium bromide in water is — 4.71 Calories, or — 19.7 kilojoules, and that the heat of solution of anhydrous sodium bromide is only — 0.19 Calorie, or — 0.8 kilojoules. Neglecting the heat of dilution of the part dissolved in transition, we may thus compute the heat of transition to be $-19.7 + \frac{61}{103} \times .8 = 19.2$ kilojoules. It may be of interest to compare this with the latent heats of melting of corresponding quantities of several single component systems; of water, the latent heat of melting of a gram-molecule is 5.8 kilojoules; and of bromine, 5.4 kilojoules; of iodine, 6.3; of phosphorus, 0.6 for each gram atom. It is apparent from this comparison that, even taking into account the fairly large molecular weight of sodic bromide (139), the heat of transition compares favorably even with the best single component systems.

It becomes now a matter of great interest to calculate as nearly as possible the effect of impurity in the sodic bromide upon the transition temperature. Sodic chloride may be taken as an example, being the most probable impurity. Van't Hoff has pointed out, and Löwenherz[2] has shown practically, that the transition temperature of a crystallized salt is lowered by the addition of a foreign substance just as a true melting point is lowered. Löwenherz found the molecular lowering to be a constant when the foreign substance added was not an electrolyte, and very nearly a constant in the case of other sodium salts. Undoubtedly the reason for this latter fact is the slight ionization of the added sodium compound in the presence of the large concentration of ionized sodium already dissolved in the saturated solution. Precisely the same conditions apply here; therefore it is safe to assume that sodic chloride, if present in the solution, would act essentially as an undissociated substance. If, now, the phenomenon of transition is viewed simply as the melting of the hydrate, a simple thermodynamic cycle leads to the approximate equation $\lambda = \frac{n}{N} \frac{RT^2}{\Delta t}$ for the heat of transition of the crystallized salt, in which n and N represent respectively the numbers of moles of impurity and solvent salt, and Δt the change in temperature of transition caused by the admixture, n being supposed to be very small in proportion to N. Substituting the value of λ resulting from the calculation given just above and the molecular weights, it is easy to find that the lowering of the transition point produced by one per cent by weight of this impurity will be about a degree. In other words, the lowering of the transition temperature caused by an impurity of sodic chloride in sodic bromide should be in thousandths of a degree approximately equal to the per cent of impurity expressed in thousandths of a per cent. That is, an impurity of, for example, 0.002 per cent would cause a depression of about 0.002°. This, of course, is only a rough calculation for several reasons. In the first place, the temperature of transition is 50.7°, not

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[2] Zeit. phys. Chem., **18**, 70 (1895).

20°, to which the reaction heat corresponds. In the next place, the transition is not a simple melting, although, indeed, it is true that the equation of van't Hoff applies as a close approximation. It is not, however, worth while to attempt a closer calculation of these quantities for the reason that further elaborate experimentation would be required in order to demonstrate the disposition of the sodic chloride in this mixture. It is not known whether or not the salts are isomorphously mixed in both solid phases, — a circumstance presenting a series of complications quite untraceable on the basis of data known at present. However, the above calculation probably shows the maximum effect to be produced by a substance with even as low a molecular weight as 58.5. Of course, substances with higher molecular weights would produce proportionately smaller effects for a given percentage composition, and therefore be even less objectionable for the present purpose. Practically we found that sodic bromide containing about .02 per cent of sodic chloride actually "melted" at a temperature only about 0.013 or 0.014 too low.

It is worth while now to compare the effect of impurity in sodic bromide with the effect of impurity in water. The lowering caused by a gram molecule of non-electrolyte in a litre of water is 1.86°, therefore an impurity of .01 per cent of the electrolyte common salt would cause a change of 0.006°. This is a somewhat smaller effect of impurity than in the case of the sodic bromide; the comparison emphasizes strongly the importance of purity in the materials in the latter case.

Preparation of Sodic Bromide.

The preparation of pure sodic bromide proved to be unexpectedly difficult, but the difficulties have been surmounted, and the result of the present work must be considered as essentially final. The problem is harder than that involving sodic sulphate, and we cannot recommend sodic bromide with quite the unqualified approval which the other salt received; nevertheless, although not so easy to repeat, the preparation is by no means impossible, and the results will certainly serve a useful practical purpose.

At first it was attempted to purify the two different specimens of commercial bromide by recrystallization as dihydrate with the hope that the chloride would be thus easily eliminated, because of the very low inversion temperature of the corresponding sodium salt. However, the result in each case was very disappointing. The bromide, although increasing in purity with each successive crystallization as shown by the steadily rising transition temperature, did not lose its impurity fast enough to make any considerable final yield a possibility. The commercial salt sometimes gives results several degrees in error, and even after eight recrystallizations one sample melted over half a degree below the true point. Such material is of course wholly out of the question. Perhaps some other isomorphous impurity beside sodic chloride was present. Whatever may have been the cause of the trouble, this crude material was entirely rejected in subsequent work, and all other samples were made from pure hydrobromic acid or bromine and pure alkali.

The next preparation of sodic bromide was made from purified sodic carbonate and hydrobromic acid. The sodic carbonate was five times recrystallized, and the bromine, from which the hydrobromic acid was made, was freed from chlorine by solution in calcic bromide and reprecipitation by water. The bromine was converted into hydrobromic acid with the help of carefully washed red phosphorus and water. Hydrobromic acid, prepared in this way, contains phosphorus and sometimes traces of arsenic; but three successive fractional distillations easily removed these impurities. The purity of the product and the residue was determined in each case by ammonium molybdate. The first residue, after the distillation of about eight kilograms of fifty per cent hydrobromic acid in the retort, yielded about thirty grams of ignited phosphomolydenum oxide. The residue from the second distillation yielded only a quarter of a gram, and the residue from the third distillation gave no precipitate whatever with the ammonic molybdate. The distillate was of course even more pure than this, because the impurities concentrate in the residue. Iodine was expelled in the first aqueous fractions by adding bromine water before each distillation. No attempt was made to eliminate the iodine by adding zinc oxide according to Stas, because this method seemed to us of doubtful efficacy. As will be shown later, the sodic bromide (sample II) prepared thus was very pure, but it had not yet attained the greatest purity possible, because the single treatment with calcic bromide is usually insufficient to remove every trace of chlorine from the commercial bromine. A number of other samples were made in this way, and according to the amount of chlorine in the original crude bromine, the substance obtained varied from a state of almost perfect purity to a state of contamination with 0.05 per cent of its weight of sodic chloride. Protracted and tedious quantitative precipitations of silver bromide, conducted with all the accuracy of atomic weight determinations, were necessary to prove this fact and explain slight irregularities in the transition temperatures of the several specimens of the various samples. These analyses need not be detailed here, because all of the specimens used in the final work to be recounted were as free as possible from chlorine, but they were amply worth the trouble which they occasioned, because they not only proved the slight impurity in the bromine, but also led to the subsequently confirmed suspicion concerning the atomic weights of sodium and chlorine.

The reason for this difficulty in preparing pure bromine is clear. The elimination of chlorine from bromine by solution in calcic bromide and precipitation with water is a question of the distribution of the chlorine between two phases, much the greater part of the chlorine going into the aqueous phase. When the amount of the chlorine in the original bromine is small, a good product may be obtained by a single treatment; but when it is large, several successive treatments must be used in order to eliminate every weighable trace of chlorine. One of the portions of sodic bromide used in the transition temperature work was made from bromine thus purified several successive times in addition to repeated distillation.[3] This sample, perhaps as pure as may be obtained by any method, is called III in the work which follows. A fourth (called IV) was prepared by another method of procedure. Potassic permanganate was recrystallized until free from chloride, the tests being made in small portions after precipitation with alcohol and filtration of the manganese dioxide. Sulphuric acid was boiled until free from chloride. With the help of these two substances, bromine was set free from a large sample of pure sodic bromide (sample II). After shaking with a solution of the bromide, the bromine was twice redistilled. This very pure preparation was run into carefully purified ammoniacal sodic carbonate in a hard glass dish (for platinum might have been slightly attacked) and the solution of sodic bromide was transferred to platinum as soon as the operation was completed. The ammonia used in this process had been prepared from ammonic sulphate purified by treatment with nitric and sulphuric acids in the manner recommended by Scott[4] to free it from amines. It was twice further distilled into pure water. The reaction may be expressed as follows:

$$3Na_2CO_3 + 2NH_3 + 6Br = 6NaBr + N_2 + 3CO_2 + 3H_2O$$

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[3] This sample had been prepared by T. W. Richards for work on atomic weights.

[4] Journ. Chem. Soc., **79**, 147 (1901).

The sodic carbonate was maintained in slight excess, and this excess finally neutralized by some of the pure hydrobromic acid described above. The salt was recrystallized twice with thorough draining. Very pure material may be made in this way, if the ammonia, bromine, and sodic carbonate are all pure.

The hydrobromic acid used for the fifth and last sample of sodic bromide was made by bubbling electrolytic hydrogen through bromine, and passing the mixture through a hot hard glass tube. The bromine used was in the first place purified by distillation from a bromide, and a further attempt was made to purify it by passing the vapor when mixed with hydrogen before heating over a counter current of a solution of a pure bromide (made from a portion of the acid itself) flowing over beads in tall glass columns. All the apparatus was constructed of glass, without rubber connections, and the current was run slowly; but it is doubtful if great gain was effected by contact with the solution. The reason for the slow purification of a vapor by this means is undoubtedly the fact that the reaction can take place only on the limited surface between the gaseous and liquid phase.[5]

A large quantity of acid thus prepared was neutralized by a new specimen of sodic bicarbonate even purer than before. The bicarbonate was made by precipitating a solution of pure often recrystallized sodic carbonate in platinum through the long continued action of carbon dioxide. The easiest method consists in placing the open dish under a bell-jar and keeping the latter full of the gas at a pressure slightly greater than that of the atmosphere. The carbon dioxide was generated by heating pure dry sodic bicarbonate in a large bomb-shaped vessel, and washed once with a dilute solution of the same salt. The sodic bromide thus prepared served for the last two final determinations given below, after having been recrystallized three times.

In order to remove all doubt as to the purity of salt, a sample prepared in this way was analyzed by the precipitation of argentic bromide with all the precautions of an atomic weight determination, and the result was compared with a similar analysis of a sample made with hydrobromic acid known to be pure. The latter was kindly given by Professor Baxter and Mr. H. L. Frevert, having been made by them for the determination of the atomic weight of cadmium. Both specimens of sodic bromide were well drained, desiccated in vacuum, fused in nitrogen, weighed and analyzed by conversion into silver bromide as usual.

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[5] As a matter of fact, one sample of acid prepared in this way was found by Professor Baxter, who kindly interested himself in the matter, to contain appreciable traces of chlorine. Evidence is given later showing that the particular sample V of sodic bromide prepared from acid thus made was free from this impurity, however.

Of our sodic bromide 5.49797 grams yielded 10.03253 grams of argentic bromide, and of the other sample 3.64559 grams yielded 6.65248 grams, all the weighings being reduced to the vacuum standard. The respective weights of argentic bromide corresponding to 1.00000 parts of sodic bromide are 1.82477 and 1.82480, showing the essential identity of the samples.

These two analyses not only serve this purpose, however; they likewise furnish new data for calculating the atomic weight of sodium, which is thus found to be 23.008, if silver is taken as 107.930 and bromine as 79.955. The new result from the chloride is also 23.008, an exactly identical figure. Thus the comparison of the results furnishes another reason for believing that the salt used in our transition temperature experiments was pure. Only their small number and our uncertainty concerning the exact atomic weight of bromine debars us from presenting these results in a special paper as a contribution to the literature on atomic weights.

METHOD OF DETERMINING THE TRANSITION TEMPERATURE.

The apparatus necessary for this work naturally divides itself into two parts, — the thermometers on the one hand, and the apparatus for containing the salt on the other. The thermometers employed have all three been standardized with the greatest care by the International Bureau of Weights and Measures in Sèvres. They have already been described in detail in the previous papers.[6] They were those bearing the designations Tonnelot 11142 and Baudin 15200 and 15275. Further description of them is unnecessary here.

To contain the salt two different arrangements were used. In one case the salt was contained in a wide test-tube placed in the bottom of a long still wider sealed tube sunk in a very deep thermostat until the 50° mark on the thermometer stem was almost level with the water. The object of this device was obviously to maintain the thermometer stem all at the transition temperature in order to avoid any considerable correction for the cooled column.

The more convenient apparatus employed in all the other determinations was modelled after that described in the paper upon the sulphate, and is illustrated in the accompanying Figure 2. The tube containing the inverting mixture was surrounded by an air-jacket (A), and the latter by water maintained at the desired temperature above the transition temperature by an electrically heated resistance wire. The thermometer itself was surrounded by a long jacket (D) of good glass through which a current of pure water was maintained, flowing from a large thermostat, in order to keep the stem of the thermometer at the same temperature as the bulb. Preceding experience had taught us to heed the cooling of this stream of water as it flowed from an elevated thermostat (C); of course, at the temperature of 50°, the cooling was even more marked than in the case of sodic sulphate at 32°. To avoid any error on this account, a small extra thermometer was inserted in the long jacket around the standard thermometer to record the temperature of its stem. This extra thermometer is not shown in the diagram. In order to be certain of the temperature of the transition to a thousandth of a degree, it is necessary to be certain of that of the stem to a tenth of a degree. The evenly distributed cooling of the flowing column in the distance represented by the length of the mercury thread amounted to no more than 0.4°, therefore the temperature of the water in the jacket observed at about the middle point of the column could be taken as the average temperature of the column. The thermostat (C) was accordingly set so that the temperature of the flowing water at this point was within a tenth of a degree of the temperature of the inverting salt. In this way the conditions for accurate thermometery were adequately obtained and all uncertain corrections were avoided.

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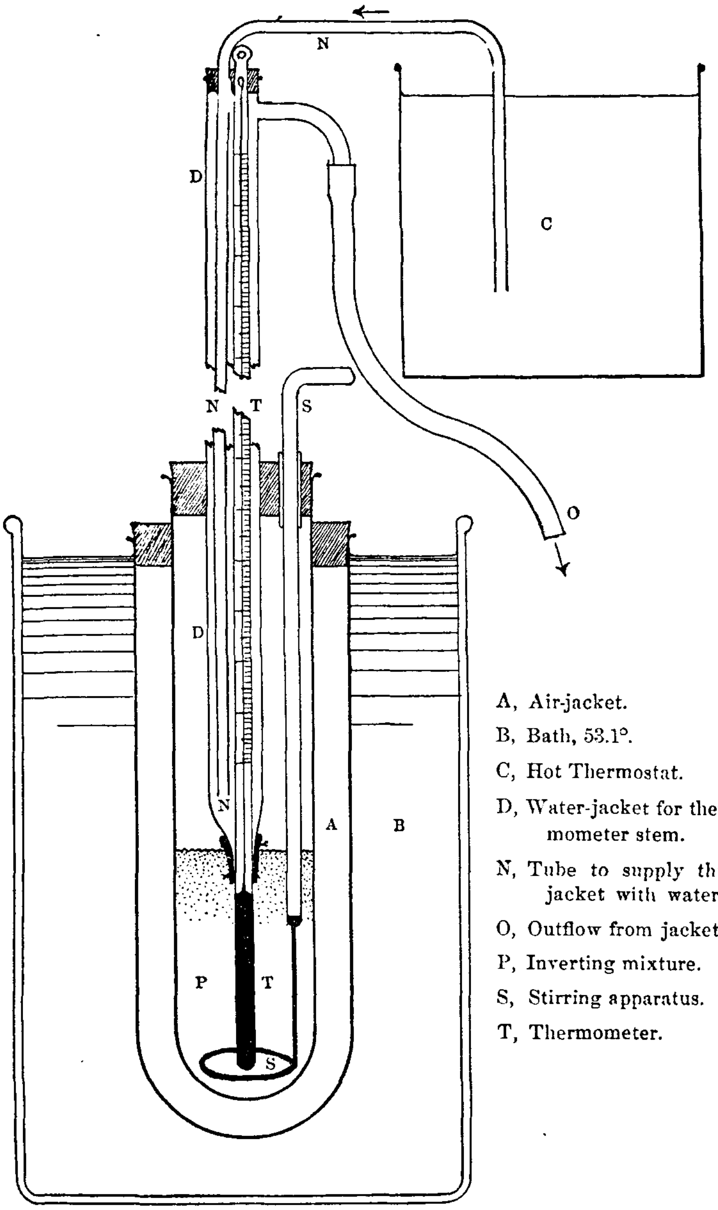
[6] Am. Jour. Science, **6**, 201 (1898); These Proceedings, **38**, 434 (1902).

According to the degree of the exposure of the top of the large test-tube containing the mixture it is necessary to maintain the water bath outside its air-jacket at a temperature somewhat above that of the mixture, in order to allow for the cooling effect of convection in the air-jacket. For the particular apparatus employed in these experiments and at the temperature of 50° this interval was found to be 2.5°. With this interval the inflow of heat was so nicely regulated that the temperature of the sodic bromide was maintained for at least two hours with wonderful constancy.

The stirrer used in stirring the salt and solution during the observation of the transition temperature was made of glass and a large sealed-in platinum wire. Platinum alone was first tried, but it was deemed to have too large a thermal conduction; and as the containing tube was glass, a further stirrer of glass could be accepted. Instead of powder-



A, Air-jacket.
B, Bath, 53.1°.
C, Hot Thermostat.
D, Water-jacket for thermometer stem.
N, Tube to supply this jacket with water.
O, Outflow from jacket.
P, Inverting mixture.
S, Stirring apparatus.
T, Thermometer.

FIGURE 2. APPARATUS EMPLOYED.

ing the salt in an agate mortar, as had been our practice in the case of sodic sulphate, we crystallized it by rapid cooling. It is necessary to have the salt finely divided by some means in order to stir it successfully. As far as we could discover, the transition temperature was not affected by the size of the solid crystals.

The ice used in these experiments was a commercial ice of very good purity. Before crushing it was well washed, and thereupon introduced into very carefully purified, boiled, and fully cooled water. Under these circumstances very little ice melted, and accordingly the water around it was very pure. Moreover, the total residue proceeding from a very considerable quantity of this ice alone amounted to only one ten-thousandth of one per cent. Even if the impurity had consisted of sodic chloride or some other substance this small amount could have caused no appreciable effect on the freezing-point; and practically we found that according to this method of procedure the commercial ice surrounded by pure water gave results exactly identical with the purest ice we could make.

As was to be expected, the preliminary experiments gave results somewhat too low, because of traces of chloride; for example, salt II, when crystallized once, gave a transition temperature of 50.661°; when twice recrystallized, 50.670°; when three times recrystallized, 50.671°. It was now almost pure, since the probable value of the true transition temperature is 50.674°. Other specimens gave preliminary values ranging from 50.663° to 50.673°. On the other hand, the salt labelled III, which was known to be of greater purity, showed no difference in the transition temperature between the first and the seventh successive crystallization, giving the constant value 50.674. Having thus both analytical and physico-chemical evidence of the purity of our material, it was therefore safe to make a final series of determinations with all possible care. The data of these are given below.

Final Determinations of Transition Temperature.

Three thermometers were used for the final work, as has already been said. The work with the Tonnelot thermometer was carried out with salt III, which had been recrystallized seven times. It is a pleasure to thank Mr. J. B. Churchill for his assistance in carefully recrystallizing the salt and in aiding in the temperature measurement in this case. The data of the measurement are given below. The salt used for the determinations made with the two Baudin thermometers was that labelled V, recrystallized three times. The temperature measurements are recorded in the usual fashion, and need no further interpretation.

FINAL MEASUREMENTS.

Thermometer.	Tonnelot. 11142.		Baudin. 15275.		Baudin. 15260.	
	In salt.	In ice.	In salt.	In ice.	In salt.	In ice.
Reading	50.647	+0.098	50.873	0.059	50.719	+0.089
Cor. calibration	+0.159	0.000	−0.059	0.000	+0.064	0.001
" int. pressure	+0.063	+0.016	+0.036	+0.013	+0.055	+0.009
" ext. pressure	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001
" exposed column	+0.005	0.000	0.000	0.000	0.000	0.000
	+50.873	+0.113	50.849	+0.071	50.837	+0.098
" exact 0° point	−0.113	←	−0.071	←	−0.098	←
	50.760		50.778		50.739	
" fundamental interval	+0.016		−0.001		+0.036	
" to hydrogen	−0.102		−0.102		−0.102	
	50.674		50.675		50.673	

Thus, the three final determinations, made with three different thermometers from two makers, and with two different specimens of salt, at widely different times, gave as results respectively, 50.674°, 50.675°, 50.673°. The remarkable agreement of these results must be partly accidental, because the extreme divergence is less than the possible divergence from accuracy of each single determination. The mean, 50.674°, may be taken as representing very nearly the true transition temperature of the salt, on the international hydrogen scale.

In conclusion, it is a pleasure to acknowledge our indebtedness to the Rumford Fund of the American Academy of Arts and Sciences for pecuniary aid in carrying on this investigation.

SUMMARY.

The results of this paper may be summed up in the following sentences :

1. Pure sodic bromide is not to be obtained by recrystallizing the ordinary commercial samples, but must be made from pure bromine and pure sodic carbonate.

2. Prepared in this way, our salt upon analysis was found to correspond very closely with the new value of the atomic weight of sodium, 23.008, if silver be taken as 107.93 and bromine 79.955. Therefore, it was presumably pure.

3. The less pure material on successive recrystallization gave in every case a slightly rising transition temperature as the crystallization proceeded. Only the purest material melted at a perfectly constant point, therefore constancy of melting point is an indicator of purity ; but it is safer to analyze the salt as well.

4. When all precautions are taken it is possible to duplicate the results for the transition temperature with samples of salt prepared in different ways and at different times without great difficulty ; a value within .01° of the truth may be easily obtained, and further precautions enable one to approach much closer. Therefore, the point is one suitable to use in the calibration of thermometers, although its determination requires more chemical skill than that involving sodic sulphate.

5. The actual value of the transition temperature on the international hydrogen scale is 50.674°.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A NEW METHOD OF STANDARDIZING THERMOMETERS BELOW 0° C.

By Theodore W. Richards and Frederick G. Jackson.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations
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A NEW METHOD OF STANDARDIZING THERMOMETERS BELOW 0° C.

By Theodore W. Richards and Frederick G. Jackson.

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Within recent years the theory of physical chemistry has been profoundly influenced by inferences drawn from the freezing points of aqueous solutions, and many experimenters have contributed data upon this subject. Too often, however, the data are of uncertain value, even when all conceivable precautions have been taken to avoid supercooling and superheating. One cause of the uncertainty is the lack of knowledge concerning the calibration of the thermometers used. Obviously the error of the hundredth of a degree in the thermometer will affect the result by a whole per cent if the fall of temperature caused by the dissolved substance is a degree; and such errors are common. The importance of a simple method of calibrating thermometers below the freezing point of water, both for the theory and practice of physical chemistry, is therefore very apparent. The object of the present paper is to describe the practical working of a method which may be easily carried out by any one, a method which has already been suggested by one of us in a previous paper.[1]

Evidently a solution of given concentration of a given substance must freeze always at exactly a given temperature. If, then, the freezing points of a series of solutions of this substance are determined once for all on a given thermometer of known precision, any other thermometer in any part of the world may be compared at these points with the standard thermometer by finding its readings in similar freezing solutions. If the indications of the standard thermometer are plotted in a curve comparing concentration with scale reading, any point on any thermometer may be verified. This verification is obviously accomplished by immersing the instrument into a mixture of cracked ice and water and adding the dissolved substance until the desired point is reached and maintained. An analytical determination of the concentration of the solution then gives at once by comparison with the standard curve the true value of the temperature which corresponds with this concentration, and hence the error of the thermometer. For example, supposing that the error of the graduation of an uncertain thermometer at reading —1.513° is desired, the thermometer is immersed in ice and water, and hydrochloric acid is added until this point is reached. The solution is then analyzed and found to be 0.4085° normal. From the standard curve given below it is quickly found that this concentration should give a reading of 1.498°, therefore the thermometer at this place was clearly 0.015° in error. The practical value of the method obviously depends wholly upon the accuracy with which the curve comparing concentration and temperature reading has been determined. This accuracy depends, first, on the accuracy of the standard thermometer; second, upon the care in adjusting of the freezing point; and third, on the accuracy of the analysis.

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[1] T. W. Richards, "The Freezing Points of Dilute Solutions," Journ. Am. Chem. Soc., **25**, 297 (1903); Zeit. phys. Chem., **44**, 570 (1903).

The thermometer used in the present investigation was one carefully standardized by the Bureau International des Poids et des Mesures in Sèvres for this express purpose. It was made by Baudin, and is numbered 15275. Its scale reads as far as 12° below zero. Because our results depend upon this single thermometer, they must be considered as only preliminary, for, no matter how careful the calibration, the results of a single thermometer are somewhat uncertain. Nevertheless this thermometer is undoubtedly so much better than those usually used that our results have practical value, even although they cannot be considered as quite the final ones. In the near future we shall hope to take this matter up again with several similar thermometers, obtaining an average value which will be more nearly certain than the present one.

With regard to the care in adjusting the freezing point, it is enough to say here that all reasonable precautions were used. Two series of determinations were conducted: one in a tall beaker, air-jacketed with another beaker, which was in turn surrounded by a mixture of ice and solution of almost exactly the same temperature as the mixture within the inner beaker. The other series was conducted in a large Dewar silvered vacuum jacketed tube, about ten centimeters in internal diameter and thirty centimeters long. This was the more convenient form of apparatus. These two series gave essentially the same results. The thermometer was left in ice for twenty-four hours in every case before it was used, and was read by means of a powerful micrometer telescope with all the precautions used in the most precise thermometry. Hydrochloric acid was the substance chosen for the solution because its concentration may be determined with accuracy and convenience in many ways.

The third consideration determining the accuracy of the curve is that just referred to, namely, the accuracy of the analysis. While evidently for most purposes volumetric work would give the result quite certainly enough, we nevertheless in the present case preferred to use the gravimetric method of precipitating the hydrochloric acid with an excess of silver nitrate. The solution was drawn from the neighborhood of the thermometer bulb, after thorough stirring, by means of a chilled pipette, whose capacity had been determined by careful calibration. The solution was measured instantly, while it was still cold, and the precipitation of the silver chloride was conducted in the usual manner. In the calculation of the results, the concentrations were computed in terms both of weight of silver chloride from ten cubic centimeters of solution, and in terms of the normality of the hydrochloric acid solution (i. e. the number of gram equivalents per liter calculated from the true volume corrected for the contraction of the glass tube at the temperature of the cold bath). The results were as follows:

Freezing Points of Hydrochloric Acid Solutions.

Depression of Freezing Point t°.	Weight of Silver Chloride from 10.00 Cubic Centimeters of Cold Solution.	Normality of Solution c.	$\frac{t}{c}$
	grams.		
0.500°	0.199	0.139	3.59
1.000°	0.397	0.277	3.61
1.500°	0.587	0.409	3.67
2.000°	0.772	0.538	3.72
2.500°	0.951	0.663	3.77
3 000°	1.109	0.773	3.88
3.500°	1.271	0.886	3.95
4.000°	1.430	0.997	4.01

The curve deviates slightly from a straight line, in such a direction that more concentrated solutions produce a slightly greater depression in proportion to their concentration than the less concentrated ones. This is shown by the increasing values in the last column. The hypothesis of ionization predicts the opposite effect.[2]

From these points a curve may be drawn which will enable any one to correct his own thermometer at any point upon repeating the experiments in question.

It may be noted in conclusion that this means of calibrating thermometers is theoretically as exact as any other. According to the Phase Rule, in a system composed of two components, hydrochloric acid and water, four conditions must be fixed in order to fix a point. In the present case the four conditions are the pressure, the concentration of the dissolved substance, and the co-existence in equilibrium of two phases, solution and pure ice.

SUMMARY.

This paper describes a simple method of calibrating thermometers at temperatures below the freezing point of water by using as standards of comparison the depressions of the freezing point caused by given additions of hydrochloric acid. The data given are for the construction of a curve enabling direct comparisons of a thermometer to be made with a minimum of labor. The method is shown to be both practically and theoretically satisfactory. This paper is preliminary in nature, and does not pretend to give final values, because only a single standardized thermometer was used.

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[2] It is needless to call attention to the fact that many other cases of this kind have been studied in detail by de Coppet, Rüdorff, and others, especially in very recent times by Harry C. Jones.

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CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

LI. — *A METHOD OF DETERMINING THERMAL CONDUCTIVITY.*

By F. A. Laws with F. L. Bishop and P. McJunkin.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations
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LI. — A METHOD OF DETERMINING THERMAL CONDUCTIVITY.

By F. A. Laws with F. L. Bishop and P. McJunkin.

Presented December 13, 1905. Received December 15, 1905.

Among the various methods for determining thermal conductivity, the wall method, a modification of which is here employed, has been perhaps most frequently used. It has been open to the serious objection that the temperatures of the surfaces of the metal may not be the same as those of the heating and cooling baths which have frequently been taken as the temperatures of the metal surfaces. This difficulty was overcome by Professor E. H. Hall by plating the surfaces of the wall with another metal and using the E. M. F. of the thermo-electric junctions so formed to determine the temperature difference. There are also difficulties in insuring a perfectly uniform flow throughout the section under investigation.

The method here employed is to liberate electrically a known quantity of heat inside a spherical shell of the substance, and maintain the outside surface at a constant temperature. The difference of temperature thus produced is measured by means of thermo-electric junctions of which one metal is that of the shell, the other metal being electroplated upon it. This method, then, will insure (1) uniform flow of heat, (2) absence of radiation corrections, (3) exact determination of the temperature difference, (4) an exact determination of the quantity of heat passing through the wall, and (5) independence of the density and specific heat of the substance.

Let the quantity of heat be represented by Q; the difference of temperature of the surfaces of the shell by $t_1 - t_2$; the radii of the shell by p_2 and p_1.

Then
$$K = \frac{Q\,(p_2 - p_1)}{4\,\pi\,(t_1 - t_2)\,p_1\,p_2}$$

Description of Apparatus.

The shell is in the form of a hollow sphere of cast iron, in the casting of which every precaution was taken to have the iron free from blow holes and of uniform quality. It is divided into two hemispheres by a plane passing through a great circle. A narrow rim on one hemisphere accurately fitting a corresponding groove in the other, insures the perfectly concentric position of the two parts at all times. A radial hole is provided for the admission of lead wires to the heating coil and thermo-electric junctions, and another for the shaft which drives the fans in the interior of the sphere.

The sphere is coated by electrolysis with copper to a uniform thickness of about 2 mm., it having been shown by Professor Hall[1] that this thickness would be sufficient to prevent the lead wires from conducting away sufficient heat to change the temperature in the immediate vicinity of the copper-iron junction.

These lead wires, eight in number, four inside and four outside, are electroplated to the copper. The two from the inside of each hemisphere are brought through a wood fibre plug split in the same plane as the sphere. These are then united with the outside wires, and the whole wrapped with a thin silk ribbon, thus forming a small, well protected cable. The ends of the wires are brought to a block of fibre $3'' \times 1'' \times \frac{5}{8}''$ provided with eight small holes arranged in two rows $\frac{1}{2}''$ apart. The wires from the outside are put in one row, and the corresponding ones from the inside in the other. The holes are then filled with mercury and a cover of fibre with a similar set of holes screwed on the block to hold the wires in place.

The leads from the galvanometer are brought to this block and so arranged that any pair of junctions can be connected and reversed without approaching the block or galvanometer. The D'Arsonval galvanometer used has a resistance of about 600 ohms and gives a deflection of about 160.0 mm., readable to tenths of a mm., with a potential difference of .0001 volt, the scale distance being 3.3 meters. It is mounted on a Julius device and carefully protected from mechanical disturbances. The galvanometer and leads to the junctions are packed in hair felt to insure a constant temperature.

The temperature of one of the junctions must be known. This is accomplished by means of a thermo-electric couple consisting of an iron and a nickel wire. One junction is fastened securely in a wood fibre plug so that it makes contact with the material of the sphere exactly where the copper joins the iron on the outside. The other junction is placed in a large beaker of oil. The iron wire is cut, and from its terminals, leads taken to a sensitive galvanometer. The beaker is heated until there is no deflection of the galvanometer, and the temperature read by means of a thermometer. The bath is heated slowly and stirred vigorously to insure uniformity of temperature and reduce the effect of the lag in the thermometer.

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[1] These Proceedings, **31**, 1895.

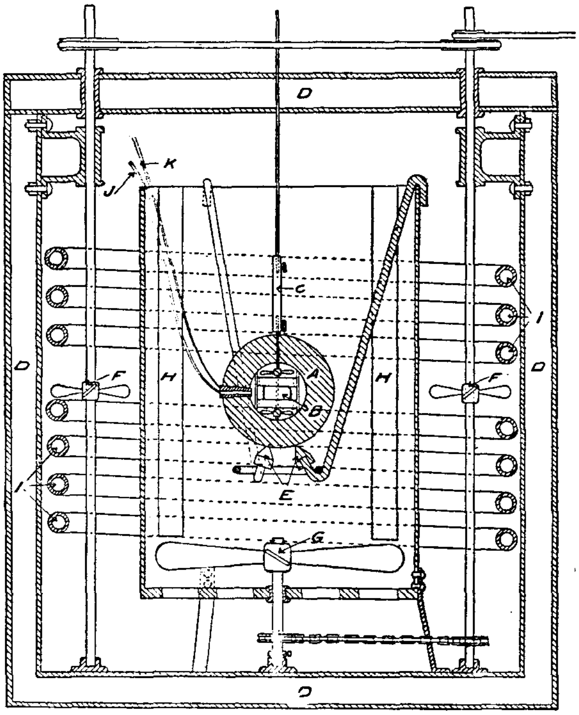

FIGURE 1.

The interior of the sphere is filled with oil. The heat is produced by means of an electric heating coil. This coil is made of No. 28 (B. & S.) German silver wire, having a resistance of about 13 ohms. It is wound upon two rings of wood fibre, each 1.5″ in diameter, $\frac{1}{8}''$ thick, and $\frac{3}{8}''$ wide. These are separated from each other and from the sphere by means of three brass rods $\frac{1}{16}''$ in diameter, which are tipped with fibre where they come in contact with the sphere, thus preventing local heating. The two circulating fans, one on each side of the heating coil, are mounted on a shaft of steel which runs in brass bearings mounted on the top of the rings of the heating coil and passing through the sphere in a bearing of wood fibre.

The shaft terminates just outside the sphere in a piece of fibre 3″ long, connected to the driving motor by means of a brass rod. This prevents undue conduction of heat from the interior of the sphere. The leads to the heating coil are rectangular iron wires passing through the sphere in the same plug as the wires to the inside thermo-electric junctions. The current and voltage are measured by an ammeter and voltmeter.

The sphere thus arranged is suspended in a bath kept at a uniform temperature. The bath is contained in two galvanized iron tanks. The outer tank is 30″ in diameter and 30″ high, surrounded and covered by a 2″ layer of hair felt to prevent undue radiation. The inner tank is 15″ in diameter and 22″ high, supported on the bottom of the larger by three iron supports 3″ high. The bottom is perforated with a large number of holes to allow free circulation of the oil. The sphere is supported at the centre of this tank on three wood fibre points, which are held in position by brass rods hung from the top.

To provide a means for circulating the oil in the bath, two steel shafts are mounted on diametrically opposite sides of the largest tank. Each shaft carries near its centre a fan having four blades 6″ in diameter.

One shaft is connected by a chain to gearing on a shaft in the centre of the small tank. This shaft carries a fan 12″ in diameter, which forces the oil up or down the inner tank. On the inside of the inner vessel are three projecting vanes, which break up the otherwise regularly circular motion of the liquid.

The fans are driven by a motor so arranged that the direction and speed of rotation can be changed. A special distilled petroleum oil was finally selected for the bath. The temperature of the oil is maintained constant by cold water flowing through a coil of lead pipe surrounding the inner tank. This is connected to a supply tank, thus insuring a constant temperature. It is also connected directly to the water mains and steam pipes, in order to provide a means for rapidly changing the temperature.

The temperature of the bath can be regulated by hand to give a constancy of 0.01° C. The temperature is observed by a thermometer read with a telescope.

Calibration Apparatus.

In order to determine the relation existing between the difference of temperature of the inside and outside junctions, and the corresponding E. M. F., the two hemispheres are removed from the apparatus and maintained at different known temperatures, while metallic contact is obtained between them by a piece of the same iron as the sphere.

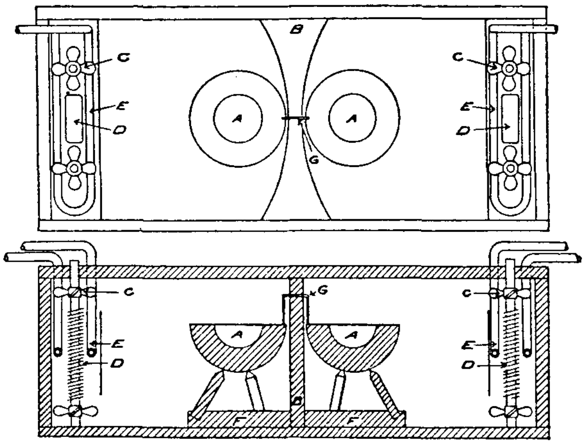

Figure 2.

The calibration tank (Figure 2) is a box 12″ × 24″ × 10″ deep lined with tin and separated into two compartments by a double tin partition, the space being filled with asbestos to prevent conduction of heat from one to the other.

Each compartment is provided with two fans to agitate the liquid, an electric heating coil, and a coil of lead pipe through which steam or cold water can be forced. Thus each compartment can be maintained at any desired temperature.

One hemisphere is placed on an insulating stand in each part of the calibration tank. These are connected by a small piece of iron from the original casting in the form of a U $\frac{1}{32}''$ in cross-section, each leg 1″ long and the base $1\frac{1}{2}''$ long. Each end of this is driven into a small hole in the iron of the hemisphere, thus insuring perfect metallic contact.

The lead wires from each hemisphere are bound together, and their ends inserted in separate holes on the wood-fibre block used for the terminals in taking measurements.

The following procedure is used in making the calibration. With the baths at the same temperature, tests are made to determine the existence of any outside E. M. F. The temperatures are then adjusted and the electromotive force determined. Next the temperatures are reversed and the electromotive force again determined.

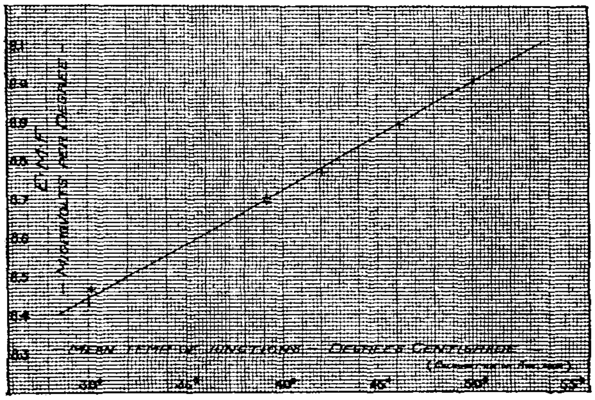

Figure 3.

The E. M. F. at the contact of the U piece and the hemispheres was at first very large, due to the fact that the U piece was simply pressed down on the iron, but after the small holes were made in the sphere and the U piecé was driven in place, it was impossible to detect any extraneous E. M. F.

The following curve shows the results of the calibration as carried out in August, 1902.

The method of calibration of course assumes that the metal employed is of uniform thermo-electric properties, and so is applicable in strictness only to pure metals.

In computing results from the curves, the E. M. F. is taken at the temperature of the lower junction and the difference of temperature computed; then the E. M. F. of the mean temperature thus obtained is used to again determine the difference of temperature. This method of approximation is carried out until the difference of temperature obtained on successive trials is the same.

The thickness and diameter of the sphere were carefully measured before it was electroplated by special micrometer calipers.

The following results show the mean of thirty-six measurements.

Thickness, 1.4327 inches; Diameter, 6.0022 inches.

METHOD OF TAKING READINGS.

The circulating fans on the interior of the sphere and the agitating fans in the bath are first set in motion. Then the different junctions are connected to the galvanometer in order to locate any outside E. M. F., if such exists. The current is now turned on the heating coil and the flow of water through the cooling coil adjusted until the temperature of the bath remains constant. Meanwhile the galvanometer is calibrated as follows: The galvanometer is shunted across a standard ohm which is in series with a large resistance and storage cell. The resistance in the storage cell circuit is varied until the desired deflection is obtained (this being determined by the difference of temperature at which the run is to be made). The E. M. F. of the storage cell is at once obtained by the Poggendorf method, using a standard Carhart-Clark cell. When the temperatures of the outside of the sphere and bath have become constant, the terminals of the galvanometer are connected to junction No. 1. As soon as the deflection is constant the readings of the galvanometer, ammeter, voltmeter, and thermometer in bath are taken, also reading of thermometer in bath with nickel-iron junction, giving the temperature of the outside junction. The galvanometer is then reversed on No. 1 and the readings again taken.

The same method is followed on each junction, and then the whole series repeated four times, after which the galvanometer is again calibrated.

RESULTS.

The following results show the mean of four series of readings on each junction; i. e., 29.633 is the mean deflection on the four junctions, while 6.472 and 89.55 are the average amperes and volts during the run. This is a typical set of readings.

MAY 14, 1898.

Deflection.	E. M. F. of Junction.	Amperes.	Volts.
29.71		6.46	89.4
29.63		6.45	89.2
29.41		6.48	89.6
29.78		6.50	90.0
29.633	9.12 micro-volts	6.472	89.55
	Corrected values	6.538	89.29
Difference of temperature, 10.46°.	Value of K, 1281 at 39.50°.		

SUMMARY OF RESULTS.

Values of K:

April	14, 1898	. . .	.1273	at	43.7°
"	14, "	. . .	.1267	"	43.3°
May	14, "	. . .	.1281	"	39.5°
June	24, "	. . .	.1281	"	39.8°
"	28, "	. . .	.1259	"	43.2°
"	28, "	. . .	.1254	"	43.2°
	Mean	. . .	.1269		
	A. D.	. . .	.0004		

There are certain errors which enter into these results which will be eliminated or corrected in subsequent experiments; namely, conduction through holes cut for lead wires, etc., variation of current and voltage, heat generated by rotation of fans on interior of sphere, uncertainty of galvanometer readings due to lack of sensitiveness of the instrument, lack of homogeneity in the shell.

Cast iron was used because it could be obtained and prepared with facility for the experiments which were designed as a test for the capabilities of the method, which is to be applied to the determination of the conductivity of pure metals.

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ON THE LANGUAGE OF VITRUVIUS.

By Morris H. Morgan.

ON THE LANGUAGE OF VITRUVIUS.

By Morris H. Morgan.

Presented December 13, 1905. Received December 23, 1905.

During the last ten years the question of the date and the authorship of 'Vitruvius de Architectura' has been revived after a long slumber. In 1896, Professor J. L. Ussing published a treatise in Danish in which his object was to show that the writer of that work was not an architect, but an amateur who lived about the middle of the third century of our era, and who was a mere compiler, drawing chiefly from Varro. Two years later, in 1898, this treatise, much enlarged, was translated into English and carefully revised by the author, and in this form it was published in London by the Royal British Institute of Architects under the title *Observations on Vitruvius de Architectura Libri Decem, with special regard to the time at which this work was written.* To prove his point, Ussing made use of two kinds of arguments, the first being based upon the language and style, and the second upon the subject matter of the work. Both the original Danish and the translation into English have attracted the attention of classical students and architects in no small degree. Still more recently a French scholar, M. Victor Mortet, has written a series of articles entitled *Recherches Critiques sur Vitruve et son Œuvre* in the *Revue Archéologique* (1902, pp. 39–81 ; 1904, pp. 222–233 ; 382–393) in which he holds that our author wrote during the reign of the Emperor Titus. His arguments depend almost altogether upon the contents of the work, not upon its language and style, which he does not treat in any detail.

In fact, it is to the nature of the contents of Vitruvius that attention has been almost entirely directed by those who have written upon the subject of his date. Scholars who have examined the question are familiar in this connection with the names of Newton, Hirt, Schultz, Osann, Detlefsen, Diels, Oehmichen, Thiel, Degering, and others to whose writings there is no need of further reference here. To be sure, Praun in his *Bemerkungen zur Syntax des Vitruv*, Bamberg, 1885, and Eberhard in his two programmes *De Vitruvii genere dicendi*, I, Pforzheim, 1887, and II, Durlach, 1888, have made careful and valuable studies in the language of Vitruvius, but neither of them endeavored to show anything about his date, accepting the common view that he wrote under Augustus.[1] Consequently when Ussing made use of arguments based upon language and style he was opening an almost new field, although for his collection of examples he relied chiefly upon Praun. His use of these arguments seems to have had a considerable effect upon scholars known personally to me; further, his conclusion was accepted by Lanciani (*Bullettino Communale*, 1899, p. 24, n. 2); and it led Wölfflin to the statement that the case must be considered as once more reopened for further discussion (*Archiv*, VII, 301). This dictum caused Degering in his article on Etruscan temples (*Gött. Nachrichten*, Phil.-Hist. Kl., 1897, 2, 137) to think that Ussing might possibly be in the right, although recently (*Rhein. Mus.* LVII, 1902, p. 8) he has supported the contrary view on grounds of subject matter. But neither he nor any one of the reviewers[2] of Ussing's treatise has published a detailed study of Ussing's linguistic and stylistic arguments with a view to determining whether they really do furnish evidence of a late date of composition. It seems worth while, therefore, to examine them closely, and this I propose to do in the following article. Ussing's contention is that the phenomena to which he draws attention 'point to the decadence of the Latin language and to its transition to the Romance tongues.' I shall inquire whether these phenomena or traces of them are found in republican Latin writers and in the Augustan and Silver ages.

But before beginning this inquiry three observations are necessary. In the first place, we must never forget that in 'Vitruvius de Architectura' we are dealing with a work which, if it was composed before the end of the Augustan age, is absolutely unique in its kind. We have no other prose work on a technical or scientific subject (unless we include agriculture among such subjects) written in Latin as early as this period, and we have no other treatise on architecture, either in Greek or in Latin, coming down to us from antiquity. And even in other fields than science, the amount of Latin prose of the Augustan age that has survived to us, is really quite small, so that for all these reasons a standard or norm of comparison for the prose of that age is hard to obtain. But secondly, I am not concerned in this article to distinguish too exactly between the prose of the Augustan and that of the Silver age, nor to show that 'Vitruvius de Architectura' was composed under Augustus rather than under Titus. Ussing argues that it is a work of the third century. If I can show that the linguistic and stylistic peculiarities upon which he relies are found in the writings of the republic and early empire, it will be enough for my present purpose. The decision between the time of Augustus and the time of Titus is a different matter, and whether it is to be reached by means of arguments drawn from the language or from the subject matter[3] does not at this moment concern me, although it will, I hope, be treated before long in another article. Thirdly, the whole gist of the linguistic part of Ussing's argument seems to consist in his belief that if a writer lived in the 'classical period' his style must therefore be 'classic.' This is a pure assumption, and it is confuted by all actual experience. Thus, a man to-day may be an excellent architect or may excel in other technical and scientific pursuits, and he may have received a good general education,—yet he may not be able to express himself in writing with polish, or with freedom, clearness, or even always with mere correctness. Very many such men are among the writers of to-day. Why should we think that there were no such men living and writing in the classical period of Latin literature? We know that there were such men. It is enough to compare the correspondents of Cicero with Cicero himself, the authors of the *Bellum Africum* and *Bellum Hispaniense* with Caesar, to read what is known of the involved and affected style of the great patron of literature, Maecenas, and to remember that *Vergilium illa felicitas ingenii in oratione soluta reliquit* (Sen. *Contr.* 3, praef. 8, p. 243 K). Having made these observations, we are ready to proceed to the consideration of Ussing's criticisms.

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[1] Such was also the attitude of Richardson in his article in the *Harvard Studies in Classical Philology*, **1**, 153 ff. The dissertation of Stock, *De Vitruvii sermone*, Berlin, 1888, is of no value for our purposes. The treatises of H. Ulrich, *De Vitruvii copia verborum*, I, Frankenthal, 1883, and II, Schwabach, 1885, I know only from the review in Wölfflin's *Archiv*, **1**, 126.

[2] The chief of these are to be found in *Berl. Phil. Woch.*, 1897, 773 ff. (by Krohn); *Revue de Philologie*, **21**, 118 ff.; Bursian's *Jahresbericht*, **108**, 1901, 118 ff. (by W. Schmidt); *Journal Royal Institute of British Architects*, 3d Ser., 1899, 149 ff. (by Brown); *Athenaeum*, 1897, p. 586.

He thus begins (p. 4): 'One of the peculiarities which occur especially in the authors of the later period of the empire, where they wanted to write nicely and philosophically, is the frequent use of abstract nouns, even in the plural. So also Vitruvius.'—Nobody would be found to deny that abstracts are common in late Latin, but what is omitted from Ussing's statement is for us the important fact, viz.: that the common use of abstracts began long before the later period of the empire. On this point, see Schmalz, *Lat. Gramm.*,[3] p. 430: 'In der Sprache des Volkes waren die Subst. abstr. gerade nicht unbeliebt, wie ein Blick auf dem Wortschatz des Plautus zeigt; aber immerhin ist *erst mit Cicero* und zwar *infolge seiner philosophischen Studien* eine Bereicherung eingetreten.' Thus, to illustrate, I may take a single example: the abstract *repugnantia* appears first in Cicero's philosophical writings (*T. D.* 4, 23; 29; *Off.* 3, 17; 34); and it is used in the contemporary Second Philippic, 19 (see Sihler *ad loc.*). In the quotation from Schmalz I have italicized certain words because I think it worth observing that Cicero was dealing with Greek ideas and Greek sources at the time when he felt the need of enriching Latin with new abstracts. May not this in large measure account for the great number of abstracts in Vitruvius? But not altogether, for it appears that the Scriptores Rei Rusticae, even the earliest from Cato and Varro to Columella, exhibit a liking for abstracts[4] which, in these truly Roman writers, cannot be attributed to exigencies due to the use of Greek sources. The fact is that as new ideas called for expression in Latin prose, the avoidance of abstract substantives in the expression of them was often really a *tour de force*, and only the best writers struggled very hard to avoid them or, when they used them, apologized for their use.[5] And finally the frequent employment of abstracts in the correspondence of Cicero shows that they were also common in the colloquial language of the educated and used as a briefer form of expression of thought than that which the master reserved for his greater works.[6]

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[3] For a few notes on this, see my article in the *Harvard Studies in Classical Philology*, **17**, 9 ff.

Ussing proceeds: 'Among abstract nouns which appear only in his writings I will mention *ignotitia* (64, 4[7]), *indecentia* (174, 9), *pervolitantia* (232, 3), *nascentia* (232, 17), *crescentia* (238, 14; 23; 239, 3), *commensus* = *mensura* (15, 25; 31, 3; 65, 25; 103, 21; 134, 11)' — Of these, it may in the first place be remarked that Ussing's statement is not exact, for three of them do appear in other writers: *ignotitia*, Gell. 16, 13, 9; *indecentia*, Cael. Aurel. *Chron.* 3, 8 (p. 254, Vicat); *nascentia*, see Rönsch, *Itala u. Vulgata*, p. 50. To be sure these are late writers, but let us, before concluding that the occurrence, say of *ignotitia*, in Vitruvius is a proof that the work which goes under his name is a late production, inquire what other abstracts there are which he could have used in the sense of 'ignorance'? There are four, *ignorantia, ignoratio, inscientia*, and *inscitia*. But all of these are new contributions to the enrichment of the language made, so far as we know, in the time of Cicero or by him. The first, as we know, did not please him and it is usually avoided (Schmalz, *Antibarbarus*,^{6} I, p. 618). Vitruvius does not use any one of the four, but has instead once *ignotitia*, a violation of the rules of composition (the only one of this sort in Vitruvius), but paralleled by *insatietas* (Plaut.), *intemperies* (Plaut., Cic.), *invaletudo* (Cic.), *inreligio* (Rhet. ad. Herenn.). Of course I am aware that the last two have been emended away, yet see Wölfflin, *Archiv*, IV, p. 403. And *ignotitia* is not surprising in a writer who has *notitia* three times (5, 12; 7, 13; 133, 27) in the sense of 'knowledge.' The second abstract, *indecentia*, would be surprising if the truth were, as one might gather from the Lexicon and from Schmalz (*ibid.* p. 660), that *indecens* first appears in the Silver age. But Vitruvius has it only three lines below (174, 12), and why is he led to employ these words? Because he is employing them technically in an anecdote illustrative of sins against propriety (*decor*) in art (173, 19), — propriety, which with him is one of the six component elements of true architecture (11, 12 ff.), and a subject to which he frequently alludes.[8] In thinking of *decor* he forms *indecentia* as naturally as Cicero, thinking of *dolor*, forms *indolentia* (*Fin.* 2, 11). The third abstract, *nascentia*, occurs in the context *non e nascentia sed ex conceptione genethliologiae rationes explicatas*, where Vitruvius is referring to those astrologers who based horoscopes not on the moment of birth but on that of *conception*. Here the Greek technical terms were γένεσις or ἔκτεξις and σύλληψις; cf. Sext. Emp. p. 737, 18 Bk.: τὴν δὲ γένεσιν τῶν ὑπὸ τὴν ἐπίσκεψιν πεσουμένων ἀρχαϊκώτερον ἤτοι ἀπὸ τῆς τοῦ σπέρματος καταβολῆς καὶ συλλήψεως λαμβάνειν ἢ ἀπὸ τῆς ἐκτέξεως. See also Hippolytus, *Ref. Haer.* 4, 3. Another word for 'birth' in this connection was ἀπότεξις (Sext. Emp. p. 737, 7), and the simple τέξις was also used (*ibid.* p. 739, 12). Vitruvius's *conceptio* is obviously a translation of σύλληψις and it was thus used by Cicero (*Div.* 2, 50). For γένεσις or ἀπότεξις what should he have used? This is a question which seems not to have occurred to those who would blame him for using *nascentia.* Cicero does indeed avoid the use of a single abstract and has the somewhat clumsy phrases *ortus eius qui nascatur* (*Div.* 2, 89), *ortus nascentium* (*Div.* 2, 91; see also *Div.* 2, 92; 94). For the Augustan period we have no evidence, so far as I am aware, unless it be found in Vitruvius. In Censorinus we have *genesis* (*Nat. D.* 13), in Tertullian *genitura* (*De Anima* 25 *fin.*). Pliny also employs both of these words, yet not in connection with astrology (*N. H.* 36, 19; 18, 202), and Augustine uses *genitura* like Pliny (*Civ. D.* 5, 3). Suetonius has *genitura* several times: once in the general sense of 'birth' (*Nero* 6), otherwise meaning 'horoscope' or 'nativity'; he also has *genesis* at least twice in this sense. For this Tertullian (*Idol.* 9) has *nativitatem.* Thus it appears that except in Vitruvius we know of no early abstract used for 'birth' in connection with the horoscope, and that the late writers who have occasion to speak of it do not use *nascentia.* Its occurrence in Vitruvius, therefore, cannot be taken as evidence of late authorship, but quite the reverse, for a late writer would have used *genitura* or *genesis.* There remain the three abstracts cited by Ussing which are really not found elsewhere,[9] *pervolitantia* and *crescentia.* The first is the expression by an abstract of the idea expressed by *pervolitat* (219, 10), both employed of the revolution of the *mundus* or *caelum.* Abstracts in *-antia* occur before Vitruvius's time: e. g. *flagrantia* (Plaut., Cic.), *incogitantia* (Plaut.), *errantia* (Acc.), *variantia* (Lucr.). The second, *crescentia*, is used three times, twice to denote the increasing length of the hours on a dial (238, 14; 239, 3), and once of the increasing length of days (238, 23). Both are employed technically and in their contexts are no more objectionable than Cicero's *indolentia* mentioned above. Of the third abstract, Ussing uses the expression '*commensus* = *mensura.*' But this seems to be a misapprehension. Vitruvius has *mensura* fourteen times, always in the simple meaning of 'measure' (see Nohl's *Index*), but *commensus* he employs ten times (*ibid.*), and never in that simple sense, but always with the idea of *comparative* or *proportionate* measurement, just as Cicero employs the verb *commetior* in *Tim.* 33: *siderum ambitus . . . inter se numero commetiuntur;* cf. *Inv.* 1, 39; *nam saepe oportet commetiri cum tempore negotium.* Thus we have in Vitruvius a new abstract employed as a technical term, and its appearance ought to be no surprise at any period in an author who has so much to say on the subject of the importance of proportionate measurements as has Vitruvius.[10]

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[4] See Cooper, *Word Formation in the Sermo Plebeius*, p. 2, and the lists, pp. 5–50.

[5] Cooper, *ibid.* p. xxxiii f.

[6] Cf. Stinner, *de eo quo Cicero in Epistolis usus est sermone*, p. 7, and such an array as that in Cooper, p. 6, where we have 24 abstracts in *-tio* occurring earliest in Cicero's letters.

[7] For convenience, I have changed Ussing's references to the pagination of Rose.

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[8] Praun, *Syntax des Vitruv*, p. 43, has also urged that in the whole anecdote Vitruvius is following a Greek source.

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[9] Of course he might have cited others: see Cooper's lists.

Continuing his remarks about abstracts, Ussing says: 'Striking plurals are *conscriptiones* (103, 14; 155, 10), *eruditiones* (2, 18; 36, 23), *scientiae* (10, 24; 62, 23; 233, 2), *sollertiae* (158, 12).' — Here we need only remark that *conscriptiones* occurs in Cicero, *Cluent.* 191, and *scientiae* in Cicero, *D. O.* 1, 61; *C. M.* 78, *conscientiae* in Cic., *R. A.* 67. In the last two passages in Cicero the plurals are no doubt influenced by the other plurals in the passage (*C. M.* 78: *tot artes, tot scientiae, tot inventa; R.A.* 67: *suae malae cogitationes conscientiaeque animi terrent*), and the same may be observed in the Vitruvian usages of this plural and of *eruditiones* and *sollertiae*[11]; cf. the similar use of *eruditiones* in Gell. *praef.* 3. But why delay over such a point? The use of the plural of abstracts, though great in late authors, is no proof of late authorship, for it is found at all periods: 'besonders bei Plautus in verhältnissmässig grosser Zahl; in klass. Zeit erweitert sich dieselbe wesentlich durch Cicero' (Schmalz, Lat. Gramm.,^{3} p. 431). Seneca (*Ep.* 114, 19) criticises the plural *famas* in Sallust and his imitator Arruntius. See also a list of the plurals used by Mela, in Zimmermann, *De Pomponii Melae sermone*, p. v ff.

Neither is a late date assured by the usage to which Ussing next draws attention: 'Sometimes these abstract nouns retain so much of their verbal character that the author finds it sufficient to add only *est* instead of *factum est*, as in *cum fuerit fundamentorum ad solidum depressio* (15, 19), and *cum erit moenium conlocandorum explicatio* (20, 24).' — See Schmalz again, p. 430, where this use is shown to be not foreign to Cicero, and cf. also Cic. *Pis.* 84 (*accessio*), *Rab.* 4 (*consensio*), *Cat.* 1, 32 (*consensio*).

Ussing's next point appears to be based upon a misunderstanding. He says: 'One of the words frequently occurring in Vitruvius is *symmetria;* according to Nohl's *Index*, it is found about a hundred times. At the time of Pliny this word is still a stranger to the Latin language; comp. *Hist. Nat.* 34, 65: *non habet Latinum nomen symmetria.* Pliny no doubt appreciated his own Latin style, but he does not carry his purifying tendencies so far as to exclude every foreign word, if it was generally adopted in the language; his apology testifies to the fact that such was not the case with *symmetria.*' — Here, as I observed, Ussing seems not to understand Pliny's meaning. He was writing of Lysippus and of the greater grace and freedom from bulkiness which this sculptor exhibited in the bodies of his statues, 'by which they were made to seem taller.' Then he adds: *non habet Latinum nomen symmetria quam diligentissime custodit,* that is: 'there is no Latin word for that symmetry which he observed so carefully.' What Pliny says is therefore no condemnation of the use of the word *symmetria*, which indeed he himself employs in three other passages (34, 58: *in symmetria diligentior*, a comparison of Myron and Polyclitus; 35, 67: *Parrhasius primus symmetrian picturae dedit*; 35, 128: *Euphranor primus videtur usurpasse symmetrian*), but a definite statement that when a Latin writer is talking about 'symmetry,' he must use the Greek word. Now 'symmetry' is one of the very points upon which Vitruvius most insists in every department of the architect's profession. Near the opening of his work, he mentions it as one of the six components of good architecture (11, 12), and soon afterwards he devotes ten lines to a definition of what it is (12, 14). Having done this, even the earliest of Latin prose writers would be fully entitled to employ the word as often as he chose. If it is not found earlier than Vitruvius, this is simply because of the accident that there is no Latin work extant in which there was so much occasion to speak of 'symmetry' in the technical sense.[12]

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[10] See also on *symmetria*, p. 474, n. 12.

[11] It must also be observed that *sollertiae* in 158, 12, means 'instances of skill'; cf. Cic. *Q. F.* 1, 1, 39: *iracundiae*, and 40: *avaritiae*. The whole passage is misunderstood by the translators. It means 'by compiling from antiquity remarkable instances of the skill shown by genius.'

Leaving the subject of abstracts, Ussing next takes up another topic in which he is equally unfortunate. 'Not infrequently,' he says, 'words are found in a different connection and different signification from that of the classical authors. Thus *notitia* in the sense of "renown" (63, 6; 133, 6), *ponere* "put forth" (64, 30), and *anteponere* "put forth at first" (33, 4 and 10); *dignum est* for *operae pretium* (46, 6); similar things are quoted from Vopiscus, Lactantius, and Augustinus; *necessitate* = *necessario* (246, 3).' — By the phrase 'classical authors' Ussing must, for the sake of his argument, be taken as meaning authors writing in the classical period, no matter what their reputation for style or lack of it may be. Therefore we are entitled to point to *notitia* meaning 'renown' in Nepos, *Dion*, 9: *Hi propter notitiam sunt intromissi.* In poetry it is found thus in Ovid, *Pont.* 3, 1, 50; 4, 8, 48. Ussing's example of *ponere*, in the sense of 'put forth,' disappears, since it is an emendation for *exponere*, adopted by Rose in his first edition but rightly abandoned in his second. As for his example of *anteponere,* it should be written as two words, *ante ponere* (so Rose[2]; cf. Cic. *Fam.* 1, 9, 21: *ut paulo ante posui*), and the Vitruvian employment of *pono* in these two places should be compared with the common colloquial usage of it, as for example in Cic. *Fin.* 2, 31; *Legg.* 2, 6; Livy, 10, 9, 12. For the use of the impersonal *dignum est* in the sense of *operae pretium,* it would not be difficult to find examples (cf. for instance Plaut. *Ps.* 1013, and, with *indignum,* Sall. *Iug.* 79, 1), but the real peculiarity in the Vitruvian usage is that *ut* with the subjunctive follows, the whole sentence being: *quae si prope urbem essent, dignum esset ut ex his officinis omnia opera perficerentur.* This impersonal usage does not indeed seem to occur before the very late authors mentioned by Ussing (cf. Dräger, II, 258). A very similar employment of the personal *digna* is, however, found in Livy 24, 16, 19: *digna res visa ut,* etc., where of course the relative construction would be as impossible as in the Vitruvian sentence.[13] Finally, of *necessitate* used in the sense of *necessario,* it must be admitted that this cannot be paralleled in or before classical times, and that the employment of the ablative of an abstract instead of an adverb is one of the characteristics of African Latin (Sittl, *die lokalen Verschiedenheiten der lateinischen Sprache,* p. 107). It has in fact been observed that many stylistic peculiarities that are found in the African writers occur also in Vitruvius (Praun, p. 13, n.). However, if the ablative of any abstract is allowable instead of an adverb it would surely be *necessitate;* cf. Caesar's *qua necessitate adductus, B. G.* 6, 12, 5; *qua necessitate permotus, B. C.* 3, 24, 4, with the pleonastic *necessitate coactus* of *Bell. Afr.* 55, 2 (cf. 21, 1; 24, 4), which is like *necessario coacti* in Ter. *Andr.* 632; *Bell. Hisp.* 24, 2; 32, 1. This pleonasm with *necessitas* is common in Vitruvius.

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[12] It may be worth observing that Vitruvius employs his new formation *commensus* in contexts along with *symmetria*, as if perhaps he felt that the Greek term needed some help from Latin: see 15, 25; 31, 3; 134, 11; 138, 23 and 27.

Ussing's next remark, as he himself seems to be conscious, is of no value as proof of late authorship: 'In a few instances *videtur* is meant to signify *placet: magnitudines balinearum videntur fieri pro copia hominum* (126, 11); *itaque minime fistulis plumbeis aqua duci videtur* (210, 13). In other places Vitruvius correctly adds *oportere,* so that the omission might perhaps rather be called a peculiarity of style in the author, as in *primo volumine putavi . . . exponere* (36, 23).' — But this use of *videtur* cannot be called a peculiarity of Vitruvius nor evidence of late authorship, for the passive of *video* in the sense of *placet* or δοκεῖ occurs three times in the *Bellum Africum* (5; 25, 1, 42, 1). Of *putari*

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[13] For the great variety of constructions with *dignus* in Vitruvius, see my article in *Harvard Studies in Classical Philology,* **17**, 1 f.

exponere it might be thought that as the verb *oportere* has occurred in the foregoing sentence and as it occurs again in the following sentence, its omission with *putavi* may be excused without danger of misunderstanding. Or perhaps we have here a use analogous to that of *cogito* in the sense of 'intend' followed by the infinitive, found frequently in the letters as well as in other works of Cicero.[14] However, as Ussing himself observes, the usage may be attributed to the author himself rather than to the habits of a late period of Latinity to which it has not been shown to belong.

Ussing's next observations would be very striking indeed, if they were found to bear examination; but this is not the case. 'Shall we consider it merely accidental that the word *narrare*, which was generally used during the classical period, does not occur at all in Vitruvius, who only uses *memorare;* or that the verb *ire* (without prefix) appears but once, whereas we frequently find *vadere*, which in Cicero means 'to depart,' and only in Virgil and Ovid signifies 'to go,' thence entering into the later prose and subsequently into the Romance languages, entirely superseding the genuine Latin word?' — The first of these observations is misleading. It is true that Vitruvius never uses the verb *narro* (in any form), but on the other hand he never uses the active voice of the verb *memoro*. He has the verb twelve times, always in the passive. Once it is used absolutely: *mors eius . . . varie memoratur* (158, 3). Five times it is used with a personal subject and the active infinitive: *is memoratur dixisse* (62, 17; cf. 161, 18; 280, 18; 42, 27; 43, 6). Six times it is used with a personal subject and the passive infinitive: *inventio sic memoratur esse facta* (86, 21; cf. 177, 2; 199, 19; 231, 15; 272, 22; 156, 5). Now suppose that *narratur* or *narrantur* were found in these eleven passages: we should at once be told that here was evidence of late authorship, for this is a usage which, beginning with Livy, is found in the Plinys, and is prevalent in late Latin (Schmalz, *Antibarbarus*,^{3} s. v. *narrare*). That it does not occur in Vitruvius, therefore, is significant of an early period, if it is significant at all. But his use of the passive of *memoro* is classical, though rare: cf. Cic. *V.* 4, 107: *ubi ea gesta esse memorantur.* It appears to be nothing more than a bigger word for *dicitur*, and Praun (p. 7) remarks: 'Vitruv hat wohl nach Art der Halbgebildeten den landläufigen Ausdruck vermieden, um durch ein selteneres Wort seiner Rede ein schöneres Kolorit zu geben.' Next let us examine the case of *vado* and *ire*. To begin with, it is not true that

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[14] See Stinner, p. 54 f.

'only in Virgil and Ovid' does *vado* signify 'to go.' For cf. Ennius, *A.* 281 M.; *vadunt solida vi; A.* 591: *ingenti vadit cursu; Auct. Herenn.* 2, 29; *cum feras bestias videamus alacres et erectas vadere;* Catullus 63, 31: *vaga vadit* (sc. *Attis*); 63, 86: (*leo*) *vadit fremit refringit virgulta pede vago;* Sallust, *Iug.* 94, 6: *Romani instare, fundere ac plerosque tantum modo sauciare, dein super occisorum corpora vadere;* Cic. *T. D.* 1, 97: *vadit enim in eundem carcerem atque in eundem paucis post annis scyphum Socrates.* In all these passages we find *vado* used in the sense of 'go' rather than 'depart,' but the 'going' indicated in them is something more than is meant by the every day sense of that word; for something rather more grand is intended. The English 'move' would be a better translation. Here it is interesting to compare with the Ciceronian passage Livy 2, 10, 5, where of Horatius Cocles he says: *vadit inde in primum aditum pontis*, and Weissenborn-Müller notes: 'er geht mit gewaltigem Schritte, μακρὰ βιβάς.' See also Livy 6, 8, 2 and 7, 24, 6. Finally we have *vado* in two letters of Cicero: *Att.* 4, 10, 2: *ad eum postridie mane vadebam cum haec scripsi; Att.* 14, 11, 2: *Lentulus Spinther hodie apud me. Cras mane vadit.* I believe that I have now cited all the Ciceronian passages in which the simple *vado* occurs, and it seems probable that when Ussing speaks of *vado* as meaning 'to depart' in this author, he is thinking of the two occurrences in the letters. But it is obvious that in them it is only the context that authorizes the translation 'depart,' which would have applied equally well to *iturus eram* for instance, if it had stood in the former of them. And on the latter Tyrrell and Purser suggest the translation 'passes on his way,' adding: 'There is a slight poetical colour about this word; cf. Stinner, p. 16.' Having thus prepared ourselves to understand the meaning of *vado*, let us turn to Vitruvius. We are told that he uses *ire* only once but *vadere* 'frequently.' The fact is that he uses a form of the verb *vado* five times. But never was there a case in which statistics were more misleading if we conclude from them, without examining the contexts, that to Vitruvius *vado* and *eo* were synonyms, and that he uses *vado* in the every day sense of *eo*. At the outset we must remember that Vitruvius is not an historian, orator, or dramatist, and that consequently we should not expect to find the verb *eo* used often by him: he has little occasion to speak of anybody as 'going' anywhere in the usual sense. This observation alone would be sufficient to account for the absence of the simple verb *eo* from his work. Now how does he employ the verb *vado?* Five times he has the simple verb. Of these occurrences, three refer to movements of the sun or moon: 220, 13: *sol autem signi spatium quod est duodecuma* *pars mundi mense vertente vadens transit;* 240, 2: *itaque quemadmodum sol per siderum spatia vadens dilatat contrahitque dies et horas;* 225, 4: *cum* (sc. *luna*) *praeteriens vadat ad orientis caeli partes.* In these three passages we have no common 'going,' but the grand movement of heavenly bodies, and it is worth observing that Cicero never uses the simple verb *eo* of movements of the sun, moon, or stars in his orations or philosophical works. He has *elabor*, *vagor*, *erro*, and the compounds *accedo*, *antecedo*, *discedo*, *recedo*, *anteverto*, *peragro*, *subsequor*, *abeo*, *adeo*, and *obeo*[15] (see Merguet's Lexicons s. vv. *sol*, *luna*, *stella*). The other two passages in which Vitruvius uses the simple verb *vado* are both in prefaces, in which, as is well known, our author often aims at a higher style than in the body of his work. The first is 132, 8: *at qui non doctrinarum sed felicitatis praesidiis putaret se esse vallatum, labidis itineribus vadentem non stabili sed infirma conflictari vita.* Here the picture of the foolish man who depends on luck rather than on learning, 'moving in slippery paths,' is appropriately colored by the use of *vadentem.* The second is 215, 25, where in the famous anecdote about Archimedes it is said: *exsiluit gaudio motus de solio et nudus vadens domum versus significabat clara voce invenisse quod quaereret.* Here the use of *vado* is like that which is found in Cicero's letters as cited above (p. 477). It appears, therefore, that there is nothing in Vitruvius's use of the simple verb which is at variance with classical examples. On the contrary, Ussing would have been more fortunate had he criticised the single occurrence in Vitruvius of the simple verb *eo*, 220, 11: *luna . . . caeli circumitionem percurrens ex quo signo coeperit ire ad id signum revertendo perficit lunarem mensem;* for we have seen that it is not Ciceronian to employ this simple verb of the movements of heavenly bodies. But how about the Vitruvian use of the compounds of these verbs? Here the statistics tell the opposite tale, for he has compounds of *eo* (*ad-*, *ex-*, *in-*, *prod-*, *red-*, *sub-*, *intro-*) fifty-six times and compounds of *vado* only twice, in each case with *per-* (221, 24: *Saturni* (sc. *stella*) . . . *pervadens per signi spatium;* 226, 21: *sol signa pervadens*). Both of these are descriptive of the movements of heavenly bodies, and the compound *pervado* is Ciceronian (e. g. *V.* 3, 66; *N. D.* 2, 145). To conclude: Vitruvius's use of *vado* and *-vado*, six times in the present participle and once in the form *vadat*, is shown by an examination of the contexts to be no proof of late authorship.

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[15] That Vitruvius also uses compounds of *eo* may be seen, for example, from two of the passages just cited.

To pass on to Ussing's next point: 'Is it accidental that, after the fashion of more recent authors, Vitruvius frequently transcribes the simple future by *erit ut?* e.g. 7, 10: *erit ut uterque liberetur.* 130, 27: *ita erit uti possit turris insuper aedificari;* 144, 9: *tunc erit ut . . . fiant.* Dräger, *Hist. Synt.* 2, p. 267, quotes a similar example from Apuleius, *Met.* 2, 3: *nunquam erit ut non apud te devertar.*' — This observation is drawn from Praun (p. 51), who cites two other cases (28, 9: *tantum erit uti . . . habeant;* 92, 16: *erit ut emendentur*), and remarks that Vitruvius has only twice used the classical (though rare) present tense *est ut.* There is however an earlier occurrence of *erit ut* than that of Apuleius; cf. *Rhet. ad Herenn.* 4, 41: *Sed non erit, tamquam in plerisque, ut, cum velimus eā* (sc. *exornatione*) *possimus uti.* We have, therefore, no evidence of 'the fashion of recent writers' in the Vitruvian passages, particularly when we consider that Apuleius is the only 'recent writer' cited in this connection, and that his use of *erit ut* is negatived. So is the use in the *Rhet. ad Herenn.*, while the Vitruvian uses are all positive. But while the present tense *est ut* is usual in periphrases, we also have *fuit ut,* Cic. *Cael.* 48, and why then should we be surprised at *erit ut* (not exactly paralleled elsewhere) in a writer like Vitruvius?

Ussing proceeds: 'With regard to the comparison of adjectives, we often find the comparative unnecessarily emphasized: *maxime facilius* (3, 23), *maxime tutiores* (22, 15), *maxime utiliores* (38, 15), *quo magis ex meliore vino parabitur* (180, 22), *potius digniores* (134, 1). Compare *nimium penitus* (211, 7). Similarly Lactant. *Instit.* 1, 21, 10: *maxime dulcior.* Commodian, *Apolog.* 5: *plus levior.* Sulpicius Severus, *Chron.* 2, 46, 5: *plus iusto inflatior.*' — Here we may begin by pointing out that the example with *potius* (134, 1) is not like the others on account of the following *quam,* the context reading thus: *iudicant . . . ipsos potius digniores esse ad suam voluntatem quam ad alienam pecuniae consumere summam.* With this cf. Nepos 9, 5, 2: *potius patriae opes augeri quam regis maluit;* Cic. *D. O.* 2, 300: *cum quidem ei fuerit optabilius oblivisci posse potius quod meminisse nollet quam quod semel audisset vidissetque meminisse.* Next for the example with *magis* we have early parallels in Plautus (e. g. *Capt.* 644; *Men.* 978, and see Wölfflin, *Comparation,* p. 46); in the classical period in the *Bellum Africum,* 48, 3: *magis suspensiore animo;* 54, 5: *magis studiosiores,* and in the time of the Emperor Claudius in Pomponius Mela 2, 86: *magisque et magis latior.* For *maxime* with a comparative I know of no instances before very late Latin, but it ought not to surprise us in Vitruvius, because, as Wölfflin has remarked (p. 47, cf. 63 ff.) in the case of the example from Lactantius cited by Ussing, these are instances in which the comparative has lost its force and is used like a positive. No reader of Vitruvius is unfamiliar with this frequently recurring phenomenon (see e. g. Praun, p. 80). Finally I fail to see how the example *nimium penitus* (211, 7) figures among emphasized comparatives. It means 'too deep.' For *penitus* modified by another adverb, see Cic. *Clu.* 4: *tam penitus; V.* 2, 169: *bene penitus;* and examples of *nimium* modifying an adverb are not uncommon (cf. e.g. Cic. *Cat.* 1, 10: *nimium diu*).

Next we find: 'The superlative is repeatedly placed parallel to a positive in such a way that the difference is effaced: 53, 12: *si sit optima seu vitiosa;* 188, 12: *quae gravissimae duraeque et insuaves sunt partes.* Of course there are cases where no harm is done by such a juxtaposition, and where it may occur even in classical authors; see Wölfflin, *Comparation*, p. 54 f.; but this is not the case here.' — The selection of the two Vitruvian examples is not very fortunate, because it might be thought, particularly in the first, that the difference is not 'effaced.' He is there recommending the use of the 'best' brick, and this is contrasted with brick which is 'faulty,' though not necessarily the 'worst.' In the other example, the foregoing clause should be observed. However, what Ussing really means to criticise is the lack of symmetry shown in the coupling of a positive with a superlative, a lack of which Cicero and writers of his taste would not be guilty, and for this purpose better examples had been 24, 6: *parvo brevissimoque;* 83, 15: *dignam et utilissimam;* and others cited by Praun (p. 79). This unsymmetrical coupling is to be sure found very often in late Latin, particularly in the Africans,[16] but we must not think that there is no trace of it in early or Augustan Latin. Thus we find: Plaut. *Rud.* 1321: *miserum istuc verbum et pessumum;* Ter. *Ph.* 226: *iustam facilem optumam;* Sall. *Or. Lepidi* 1: *maxumi et clari estis;* Dec. Brutus ap. Cic. *Fam.* 11, 19, 2: *seditiosum et incertissimum.* And a little later, in Velleius 2, 69: *acri atque prosperrimo bello.* We have even the comparative and superlative joined in *Bell. Afr.* 56, 2: *inlustriores notissimique*, formerly emended away by Wölfflin, but allowed in his edition of 1896.

The next set of evidences which Ussing presents is as follows: 'Among the adverbs may be mentioned *aliter*, not in the sense of "otherwise," but "differently from one another"; 33, 24: *in eo hominum congressu cum profundebantur aliter e spiritu voces;* cf. 218, 23: *itaque longe aliter distant descriptiones horologiorum locorum mutationibus; forte* =

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[16] See Sittl, *die lokalen Verschiedenheiten*, p. 101 ff.

fortasse: 133, 3: *Sed forte nonnulli haec levia iudicantes putant,* etc.; *parve:* 229, 14: *parve per eos flectitur delphinus; temperate* (with genitive as *parum*); 18, 6: *volucres minus habent terreni, minus umoris, caloris temperate, aëris multum,* cf. 45, 20: *umoris autem temperate;* 57, 4: *umoris temperate;* 57, 21: *terreni temperate.*' — Here it must first be observed that although *aliter* is strangely used by Vitruvius in the two passages cited,[17] yet since no parallel is quoted by Ussing or Praun[18] from a late author, this again must be set down as a peculiarity of the style of Vitruvius[19] (see above, p. 475). Of *forte* in the sense of *fortasse,* I know no occurrence in prose before or in the Augustan age. Besides 133, 3 (cited by Ussing), we find it in 116, 7: *dicet aliquis forte.* It also occurs unobjectionally with *si* in 24, 10 and 184, 22; and not in the sense of *fortasse* twice; 168, 13 and 176, 12. In two out of six occurrences Vitruvius violates the approved usage and writes like a late prose author. But it should not be forgotten that a poet of the best period used *forte* thus: cf. Hor. *Epod.* 16, 15: *forte quid expediat quaeritis.* As for 'the adverb *parve,*' no student of Vitruvius should be willing to base any statement about style on the obviously corrupt passage in which it appears in the manuscripts (see Rose's apparatus criticus, and Kaibel, *Hermes* 29, 95; Thiele, *Himmelbilder,* 55). Of the Vitruvian usage of *temperate* (in itself a perfectly good Ciceronian adverb) with the genitive, three things are to be remarked: first, that it cannot be used as evidence of late authorship, because no late author is cited as employing it; second, that it is not in meaning the equivalent of *parum,* for in 57, 4 the words *umoris temperate* are followed by *parum terreni* (cf. also 45, 20); third, that the genitive with *temperate* is evidently due to the influence of the other perfectly regular genitives with *minus*, *parum*, *minimum*, *multum*, which are found in the contexts of the four passages under consideration.

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[17] And in 14, 24: *cum ad usum patrum familiarum aut ad pecuniae copiam aut ad eloquentiae dignitatem aedificia aliter disponentur.* Here the best MSS. have *alte,* but the emendation (found indeed in L) is certain. Vitruvius has *aliter* elsewhere 15 times in the usual applications.

[18] Or cited in the *Thesaurus,* where Vitr. 33, 24 is not included at all, and where the peculiarity of 218, 23 is overlooked; see *Thesaurus,* s. v. *alius,* p. 1653, 52.

[19] The nearest resemblance is Seneca *Q. N.* 4, praef. 22 as it is quoted in the *Thesaurus,* p. 1656, 40: *uno enim tempore (Sicilia) vidit Pompeium Lepidumque ex maximo fastigio aliter ad extrema deiectos, cum Pompeius alienum exercitum fugeret, Lepidus suum.* Editions here with MSS. cited in them have *aliter aliterque.* Some good reason for the reading in the *Thesaurus* will, I suppose, be given by Gercke who made the excerpts from this work of Seneca's for it, in his forthcoming edition of the *Q. N.* But it seems to me that, with this reading, the passage is erroneously placed in the *Thesaurus* under the caption *aliter et (-que).* Another use of *aliter* in the sense of 'differently' is found in Pomp. Mela 1, 57: *multo aliter a ceteris agunt.*

Still speaking of abverbs, Ussing continues: '*iuxta* = *secundum*, "according to," 10, 23: *iuxta necessitatem*. The same occurs in Justinus and later. *Trans* without an object, "on the other side," 220, 1: *circumacta trans locis patentibus ex obscuris egreditur ad lucem*, elsewhere in clerical authors, cf. *Archiv*, IV, p. 248. *Trans contra*, "opposite to," 219, 7 and 225, 13, as in Aurelius Victor and Boethius, cf. *Archiv*, V, p. 319 ff.'—The context in which the strange phrase *iuxta necessitatem* occurs, is as follows: *cum . . . ratio propter amplitudinem rei permittat non iuxta necessitatem summas sed etiam mediocres scientias habere disciplinarum*. This is certainly a badly expressed sentence, and we may observe the usage of *permitto* with the infinitive as found in Livy, later historians and ecclesiastical writers, which would be stamped as vulgar did it not occur once in Cicero (*Verr.* 5, 22), and also an accumulation of plurals of abstracts such as a polished writer would have avoided. The phrase *iuxta necessitatem* occurs nowhere else to my knowledge, but the word *necessitas* is a favorite one with Vitruvius (twenty-seven times, according to Nohl's *Index*; cf. especially the phrase *ad necessitatem* in 260, 21 and 266, 3), and the use of *iuxta* in the sense of 'conformably to,' 'as the result of,' 'gemäss,' besides here, is found first not in Justinus but in Livy 39, 9, 6: *huic consuetudo iuxta vicinitatem cum Aebutio fuit* (see Schmalz, *Lat. Gramm.*,^{3} p. 263). In Vitruvius the phrase must mean 'of necessity,' 'necessarily,' but to say just what it modifies is a difficult matter.[20] In his observation about *trans*,

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[20] Generally it has been taken with *summas*, but, so taken, Vitruvius would be saying that an architect need not possess 'necessarily the highest' but only a moderate knowledge of all the arts and sciences which he has mentioned in §§ 3–16. What follows, however, would seem to show that he feels that practically the architect cannot be expected to have even a moderate amount of knowledge of them all. The reading of S^c is perhaps therefore worth consideration, especially in view of Degering's estimate of the value of this manuscript (*Berl. Phil. Woch.* 1900, p. 9 ff.); for here we find *iuxta necessitatem* standing not before *summas* but before *mediocres*: *non summas sed etiam iuxta necessitatem mediocres*. And we may go further, for my friend Professor A. A. Howard has suggested that a second *non* appears to be lacking in the clause *sed . . . mediocres*. If Vitruvius was written in lines of from 17 to 20 letters, like Livy, perhaps here originally stood:

NONSVMMASSEDETIAM
NONIVXTANECESSITATEM
MEDIOCRESSCIENTIAS

Then the accidental omission of the second line by the scribe of the archetype of

Ussing has certainly pointed to a misuse of that word which is not found elsewhere before the ecclesiastical writers. This preposition was originally a participle (Thielmann, *Archiv*, IV, 248), not an adverb like other prepositions, and we have no early parallel of its employment as an adverb, though we might expect to find it in the less careful writers from analogy with the adverbial use of other prepositions. In Vitruvius, *trans contra* seems to be a translation of καταντικρύ, especially in 219, 7, where he had in mind the pseudo-Aristotelian *de mundo*, 2, or a similar account of the πόλοι. It may also be observed that Vitruvius uses *intra* as an adverb half-a-dozen times (see Nohl's *Index*), a usage commonly called post-Augustan, but found in *Bell. Hisp.* 35, 2 (Köhler, *Act. Erlang.* I, p. 400); also *adversus* five times as an adverb, — found thus in prose not elsewhere before Nepos (*Thesaurus*, s. v. p. 851, 48 ff.). And we must be slow to stamp *trans contra* as a necessarily late doublet lest we meet with the fate of that 'grammaticus haud incelebri nomine' in Gellius (19, 10), who sneered at *praeterpropter* only to be confounded by learning that it had been used by Ennius, Cato, and Varro.

Next Ussing turns to prepositions, saying: 'In the use of prepositions we are struck by several peculiarities which indicate the dissolution of the language: *ab*, indicating the cause, "because of," in 58, 1: *ab pondere umoris non habent rigorem . . . ab lentitudine firmas recipiunt catenationes*; 59, 6: *ab suci vehementi amaritate ab carie aut tinea non nocetur. Ab*, "compared with," has been — no doubt correctly — substituted by Rose for *ad* in 142, 2: *non enim atria minora ab maioribus easdem possunt habere symmetriarum rationes*, a habit which Wölfflin in *Archiv*, VII, p. 125, has proved to exist in the ancient Latin translations of the Bible, Itala, and Vulgata, and which is analogous to the use of other prepositions such as *prae, super* or *supra, ultra*.' — These criticisms may be briefly dismissed. A glance at the *Thesaurus* s. v. *ab*, pp. 33–34, will be enough to show that the use of this preposition to denote cause is no evidence of the 'dissolution of the language' unless the language began to dissolve with Lucretius, Varro, Livy, and the Augustan poets. The other criticism, about *ab*, 'compared with,' is taken from Praun (p. 79), who, by an oversight foreign to his usually careful work, has misinterpreted the passage. There is no idea of comparison here, for *ab maioribus* does not depend upon *minora*. The sentence means: 'In the case of smaller

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our manuscripts and its insertion in the margin might give rise to the differences found in HG on the one hand and Sᶜ on the other. The restoration of this second *non* gives to the passage the meaning which Eberhard (*de Vitruvii genere dicendi*, I, p. 9) desired to find in it, though with his reading this would not be possible.

atriums the symmetrical proportions cannot be the same as in larger.' See the *Thesaurus* s. v. *ab*, pp. 39, 55.

'*Ad* is placed instead of the dative or parallel with it, as in 91, 3: *metopae quae proximae ad angulares triglyphos fiunt;* 182, 4: *hae regiones sunt proximae ad septentrionem* (equally by Euodius in Augustine *Ep.* 158, 2: *ad finem vitae proximus*); 147, 1: *lavationi rusticae ministratio non erit longe*, but soon after: *ad olearios fructus commoda erit ministratio.* Equally in 256, 16: *ita hortis ad inrigandum vel ad salinas ad temperandum praebetur aquae multitudo;* 251, 18: *ut ad solvendum non esset*, in lieu of the generally applied *solvendo.* "On the whole," Praun observes on p. 65, "the preposition *ad* with the gerund or the gerundive has extended its sphere at the expense of the other constructions, the genitive, the dative, and *in* with the ablative." ' — The use of *proximus* with *ad* and the accusative is found much earlier than Euodius; cf. Varro, *L. L.* 6, 8: *ad nos versum proximum est solstitium;* Lucr. 2, 135: (*ea corpora quae*) *proxima sunt ad viris principiorum;* Pliny, *N. H.* 2, 64: *ad terrae centrum humillimae atque proximae.* We have also *proprius ad* in Cicero, *Fin.* 4, 64. It must not be thought that this is the only construction with *proximus* found in Vitruvius. He has the simple dative twenty-one times, and *ad* with the accusative only three times (add 135, 11 to Ussing's examples). In his second set of examples under this head of the use of *ad*, Ussing (following Praun, p. 89) seems to think that we have two constructions with *ministratio erit*, first the dative and then *ad* and the accusative. But this latter belongs to *commoda*, and the construction is that which is found twice on the preceding page (146, 6: *ad omnes res commoda;* 146, 14: *ad usum commoda*). Though elsewhere rare, yet we have in Caes. *B. C.* 3, 100, 3: *tempore anni commodiore usus ad navigandum*, and in Ovid, *F.* 2, 288: *nec satis ad cursus commoda vestis erat.* It cannot therefore be held to be a sign of the 'dissolution of the language.' In the third set of examples (256, 16) Ussing with Praun (p. 64) seems to have taken *hortis* as a dative, and to have thought that with *praebetur* we have both a dative and an accusative with *ad.* But it seems far more probable, if not certain, that we have here two locative constructions: *hortis*, 'in gardens' (for Vitruvius's use of the locative ablative of many appellatives, see Nohl, *Anal. Vitr.*, p. 10, and observe that only eight lines below our passage he has the locative ablative *locis* with *praebendum*, 256, 24: *sin autem magis altis locis erit praebendum*), and *ad salinas*, 'at saltworks.' It is true that I do not find the locative phrase *ad salinas* in any other writer, but this is mere accident, for it is an expression which belongs in the class of other locative phrases with *ad* cited in the *Thesaurus*, p. 522 f.[21] And Vitruvius has this use of *ad* elsewhere: e. g. *ad villas* (148, 9), *ad circum, ad campum, ad portum* (30, 12 f.). It is worth noting that by another accident *ad campum* (sc. *Martium*) seems not to occur elsewhere in literature, but that it is found in the *Monumentum Ancyranum*, 2, 40. The variation in the locative expressions, from *hortis* to *ad salinas* is Vitruvian: see e.g. the considerable variety in 30, 7–22; also *in gymnasio . . . foro* (174, 10); *ad villas . . . in urbe* (148, 9-11); *in montibus aut ad ipsos montes* (188, 18). Next Ussing's fourth example under this head, *ut ad solvendum non esset*, presents the unique *ad solvendum* instead of the common dative *solvendo* (found for instance in Cic. *Phil.* 2, 4; *Off.* 2, 79; *Att.* 13, 10, 3; *Fam.* 3, 8, 2; and in the jurists). What should be inferred from this? That our Vitruvius is a late writer? Not at all, for no late writer is cited as using *ad solvendum*. It is a peculiarity in Vitruvius and nothing more. Of the same sort is that peculiarity in Cicero's letters when he uses twice *esse ad scribendum* (*Att.* 1, 19, 9; *Fam.* 12, 29, 2) instead of the common *scribendo adesse* (for which see the *Thesaurus*, s. v. *assum*, 918, 43 ff., and Cicero himself in the second passage just cited). And a glance at the context of Vitruvius shows why he used the peculiar *ad solvendum*. It runs thus: *Sic Paeonius ducendo et reducendo pecuniam contrivit ut ad solvendum non esset.* Obviously the usual dative *solvendo* was avoided for fear of obscurity on account of *ducendo* and *reducendo*. Finally, with Praun's general observation cited by Ussing we need not trouble ourselves here, for of course Praun never meant it to be taken as evidence of the late authorship of Vitruvius.

'*De* instead of the simple ablative in 1, 16: *parenti tuo de eo fueram notus.* Likewise *e* in 3, 22: *circini usum, e quo maxime facilius aedificiorum expediuntur descriptiones.*' — But casual *de* is in itself no proof of recent authorship, and the use of it as denoting 'den Erkenntnisgrund' is one of Dräger's categories (I, p. 630) illustrated by him with examples from Plautus and Cicero, to which may be added *Rhet. Herenn.* 4, 44 *res tota parva de parte cognoscitur.* Furthermore, in the passage cited from Vitruvius, the simple *eo* could hardly have been written without danger of obscurity on account of *parenti tuo.* The use of *e* with the ablative instead of a simple instrumental may seem lumbering and awkward in 3, 22; but that it was not unknown to the classical period is obvious from its appearance in Cicero, *Rep.* 2, 58: *ex aere alieno commota civitas* as well as several times in *Bell. Afr.* as cited by Köhler, *Act. Erlang.* I. p. 439. See also Pomp. Mela, 2, 21.

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[21] Cf. also Livy's *circa Romanas salinas* 7, 19, 8; also *ad gallinas*, Plin. *N. H.* 15, 137; Suet. *Galba*, 1.

Passing next to conjunctions Ussing says: 'With regard to conjunctions, Dräger (II, p. 153) has already pointed out that *aut* and *sive* are used quite indiscriminately by Vitruvius. A critic in the *Athenaeum*, Jan. 1, 1898, says: "the misuse of *aut* or *sive* is no great matter." I had not expected this declaration from "a skilled reader." Most Latin scholars would have the contrary view.' — But the remark of the critic in the *Athenaeum* must not be judged apart from its context. He does not mean that the confusion of *aut* and *sive* is no great matter as a point of style, or that it would be found in a polished writer. His whole contention is that one should expect to find such errors in unpolished writers, and that consequently this error cannot be used in settling the date of Vitruvius. And this contention is borne out by the facts found in the *Thesaurus* in the treatment of the use of *aut*. Dräger, also, in the passage cited by Ussing, shows how the Elder Pliny employs *aut* and *sive* as synonyms, so that this confusion cannot be held to be evidence of very recent authorship. And for the Vitruvian employment of *aut . . . sive* or *sive . . . aut* in the same sentence, parallels are quoted from the *Aetna*, from Manilius and from Celsus in the *Thesaurus* (s. v. *aut*, p. 1571, 11 ff., and 78; cf. the somewhat similar *seu . . . aut* in Plautus, *Ps.* 543, cited on p. 1570, 56), with the following general remark on such combinations in prose writers, p. 1571, 55: 'increbrescunt apud eos qui poetarum sermonem etiam alias imitantur et apud minus cultos (Vitr. Cels.).'

Ussing proceeds: 'Equally unclassical is the use of negatives in sentences consisting of two alternatives. The word *neve* does not occur in Vitruvius. He always puts *ne . . . neque* instead of *ne . . . neve*, as 5, 16: *ne sit cupidus neque in muneribus accipiendis habeat animum occupatum.* As for negations, it is also to be observed that he likes to place them foremost in the sentence. He says *non putavi praetermittendum* (1, 14) instead of *putavi non praetermittendum; non puto dubium esse* (124, 1), etc. This is done occasionally in other authors, but in Vitruvius very frequently. A striking example is 48, 22: *non enim quae sunt e molli caemento subtili facie venustatis, non eae possunt esse in vetustate non ruinosae.*' — With regard to Ussing's first point, it is sufficient to quote Schmalz, *Lat. Gramm.*,[3] p. 358: 'Selten ist die Anknüpfung mit *nec* statt mit *neve*; bei Cicero wird *nec* nach *ne* nie angetroffen (vgl. C. F. W. Müller zu Cic. *Off.* 1, 91), auch nicht bei Caesar und Sall., aber bei Nepos, bei Vitruv, und Sen. Phil., welche *neve* gar nicht kennen, bei Liv., Flor., nach Liv. vereinzelt, häufig bei Dichtern, so schon bei Plaut., bei Verg., Hor., Ov. u. a.' It is obvious that we have here what may be called a distinct division on a point of style. Though the Ciceronian must be taken to be the better, yet we see that late authorship cannot be proved from the other usage. On the second point, the setting of negatives foremost in the sentence, no evidence is presented that this was a habit of late authors. In phrases like *non putavi praetermittendum* Praun, who cites (p. 27) eleven occurrences of it in Vitruvius, holds that the attaching of the negative to *puto* is the Greek idiomatic use as in οὐ φημί. He might have compared οὐκ οἴομαι, οὐ νομίζω, etc.; see Kühner-Gerth, *Gr. Gramm.* II, p. 180. And W. Schmidt in *Jahresbericht Altertumsw.*, CVIII, 1901, p. 119 draws attention to Caesar *B. G.* 2, 31, 2: *qui ad hunc modum locuti: non se existimare Romanos sine ope divina bellum gerere.* But I think it probable that this position of *non* was, in the less polished speech, commoner than is usually supposed, for it appears not only in the *Bellum Africum* 59, 1: *Non arbitror esse praetermittendum quemadmodum*, etc., and 84, 1: *Non videtur esse praetermittendum de*, etc., but also there is a similar use in the eighth book of the Gallic War, by Hirtius, 48, 10: *quod ego non existimavi mihi esse faciendum, propterea quod*, etc. Finally, in Ussing's last example we have in *non enim quae . . . non eae possunt* nothing but the rhetorical figure of anadiplosis, found (to compare great things with small) in Demosthenes 9, 31: ἀλλ' οὐχ ὑπὲρ Φιλίππου καὶ ὧν ἐκεῖνος πράττει νῦν, οὐχ οὕτως ἔχουσιν. And the recurrence of *non* once again in *non ruinosae* may be compared with Cic. *Fam.* 13, 18, 2: *non potest mihi non summe esse iucundum* (see also Dräger I, p. 135). Neither of these usages is any proof of late authorship.

Taking up a new topic, Ussing says: 'It is a well-known fact that in the Silver age the conjunction *num* is gradually replaced by *an*, and later on disappears entirely from the language. In Vitruvius *num* does not exist at all, neither do we find (the single) *an*, *ne*, nor *nonne*. The only particle by which he introduces a dependent interrogative clause is *si*, e. g. 53, 14: *si est firma probatur;* cf. 32, 4: *quaesiit si essent agri;* 133, 20: *quaerebant si honeste essent educati;* 156, 20: *quaesiit si quem novissent;* 183, 10: *de aqua . . . quibusque rebus si erit salubris et idonea probetur explicabo.* Only in double clauses we find *utrum . . . an*, as 18, 26: *dubitantes utrum morbo an pabuli vitio laesa essent.* But *si* occurs equally, cf. 53, 12: *de ipsa autem testa, si sit optima seu vitiosa ad structuram, statim nemo potest iudicare*; 173, 17: *neque animadvertunt si quid eorum fieri potest necne. Si* in this sense already occurs in Plautus; so we do not wonder that it is found in Vitruvius, but we wonder that it is the only interrogative conjunction he knows, as it is the only one which has migrated into the Romance languages. Whether this *si* is due originally to an influence from the Greek language, I dare not decide.' — The examples for this paragraph are taken by Ussing from Praun (p. 74 f.), but the inferences drawn from them by these two scholars are different. Ussing holds that the almost exclusive use of *si* in indirect questions instead of other particles is evidence of late authorship; Praun, that such was 'die Richtung der Volksprache' in the classical period. This phenomenon of the almost exclusive use of *si* with which Ussing concludes his paragraph is really the only point in it that has any force, for the preceding details are unimportant. Thus, there is nothing surprising in the absence of *num* from Vitruvius, since it is not found in Catullus, Tibullus, or Pliny the Elder (Schmalz, *Lat. Gramm.*[3] p. 360). On the other hand, *num* does not 'entirely disappear' from late authors, for it is found in an indirect question in Orosius 1, 19, 9. Boethius has *numne* (*Herm. Sec.* p. 46, line 12 Meiser), and Arnobius has *numquid* 46 times (Schmalz, *ibid.*). The word *nonne* in indirect questions is exclusively Ciceronian (Schmalz, p. 361). As for *-ne*, Caesar and Sallust have it only half-a-dozen times each, whereas Tacitus has it nearly thirty times, so that nothing about the date can be argued from its absence from Vitruvius. We should not be surprised at missing *an* in Vitruvius in the simple indirect question with *quaero* or other verbs meaning 'ask,' because it is not commonly found in the ante-classical or classical period except in connection with *scio* and verbs of doubting (*Thesaurus*, s. v. *an*, p. 7 ff.). What then is left of Ussing's observation? Nothing but eight examples in which *si* is said to be used in indirect questions in Vitruvius (seven quoted by Ussing, to which add 162, 17: *quaeratur solum si sit perpetuo solidum*). But a closer examination of these examples will show that half of them may be eliminated at once. I mean the two with *probari* and those with *animadvertere* and *iudicare*. In all of these except one (53, 12) we have the indicative in the clause with *si*, and none are indirect questions but all are conditional protases used instead of indirect questions (see Praun, pp. 70 and 72 on the two examples with *probari*). This leaves only the four cases with *quaero*, which certainly cannot be called into evidence for late authorship, since *quaero si* is found in the Augustan period, for instance in Propertius (2, 3, 5) and Livy (29, 25, 8; 39, 50, 7). The only truthful observation, therefore, which can be made about Vitruvius's habits in expressing indirect questions is that he seldom employs the 'sentence-question'[22] and only in the phrase *quaero si.*

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[22] For other kinds of indirect questions in him, see Praun, p. 75 f.

Ussing next passes to Hellenisms: 'The most ancient Roman authors not unfrequently borrowed words from Greek to express ideas or to name objects for which their own language lacked words, but they did not borrow forms or constructions. The age of Cicero and Augustus tried to remove the Greek words and to keep the language pure, but these attempts did not entirely succeed, and in the Silver age we find repeatedly that where it became necessary to use Greek words, the authors liked to show their knowledge in retaining the Greek flexions, as *os* in the nominative instead of *us*, *u* in the genitive, etc. In the course of time such Hellenisms increased, and the great number of them which occur in Vitruvius also help to indicate the period when he lived.' — Here the confession of Ussing, that the attempts of Cicero and Augustus to remove Greek words and to keep the language pure 'did not entirely succeed' is fatal to his argument. We must remember that we are dealing with an author who stands alone in his kind. It is true that Ennius, Plautus, and Terence, when they used Greek words, generally Latinized them in form, but we know that Accius preferred to retain the Greek terminations (Varro, *L. L.* 5, 21; cf. 10, 70), and we see that Lucilius, Catullus, and Varro as well as the Augustan poets employed many Greek forms, while the number of Greek words in *Bell. Afr.*, *Bell. Hisp.*, Celsus, Pliny the Elder, and Petronius shows that we have not to wait until late Latinity for the appearance of this tendency. I need say nothing of Cicero's letters, which in spite of his own dictum in the *Tusculans* (1, 15), *scis me Graece loqui in Latino sermone non plus solere quam in Graeco Latine,* prove that 'Greek words and phrases were the *argot* of literary Rome.'[23] If Cicero uses Greek as 'part of the terminology of rhetoric and politics, not merely calling it in to supply a deficiency in the Latin language but dropping into it when he might as easily have used Latin,' we ought not to be surprised at finding Vitruvius doing the same in treating a subject on which not many Romans had written before him. When we find Greek terminations in Vitruvius, we must remember that Cicero wrote *tyrannida* in *Att.* 14, 14, 2, though *tyrannidem* in *Off.* 3, 90, and that this Greek ending is not confined to letters to Atticus but is found in *hebdomada* in *Fam.* 16, 9, 3. And in *Or.* 191 we have *paeana,* though *paeanem* stands in *D. O.* 1, 251. Neither should it be thought that Vitruvius uses only Greek terminations for Greek words. For example: Nohl's *Index to Vitruvius* gives under the letters *a, b,* and *c,* 973 words (excluding proper nouns and adjectives, and Greek words quoted as such, like *id* αβατον *vocitari iusserunt*). Of these 973 words, 101 are adopted from Greek, including of course forms of such words as *athleta*, *barbarus*, *basilica*, *camera*, *centaurus*, *chorda*, which were fully naturalized in the Latin of the classical period. Now of these 101 words it appears that 71 are used by Vitruvius with Latin terminations. Of the remaining thirty, eighteen are technical terms belonging to the vocabulary of architecture, and hence naturally Greek, such as *amphithalamos* (nom.), *baseos* (*gen.*), *cathetoe* (nom.). This leaves of the 101 words, only twelve untechnical terms in which Vitruvius employs Greek terminations. They are: *acroasin* (Cicero and Varro have *acroasi*), *aethera* (Cic.), *agrammatos*, *amusos*, *aniatrologetos*, *arctoe* (Cic.), *arithmeticen*, *arteriace* (Plin., Cels.), *asty*, abl. (Ter. and Nepos have *astu*), *catacecaumeniten* (Plin. has *catacecaumenitae*), *colossicotera*, *cratera*, acc. (Virg., Ov.). Therefore, of the 101 words only seven are found in Vitruvius with Greek terminations which are not similarly found in other authors, the latest of whom is Pliny, and these seven are all unusual words, all but one in fact (*acroasis*) making their appearance in Latin for the first time in Vitruvius. This examination, therefore, incomplete as it is, may probably serve to show that Hellenisms in terminations are no more common in Vitruvius than in writers of the classical period.

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[23] Tyrrell, *Correspondence of Cicero,* I, p. 66.

'He uses Greek words not only when he may possibly quote from a Greek source, but also in his own argumentations, and connected with Greek flexions, as 132, 27: *philologis et philotechnis rebus;* 247, 19: *collossicotera;* 8, 14: *aniatrologetos.* He does not even seem afraid of *-ois* instead of *-is*, as *pentadorois*, 39, 7.' — In the first of these examples we have a word not found elsewhere, *philotechnis.* It is not difficult of interpretation and seems a natural term to connect with *philologis.* To Vitruvius *philologia* means 'literature' or 'literary studies' in a wide sense (156, 7; 157, 20; 203, 14); so it did to Cicero (*Att.* 2, 17, 1). And just as to Cicero there was within *philologia* such a thing as τεχνολογία (*Att.* 4, 16, 3: *reliqui libri* τεχνολογίαν *habent*, here used of the technical discussion of statecraft in the latter portion of the *De Republica*), so to Vitruvius *philotechnicae res* are the artistic (particularly in his case the architectural) parts of literary pursuits. Thus also we find φιλότεχνοι (*lovers of art*) and φιλόσοφοι distinguished in Plato *Rep.* 476 A. The ideas, therefore, which Vitruvius expresses in this passage were not foreign to the classical period, and the word *philotechnis*, not occurring elsewhere, cannot be taken as evidence of late authorship. Neither can *colossicotera.* I am not aware that the positive of this adjective is found elsewhere, either in Greek or Latin in any other than its literal sense as applied to a 'colossal' statue. Vitruvius has it thus in 50, 3: *statuam colossicam*, and 251, 3: *colossici Apollinis*. Yet in the more general and derived sense Vitruvius (and no late author) has it twice in the comparative degree, — in the passage cited above (247, 19) where it is used of weights too enormous to be raised by the *sucula: sin autem colossicotera amplitudinibus et ponderibus onera in operibus fuerint, non erit suculae committendum;* and in 81, 1 where it is applied to buildings which are, as we might say, somewhat gigantic: *opera . . . ipsa colossicotera.* Here again we must remember what has been said of Greek as the literary *argot* of the classical period. Cicero in his letters does not shrink from introducing Greek comparatives into Latin sentences: e. g. *Att.* 12, 45, 2: *nam ceteroqui* ἀνεκτότερα *erant Asturae; Att.* 4, 2, 7: *cetera quae me sollicitant* μυστικώτερα *sunt.* Other such comparatives are πολιτικώτερα (*Att.* 14, 14, 1), φιλολογώτερα (*Att.* 13, 12, 3), ἐκτενέστερον and φιλοστοργότερον (*Att.* 13, 9, 1). Caesar also used them, as we see from Cic. *Q. F.* 2, 15 (16), 5: *reliqua ad quendam locum* ῥᾳθυμότερα*: hoc enim utitur* (sc. *Caesar*) *verbo.* The word *aniatrologetos* (8, 14) is also a ἅπαξ (cf. ἰατρολογέω and ἰατρολογία). It is worth observing that the whole passage is full of Greek names and words: *architectus, grammaticus, Aristarchus, agrammatos, musicus, Aristoxenus, amusos, Apelles, graphidos, plastes, Myron, Polyclitus, plasticae, Hippocrates* — all these occur in the same section. And we may note that in our word the ending *-os* is due to an emendation by Giocondo, the manuscripts giving *-us.* Finally, of the ending *-ois* as found in *pentadorois*, there is no manuscript evidence that Vitruvius used it, but if he did, he was perfectly excusable since the whole passage bristles with Greek, and Pliny *N. H.* 35, 171 shows that, if he was not drawing from Vitruvius, he had the same Greek source before him. Vitruvius is describing the kinds of bricks used by the Greeks in their buildings: *ex his unum* πενταδωρον, *alterum* τετραδωρον *dicitur.* δωρον *autem Graeci appellant palmum quod . . . palmam. Ita quod est quoquoversus quinque palmorum pentadoron, quod est quattuor tetradoron dicitur, et quae sunt publica opera* πενταδωροις, *quae privata* τετραδωροις *struuntur.* I print the passage as Rose gives it. The manuscripts have only Latin letters.[24] For πενταδωροις and τετραδωροις they give *pentadoros* HS, *pentatoros* G; *tetradoros* GS, *tetradoro* H. If Vitruvius himself used Latin letters here, it is obvious that he may have written *pentadoris* and *tetradoris* with Latin terminations, so that in either case nothing is left of Ussing's argument, since even Cicero does not hesitate to treat a Greek dative like a Latin ablative (cf. *Att.* 5, 21, 14: *de ἐνδομύχῳ probo idem quod tu*).

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[24] This is also frequently the case in cod. M of Cicero's letters to Atticus, where our editions give Greek letters; see Tyrrell and Purser to *Att.* 2, 20, 1 and 14, 3, 2.

'A characteristic Hellenism is the use of the genitive corresponding to the comparative *than*, as 105, 23: *superiora inferiorum fieri contractiora;* 22, 2: *ut ne longius sit alia ab alia sagittae missionis.* This Grecism is found in Apuleius, as in *Met.* 3, 11: *statuas et imagines dignioribus meique maioribus reservare suadeo*[25]; *De Dogm. Plat.* 1, 9: *animam . . . omnium gignentium esse seniorem.* In Tertullian, *Apol.* 40: *maiorem Asiae et Africae terram;* in the Latin translations of Irenaeus and Hermes Pastor; very frequently in the oldest Latin translation of the Bible (Itala), as I *Maccab.* 6, 27: *maiora horum facient.* The Vulgate here has the regular construction: *maiora quam haec,* and mostly so, but occasionally the genitive has been retained, comp. Wölfflin, *Archiv,* VII, p. 117 ff. The above-mentioned reviewer in the *Athenaeum* says that this "slipshod Greek genitive is not avoided by Plautus and Ennius." I should have been much obliged to him for indicating the places. I thought I knew my Plautus pretty well, but I have never found it.' — Here we should have the strongest evidence of late authorship which we have thus far reached if we could really feel sure that Vitruvius used the Greek construction of the genitive of comparison. That he did so, seems to have been doubted by no recent writer on the subject of this genitive, and it is defended either on the ground that he was following Greek sources (Wölfflin, *Archiv,* VII, 118; Sittl, *die lokalen Verschiedenheiten,* p. 114), or by pointing to traces of this use in even earlier writers. These traces were of course what the reviewer in the *Athenaeum* had in mind, and that he is somewhat unjustly treated by Ussing will be granted by anybody who will take the trouble to read Schmalz, *Lat. Gramm.*,³ p. 253, n. 1. Even Wölfflin in the very article cited by Ussing points to these traces in Plautus. But in Vitruvius it must be confessed that we have no longer 'traces,' and that, if we take the passages as they are usually taken, without further investigation, the real Greek genitive of comparison is found in him for the first time in Latin.[26] Is it, however, certain that the two passages cited by Ussing[27] are properly taken? The first of them must be seen in full before it can be studied. It runs thus: *Ergo si natura nascentium ita postulat, recte est constitutum et altitudinibus et crassitudinibus superiora inferiorum fieri contractiora.* Now in an earlier sentence Vitruvius had written *uti firmiora sint inferiora superioribus* (75, 16). Here is the usual ablative of comparison. Why does he not employ it in our passage? He purposely avoids it, I think, because after *altitudinibus* and *crassitudinibus* another ablative, *inferioribus*, would be awkward and perhaps obscure. So in Sall. *H.* 2, 37: *vir gravis et nulla arte cuiquam inferior*, another ablative instead of the dative is inconceivable. But it does not follow that in Vitruvius *inferiorum* is a genitive of comparison. Every careful reader must already have seen that we are dealing with a brachylogy, and that *altitudinibus et crassitudinibus* are to be taken a second time so that *inferiorum* does not depend upon *contractiora.* In first drawing attention to this example Praun did not cite it completely but omitted the two ablatives, and in this mangled condition it has since been quoted as a case of the genitive of comparison — which it is not. There remains then only one case to be considered (22, 2), and here I do not believe that Vitruvius wrote *sagittae missionis* but rather *sagittae missione.*[28] Errors in writing the genitive in *-is* instead of the ablative in *-e* or *-i* are not uncommon in the manuscripts of Vitruvius especially where another genitive precedes. Thus we find *rationis* (2, 23) for *ratione*, where *sollertiae* precedes in the manuscripts. We have also *solis orbis* for *solis orbi* (224, 28); *decussis* for *decussi*, where *additis* precedes (67, 13). And we also find the plural in *-es* for the ablative singular, as: *necessitates* for *necessitate* (54, 14), *partes* for *parte* (94, 29), *frontis* for *fronte* (82, 12), and *frontes* for *fronte* where *ornationis* precedes (119, 17). So it appears that there is little or no good evidence that Vitruvius used the genitive of comparison at all.

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[25] The reading here of *mei* depends upon the 'manus recentissima' of cod. F (Vliet, p. xiii). The manuscripts themselves have *meis,* and Vliet reads *meritis.*

[26] I cannot accept Varro *R. R.* 2, 5, 10, cited by Schmalz, as a certain case. See Keil's note on it.

[27] A third, cited by Praun (p. 79) and Wölfflin (*Archiv,* 7, 118), is not a genitive of comparison as has already been noted by Nohl (*Wochenschrift f. kl. Phil.* 3,

Ussing next observes: 'It has often been said that Vitruvius "translated largely from the Greek." I am not sure that he has translated more than the chapters of Athenaeus which will be mentioned below.[29]

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p. 563. It is 231, 1: *Ad anguis inferius ventris sub caudam subiectus est centaurus*, which means 'Beneath the Snake's belly, under its tail, lies the Centaur'; cf. Aratus 447; *οὐρῇ δὲ κρέμαται ὑπὲρ αὐτοῦ κενταύροιο.* Here *ad inferius* = *ad inferiorem partem ventris*; for the use of *ad*, see the *Thesaurus*, s. v. p. 519, 23; 525, 6–36.

[28] As it was printed by Schneider. Perhaps, as the codd. have *sagitta emissionis*, we should keep the longer word as in 283, 18: *sagittae emissionem*, — reading it, however, in the ablative with the earliest printed editions.

[29] Here Ussing is referring to pages 29–41 of his article where, accepting the view of Diels that this Athenaeus Mechanicus was a post-classical writer, he argues that Vitruvius, drawing from him, must be even later, and rejects Thiel's

He seems more likely to have drawn his knowledge from Latin sources, but his style is appreciably influenced by Greek. Among these influences we will mention his preference for *non minus* (οὐδὲν ἧσσον) instead of *item;* cf. 103, 24; 187, 12; 218, 7, etc. Further, the superfluous use of *etiam* (καί) in comparisons, as 216, 4: *aequo pondere quo etiam fuerat corona.* Equally the striking omission of the demonstrative pronoun before the relative, as 30, 6: *aedibus sacris quorum deorum maxime in tutela civitas videtur esse,* and 30, 11: *Herculi in quibus civitatibus non sunt gymnasia;* and the still more striking attraction of the relative in 34, 27: *spatio relicto quanto arborum longitudines patiuntur.'* — This paragraph does not seem quite apropos of the argument, for it merely suggests Greek sources for certain usages in Vitruvius without indicating that they are found in late Latin. I am not aware that *non minus* in the sense of *item* is so found. It appears to be like *nec minus* as used in Varro *R. R.* 1, 13, 3; 3, 1, 6; Propertius 1, 3, 5.[30] The 'superfluous *etiam* (καί)' calls for no further comment here; and for the substantive standing in the relative clause without a demonstrative in the main clause, as well as for the attraction of the relative, see Schmalz's Lat. Gramm.,^{3} pp. 372 and 373. These usages are not evidence of late authorship.

Neither is there such evidence in the following paragraph: 'In the Syntax of Vitruvius, one of the things that attract our attention is his way of expressing measures. He often uses the regular construction with the accusative, as *latitudine maior quam pedes* XX; but he equally employs the genitive, a construction which also appears in more ancient authors, as Varro ap. Pliny *N. H.* 36, 92: *pyramides . . . imae latae pedum quinum septuagenum, altae centenum quinquagenum;* Columella 2, 10, 26: *areas latas pedum denum, longas pedum quinquagenum facito;*

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theory of a common source for both in Agesistratus (whom, however, Rose^{2} has indicated for Vitruvius, 275, 16). How unsuccessful Ussing is in this argument has been shown by Schmidt (*Bursian's Jahresbericht,* **108**, 1901, p. 120). In another part of Ussing's book (p. 28) there is a very just observation which he would have done well to bear in mind throughout: 'As if it were possible to write about the very same things without occasionally using the same words; or as if there must not necessarily be found a similarity in those who proceeded from the same school, and had drawn their knowledge from the same book.' A principle of common sense which 'source-hunters' often ignore!

[30] *Non minus* in this sense is found more than thirty times in Vitruvius; besides he has *non minus etiam* nine times (cf. *nec non etiam,* Varro *R. R.* 1, 1, 6; 2, 10, 9; 3, 16, 26; and Schmalz, Lat. Gramm.^{3} p. 351, on such pleonasms in uncultivated style).

Plin. *N. H.* 18, 140; 36, 7. Thus Vitruvius 77, 9: *uti lata et longa sit columnae crassitudinis unius et dimidiae;* cf. 77, 18; 100, 24; 94, 14; 205, 20; 207, 25, etc. But instead of this *genitivus qualitatis* Vitruvius also uses the ablative; cf. 39, 1: *longum sesquipede, latum pede;* 94, 28: *crassitudines extenuentur his rationibus uti si octava parte erunt quae sunt in fronte, hae fiant* x *parte.'* — To these examples of the ablative may be added[31] 170, 1: *alte circiter pedibus tribus;* 99, 24: *altae dimidia parte;* 99, 26: *altam suae crassitudinis dimidia parte.* But they cannot be taken as evidence of very late authorship, for Columella has this ablative in 5, 9, 3: *digitis quatuor alte; Arb.* 1, 6: *tribus pedibus alte;* and both the genitive and ablative in 3, 13, 5: *quidam dupondio et dodrante altum sulcum, latum pedum quinque faciunt.*

Coming next to locative constructions, Ussing says: 'A similar wavering is found in the local determinations. Country names are put in the ablative without prepositions, as 43, 27: *Achaia Asia;* 134, 14: *aliter Aegypto, aliter Hispania, non eodem modo Ponto;* 182, 3: *Ponto et Gallia;* 176, 15 ff., frequently. Even the genitive appears, 59, 3: *Cretae et Africae.* Names of towns in the ablative instead of the genitive, 49, 8: *Arretio;* 101, 22: *Sunio;* 195, 19: *Zacyntho.* This harmonizes with the use of *eo* instead of *ibi,* 120, 16: *eo tragici et comici actores in scaena peragunt;* 284, 11: *arboribus excisis eoque conlocatis.* (If the same is found in Cicero's *Ep. ad Brutum* 1, 2, 1, it may as well be considered as a testimony against the genuineness of these epistles).' — A full collection of Vitruvius's use of country names in the ablative without a preposition has been published by Nohl in his *Analecta Vitruviana,* p. 9. From this it appears that 21 names are thus used.[32] This is a large number, but the usage itself cannot be accepted as proof of late authorship because we find in Virgil *Ponto* (*Ecl.* 8, 95 f.), *Latio* (*A.* 1, 265; 6, 67), *Lycia* (*A.* 12, 344), *Italia* (*A.* 1, 263), and in Pliny *Hispania* (*N. H.* 8, 226), and *Aegypto* (13, 56; 18, 123; 19, 79).[33] For the rest, Vitruvius uses also the regular construction of *in* with the ablative in the case of 25 country names, some being the same as those which he has used without the preposition. When Ussing remarks 'Even the genitive appears,' he must mean the 'locative' for there would be nothing surprising in the employment of a true genitive construction. I do not believe that the true locative of any country name is found in Vitruvius, since I think that all the forms which seem to be such may be explained on other grounds, just as the apparent locatives of country names in Pliny have been explained away.[34] Only six cases call for consideration. Of these *Asiae* (190, 14) and *Phrygiae* (196, 14) are chorographic genitives (see Schmalz, *Lat. Gr.*,^{3} 234 f.), such as are found in Caesar, Livy, and Pliny. In 195, 15 *Aethiopiae* is now read *Aethiopia*, but, if the manuscript reading is kept, we have a genitive depending on *lacus*. In 198, 8 *Boeotiae* (a genitive) has been emended to *Boeotia* on account of the following ablatives. In the example cited by Ussing, *Cretae et Africae* (59, 3), one of two explanations may be given. Although the name *Cretae* is generally treated like that of a country and consequently appears in Cicero with *in* and the ablative, yet as an island it is used in the locative by Varro (*R. R.* 1, 7, 6) and Virgil (*A.* 3, 162). If Vitruvius used it thus, then the following *Africae* is an assimilation for concinnity, like Sallust's *Romae Numidiaeque* (*J.* 33, 4). But both *Cretae* and *Africae* may be genitives depending on *regionibus*, for the whole sentence reads: *nascuntur autem eae arbores maxime Cretae et Africae et nonnullis Syriae regionibus.* There remains only 200, 24: *sunt autem etiam fontes uti vino mixti, quemadmodum est unus Paphlagoniae, ex quo*, etc. Here *Paphlagoniae* is to be taken as a genitive. But even if locatives of country names were actually found in Vitruvius, we could parallel them from the classical period, since we have *Peloponnesi* in Varro (*R. R.* 2, 6, 2), *Chersonesi* in Nepos (1, 2, 4), and *Galliae* in Hirtius (*B. G.* 8, 1, 2).[35] As for names of towns in the ablative instead of the locative, Nohl's treatment (*Analecta*, p. 10) is not exact, for he does not distinguish between towns and islands. The names of towns actually thus used by Vitruvius are *Arretio*, *Chio* (283, 3 where the word *murum* shows that the town is meant), *Halicarnasso*, *Lyncesto*, *Paraetonio*, *Sunio*, *Tarso*, *Teo*, *Teano*, — that is, nine in all.[36] It is true that this misuse becomes common in late Latin (*Archiv*, XIII, 315 f.), but still we find occurrences of it early enough to show that in Vitruvius the phenomenon is due to his lack of finish and that it cannot be taken as evidence of late authorship. Thus, Cato has *Venafro* (*R. R.* 135, 1), and Varro has *Amiterno* (*L. L.* 6, 5). On the whole, with regard to these three categories we must treat them as errors of style, just as Pliny's frequent use of *in* with the ablative of a town name (*Archiv*, XIII, 337) is treated. Nobody thinks of stigmatizing the Natural History as a piece of late workmanship because of them, particularly in view of the practice of the Emperor Augustus, who used prepositions with names of towns in order to avoid obscurity (Suet. *Aug.* 86). We come next to Ussing's remark about the use of *eo*. Here it is not necessary to try to defend Vitruvius by means of the disputed passage in Cicero, *Ep. ad Brutum*, nor even to refer to the undisputed erroneous use of *eo* in Celsus 8, 9, 1: *ibi pus proximum erit eoque uri debebit*. It is enough to show that Vitruvius's use is correct. This has been done for 120, 16 by Rose in a footnote in his second edition, where he refers to *perago* used twice with *ad* and the accusative on a later page. In 284, 11 *eo* is due to the meaning of *conlocatis* which here does not mean simply 'to place' but rather 'to bring together'; consequently *eo* is properly used, as with *ad* and the accusative in Plaut. *Men.* 986: *in tabernam vasa et servos conlocavi*, a construction found also in Vitruvius himself, 272, 9: *in eos cuneoli ferrei . . . conlocantur*. Of course Vitruvius has also the other use of *conloco*, with *in* and the ablative, or with *ibi* or *ubi*, examples of which may readily be found in Nohl's *Index*. With these two uses with *conloco* may be compared the same two with *coacervo*; for instance, *Bell. Afr.* 91, 2: *eo coacervatis*, and Cic. *R. A.* 133: *coacervari una in domo*. As for the proper meaning of *eo* ('thither,' not 'there') Vitruvius is perfectly aware of it and so employs it in seven other passages.[37]

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[31] It is the more necessary to present these additional cases because the two which Ussing cites are not very convincing. The second lacks any adjective like *longus, latus,* or *altus,* and is therefore an ordinary genitive of quality; the first is easily emended away, as *pede* in 278, 7, is now emended to *pedem,* and as in Plin. 35, 171 *longum sesquipedem* is now read instead of the older reading *sesquipede* of the inferior manuscripts.

[32] For *Lucania,* however (198, 9), *Lucanis* of the manuscripts should be retained; see my note in *Harvard Studies in Classical Philology,* **17**, p. 6.

[33] Cf. Funaioli, *Archiv,* **13**, 327 ff.

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[34] Funaioli, *Archiv*, **13**, 581 f.

[35] Here I think that *Galliae* must certainly be taken as a locative on account of *rebus gestis Alexandriae* just below. Still, see *Archiv*, **13**, 331.

[36] For the passages, see Nohl. On the other hand Vitruvius has the locative of stems in *-o-* six times, and always in stems in *-a-*.

Passing now to other topics Ussing says: '*Noceri* is constructed personally in the passive voice, 45, 22: *neque ab ignis vehementia nocentur*; 59, 7: *larix ab carie aut tinea non nocetur*. Similarly Apuleius, *de Dogmate Platonis*, 2, 17.' — These two examples are not sufficient evidence of late authorship, for Vitruvius always uses this verb properly in the active voice (six times absolutely and eight times with the dative[38] case), and also has it once impersonally in the passive (59, 14). The two examples are rather to be treated among those violations of regular usage which crop out here and there even in the best writers. It is true that I know of no similar certain case of *noceri* before Ulpian, *Dig.* 43, 19, 3, 2; for Sen. *Ira*, 3, 5, 4, cited by Neue (*Formenlehre* III, 5), is not a personal use, and Nepos, 7, 4, 2, is open to doubt. But for examples of other verbs which take the dative in the active voice and which occur occasionally in the personal use in the passive, cf. *crederetur*, Cic. *R. A.* 103 and *credor* used thus by Ovid, *Tr.* 3, 10, 35, and *M.* 7, 98; *obstrepi*, Cic. *Marc.* 9; *antecelluntur*, *Rhet. Herenn.* 2, 48; *invideor*, Hor. *A. P.* 56; *imperor*, Hor. *Ep.* 1, 5, 21; and the numerous instances of the passive participle of *persuadeo*, Wölfflin, *Rhein. Mus.* XXXVII, 115 f.

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[37] He has *eo loci* also twice correctly. If he has it twice besides in the sense of *ibi* (233, 17; 235, 14), so have Cicero (*Sest.* 68) and Pliny (*N. H.* 11, 136).

[38] If the work were late we might expect to find the accusative; see Kühner, *Lat. Gr.* 2, p. 76, 5 *fin.*

'*Est causa cognoscere*, 59, 17, instead of *cognoscendi* is a construction now and then occurring in the poets; cf. Madvig, *Lat. Gr.* § 419. It has been noticed that the genitive of the gerund is very rare in Vitruvius, whereas the ablative is exceedingly frequent; cf. Praun, p. 57 ff. It is, as we know, the ablative form which passes into the Romance languages Italian and Spanish.' — There is nothing in *est causa cognoscere* that points to late authorship, for nothing like it is cited in any other author, late or early. The peculiarity of it does not consist in the construction used with the word *causa*, for the infinitive with this word occurs in poets (Verg. *A.* 10, 90; Tib. 3, 2, 30; Lucan, 5, 464), and for the general principle involved see Schmalz, *Lat. Gramm.*^{3} p. 293. The peculiarity lies in the *meaning* of the word *causa*, for, as Praun has remarked (p. 20), *est causa* here is equivalent to *operae pretium*, and no parallel for this, early or late, is cited. It must therefore be considered as a peculiarity of the author.[39] With regard to the rest of Ussing's paragraph, two observations should be made. First, that the rare use of the genitive of the gerund in Vitruvius (only five occurrences, Praun, p. 57 f.) is partly due to the fact that he never uses it with an adjective or with *causā* or *gratiā* (Praun, *ibid.*). But with adjectives this construction is very rare in old Latin, not common in the classical writers, and of slow growth before Tacitus, who greatly developed it (Lane, *Lat. Gramm.* § 2258; Schmalz,^{3} p. 304). See also Praun's remarks (p. 65) on the use of the gerund or gerundive construction with *ad*, instead of in the genitive, as found in writings of less formal and polished style. Secondly, regarding the prevalence of the ablative construction in Vitruvius, this is the commonest of all the gerund and gerundive constructions at all periods. Praun (p. 59) cites Valerius Maximus as a special lover of it, so that we need not come down to late Latin to find it. Even the modal use, which is such a favorite with Vitruvius, is found, once in Cicero, and examples occur in Caelius, Sallust, and the *Bellum Hispaniense*, until finally Ovid and Livy made it common (Schmalz,^{3} p. 305).

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[39] Rose^2 emends to *causam*.

Next there follows in Ussing a long paragraph which I do not think it worth while to reproduce here. It deals with the undoubted fact that in Vitruvius the mood employed in indirect questions is very apt to be the indicative.[40] After referring to this usage in Plautus, Ussing says: 'No classical prose writer would indulge in putting the indicative in a dependent clause which really expresses a reflection or a doubt.' He does not say that late writers do so, but of course it is well known that such is the fact (for instance, see the literature cited in Sittl, *Die lokalen Verschiedenheiten*, p. 134), and his argument therefore is that this phenomenon in Vitruvius is evidence of late authorship. In this paragraph Ussing says nothing about the appearance, here and there, of this indicative in several prose writers who not only belong to the 'classical period' but who are also so strict in their standards of style that they are entitled themselves to be called 'classics.' That is, Ussing adopts the attitude of those earlier generations of scholars who, from the time of Lambinus down to near the present day, did not scruple to emend away all offences against the strict norm of classical style. Such is not the attitude of most scholars now; individualities in writers are recognized, and departures from the strict norm are often welcomed, rather than rejected, as indications either that the literary language had not yet attained to exactness in following rules or that the writer in question is employing the phraseology of colloquial speech, which then, as always, was less careful than the literary style. In this spirit we ought to consider the appearance of the indicative in indirect questions in Vitruvius. The best general statement with regard to this employment of the mood has been made by Schmalz (*Lat. Gramm.*,^{3} p. 359). The usage crops out in the *Rhetor ad Herennium*, in Varro, in Cicero's early writings and in his letters, and in letters to him. It is avoided by the historians though not by the poets of the Augustan age, and it is found in Petronius and Pliny the Elder. The closest parallels to the indicative in clauses expressing 'a reflection or a doubt' as in Vitruvius, are to be found in the seven examples cited by Marx from the *Rhetor ad Herennium* in his edition of that book, p. 176 f.

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[40] The fullest collections are to be found in Praun, p. 71 ff., and Richardson, *Harvard Studies in Cl. Phil.* **1**, p. 157.

In Ussing's next paragraph there is but one sentence that calls for attention: 'It is certainly unclassical to employ the subjunctive in an indefinite relative clause, as 158, 5: *quorum utrum ei acciderit, merenti digna constitit poena.*' — While the subjunctive in this use probably does not occur in the classical period, yet it is found not infrequently in the Elder Pliny (Frobeen, *Quaestiones Plinianae*, p. 33), so that, if it were found in Vitruvius, the phenomenon would be no proof of late authorship. But in fact, I do not believe that *acciderit* is a subjunctive. The truth probably is that *constitit* comes not from *consto* (as Nohl takes it in his *Index*), but from *consisto*, the perfect of which is not infrequently used in a present sense. For this use, see the grammars of Kühner (II, p. 95) and Lane (§ 1607), and for numerous examples, Munro's note to Lucretius 1, 420, where he cites Cicero's letters, the two Senecas, Vergil, Ovid, and Horace. This present meaning of *constitit* makes *acciderit* allowable as a future perfect. Of course, however, the really remarkable thing in the sentence is the employment of *utrum* where there is a choice of more than two things (see the context). For this use I know of no parallel, early or late.

Ussing's last observation is as follows. 'Finally we shall briefly mention the position of the words. We have already noticed the inclination to put the negation foremost in the sentence. Similarly the auxiliaries, *esse*, *posse*, and *velle*, etc., are preferably placed before the infinitive to which they belong, as 10, 10: *ut possint . . . disciplinas penitus habere notas*; 91, 5: *qui metopas aequales volunt facere.* In sum, the governing verb is very often put before its object, whether a word or a whole sentence.' — And he begins his summary, which immediately follows, with this sentence: 'These features and many others point to the decadence of the Latin language and to its transition to the Romance tongues.' — As for this argument, I am not aware that sufficient collections have ever been made regarding the position of the auxiliary verbs to warrant the use of it in fixing the date of a literary work. This was the reason why Sittl published nothing on the order of words in his treatise on the African writers, where he says: 'Die Beobachtung der Wortstellung ergibt ebenfalls viel interessantes, aber da hier über die nichtafrikanische Literatur fast keine Beobachtungen vorliegen, wage ich es vorläufig noch nicht, unseren Provinzialen etwas zu vindizieren' (*Die lokalen Verschiedenheiten*, p. 135). If now we examine the case of *volo* in Vitruvius, we find him placing it 22 times before the infinitive and 6 times after the infinitive. But the *Rhetor ad Herennium* has it 42 times before and 18 times after (see Marx's Index); in the *Bellum Africum* the use is equally divided, seven of each (Wölfflin's Index); so in Varro's Menippeans, four of each (Riese's Index), while in his *Res Rusticae* it stands first 22 times and after the infinitive 33 times. With regard to *possum*, Lupus has observed that in Nepos the infinitive very often follows it and other verbs (*Der Sprachgebrauch des Nepos*, p. 191). In Vitruvius, the verb *possum* is used with the infinitive 300 times (Nohl's *Index*). But in exactly half of these, there is a negative attached to *possum*, and it is this expression of *impossibility* which Vitruvius prefers to place before the infinitive. He has 126 instances of it thus placed and in only 24 does it follow the infinitive. Of the other 150 cases where there is no negative with *possum*, the infinitive precedes 76 times and follows 74 times. In view of such varieties, I do not see how the position of these auxiliaries can be used in discussing the date of Vitruvius until their position in other authors has been carefully studied.

Thus the linguistic and stylistic phenomena noted by Ussing have been examined, and in summarizing them it appears that there are only a very few which cannot be paralleled either exactly or in principle during the Republican, Augustan, or Silver ages of Roman literature. These few are: the *impersonal* use of *dignum est ut* (p. 475), *necessitate* as an adverb (p. 475), *forte* meaning 'perhaps' (p. 481), and *trans* as an adverb (p. 482). And something has been said in explanation of all these except the last. The many heads of Ussing's indictment are therefore reduced to the minimum. But what if it be argued that, although instances of the several phenomena may be found in various authors of the earlier time, yet since they are not all found in any one author except Vitruvius, this accumulation of them in him points to late authorship? The answer to this cumulative argument is that it begs the whole question. For, as I have pointed out above (p. 468), no other technical treatise written in the better age is extant, and therefore we are not entitled to say that such treatises did not abound in examples of the phenomena which appear in Vitruvius. As for the resemblances between the language of Vitruvius and that of the Romance nations, Krohn[41] has already observed that these are *a priori* only natural. Latin was not transmitted to Romance lands by the polished works of Cicero, but by the every day writings and the colloquial speech of people like Vitruvius, — professional men, *publicani*, business men, and soldiers. The resemblances, therefore, are not necessarily evidence of late authorship. In conclusion, I may add that it seems improbable that anybody who thinks that Vitruvius is like the late Latin authors, can have actually read him through with much care. They, whatever their faults of grammar and style, are smooth and easy reading by comparison with him. He has all the marks of one unused to composition, to whom writing is a painful task. A forgery or a late compilation of an earlier work would presumably proceed from a hand used to literary performances.

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[41] *Berl. Phil. Woch.*, 1897, p. 774.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

ON THE MANNER OF GROWTH OF A CURRENT IN THE COIL OF A NEARLY-CLOSED ELECTROMAGNET AS INFLUENCED BY THE WIDTH OF THE AIR GAP.

By B. O. Peirce.

ON THE MANNER OF GROWTH OF A CURRENT IN THE COIL OF A NEARLY-CLOSED ELECTROMAGNET AS INFLUENCED BY THE WIDTH OF THE AIR GAP.

I

By B. O. Peirce.

Presented December 13, 1905. Received December 27, 1905.

In the course of a set of experiments on the time-lag in the magnetization of iron, which are being carried on in the Jefferson Laboratory, it has been found desirable to use, in series with the testing apparatus, one or more large electromagnets to increase the inductance of the whole circuit so greatly that the effects of sudden small changes in the inductance of the testing coils may be of slight importance; and it has been necessary to study specially some of the properties of these particular magnets, since, in their cases, certain of the conditions which underlie the convenient methods usually employed in treating practical problems are not very exactly fulfilled. In order to be able to predict the behavior of a nearly-closed magnet under the conditions of the work, one needed to know the effective inductance of the magnet under given conditions, and the manner of growth of the current in the coil circuit as dependent upon the applied electromotive force, the final value of the current, and the width of the air gap. These matters, with some others, were studied, and, apropos of the extremely interesting experiments recorded in Dr. Thornton's recent article[1] on "The Magnetization of Iron in Bulk," and of the work[2] of Messrs. J. and B. Hopkinson and E. Wilson on "The Propagation of Magnetization of Iron as Affected by the Electric Currents in the Iron," I propose to give briefly in this paper the results of tests made on one of the electromagnets I have used, as illustrating some characteristics of nearly-closed, massive iron cores. With these results I wish to compare some others obtained from a large closed electromagnet with finely laminated core.

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[1] The Philosophical Magazine, **8**, 1904.

[2] The Philosophical Transactions, **186**, 1895. Proceedings of the London Institute of Electrical Engineers, 1895.

The general shape of the magnet in question is shown in Figure 1. The outside dimensions of the frame proper are about 101 cm. × 80 cm. × 40 cm. The base is of cast iron and of rectangular cross-section (20 cm. × 40 cm.), the cylindrical arms are of soft wrought iron 25 cm. in diameter, the rectangular pole pieces are 4.5 cm. thick, and the area of each of the opposed faces is about 580 square centimeters. The four coils, which were commonly used in series, have together 2823 turns and a resistance at 20° C. of about 12.4 ohms; the magnet weighs about 1500 kilograms.

The electromagnetic induction within so large a solid core as that of this magnet practically attains its final value, as is well known, only after an appreciable length of time. This time, for a given value of the electromotive force in the coil circuit, depends upon the amount of non-inductive resistance in the circuit outside the magnet, and, for a given value of the final exciting current in the coil, depends upon the applied electromotive force; under favorable circumstances it may be 200 seconds. At the outset Mr. J. Coulson and I obtained a large number of hysteresis curves for the core under given conditions, but for steady currents in the coil. The curves in Figure 2 are four of a series for different widths of the air gap, each obtained after the magnet had been put a number of times in succession through a Ewing's cycle with 6 amperes as the maximum current. The ordinates show the flux density at the centre of the gap in thousands of units, and the abscissas the current in amperes for the positive descending quarter of the cycle; the rest of the figures are omitted to avoid confusion. The flux was measured by pulling a small thin coil of known dimensions, attached to a calibrated

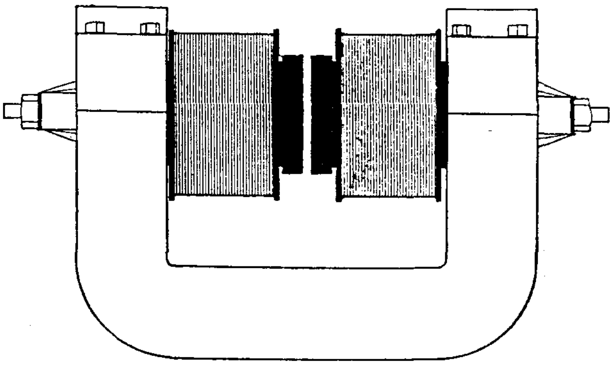

FIGURE 1.

ballistic galvanometer, out of a pocket at the middle of the gap. The cylindrical arms of the magnet were held firmly by massive yokes outside the frame, but it was necessary to insert strips of non-magnetic material in the gap to prevent it from gradually closing by the bending of the frame when strong currents were used. These "chocks" were usually so inserted, by aid of gauges made for the purpose, that just one half of the gap — divided by a vertical line from the other half — was free. Counting from the top of the diagram, the full curves correspond to gap-widths of 1.6 mm., 6.6 mm., 9.8 mm., and 19.7 mm. respectively; between the first two others are bits of two curves belonging to gap-widths of 3.2 mm. and 4.7 mm.

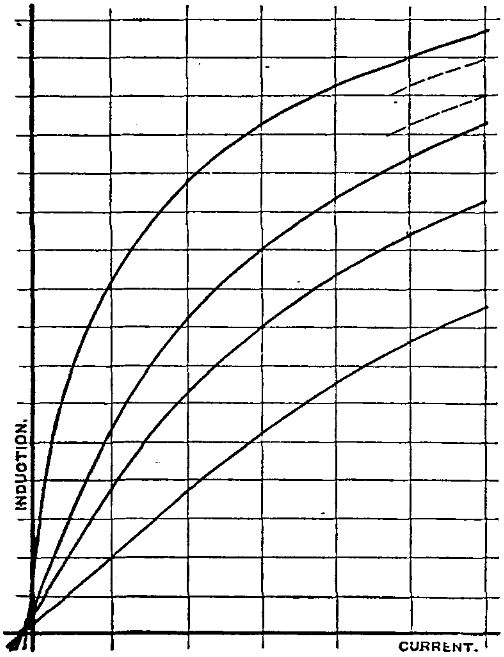

Figure 2.

The length of the line of induction which goes through the centre of the pole pieces is about 250 cm., and it would be easy to find the form of a curve, similar to those of Figure 2, for a closed gap by shearing[3] the upper curve of the diagram in the usual manner. When the gap was closed, a number of turns of insulated wire were wound directly about the core, and the ends of this coil were connected through an oscillograph which made its records on the same piece of paper which recorded the indications of another oscillograph in the main circuit. The main circuit contained also a massive rheostat of 200 ohms, total resistance, and the current in the circuit was made to grow from zero by steps so far apart in time that, in every interval, the current in the secondary circuit had time to die sensibly out. After a maximum current of the desired intensity had been attained, the current was then reduced by steps to zero, and was then built up in the opposite direction to the same maximum value by steps. The areas under the oscillograph curves of the secondary circuit evidently furnish means of obtaining a hysteresis curve for the core of the magnet, but this well-known method of procedure is in my hands and for this particular magnet not quite so satisfactory as the one indicated above.

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[3] J. Hopkinson, The Philosophical Transactions, 1885; J. and E. Hopkinson, The Philosophical Transactions, 1886; E. Hopkinson, Report of the Brit. Assoc. Adv. Sci. 1887; Ewing, Magnetic Induction in Iron and other Metals, ch. x; Du Bois, Philosophical Magazine, 1890.

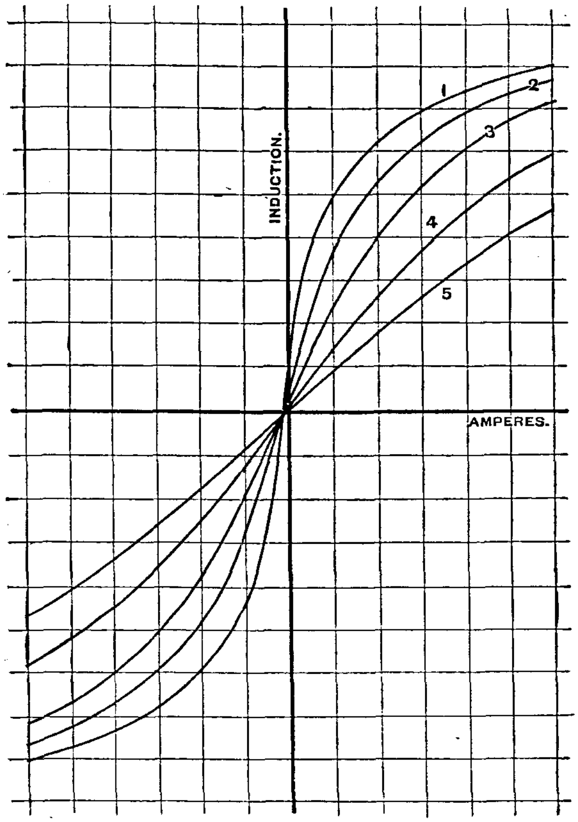

FIGURE 3.

The leakage in a magnet of the form of this one is of course considerable, even when the gap is closed, and, for a given width of gap, the ratio of the induction through a circumference of, say, 48 cm. diameter in the plane of the gap with its centre (O) at the gap-centre, to the flux density at O, depends slightly upon the intensity of magnetization of the iron. With a gap-width of 28.3 mm. this ratio increased by about 3 per cent as the strength of the current in the coils rose from a small value to 6 amperes, and was sensibly the same for ascending and descending branches of a hysteresis cycle.

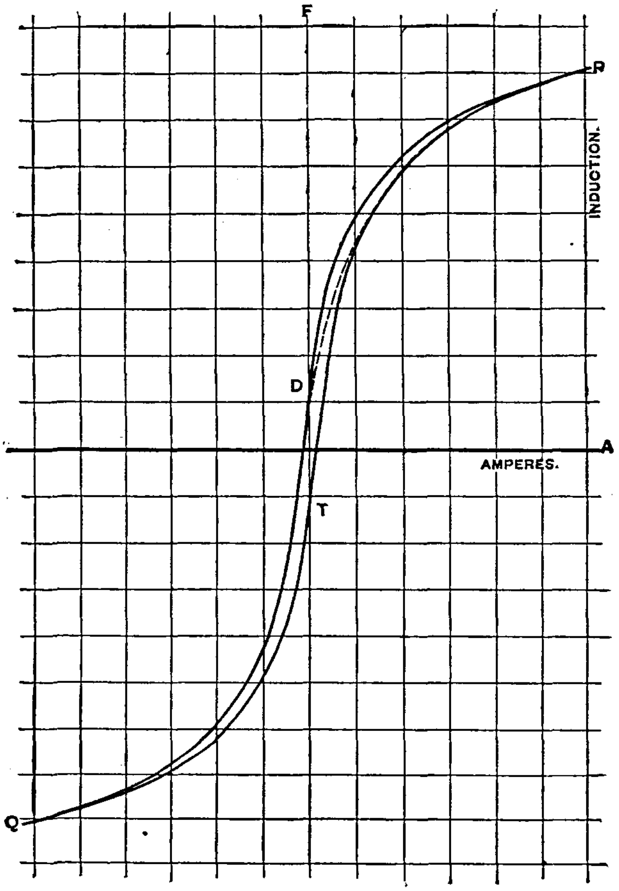

FIGURE 4.

The ordinates of each of the curves in Figure 3 show, in millions of units, the mean total flux of induction, through each turn of the coils of the magnet, for a given gap-width, corresponding to currents represented in amperes by the abscissas. These curves are specimens of a set obtained experimentally: only half of each cycle — representing about 20 determined points, every one of which lies sensibly on the curve — is given lest the complete diagrams prove confusing. Reckoned from the top of the diagram, the curves correspond, respectively, to the gap-widths 1.6 mm., 4.7 mm., 9.7 mm., 19.7 mm., and 28.4 mm. Figure 4 shows a similar cycle corresponding to a gap-width of about 1.6 mm. for a maximum current of about 6.2 amperes: the magnet was put repeatedly through the cycle before the observations were made. The dotted curve *D P* shows a rising branch of the cycle from *D* to *P*.

It is evident that the manner of growth of a current in the coil of the magnet is influenced by eddy currents in the core, by the residual effects of past magnetic experiences and the corresponding form of a hysteresis cycle for rapid changes of the magnetizing field, and by magnetic lag, if such there be, as well as by the causes enumerated above. It is always difficult to distinguish between the effects of all these causes, and it will be well to get such help as we can from a theoretical discussion of the effect of eddy currents alone in a core of definite constant permeability.

Eddy Currents in a Core of Fixed Permeability within a Long Solenoid.

Several writers have discussed the application of Maxwell's general equations[4] to the determination of the growth of currents in coils of wire which surround solid metal cores of various forms, and Heaviside printed more than twenty years ago[5] an extremely interesting series of fifteen papers on problems connected with the induction of currents in the solid core of a long solenoid. Although it is practically impossible to subject to accurate computation the growth and the decay, under given conditions, of currents in the coils of a magnet like that shown in Figure 1, it will be instructive to consider some analogous problems in the case of a long solenoid, the solid core of which is supposed to have a fixed permeability, and to be of the same diameter as that of the iron cylinders within the coils of the magnet in question. To facilitate comparison between the numerical results of this paper and those obtained in similar cases by Heaviside, it will be convenient to use his notation, at least in part.

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[4] Treatise on Electricity and Magnetism, 2, ch. ix.

[5] The Electrician, 1884–85; Electrical Papers, 1, 353–416.

A long, solid, circular, iron cylinder, of specific resistance ρ and of radius a, is closely surrounded by a uniformly wound coil of wire which has N turns per centimeter of the length of the core; the outer radius of the coil is $(a + b)$ and the axis of the core is the z axis. A current (C) in the coil is accompanied by a magnetic field (H) in the core which has the direction of the z axis, and any change in the intensity of C induces in the core temporary currents, the lines of which are circles parallel to the xy plane with centres on the z axis. At any instant the value of H and that of q, the vector which gives the density of the current at any point in the core, are functions of the distance (r) from the z axis alone, and are independent of z; hence Maxwell's current equation,

$$4 \pi q = \text{Curl } H, \tag{1}$$

reduces to the simple form

$$4 \pi q = - \frac{\partial H}{\partial r}. \tag{2}$$

The currents in the core do not affect the intensity (H_a) of the magnetic field at the boundary of the coil, so that at every instant

$$H_a = 4 \pi N C. \tag{3}$$

Since in columnar co-ordinates

$$\nabla^2 (V) \equiv \frac{1}{r} \cdot \frac{\partial}{\partial r} \left(r \cdot \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \cdot \frac{\partial^2 V}{\partial \upsilon^2} + \frac{\partial^2 V}{\partial z^2}, \tag{4}$$

where V is any scalar function, the general equation,

$$\frac{4 \pi \mu}{\rho} \cdot \frac{\partial H}{\partial t} = \nabla^2 (H), \tag{5}$$

becomes

$$\frac{4 \pi \mu}{\rho} \cdot \frac{\partial H}{\partial t} = \frac{1}{r} \cdot \frac{\partial}{\partial r} \left(r \cdot \frac{\partial H}{\partial r} \right), \tag{6}$$

where μ is the permeability of the iron, supposed constant.

When there are no currents in the core, the intensity (H) of the magnetic field in the core has at every point the boundary value (H_a), but when positively directed eddy currents exist, the intensity of the field is greater near the axis and sinks gradually, as r increases, to H_a at the boundary. If, then, L is the inductance of the coil per centimeter of the length of the core, when the core is without currents and there are no other circuits in the neighborhood so that the magnetic field within the coil is uniform, this inductance will be increased by an amount L' when, in consequence of Foucault currents running right-handedly around the axis, the intensity of the field within the core is raised above $4\pi N C$. The contribution L' comes from a field every line of which threads every turn of the coil. We have, therefore,

$$L' = N\int_0^a \mu\,(H - H_a)\,2\pi r\,dr = 2\pi\mu N\int_0^a H r\,dr - 4\pi^2 a^2 N^2 \mu\, C, \qquad (7)$$

and if the coefficient of C in the last term be denoted by L_1, the whole induction flux through the turns of the coil per centimeter of the length of the solenoid is

$$p \equiv (L - L_1)\,C + 2\pi\mu N\int_0^a H r\,.\,dr\,; \qquad (8)$$

if w is the uniform resistance of the coil per centimeter of the length of the core and E the impressed electromotive force in the coil circuit, reckoned in the same way,

$$E - \frac{dp}{dt} = w\,C, \qquad (9)$$

or

$$E = wC + (L - L_1)\frac{dC}{dt} + 2\pi\mu N\int_0^a \frac{\partial H}{\partial t}\,.\,r\,.\,dr\,; \qquad (10)$$

or, by virtue of (6),

$$E = wC + (L - L_1)\frac{dC}{dt} + \tfrac{1}{2}N\rho\, a\left(\frac{\partial H}{\partial r}\right)_{r=a}. \qquad (11)$$

To fix one's ideas, one might imagine every centimeter of the length of the coil (measured parallel to the core axis) to be a separate circuit containing an applied electromotive force of E absolute units (the same for every such circuit) and having a total resistance w made up of the resistance (w') of the wire actually wound on the core, and the resistance (w'') of the external part of the circuit which is non-inductive.

If, in order to determine a set of normal special solutions of the linear equation (6), we assume H to be the product of a function (T) of t alone, and a function (R) of r alone, and substitute this product in the equation, we arrive at the well-known normal form

$$e^{-\alpha^2 t}\,[A\cdot J_0\,(nr) + B\cdot K_0\,(nr)], \qquad (12)$$

where either a or n may be assumed at pleasure and the other computed by means of the equation

$$\rho\, n^2 = 4\,\pi\,\mu\,a^2. \tag{13}$$

Since the core of the solenoid is solid, Bessel's Functions of the second kind will not be needed in the problems of this paper, and we may assume that H is expressible in an infinite series of terms of the form

$$A \cdot e^{-a^2 t} \cdot J_0\,(n\,r). \tag{14}$$

If, after the current in the coil has been for some time steady and the core has become uniformly magnetized, the coil circuit be suddenly broken so that the duration of the spark is less than a thousandth of a second, the intensity of the magnetic field at the boundary of the core where $r = a$ falls suddenly to, and remains thereafter at, zero, and the normal form (14) will satisfy the condition $H_a = 0$, if such a value be chosen for n as shall make $n\,a$ a root of the equation

$$J_0\,(x) = 0. \tag{15}$$

A sufficient number of these roots for the purposes of this paper can be found[6] in almost any book on Bessel's Functions, with the corresponding values of $J_1\,(x)$: the first twelve are given in Table I.

The pth root in order of magnitude of the equation $J_1(x) = 0$ is denoted by x_p.

TABLE I.

p	x_p	$J_1\,(x_p)$	p	x_p	$J_1\,(x_p)$
1	2.404826	+0.519148	7	21.211637	+0.173266
2	5.520078	−0.340265	8	24.352472	−0.161702
3	8.653728	+0.271452	9	27.493479	+0.152181
4	11.791534	−0.232460	10	30.634606	−0.144166
5	14.930918	+0.206546	11	33.775820	+0.137297
6	18.071064	−0.187729	12	36.917098	−0.131325

If the intensity of the uniform magnetic field in the core before the break was H_0, we have[7] at any time (t) after the break and at any distance (r) from the axis of the core,

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[6] Byerly, Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics, p. 286; Gray and Mathews, Treatise on Bessel's Functions, p. 244; Peirce and Willson, The first 65 roots of the Equation $J_0\,(x) = 0$, The Bulletin of the American Mathematical Society, 1897.

[7] Byerly, Treatise on Fourier's Series, etc., p. 229; Heaviside, Electrical Papers, **1**, 391.

$$H = \frac{2\,H_0}{a} \sum_p \frac{J_0\,(n_p\,r)}{n_p \cdot J_1\,(n_p\,a)}\, e^{-\alpha_p^2 t}, \tag{16}$$

and, since

$$\frac{d\,J_0\,(n\,r)}{d\,r} = -\,n \cdot J_1\,(n\,r),$$

$$q = \frac{+H_0}{2\,\pi\,a} \sum_p \frac{J_1\,(n_p\,r)}{J_1\,(n_p\,a)}\, e^{-\alpha_p^2 t}. \tag{17}$$

In the case here treated the diameter of the core is 25 centimeters ($a = 12.5$), and for the kind of iron used, at room temperatures, we may write

$$\alpha^2 = \frac{5.12\,(n\,a)^2}{\mu}. \tag{18}$$

The negative of the time rate of change of the total flux of induction through a cylindrical surface of radius r coaxial with the core and lying within it, is at every instant proportional to the expression for q given above. The values of $J_0\,(x)$ and $J_1\,(x)$ for every hundredth of a unit between $x = 0$ and $x = 15.50$ are given in Meissel's Tables[8] to twelve decimal places, and after the proper value of μ has been introduced into (18) and the value for r chosen, it is not very difficult, except in the case of small values of t, to compute the value of the series (S) in (17) for different epochs. If, for example, μ is 40, and if we consider a point at the boundary of the core, S has the values given in the next table. The time is of course measured in seconds.

TABLE II.

t	S	t	S
0.25	1.3120	3.00	0.1085
0.50	0.8413	4.00	0.0518
0.75	0.6285	5.00	0.0247
1.00	0.4974	6.00	0.0118
2.00	0.2227	8.00	0.0026

Figure 5 shows four curves in which S is plotted against t for $r = a$ and $\mu = 20$, 40, 80, and 160 respectively. If the circuit of a few turns of fine insulated wire wound directly on the core were closed through an oscillograph, and if μ were independent of H and there were no time lag in its magnetization, the records should show curves like these. For large values of μ the current would decay very slowly. The actual values of the ordinates of any curve would depend of course upon the number of turns in the secondary, the resistance of its circuit, and the magnetic constants of the solenoid. If for a fixed point in the core a curve be drawn, by plotting q against t, for each of a number of different values of μ, the ordinates of all these curves will have equal values at points where t/μ has the same value. The maximum value of q, if it has one, is independent of μ.

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[8] Meissel, Tafel der Bessel'chen Functionen, Berliner Abhandlungen, 1888; Gray and Mathews, Treatise on Bessel's Functions, pp. 247–266.

At a point distant one tenth of the radius of the core from the axis

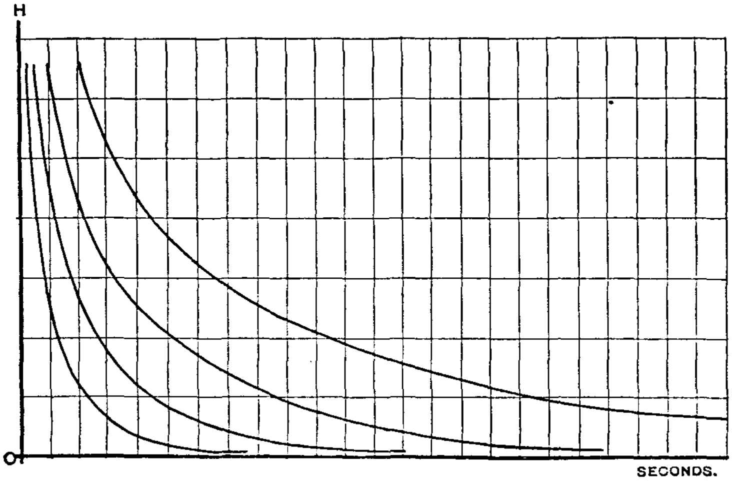

Figure 5.

$$q = \frac{H_0}{2\pi a} \sum \frac{J_1\left(\tfrac{1}{10} n_p a\right)}{J_1(n_p a)} e^{-\alpha_p{}^2 t} = \frac{H_0 S'}{2\pi a}, \qquad (19)$$

and ignoring algebraic signs, we have

TABLE III.

p	$\log \frac{J_1(\frac{1}{10} n_p a)}{J_1 n_p a}$	p	$\log \frac{J_1(\frac{1}{10} n_p a)}{J_1 n_p a}$
1	9.3617	7	0.5141
2	9.8924	8	0.5009
3	0.1611	9	0.4473
4	0.3264	10	0.3395
5	0.4308	11	0.1378
6	0.4911	12	9.6398

Table IV gives to four decimal places the values of S', for $\mu = 40$ and $r = 1.25$, at a number of different epochs after the coil circuit has been broken.

TABLE IV.

t	S'	t	S'
0.17	0.0004	1.50	0.0736
0.25	0.0062	2.00	0.0520
0.50	0.0597	2.50	0.0361
0.60	0.0770	3.00	0.0250
0.75	0.0912	4.00	0.0119
0.90	0.0948	5.00	0.0057
1.00	0.0940	6.00	0.0027
1.25	0.0853	8.00	0.0008

If a small closed testing coil of fine insulated wire could be so imbedded in the iron as to surround a portion of the core coaxial with the whole and of one tenth the whole diameter, the intensities of the current in the coil at different times should be proportional to S'. Figure 6 shows four curves[9] in which S' is plotted against t for $\mu = 20$, 40, 80, and 160.

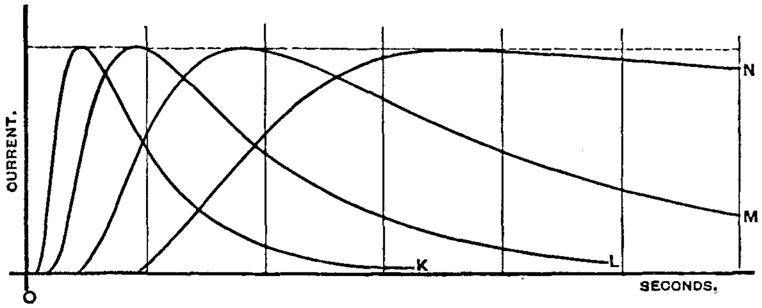

FIGURE 6.

The maximum value of q at $r = a/10$ may be found by equating to zero $\partial q / \partial t$ obtained from (19)

$$\frac{\partial q}{\partial t} = -\frac{2.56\, H_0}{\pi \mu a} \sum \frac{(n_p a)^2 J_1 (\frac{1}{10} n_p a) \cdot e^{-a_p^2 t}}{J(n_p a)} \tag{20}$$

If for $r = a/10$, q be plotted against t for a number of values of μ, t/μ will have the same value at the highest point of all the curves.

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[9] Each curve practically coincides with the axis of abscissas for a time, and then suddenly bends sharply away from the axis. It is not easy to indicate these sharp bends in a small figure.

In order to determine the manner of growth of the current in the coil of the solenoid when the circuit is suddenly closed, it will be well to follow the usual procedure in treating analogous problems in heat conduction, and inquire first how the coil current would decay if, after the core has been uniformly magnetized by a steady coil current, the electromotive force were suddenly cut out of the coil without opening the circuit. The solution of this problem furnishes immediately the solution of the one first stated. Heaviside[10] has treated by this method a solenoid with iron core 2 centimeters in diameter. If the coil is fairly thin, so that substantially the whole flux of induction which threads its turns is the flux in the core itself, we need not distinguish between L and L_1 in equation (11) and since there is no electromotive force in the coil

$$C + \frac{N\rho a}{2w}\left[\frac{\partial H}{\partial r}\right]_{r=a} = 0, \tag{21}$$

and by virtue of (3)

$$H_a + \frac{4\pi N^2 \rho a}{2w}\left[\frac{\partial H}{\partial r}\right]_{r=a} = 0, \tag{22}$$

or

$$H_a + s \cdot \left[\frac{\partial H}{\partial r}\right]_{r=a} = 0. \tag{23}$$

The constant s is to be determined from the constants of the solenoid and of the core, and we shall find it instructive to study the effect of a change in s in a problem otherwise given. We will assume at first that N, the number of turns of wire in the coil per centimeter of the length of the core, and w, the resistance in absolute units of the coil per centimeter of its length, are such as to make s unity; in the second case the value of s shall be 2.5. The first is somewhat less, the second much greater than the value which would most closely correspond to the magnet shown in Figure 1. The field intensity, H, in the core must satisfy equations (6) and (23) at every instant, and, when $t = 0$, must be equal to H_0 for all values of r.

The special solution,

$$A e^{-\alpha^2 t} \cdot J_0(nr), \tag{24}$$

of (6), in which $\alpha^2 = \dfrac{\rho (na)^2}{4\pi\mu a^2}$, satisfies (23) provided that $n\,a$ is a root of the equation

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[10] Heaviside, Electrical Papers, **1**, 394.

$$J_0\,(n\,a) = \frac{n\,a\,s}{a}\,J_1\,(n\,a), \tag{25}$$

and if we use the successive values [11] of n

$$1 = \sum \frac{2 \cdot J_0\,(n_p\,r)}{n_p a\,(1 + s^2\,n_p^{\,2})\,J_1\,(n_p\,a)}, \tag{26}$$

so that

$$H = \frac{2\,H_0}{a} \sum \frac{e^{-a_p^{\,2} t} \cdot J_0\,(n_p\,r)}{n_p\,(1 + s^2\,n_p^{\,2}) J_1\,(n_p\,a)}, \tag{27}$$

and

$$H_a = \frac{2\,H_0\,s}{a} \sum \frac{e^{-a_p^{\,2} t}}{1 + s^2\,n_p^{\,2}}. \tag{28}$$

In the case here treated $n\,a$ is a root of the equation

$$J_0\,(z) = \frac{2\,z}{25}\,J_1\,(z), \tag{29}$$

and it is not difficult to prove by the aid of Meissel's Tables that the first five roots have approximately the values given below.

TABLE V.

$n_1\,a = 2.2218$
$n_2\,a = 5.1171$
$n_3\,a = 8.0624$
$n_4\,a = 11.0476$
$n_5\,a = 14.0666$

Since

$$4\,\pi\,q = -\frac{\partial H}{\partial r} \tag{30}$$

and

$$\frac{d\,J_0\,(n\,r)}{d\,r} = -\,n \cdot J_1\,(n\,r), \tag{31}$$

$$q = \frac{H_0}{2\,\pi\,a} \sum \frac{e^{-a_p^{\,2} t}\,J_1\,(n\,r)}{(1 + s^2\,n^2)\,J_1\,(n\,a)}. \tag{32}$$

The function defined by (27) satisfies (23) when $r = a$ for all values of t, and equation (6) for all points within the core for all positive values of t; when $t = 0$, the function is in the core everywhere equal to H_0, and when t is infinite, it vanishes. It is easy to see, therefore, that a function H defined by the equation

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[11] Byerly: Treatise on Fourier's Series, etc., p. 229.

$$H \equiv H_\infty - \frac{2\,H_\infty}{a} \sum \frac{e^{-a_p^2 t} \cdot J_0\,(n_p\,r)}{n_p\,(1 + s^2\,n_p^2) \cdot J_1\,(n_p\,a)}, \tag{33}$$

where H_∞ is a given constant, satisfies equations

$$H_a + s\,[D_r\,H]_{r=a} = H_\infty$$

and (6), is everywhere equal to zero when $t = 0$, and, when t is infinite, is everywhere equal to H_∞. Since a function which satisfies these conditions is unique, (33) represents the strength of the magnetic field within the core of the solenoid while it is being magnetized from a neutral state to the uniform intensity H_∞ by a current (C) in the coil due to an electromotive force impressed in it. If N is the number of turns of wire in the coil, E the applied electromotive force in the coil circuit, and $w = w' + w''$ the resistance of the coil circuit, all three per centimeter of length of the solenoid, the coil current is given (11) by the equation

$$C = \frac{E}{w} - \frac{\frac{1}{2}\,N \rho\, a}{w} \cdot \left[\frac{2\,H_\infty}{a} \sum \frac{e^{-a_p^2 t}}{1 + s^2\,n_p^2} \right]. \tag{34}$$

It is possible to hasten the growth of a current of given final value in a simple circuit with fixed inductance (L) independent of the current strength, by increasing the applied electromotive force (E), and adding to the circuit a corresponding amount of resistance wound non-inductively; for this process decreases the time-constant L/r without changing E/r. The same statement is true in practice for almost every sort of electromagnet.[12] It is not easy to see immediately from equation (34), however, just what the effect on C is of a given change in w, for both n and a involve w implicitly through s.

Since $H_\infty = 4\,\pi\,N\,C_\infty$, we may rewrite (34) in the form

$$C = C_\infty \left[1 - \frac{2\,s}{a} \sum \frac{e^{-a_p^2 t}}{(1 + s^2\,n_p^2)} \right]. \tag{35}$$

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[12] After this paper was in type I became acquainted with the results of the elaborate study made by Professor T. Gray into the manner of growth of currents in the coils of electromagnets with finely divided cores. The beautiful curves which he gives in Volume 184 of the Philosophical Transactions of the Royal Society illustrate very strikingly the fact here mentioned.

For $\mu = 40$, and $s = 1$, the series and the parentheses which appear in this equation have at different times the approximate values given in Table VI.

TABLE VI.

t	Σ	$\frac{2s}{a}\Sigma$	$\left[1-\frac{2s}{a}\Sigma\right]$
0.00	6.2500	1.0000	0.0000
0.25	1.2985	0.2078	0.7922
0.50	0.8783	0.1405	0.8595
0.75	0.6743	0.1079	0.8921
1.00	0.5454	0.0873	0.9127
1.25	0.4530	0.0725	0.9275
1.50	0.3814	0.0610	0.9390
2.00	0.2748	0.0440	0.9560
3.00	0.1456	0.0233	0.9767
4.00	0.0775	0.0124	0.9876
6.00	0.0219	0.0035	0.9965
8.00	0.0062	0.0010	0.9990
10.00	0.0017	0.0003	0.9997

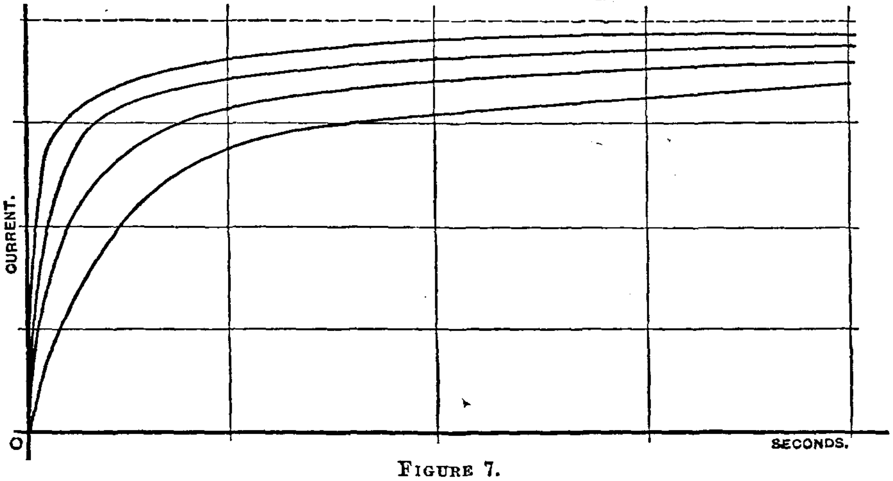

FIGURE 7.

Figure 7 shows C plotted against t for $\mu = 40$, 80, 160, and 320.

Since $4\pi q = -\partial H/\partial r$, it follows from equation (33) that the value of q in the case of the growing C is the negative of the value given by equation (32) for the case of decaying C. Figure 8 shows the value of q at different times for $\mu = 40$, 80, and 160, $r = a/10$, and $s = 1$.

When $s = 2.5$, the equation corresponding to (29) is

$$5 \cdot J_0(z) = z \cdot J_1(z), \tag{36}$$

and the first five roots have approximately the values given below.

TABLE VII.

$n_1 a = 1.9898$

$n_2 a = 4.7132$

$n_3 a = 7.6177$

$n_4 a = 10.6223$

$n_5 a = 13.6786$

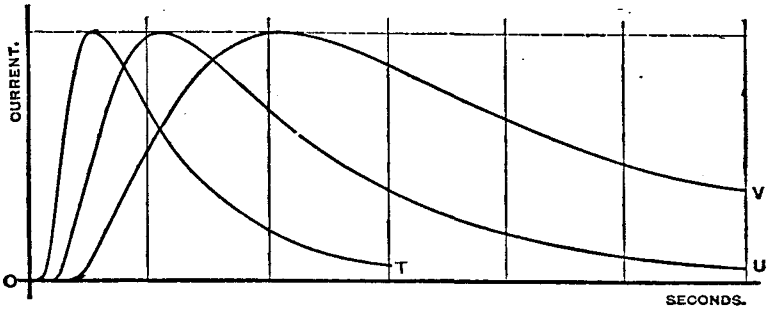

Figure 8.

and for $\mu = 40$, the series of (35) has approximately the values given in Table VIII.

TABLE VIII.

t	Σ	$\frac{2s}{a}\Sigma$	$\left[1-\frac{2s}{a}\Sigma\right]$
0.00	2.5000	1.0000	0.0000
0.25	1.0729	0.4292	0.5708
0.50	0.8052	0.3221	0.6779
0.75	0.6542	0.2617	0.7383
1.00	0.5511	0.2204	0.7796
1.25	0.4732	0.1893	0.8107
1.50	0.4111	0.1644	0.8356
2.00	0.3151	0.1260	0.8740
3.00	0.1889	0.0756	0.9244
4.00	0.1137	0.0455	0.9545
5.00	0.0685	0.0274	0.9726
6.00	0.0413	0.0165	0.9835
8.00	0.0150	0.0060	0.9940
10.00	0.0054	0.0022	0.9978

Suppose that in the case of the solenoid the resistance (w') of the coil alone, per centimeter of the length of the core, is such that when there is no outside resistance in the coil circuit, s is 2.5, and let us study the effect upon the manner of growth of C, of adding resistance wound non-inductively to the coil circuit to such an amount (w'') per unit of length of the core that s becomes 1.

From (35), we get

$$\frac{C_\infty - C_t}{C_\infty} = \frac{2s}{a} \sum \frac{e^{-\alpha_p^2 t}}{1 + s^2 n_p^2}, \tag{37}$$

and Tables VI and VIII show that the second member of this equation is greater for every value of t after the beginning when s is 2.5 than when s is 1, whatever the value of μ may be. The first member denotes the fractional part of the final current which the actual current has at the time t still to attain; if, then, the intensity of the final current ($C_\infty = E/w$) be fixed, and if the current be built up in the coil circuit, first, when $w = w'$ and $s = 2.5$ and the value of E is correspondingly low; and second, when $w = w' + w'$, $s = 1$, and E has a correspondingly high value, the actual current will lag behind the final current by a smaller amount at every instant in the second case than in the first. Again, if E be fixed and if different values be given to w so that C_∞ has different values the actual current lags behind the final current by a smaller fraction of the latter at every instant when s is 1 than when $s = 2.5$, that is when w is large than when w is small.

The quantity s, when the geometrical conditions are fixed, is inversely proportional to w, but is independent of μ, as is also n; a^2 is, however, inversely proportional to μ, and for two different values of μ, $e^{-a^2 t}$ would have the same numerical value at times which are to each other as these values of μ. It is sufficiently well proved that the effective permeability of the iron core of an electromagnet, when a current is rising rapidly in the coil, is not always the same as the permeability belonging to the instantaneous value of the current as determined from a statical hysteresis diagram. If in any case where w is fixed the effective value of μ should be greater or smaller for an increase in the value of the applied voltage E, the growth of the current would be relatively retarded or accelerated. We shall find it well to return to this subject later on.

The Effect of Variation in the Permeability of the Core of an Electromagnet upon the Manner of Growth of a Current in the Coil.

We may add to the foregoing theoretical discussion of the effects of Foucault currents in a solid core the permeability of which has a fixed value, a few words upon the manner in which a current might be established in the coil of the magnet described in this paper if there were no eddy currents in the core, as, of course, there really would be in an actual case, unless the core were divided.

After the form of the dotted curve $D\,P$ in Figure 4 had been determined with considerable accuracy (the observations of different days agreeing with each other almost exactly), the curve was plotted on a very large scale by means of a needle point upon thin sheet zinc and an accurate template was then cut out; with the help of this and a metal straightedge, I measured as carefully as I well could the slope (λ) of this curve for a large number of different values of the current (i). Since 2.823×10^9 times an ordinate of the $D\,P$ curve shows the total flux (F) through the coil for a current represented by the corresponding abscissa in amperes, then, if there were no eddy currents in the core and no time-lag in the magnetization of the iron, the building up of a current in the coil under the given circumstances on the application of a steady voltage E in a circuit of total resistance r ohms would be dominated by the equation

$$E - r\,i = 28.23\,\lambda \cdot \frac{d\,i}{d\,t}, \tag{38}$$

(or)

$$\frac{d\,t}{d\,i} = (28.23)\,\frac{\lambda}{E - r\,i}, \tag{39}$$

in which the second member is now a known function of i. I plotted this function and determined by aid of an Amsler's Planimeter the values of t for a number of values of i in the actual case, where E was 84.0 and $r = 13.55$. Figure 9 shows the building-up curve (Q) which this process yields, and also the actual curve[13] carefully reproduced from an oscillograph record.

If we were to define the inductance of the magnet in a condition represented by a point R on the dotted line $D\,P$ in the statical hysteresis diagram, as the ratio of the total flux which then passes through the coils to the intensity of the current belonging to the point, its value would decrease when the current increased, in the manner shown in the subjoined table.

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[13] J. and B. Hopkinson, The Electrician, September 1892; J. Hopkinson, Wilson and Lydall, Proceedings Royal Society, Vol. **53**.

TABLE IX.

Current.	"Inductance" in Henries.	Current.	"Inductance" in Henries.
0.5	169.4	3.5	57.4
1.0	124.2	4.0	51.9
1.5	99.8	4.5	47.5
2.0	84.0	5.0	43.7
2.5	72.3	5.5	40.5
3.0	64.0	6.0	38.1

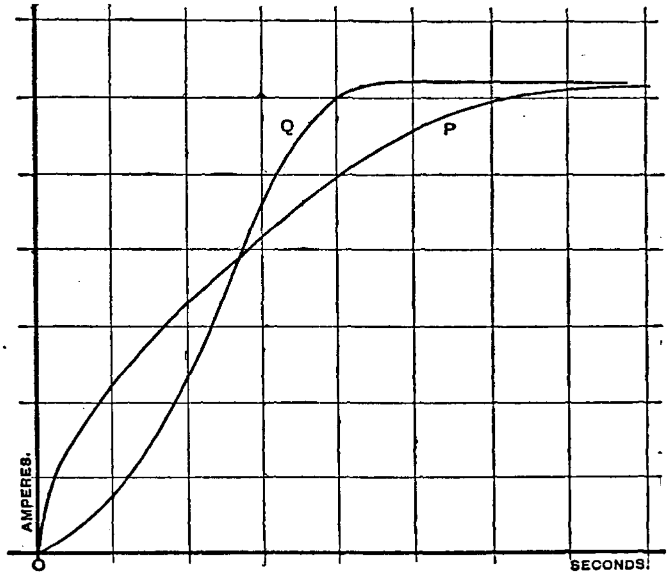

FIGURE 9.

If, however, we write

$$\frac{d\,(L\,i)}{d\,t} = E - r\,i \tag{40}$$

and plot from the actual building-up curve given in Figure 9, $E - r\,i$ as a function of t, we may get the area under this curve by means of a planimeter for a series of values of t, and determine L from the equation

$$L\,i = \int_0^t (E - r\,i)\,dt. \tag{41}$$

The "apparent inductance" thus defined is affected by Foucault currents; its values for several points of the curve are given in Table X.

TABLE X.

Time.	Current.	L.	Time.	Current.	L.
0.1	0.55	80.2	3.5	4.64	44.1
0.5	1.55	70.4	4.0	5.03	40.9
1.0	2.25	64.0	4.5	5.35	37.9
1.5	2.84	59.1	5.0	5.62	35.1
2.0	3.33	54.9	5.5	5.84	32.4
2.5	3.79	51.9	6.0	5.99	30.0
3.0	4.23	47.4			

L plotted against i yields a curve that is fairly straight.

The theoretical curve (Q) is very like some[14] of the actual curves which have been obtained from magnets with finely divided cores, whereas the actual building-up curve represented in the diagram is similar to those[15] which almost any electromagnet with solid core can be made to yield. The eddy currents, the changes in effective permeability, and the other disturbing influences taken together, do not in such a case cause the curve to deviate very widely in shape from that which one could get from a simple circuit with fixed inductances and no eddy currents: the resemblance can almost always be made close by proper choice of the electromotive force of the exciting battery.

After I had determined hysteresis diagrams (some of which are shown in Figure 3) for the magnet for a large number of gap-widths up to 28 mm., it seemed likely that for an air gap about 35 mm. wide the hysteresis diagram would not be very different from a single straight line. For this gap-width the induction flux through the coil should be practically proportional to the strength of the current and amount to about 6.45×10^6 for a current of one absolute unit so that the inductance of the coil circuit should be about (28.23) (0.645) henries for a wide range of currents. The form of the building-up curve of a current in the coil of an electromagnet generally depends very much upon the magnetic state of the iron at the outset. If a steady current which has been running through the coil for some time be interrupted, and if then after a little the circuit be closed again, the manner of growth of the new current is generally very different if this current has the same direction as its predecessor or the opposite direction: that is, if the magnetism of the core follows the hysteresis diagram in a direction corresponding to $D P$ in Figure 4 or in the direction $D Q$. An easy way of testing whether the hysteresis curve of a *large* electromagnet has an insignificant area is to obtain large oscillograph records of the building-up curves of direct and reversed currents and to compare the two. I took, therefore, a series of building-up curves for a gap-width of 35 mm., using currents of 2.75 amperes and 5.60 amperes, and found that in both cases the curves were wholly indistinguishable[16] even when enlarged and superposed on the screen, whether the current in question had the same direction as its predecessor or the opposite direction. With this gap-width, therefore, the magnet is an example of a circuit "containing iron" with an induction flux for *steady currents* almost exactly proportional to the strengths of these currents, and in this sense with a fixed inductance. If there were no eddy currents, and no time-lag in the magnetization of the core, the growth of the current in the coil should follow the law

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[14] See, for instance, the fine diagram given by Dr. Thornton, Phil. Mag., 1904, p. 625.

[15] J. Hopkinson and Wilson, Phil. Trans, 1895, pp. 275, 280.

$$C = \frac{E}{r}(1 - e^{-rt/L}),$$

where L is this fixed inductance. Figure 10 shows the actual oscillograph records for 2.75 amperes and 5.60 amperes in full line, and the theoretical curve in dotted line for $E = 80$. In this case, where the "statical effective" value of μ is independent of the current, but where eddy currents and what we may term time-lag in the taking up of the magnetism by the iron may enter, it is interesting to see that in spite of the retardation due to eddy currents, the current in the main circuit builds up more quickly than would correspond to the statical value of μ when the current is 2.75 amperes and that it starts to do so when the current is 5.6 amperes.

The building-up curves shown in this paper are careful reproductions of oscillograph records, of which I have several hundreds. Some of these were obtained with the aid of a Duddell Double Oscillograph, the drum of which could be turned either by an electric motor from the alternating street circuit or by clock-work, but most of them I got with the help of two single instruments made by Mr. J. Coulson, who helped me to take the photographs, and these served their purpose admirably. One of them, which was used in measuring comparatively small induction currents and needed to be very sensitive, was not quite aperiodic when suddenly deflected to the end of its scale, but this fact did not affect records of the kind used here. This instrument consisted merely of a mirror galvanometer in which the extremely minute magnet and mirror were fastened to a piece of fine stretched gimp damped in oil. I had four drums for carrying the sensitized paper, or the film when this had to be used, and any one of these could be driven very uniformly at almost any rate up to a rim velocity of a meter per second by means of chronograph clock-work. Most of the paper used was in strips from 16 to 20 centimeters wide and some of the curves are about a meter long. The deflections of each oscillograph were strictly proportional to the current in its coil. A large zinc template, which carefully kept the irregularities of the record, was made for each curve; this template was used in drawing a large diagram and the figures here given are copies of these diagrams very much reduced. A few of the larger records were redrawn

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[16] For a similar case, see Professor T. Gray, Phil. Trans., Vol. 184.

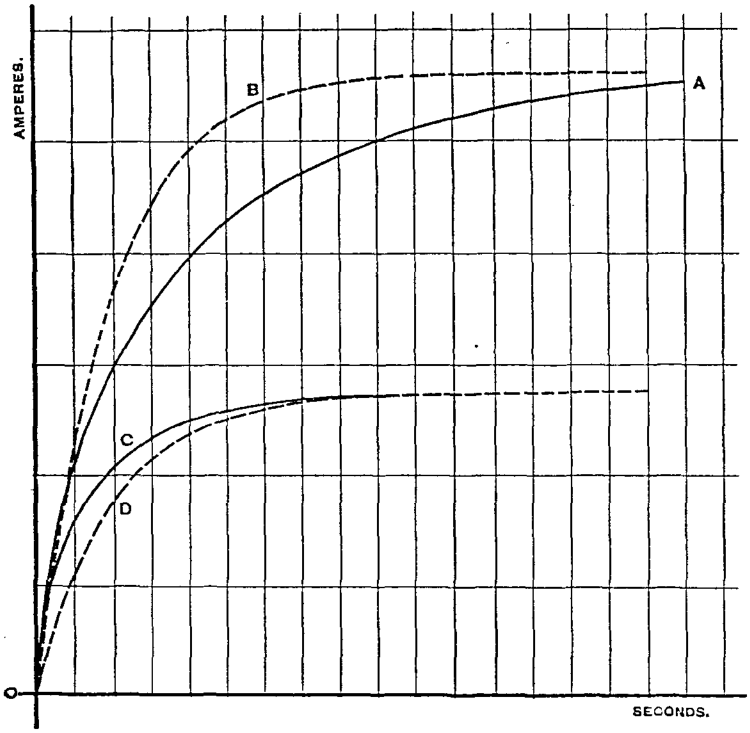

FIGURE 10.

from measurements made on the photograph itself, but all the smaller ones, and most of the others, were pricked through under a lens by a very fine needle point and the record itself was then placed directly in front of the condensing lenses of a large projecting apparatus and thrown

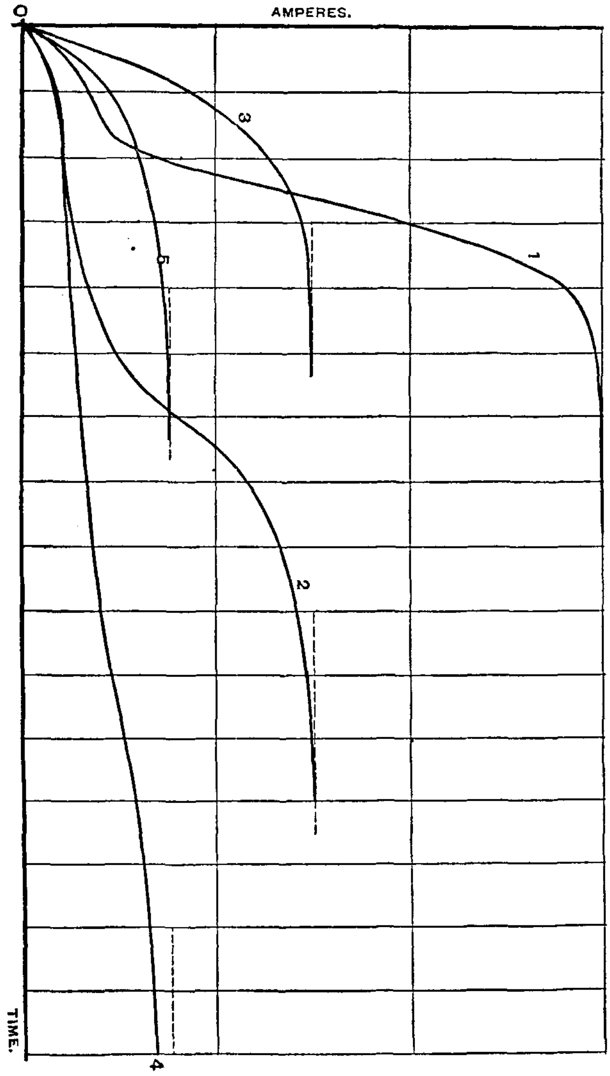

Figure 11.

up on paper tacked to a screen; it was generally possible to see the whole of the diagram on the screen besides the bright images of the needle holes, and to reproduce this diagram much enlarged upon the paper. With a given battery, and a given current with a given condition of the iron, it was always easy to get any desired number of records which, when superposed upon the screen, were practically indistinguishable.

Figure 11 shows a series[17] of building-up curves from a 15-kilowatt transformer very kindly placed at my disposal by Mr. S. E. Whiting. The finely laminated core of this transformer has a cross-section area of 108 square centimeters. The same magnetizing coil of about 340 turns was used throughout, but the electromotive force of the storage battery in the coil circuit could be changed so as to give the current the desired strength. In curve (1) the final intensity of the current was about 3 amperes, in curves (2) and (3) it was 1.5 amperes, and in curves (4) and (5), 0.75 amperes. With each value of the electromotive force the steady current was sent through the coil, first in one direction and then in the other, for a number of times, and was then interrupted for a few moments previous to taking the photographic record from the oscillograph. Curves (1), (2), and (4) were obtained when the current in the coil had a direction opposite to that of the next preceding current; curves (3) and (5), when the current had the same direction as the preceding one. For a current of 0.37 amperes the reversed-current-curve had lost its points of

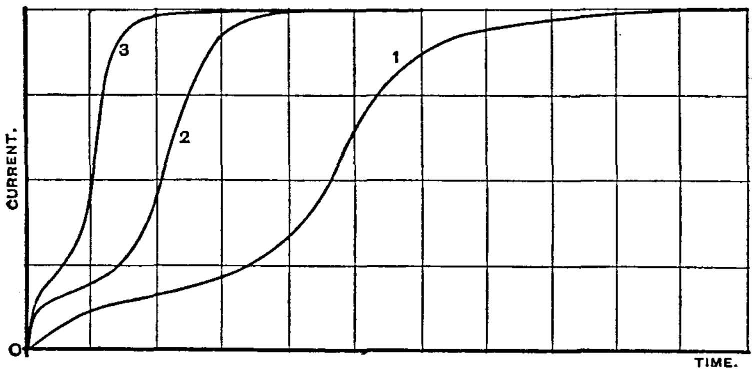

FIGURE 12.

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[17] For a large number of similar curves, see Professor T. Gray's paper in the Philosophical Transactions, Vol. 184.

inflection, and had become everywhere convex upward. With a voltage of only 6 and the low resistance primary of the transformer as exciting coil, the building-up time was an extremely short fraction of a second, and the building-up curve looked like a very straight, and nearly vertical, sign of integration.

If a number of equal coils of wire of a given size, each of resistance r, be wound together uniformly about a wooden ring so as to have equal self-inductances, and if a storage battery of small internal resistance be made to send a current through (say) n of the coils in series, the inductance of the circuit will be nearly proportional to n^2 and the time-constant which is independent of the applied electromotive force and therefore of the current, will be nearly proportional to n. If the core of the ring be made of iron, the problem will, of course, be complicated in many ways, but in this connection Figure 12 is interesting; curves 3, 2, and 1 show the manner of growth of a current in coils of about 85, 170, and 340 turns about the core of the transformer just mentioned, in terms of its final value. The electromotive force was in reality the same for all three cases, and the currents were 6 amperes, 3 amperes, and 1.5 amperes.

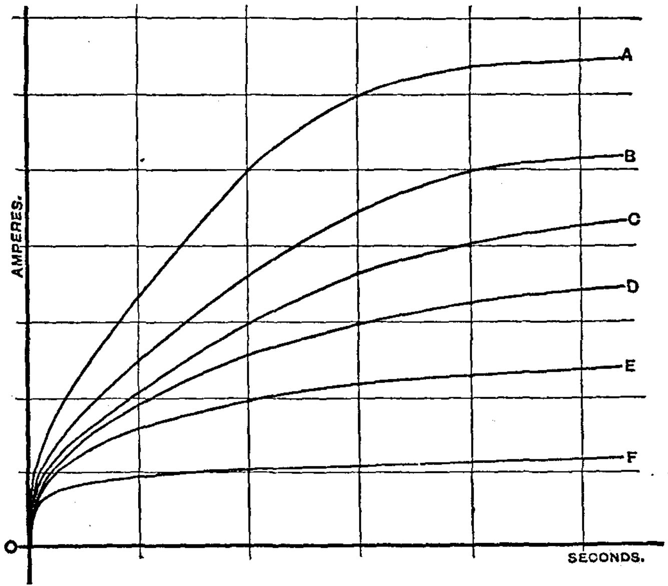

FIGURE 13.

The Manner of Growth of a Current in the Coil of an Electromagnet which has a Solid Core.

We may now consider some oscillograph records which show the manner in which under given circumstances a current will grow in the coil of the magnet represented by Figure 1. It is evident from what precedes that when other conditions are determined, the magnetic state of the core at the time when the coil circuit is closed will generally influence the result greatly. By sending through the coil a long series of steady currents of gradually decreasing intensity alternately in one direction and the other, one may reduce very low the residual magnetism in the iron even when the gap is closed.

Figure 13 shows building-up curves from a nearly neutral core, under a voltage of about 84, when the air gap was closed, for currents of six different intensities from 1.2 amperes to 6.5 amperes.

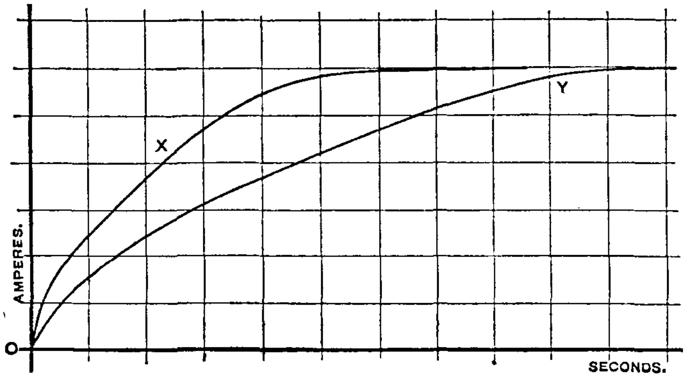

Figure 14.

Figure 14 shows curves obtained with a voltage of about 80 under two different conditions of the core. After the iron had been pretty well demagnetized, the growth of the current when the coil circuit was suddenly closed followed the law indicated by the upper curve: if, when the current had become steady, the circuit was broken and, after fifteen or twenty seconds, closed with the direction of the current reversed, the march of the current followed the lower curve. The width of the gap was 1.6 mm. in this experiment. To study the influence of the width of the air gap upon the shape of the building-up curve of the current in the coil circuit I used first a storage battery of relatively high voltage

(270) and obtained a long series of records for gap-widths up to about 30 mm. When the gap was closed and the current had been sent first in one direction and then in the other alternately for a number of times, I got the results given in Table XI for the growth of a current (*A*) which had the same direction as the next preceding current, and the results given in Table XII for a current (*B*) which had a direction opposite to that of the preceding current.

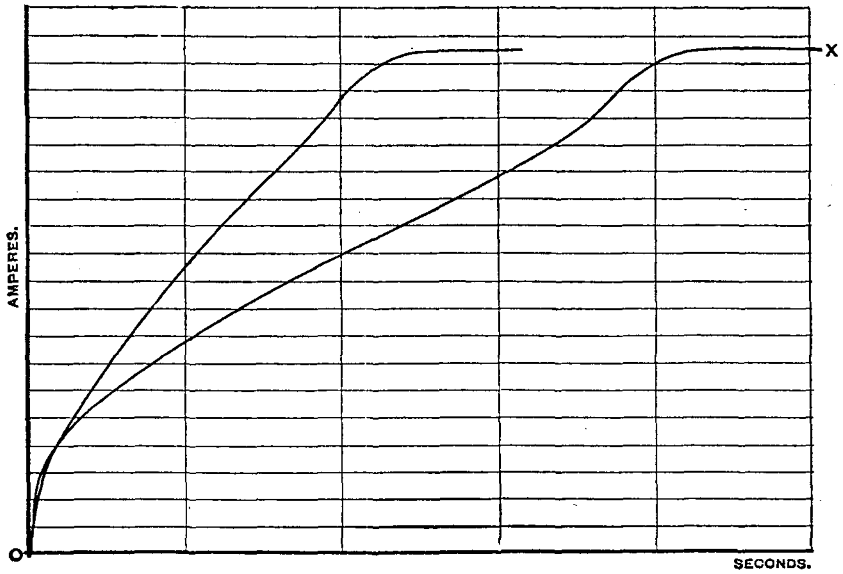

Figure 15.

TABLE XI.

A. (Direct Current.)

Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.	Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.
0.10	3.20	1.50	13.95
0.20	4.36	1.75	15.41
0.30	5.37	1.80	15.84
0.40	6.33	1.90	16.42
0.50	7.21	2.10	17.67
0.75	9.07	2.20	18.16
1.00	10.96	2.40	18.56
1.25	12.55	2.50	18.60

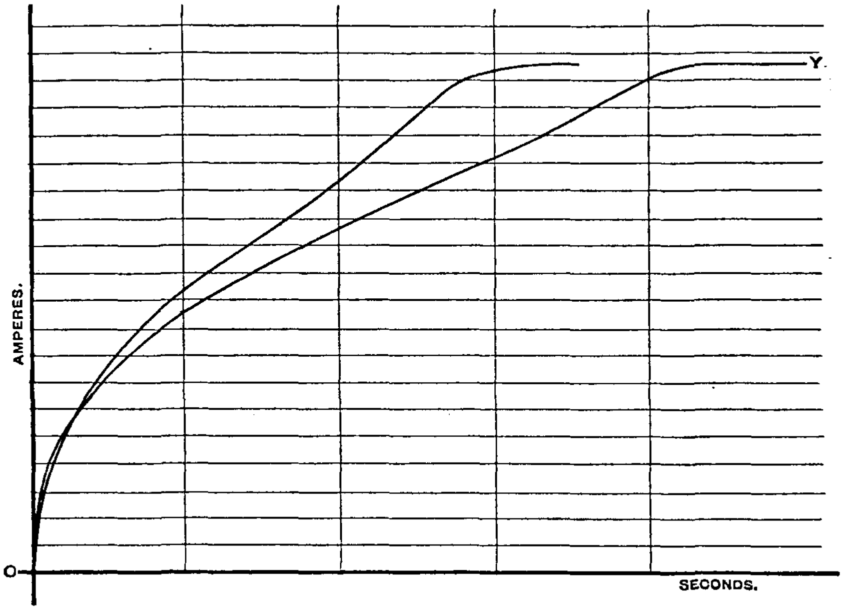

FIGURE 16.

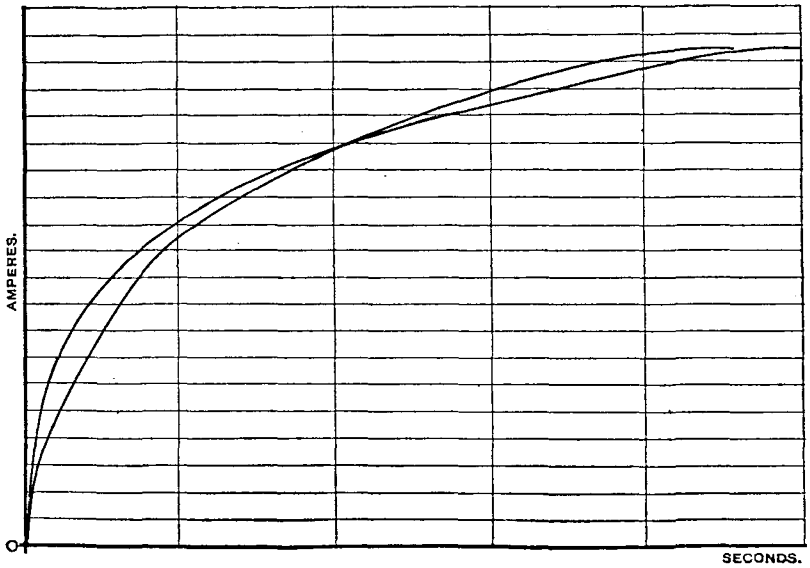

FIGURE 17.

When the gap is relatively narrow the shapes of the curves for direct and reversed currents differ little from those of the corresponding curves when the gap is closed, though the time required after the closing of the circuit for the current to attain (say) 99 per cent of its final value slowly increases with the gap-width. Figures 15, 16, 17 show curves for direct and reversed currents when the width of the air gap was 1.6 mm., 6.6 mm., and 28.3 mm. For gaps much wider than the last, the two curves of each diagram fall closely together. When with the same battery a sufficiently large non-inductive resistance was introduced into the circuit to reduce the current to 8.5 amperes, the building-up curves for gaps of width 1.6 mm. (full lines) and 25.0 mm. (broken lines) had the forms shown in Figure 18.

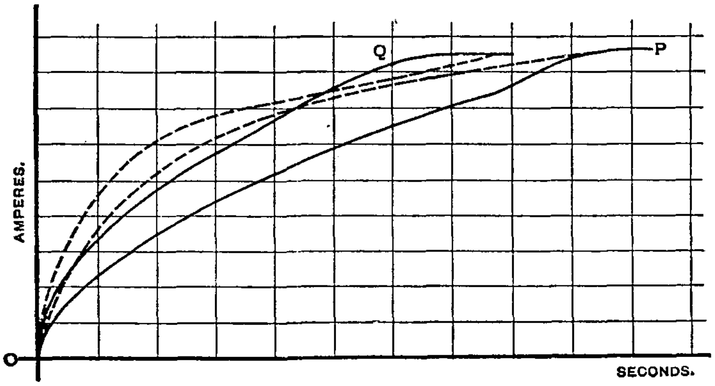

Figure 18.

TABLE XII.

B. (Reversed Current.)

Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.	Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.
0.10	3.34	3.20	14.76
0.20	4.36	3.40	15.43
0.30	4.94	3.50	15.84
0.40	5.48	3.60	16.48
0.50	5.87	3.70	17.15
1.00	7.99	3.80	17.58
1.50	9.75	4.00	18.31
2.00	11.19	4.30	18.56
3.00	14.12	4.50	18.60

When the voltage of the storage battery was reduced to 90 the building-up curve for a reversed current of 6.9 amperes had the form of the right-hand curve (1) in Figure 19 when the gap was closed, but of the next curve (2) when the air gap was 13 mm. wide. The curves cross eleven seconds after the start. For the current to attain half its final strength, 3.5 seconds are required when the gap is closed, but only 1.5 seconds when the width of the gap is 13 mm.; the current attains 99 per cent of its final value more quickly when the gap is closed than when it is open.

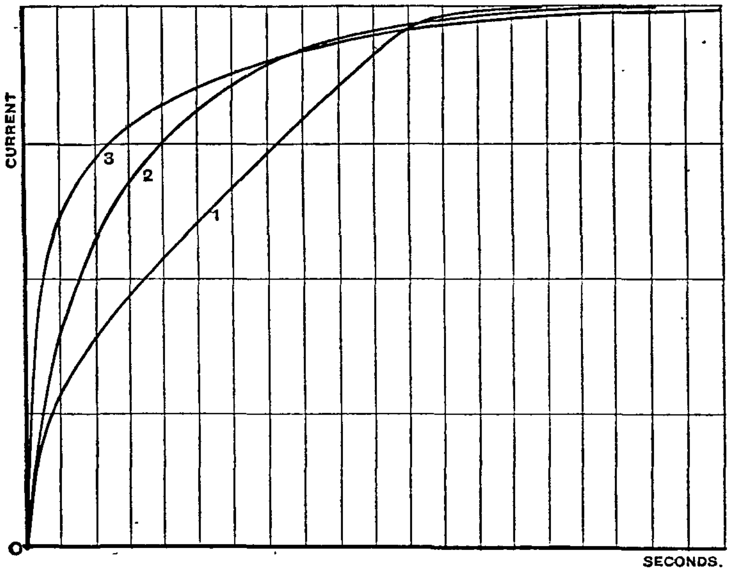

FIGURE 19.

The leftmost curve (3) of Figure 19 belongs to a reversed current of 1.3 amperes when the gap was closed; the ordinates are exaggerated so as to make the final value the same as for the larger current.

When a secondary coil, consisting of a few turns of insulated wire wound around the whole core of the magnet, was connected with an oscillograph which made its record on the same sheet of paper as the oscillograph connected with the main circuit, it was easy to get the rate of growth of the induction flux in the core. Figure 20 shows building-up curves when the gap was closed for a current of 1.3 amperes furnished (*A*) by a battery of 40 storage cells and (*D*) by a battery of 10 storage cells; in the first case it was necessary of course to add non-inductive resistance to the circuit to reduce the current to this value. The time required for the current to attain practically its final value is far less in the first case than in the second. The dotted curves show the records of the oscillograph in the secondary circuit on an arbitrary scale. The two curves are quite unlike in shape, but the areas under them, as measured by a planimeter, are almost exactly the same. The general forms of the dotted curves here shown are like all the scores of others which I have obtained under all sorts of conditions of current strength and resistance.

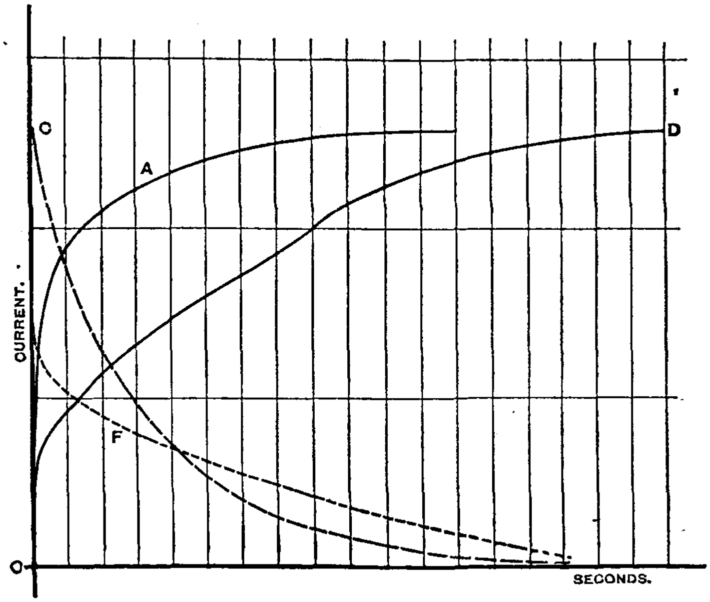

Figure 20.

The pole pieces of the magnet shown in Figure 1 are fastened to the cylindrical arms by long bolts which extend outside the frame and carry nuts which press upon the yokes and serve to keep the jaws apart. When one of these bolts was removed it left a long axial hole of about an inch in diameter through the arm, and into this I put a long rod of soft iron upon which a layer of fine insulated wire had been wound pretty uniformly, and this coil was connected with the secondary oscillograph already mentioned. This coil and its long core filled the cavity fairly completely except at the outside ends, but doubtless the joint at the inner end of the core affected the results somewhat, and in a manner not easy to treat mathematically; nevertheless the indications of this coil were interesting and characteristic in their general features, as we shall see

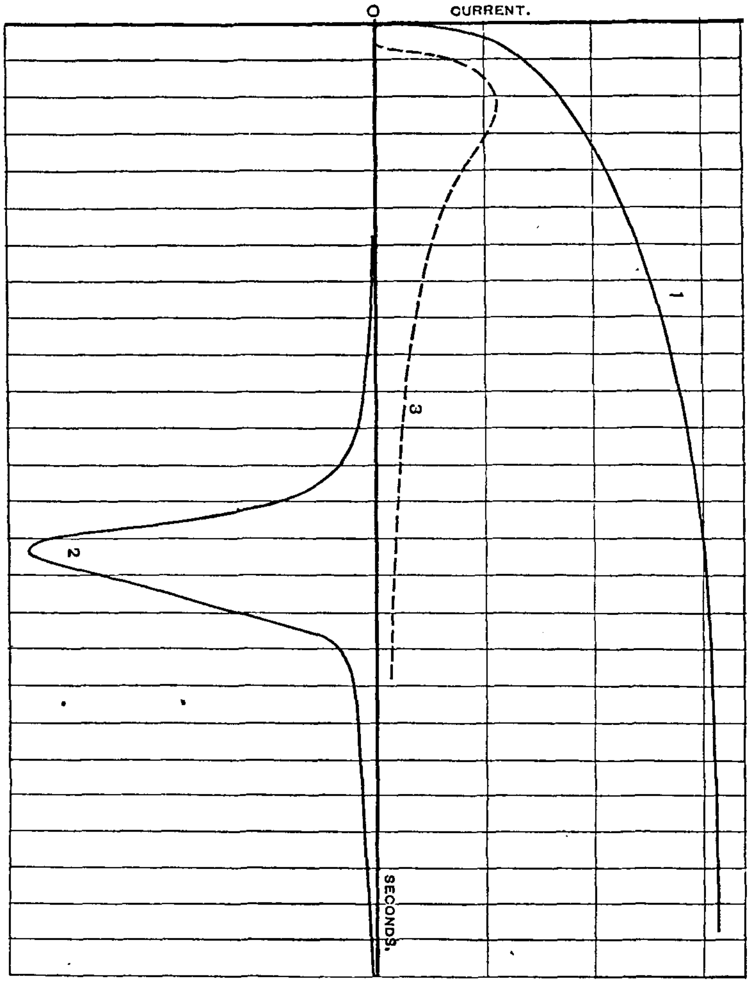

Figure 21.

hereafter, of the behavior of the eddy currents in the core near its axis. When a current was started in the coil circuit, the mirror of the oscillograph attached to the secondary coil remained sensibly at rest for an interval the length of which depended upon the final value of the coil current, the electromotive force in its circuit, and the magnetic condition of the core at the start; then a temporary current which had a sensible value for perhaps ten or fifteen seconds passed through the secondary, though if a sensitive ballistic galvanometer instead of the oscillograph had been connected with the coil it would have been made clear that the temporary current had not wholly disappeared at the end of this interval. When, after the current in the main circuit had been apparently steady for a minute or two the coil circuit was suddenly broken, the current in the secondary became almost immediately evident, soon attained its maximum value, and then died slowly away so that the mirror seemed to reach its zero again after about thirty seconds.

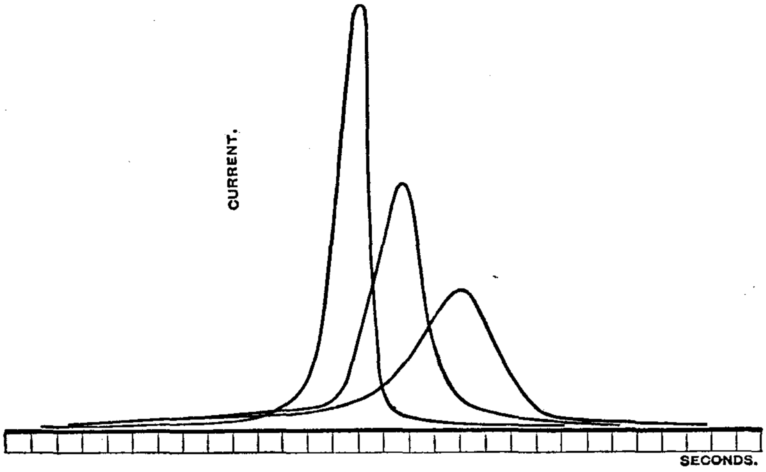

Figure 22.

Figure 21 shows a typical oscillograph record accurately reproduced. It was obtained with the air gap closed and with a current of 3.12 amperes in the main circuit from a storage battery of about 84 volts. For five or six seconds after the start there was no sensible indication in the secondary coil, though by that time the main current (1) had attained about three fourths of its final value. The greatest value of the current in the secondary did not occur until rather more than fourteen seconds after the current in the primary had begun, and the secondary current had apparently died out in less than thirty seconds from the beginning. The ordinates of the curve of the current in the main circuit represent amperes;

the ordinates of the curve (2) which represents the current in the secondary, are on an arbitrary scale. If at the time $t = 0$, the main circuit through which a steady current of 3.12 amperes had been passing were suddenly broken, the current in the secondary would have grown and died out after the manner indicated by the dotted curve (3). For about half a second after the main circuit was interrupted, there would have been no sensible current in the secondary, but then it would have suddenly appeared in a manner that strongly suggests the theoretical curves shown in Figures 6 and 8. Figure 22 illustrates the fact that the form of the secondary current curve depends very much upon the intensity of

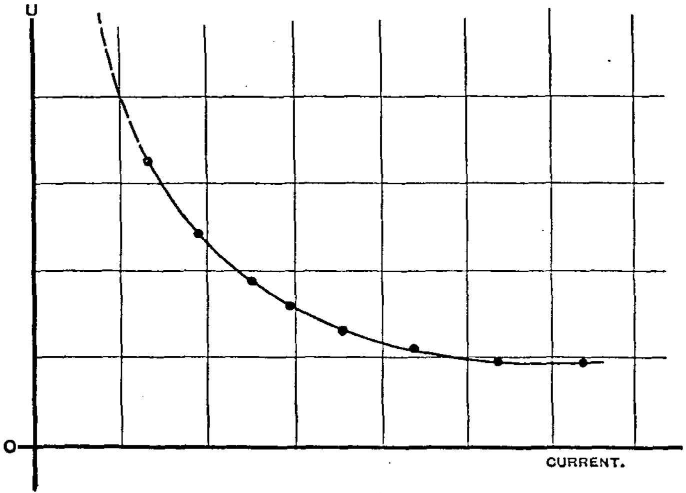

FIGURE 23.

the current in the main circuit, and that sometimes a comparatively slight change in the latter will alter materially the maximum intensity of the secondary current. The diagram is a careful reproduction of the records of the secondary oscillograph for currents of 3.76 amperes, 3.12 amperes, and 2.60 amperes respectively. The scale at the bottom represents seconds; the records, which are to be read from right to left, are displaced with respect to each other merely to prevent confusion in the figure.

The lag in seconds of the crest of the current in the secondary circuit behind the closing of the main circuit depends, naturally enough, upon the final intensity of the main current and upon the magnetic condition of the core at the outset. If a series of currents of the same final intensity be passed first in one direction and then in the other through the main coil, and if after the circuit has been broken for half a minute the current be sent through the coil again, the retardation of the crest of the secondary will be perhaps twice or thrice as great if the direction of the new current be opposed to that of the next preceding one, as it would be if the new current had the same direction as its predecessor.

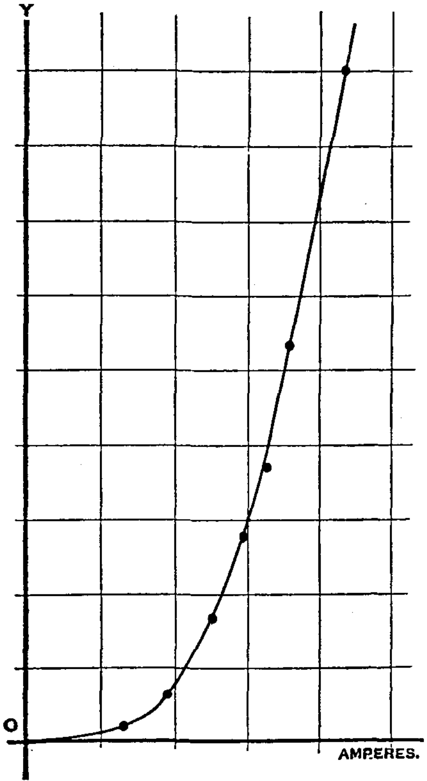

FIGURE 24.

In Figure 23 the abscissas represent currents in amperes and the ordinates the lag, in tens of seconds, of the crest of the secondary current behind the closing of the primary circuit when the voltage was about 84 and when the current considered had the opposite direction to the one before it. The gap was in this case closed. For a current of 1 ampere, as the diagram shows, the lag is about 40 seconds, while for a current of 5 amperes it is only 10 seconds. In Figure 24 the ordinates represent the relative heights of the crests of the secondary currents corresponding to primary currents the intensities of which, in amperes, are represented by the abscissas.

In the next portion of this paper I hope to discuss some of the results given here, with a large number of others which throw some light upon the march of the eddy currents in a massive iron core of the kind here used. In this connection the records of the secondary coils wound upon the separate members of a set of loosely-fitting, coaxial, cylindrical shells of soft steel, made part of the core of the magnet represented in Figure 1, will be interesting.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

ON THERMO-ELECTRIC HETEROGENEITY IN CERTAIN ALLOYS, ESPECIALLY GERMAN-SILVER.

By Edwin H. Hall, L. L. Campbell, and S. B. Serviss.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

ON THERMO-ELECTRIC HETEROGENEITY IN CERTAIN ALLOYS, ESPECIALLY GERMAN-SILVER.

By Edwin H. Hall, L. L. Campbell, and S. B. Serviss.

Presented December 13, 1905. Received January 2, 1906.

Purpose and General Plan of the Investigation.

In a paper on "Thermal and Electrical Effects in Soft Iron," published in these Proceedings for May, 1905, by the authors of the present paper, together with Mr. E. P. Churchill, brief account was given, at pages 42–45 and 54, of the construction and calibration of two sets of copper and german-silver thermo-electric couples, which will hereafter be referred to as A, B, C, D and A_1, B_1, C_1, D_1, respectively. The study of these couples, all of which were made from one original piece of copper and one original piece of german-silver, was confined to the temperature range 0° C. to 100° C., as the experiments on soft iron, to which experiments the thermo-electric couples were contributory, lay within this range.

It was intimated, however, in the paper already mentioned, that similar experiments on iron through the interval from 100° C. to 200° C. would presently be undertaken, and with a view to this work a study of copper and german-silver thermo-electric couples through this higher range became necessary. It was hoped, too, that these proposed calibration tests could be carried out with such accuracy and thoroughness as to add something valuable to our knowledge of the general behavior of such thermo-electric couples, the permanence or variability of quality in any particular couple, for example, or the causes and magnitude of the differences between couples made, as nearly as may be, of the same materials and in the same way. These secondary purposes of the investigation have been in some measure fulfilled, through variations of experimental conditions which were not at first planned and which seem to us worthy of being described with some detail.

More strictly, the higher limit of temperature now proposed was the boiling point of naphthalin under atmospheric pressure, about 218° C.

Furthermore, following the advice of Professor J. M. Crafts, who gave us every facility to copy the apparatus which he has used in his comparative study of the behavior of boiling water and boiling naphthalin under various pressures, we determined to employ for calibration purposes all the way from 100° C. to 218° C. the saturated vapors of water and naphthalin, respectively, using the water at high pressure and the naphthalin at low pressure for the temperatures intermediate between the limits.

General Summary.

1. All of the german-silver wires used in the experiments of this paper, with the exception of those taken for a certain experiment to be described below, were cut from one continuous strand described by the dealer from whom it was procured as containing 18 per cent of nickel. Special chemical analysis of a specimen of this wire, taken unannealed, gave as the composition:

Copper,	58.76	per	cent.
Zinc,	22.54	"	"
Nickel,	17.44	"	"
Iron,	1.29	"	"

Analysis of the annealed wire resulted as follows:

Copper,	63.19	per	cent.
Zinc,	18.48	"	"
Nickel,	17.59	"	"
Iron,	0.71	"	"

The diameter of this wire was about 0.02 cm.

2. Pieces of this wire were annealed by heating to incandescence by an electric current and keeping them in this state for some seconds in free air, the ends of each piece during the annealing being kept at the same level, so that the heated wire had approximately a catenary shape. After cooling, the wires were found to have suffered very little, if any, permanent extension. The strength of current used was usually about 3.5 amperes, but it was not very carefully measured. The state of incandescence probably lasted two or three times as long in some cases as in others.

3. Couples made from copper, also 0.02 cm. in diameter, and pieces of german-silver annealed as in (2), appeared to differ from each other in sensitiveness, when new, rather less than 0.5 per cent; but after

four such couples had been used for some time, with repeated heatings of the junctions to a temperature of 218° in air (sometimes contaminated by vapors of various kinds) the difference appeared to be greater, amounting in one case to slightly more than 2 per cent. These couples had been subject to many vicissitudes of handling; but no satisfactory explanation of the increase in their range of sensibility can at present be given; it is possible that the first estimate of this range was too small and the last too large.

4. The german-silver wires used in these couples were about 1.5 m. long. When one junction of such a couple was immersed to a depth of 50 cm. in air heated to the neighborhood of 180° C. and the other to a like depth in air at a temperature near 140° C., the intermediate part of the german-silver wire being exposed to the air of the room with no other shield than the wall of a glass tube within which it was enclosed, the mean electromotive force per degree of the couple sometimes changed as much as 5 per cent when the temperature interval was reversed, the junction which had been in the 180° place being put into the 140° place and *vice versa.* All attempts to explain such discrepancies as due to local electromotive forces in parts lying outside the german-silver failed. Attempts to account for them by means of local peculiarities in the german-silver wire led to the experiments described in the next three sections of this summary.

5. Four couples, each made from a single piece of annealed german-silver by leaving one part of it straight while the other part was formed into an elongated spiral wound on a cylinder about 1 cm. in diameter, were tested. The mean electromotive force of the four appeared to be about 0.07 per cent of the electromotive force of a couple made of german silver and copper, the spiral wire being thermo-electrically a little farther away from copper than the straight wire is. No great confidence should be placed in the numerical accuracy of this result.

6. Four couples, each made from a single piece[1] of annealed german-silver by leaving one part of it unchanged while the other part was stretched permanently 10 per cent, were tested. As a mean result it was found that such couples have an electromotive force about 0.6 per cent as great as that of a copper and german-silver couple, the stretched german-silver being thermo-electrically farther removed from copper than the unstretched wire is.

---

[1] These four pieces were from the same spool of wire (all called 18 per cent nickel) as the others, but not from the same strand.

If in the testing of the (copper)-(german-silver) couples a part of the german-silver wire lying in the place of steepest temperature gradient, just outside the entrance to one of the heating vessels, happens to have been severely stretched, it may give rise to such a discrepancy of behavior as that noted in (3) above. But, as such discrepancy was found in couples the german-silver of which had been carefully guarded from any serious stretching, it seemed necessary to look further for a satisfactory explanation.

7. The application of the sharp edge of a lump of ice to a straight piece of german-silver wire exposed to the air of the room, this application being many centimeters distant from either end of the wire, developed a perceptible local electromotive force, which frequently changed its sign as the point of contact was moved, 1 cm. at a time, along the wire. The cooling produced by the ice was plainly perceptible at a distance of 1.5 cm. along the wire from the point of contact, though the cooling at 1 cm. from the contact point is perhaps not more than one third as great as at this point. We may regard the ice test as showing that the mean thermo-electric quality of the wire for a distance of perhaps 1 cm. in one direction from the point of contact is perceptibly different from that of the corresponding piece lying on the opposite side of the point of contact.

Annealing was found to increase the range of thermo-electric heterogeneity of the wire and to change the local distribution of this heterogeneity.

Stretching annealed wire to a permanent extension of 10 per cent made very little change in the range or distribution of the thermo-electric heterogeneity. Reannealing a wire which had been thus stretched changed again the distribution of its peculiarities and increased still further the range of these peculiarities. In a wire so treated, contact at one point with an edge of ice developed an electromotive force about 0.2 per cent as great as that which would have been developed by touching the ice to a junction of copper with german-silver.

These tests show that there may be in straight, recently annealed german-silver wire local differences of thermo-electric quality which may cause serious error if a copper and german-silver couple is used to measure a small difference between two high temperatures, the intermediate part of the german-silver wire being exposed to air at the ordinary temperature of a room. To avoid error from this source the practice of reversing the temperature interval of the junctions should be followed whenever this is practicable.

8. By keeping the intermediate parts of the german-silver wires not far below the temperature of the cooler junction in the calibration tests, the change of sensitiveness observed on reversing the temperature interval of the junctions, about 40°, was reduced from an average of about 3 per cent to an average of about 0.8 per cent.

9. As a result of all the calibration work done on thermo-electric couples of the kind described above it appears that, if we take 2100×10^{-8} volt as the mean "thermo-electric height," electromotive force with 1° difference of temperature of junctions, at 100° C., and if we take 4.8×10^{-8} as the rate of increase per degree rise of temperature, we shall get values which will not at any point between 0° and 218° differ as much as 1 per cent from the most probable value which we have for that point.

10. A piece of manganin wire subjected to the ice test described in (7) behaved very much as the german-silver wire did, in magnitude of local electromotive forces developed, in effect of annealing, and in non-effect of stretching.

11. A piece of constantin wire subjected to the ice test showed much smaller local electromotive forces than did german-silver and manganin, perhaps one fifth as great, and annealing, although it affected the distribution of the local peculiarities, did not sensibly increase their magnitude.

Annealing this constantin wire, which had been hardened by drawing down from 0.051 cm. to 0.036 cm. diameter, changed its "thermo-electric height" with respect to copper about 1.5 per cent, the annealed wire being nearer to copper thermo-electrically than the hard-drawn wire.

12. As the thermo-electric height of manganin with respect to copper is two or three times as great as that of german-silver with respect to copper, at ordinary temperatures, as, further, annealing seems to make less proportional difference in constantin than in copper, and as, finally, the thermo-electric heterogeneity of constantin appears to be relatively less than that of german-silver, it would seem that couples of constantin with copper are to be preferred to couples of german-silver with copper for ordinary use.

It must be remembered, however, that the experiments here described relate to one kind only of german-silver and to one specimen only of constantin.

DETAILED ACCOUNT OF THE WORK.

Couples A_2, B_2, C_2, and D_2.

Four new copper and german-silver couples, which will be called A_2, B_2, C_2, and D_2, were made July 5, 1905, from the same pieces of copper and german-silver which had furnished the materials for the couples previously mentioned. Each wire was about 0.02 cm. in diameter. Each german-silver wire, about 160 cm. long, was annealed as the wires for previous couples had been, by carrying for a few seconds a current of about 3.5 amperes, which heated it to incandescence except at the ends, which were afterward cut off, leaving the wire about 150 cm. long. Each annealed wire was made to extend through two glass tubes, each about 50 cm. long and 0.3 cm. in diameter of bore, and each end of the wire, after being cleaned with emery paper, was then tightly twisted with a copper wire for a distance of about 0.5 cm. The junctions thus made were heated in an alcohol flame and flowed with a silver solder, the borax used in this operation being afterward removed, so far as was practicable, from the wires.

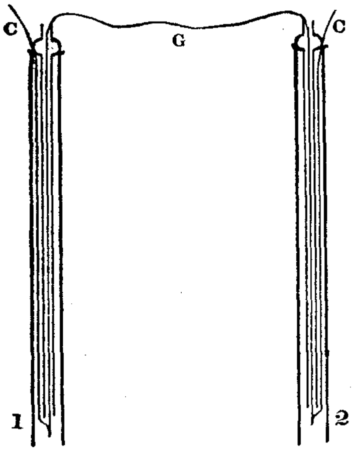

FIGURE 1.

The small glass tubes were next placed in glass tubes about 54 cm. long and 1 cm. in diameter, as in Figure 1, where C and C represent the copper wires and G represents the german-silver wire. One of these larger glass tubes, with its contents, was now placed in a vessel of peculiar construction, due to Professor Crafts, where it could be raised to the temperature of the vapor given off from distilled water boiling at any required pressure, but without coming into direct contact with this vapor. The other large glass tube, with its contents, was similarly placed in a similar vessel containing the saturated vapor of naphthalin, the purest that Eimer and Amend could furnish, at any required temperature. The steam temperatures were found from the steam pressures, measured by a mercury gauge, by means of Peabody's Tables, which are based, of course, on Regnault's observations. The naphthalin temperatures were calculated from the naphthalin vapor pressures by means of a formula furnished us by Professor Crafts, which formula has not yet been published.

Observations.

Observations were sometimes made with junctions 1 (see Figure 1) in the naphthalin pot and junction 2 in the steam pot, and sometimes with the opposite arrangement. The first was called "the parallel" arrangement; the second, the "crossed" arrangement. A complete run for any one day included a set of observations made on each couple, with the parallel arrangement, and another similar set made with the crossed arrangement, the temperatures being kept, as nearly as might be, constant during the whole run.

If on one day the observations were taken first with the "parallel" arrangement and then with the "crossed" arrangement, on the next day of work, as a rule, the observations were taken first with the "crossed" arrangement and then with the "parallel" arrangement.

The thermo-electric current from each of the four couples was observed by means of an armored Siemens-Halske galvanometer, the sensitiveness of which was at first taken before beginning the observations with the four couples, between the two sets of observations, and after the second set. This sensitiveness was found to vary but little from hour to hour, or even from day to day; and after a time the between-sets test was often omitted. All the resistance coils used during this work were carefully compared with international-standard-ohm coils, and were found correct to within one tenth of one per cent.

The usual order of observations was as follows, (3–2), for example, meaning three galvanometer deflections in one direction and two in the other direction: A (3–2), B (3–2), C (3–2), D (5–4), C (3–2), B (3–2), A (3–2), a reversal of the couples from "parallel" to "crossed" or *vice versa*, then a wait of one hour for equilibrium of temperature to re-establish itself, then, as before, A (3–2), B (3–2), C (3–2), D (5–4), C (3–2), B (3–2), A (3–2).

Observations of the pressure gauges, when artificial pressure conditions were used, and of the barometer, accompanied these galvanometer observations and gave data for calculating the temperatures of the junctions.

The following summary of the results of the observations made July 21, 1905, is a fair sample of the summaries usually obtained. The numbers given under "Sensitiveness" are galvanometer deflections observed on reversal of a current of known constant strength used in getting the sensitiveness of the galvanometer. The numbers given under A_2 or B_2, etc., are the galvanometer deflections observed on reversal of a commutator in the thermo-electric circuit containing the couple referred to:

Time.		Sensitiveness.
1:15 P. M.		21.20
3:05 "		21.20
5:25 "		21.19
	Mean	21.20

		A_2	B_2	C_2	D_2	Temperatures.	
1:40 P. M.	"Parallel"	19.84	19.66	20.03	19.80		
2:55 "		19.86	19.67	20.04		180°.36	140°.12
		19.85	19.67	20.04	19.80		
4:05 "	"Crossed"	19.68	19.85	19.36	19.74		
5:20 "		19.71	19.85	19.38		180°.36	140°.07
		19.70	19.85	19.37	19.74		
	Means	19.77	19.76	19.71	19.77	180°.36	140°.10

Mean of all = 19.75

Diff. = 40°.26

These data yield the following values:
Mean total e. m. f. $= 958 \times 10^{-6}$ volt,
" e. m. f. per degree, between 140°.10 and 180°.36 $= 2378 \times 10^{-8}$ volt.

Putting aside, for the present, consideration of the absolute values here indicated, we may observe a number of comparative features which are of interest:

First, that the smallness of the difference between the earlier and the later deflections obtained with the same couple in each of the two groups "parallel" and "crossed" is good evidence that practical thermal equilibrium was attained before the beginning of each of these two groups of observations. Even the slight tendency here observed toward increase of the deflections with progress of time is greater than was usually evident.

Second, that in each of the two groups, "parallel" and "crossed," considerable differences between the deflections yielded by different couples are to be seen, the extreme difference, in the "crossed" group, being more than 2 per cent of the mean deflection. These differences are not to be accounted for by differences of resistance of the different couples, for each had a resistance very near 16 ohms, while the resistance of the rest of the circuit, into which each couple was introduced in turn, was more than 1000 ohms. Nor are the differences in question to be regarded as purely accidental, in the sense of having no tendency to recurrence. Taking the observations of the five days on which the four couples here considered were tested with temperatures near 140° and 180° we get, as means:

	A_2	B_2	C_2	D_2
"Parallel"	19.82	19.64	19.93	19.78
	> 0.03	> 0.30	> 0.36	> 0.05
"Crossed"	19.79	19.94	19.57	19.83
	19.81	19.79	19.75	19.81

Here we have as the order according to magnitude of deflections: "parallel," C > A > D > B, "crossed," B > D > A > C, just as on July 21, the case cited in detail above. The mean range of variation through the five days now considered was about 1.6 per cent for the "parallel" and about 2 per cent for the "crossed" arrangement.

Third, that a couple which gives a deflection larger than the mean with the "parallel" arrangement is likely to give a deflection smaller than the mean with the "crossed" arrangement, and *vice versa*, so that differences of the kind mentioned in the preceding paragraph are very small among the individual means, of "parallel" and "crossed" taken together, yielded by the various couples. The greatest difference on July 21 was less than ⅓ per cent of the mean deflection, and the greatest difference in the five-day means given above is the same, 6 parts in nearly 2000.

Fourth, that the mean "parallel" deflection for July 21, which is 19.84, exceeds the mean "crossed" deflection for the same day by nearly 1 per cent of the mean deflection. It is to be observed, however, that the mean "parallel" deflection and the mean "crossed" deflection of the five-day summary are almost exactly equal, being respectively 19.79 and 19.78.

As an example of the daily summaries obtained when the junction temperatures were 100° and 218,° approximately, the following, for July 25, will serve:

Time.		Sensitiveness.
11:45 A. M.	. . . .	21.20
2:30 P. M.	. . . .	21.21
5:05 "	. . . .	21.18
	Mean	21.20

Time.		A_2	B_2	C_2	D_2	Temperatures.
12:35 "	" Crossed "	19.63	19.69	19.52	19.71	
1:24 "		19.60	19.71	19.53		217°.9 99°.9
		19.62	19.70	19.53	19.71	
4:15 "	" Parallel "	19.69	19.64	19.71	19.70	
4:57 "		19.68	19.62	19.72		217°.9 99°.9
		19.69	19.63	19.72	19.70	
	Means . .	19.65	19.67	19.62	19.71	217°.9 99°.9

Mean of all = 19.66 Diff. = 118°.0

These data yielded the following values:
Mean total e. m. f. = 2807 × 10^{-6} volt.
" e. m. f. per degree, between 99°.9 and 217°.9 = 2379 × 10^{-8} volt.

Examining this summary as we have examined that of July 21, we find:

First, that, as before, practical equilibrium of temperature was maintained throughout the course of observations.

Second, that in each of the two groups, " parallel " and " crossed," the differences between the deflections yielded by different couples are considerably less than were the corresponding differences on July 21, the greatest difference now found being rather less than 1 per cent of the mean deflection. This greater uniformity is probably due to the greater interval of temperature used on July 25, 118° instead of the 40.3° of July 21. Taking a summary of the deflections yielded by the four couples on the four days when they were tested with temperatures near 100° and 218° we get:

	A_2	B_2	C_2	D_2	Mean.
" Parallel "	19.60	19.56	19.63	19.62	19.60
	>0.02	>0.13	>0.13	>0.03	>0.02
" Crossed "	19.62	19.69	19.50	19.65	19.62
	19.61	19.63	19.57	19.64	19.61

Here, too, the greatest difference observed in either of the two groups, "parallel," and "crossed," is a very little less than 1 per cent of the mean, 19 parts in nearly 2000.

Third, that, as before, a couple which gives a deflection larger than the mean with the "parallel" arrangement, is likely to give a deflection smaller than the mean with the "crossed" arrangement, and *vice versa*, so that differences between the individual means of "parallel" and "crossed" taken together, yielded by the various couples, are small, the greatest being 9 parts in nearly 2000 for July 25 and 7 parts in nearly 2000 for the four-day summary.

Fourth, that the mean "parallel" deflection for July 25, 19.68 +, is a little greater than the mean "crossed" deflection, 19.64, for the same day. In the four-day summary, however, we find the mean "parallel" deflection to be 19.60, while the mean "crossed" deflection is a very little greater, 19.62.

On one day, July 5, and on that day only, observations were made with the interval of temperature from 100° to 136°, approximately. The summary for the individual couples with this interval for this one day is as follows:

	A_2	B_2	C_2	D_2	Mean.
"Parallel"	15.21	15.19	15.34	15.27	15.25
	>0.09	>0.16	>0.07	>0.02	>0.04
"Crossed"	15.30	15.35	15.27	15.25	15.29
	15.26	15.27	15.31	15.26	15.27

It seems hardly worth while to discuss this summary in detail at present, though it will be referred to later.

Taking the five-day summary for the interval 140° to 180° with the four-day summary for the interval 100° to 218°, and still postponing discussion of the absolute values of electromotive force found with these intervals, we reach the following conclusions from the data and the discussion already given:

1. That with the four couples A_2, B_2, C_2, D_2, made in the same way from the same materials, a maximum range of electromotive force amounting to about $\frac{1}{3}$ per cent of the mean electromotive may be expected in a thorough test with any interval of temperature between 100° and 218°. The one day's work with the interval 100° to 136° is in complete accord with this conclusion.

2. That the couple C_2 is sufficiently exceptional in its behavior to demand especial attention. This couple gave the lowest mean deflection for the 100°–218° interval and also for the 140°–180° interval; for the 100°–136° interval it was still exceptional, but it gave here the largest mean deflection. On the whole, too, the difference between "parallel" and "crossed" was larger with this couple than with any other.

All attempts to account for such variations and peculiarities by causes lying outside the german-silver wire were unsuccessful. Looking for possible causes of difference in the german-silver wires we naturally gave much thought to the rather sharp bends which, with the arrangement shown in Figure 1, inevitably occurred just above the tops of the glass tubes through which these wires ran. It was thought that these bends, by introducing some change of quality in the wires at a place where the gradient of temperature is large, might account for the differences noted. Tests made by heating or cooling the various wires at or near the bends in question did not, it is true, fully justify this theory; but in order to give it a fuller trial it was determined to make a new set of four couples and to support the german-silver wire of each couple in such a way that it would be subject to no violent or greatly variable bends in its course from one heating pot to the other.

Couples A_3, B_3, C_3, and D_3.

The four new couples, made from the same materials as the others and in substantially the same way, will be called A_3, B_3, C_3, and D_3, respectively. The new method of mounting is indicated by Figure 2, in which S is a wooden splint to which are lashed the horizontal ends of the small glass tubes containing the german-silver wire, these ends being connected by a short rubber tube. Corks fitting snugly on the outer glass tubes were lashed to the upper parts of the smaller tubes, so that when the splint S was raised the whole system of protecting tubes was raised with it. The greater part of each small tube above the larger glass tubes was exposed freely to the air of the room. The upper part of the larger tubes was surrounded by loose asbestos, which, as before, lay on the covers of the pots to a depth of about 3 cm.

The result of this attempt to improve the conditions of experiment was disappointing. With the new arrangement and the new set of couples the differences between the behavior of different couples and the differences between "parallel" and "crossed" with the same couple were greater in general than we had found with the old set of couples, A_2,

B_2, etc. Going into details, and looking all the time for the causes of descrepancies, we found that couple C_3, like couple C_2 in the quartet of couples previously studied, was the worst of its set, giving the lowest electromotive force and showing the greatest difference between the "parallel" and the "crossed" condition.

As C_3 occupied in the pots the same place which C_2 had before occupied, we naturally suspected that there might be something peculiar in the C part of one or the other of the pots. Moreover, moisture was sometimes seen in the upper part of the glass tube containing the german-silver of C_3, which suggested that there might be a small leak of steam into the tube containing one junction of this couple. Accordingly, on September 25 we changed all the couples about, putting A_3 where D_3 had been, and *vice versa*, and B_3 where C_3 had been, and *vice versa*. The effects, such as they were, of this change may be inferred from the following comparative set of numbers, those for September 23 being obtained before, and those for September 25 after, the transposition just described:

	A_3	(P — C)	B_3	(P — C)	C_3	(P — C)	D_3	(P — C)
Sept. 23	20.30	(+0.27)	20.17	(+0.16)	20.02	(—0.19)	20.23	(—0.04)
" 25	20.30	(+0.36)	20.14	(+0.36)	20.08	(+0.52)	20.14	(—0.25)

The deflections here given for the two days are reduced to like conditions of galvanometer sensitiveness and temperature interval, so that they are strictly comparable. Under (P — C) is given the "parallel" deflection *minus* the "crossed" deflection. The mean deflection is 20.180 for September 23 and 20.165 for September 25, an insignificant difference. Couple C_3 gives the smallest deflection on each day; its (P — C) was larger on the second day than on the first, and of the opposite sign, but its (P — C) for September 23 was exceptionally small and unusual in sign. Couple B_3, which had been put into the place of C_3, showed no change that appeared very significant.

On September 26, also, the couples were used in their interchanged positions, but with an interval of temperature different from that used September 23 and 25, and on this day also C_3 gave as usual a deflection about ½ per cent below the mean given by the four couples, while B_3 gave, as usual, a deflection very near this mean.

On the whole, then, it seems probable that the low electromotive force of the two C couples was an accidental coincidence, not due in any discoverable measure to their position in the apparatus. Nevertheless, to avoid possible trouble from a possible minute leakage of steam, it was determined to seal at the bottom the large glass tubes which served as a shield for the junctions.

At the same time a change was made in the manner of connecting the small glass tubes at their tops. The couples were removed temporarily from these tubes; the splints and rubber tube connectors shown in Figure 2 were abolished; and the two small glass tubes for each couple were now joined directly to each other in one piece, as in Figure 3. Then the couples were replaced in the tubes. In this change little or no care was taken to distinguish between the two ends of each couple, and it appears that D_3 was turned about in its replacement, so that for this couple "parallel" afterward corresponded to the same orientation of the couple to which "crossed" had previously corresponded; for the (P — C) of this couple was consistently + after the change, whereas it had usually been — before. This change was made between the observations of September 27 and those of September 28. A comparative summary for September 26 and September 28 is here given, the interval of temperature on both days being approximately 140°–180°.

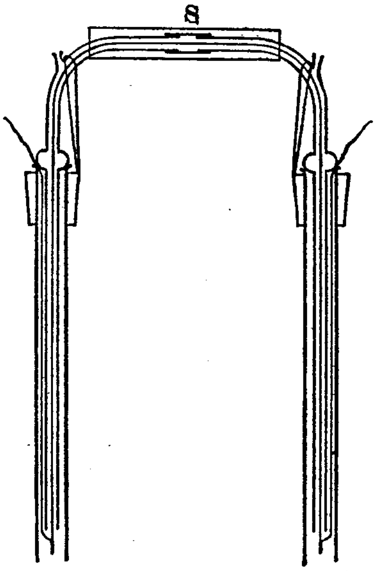

FIGURE 2.

	A_3	(P — C)	B_3	(P — C)	C_3	(P — C)	D_3	(P — C)
Sept. 26	19.88	(+0.74)	19.66	(+0.71)	19.61	(+1.03)	19.65	(—0.34)
" 28	20.00	(+0.62)	19.83	(+0.08)	19.74	(+0.61)	19.84	(+0.69)

On the whole, the results of the overhauling given to the apparatus in the change from the arrangement of Figure 2 to that of Figure 3 were unimportant so far as harmony of performance is concerned.

On September 27, still under the impression that the differences of behavior of the different couples and the (P — C) values of the individual couples might be accounted for by the gradients of temperature in the bent parts of the german-silver wire above the pots, we tried the experiment of raising all of the couples about 20 cm. higher than usual, with the object of getting the bent part of the german-silver wires far enough away from the top of the pots to have them in a region of nearly uniform temperature. With the same object in view we placed shields of asbestos to protect these parts of the wire from the heat given off by the pots and the tubes above them. All this was done in the hope of more uniform performance by the different couples. How little ground there was for this hope may be seen from the following summary of the series of observations made with this arrangement, although the observations at the last went so badly that the series was not completed in the usual way:

	A_3 (P − C)		B_3 (P − C)		C_3 (P − C)		D_3 (P − C)		Mean (P − C)	
P	20.61	+	20.21	+	19.90	+	19.63	+	20.09	+
		1.84		2.01		1.57		0.87		0.157
C	18.77		18.20		18.33		18.76		18.52	
	19.69		19.21		19.12		19.20		19.31	

These observations of September 27, taken in connection with what had gone before, told strongly against the theory which attached great importance to the bent parts of the german-silver as a source of variation and irregularity, and after the experiments of September 28 and 29 had shown that moisture had probably exerted no appreciable influence on the performance of the couples, it seemed probable that the cause of the irregularities noted must be looked for in differences of condition in the straight parts of the german-silver wires. Accordingly we decided to look for such differences by direct experiment; but we thought it best to make, before this study was undertaken, a direct experiment on the change which moderately sharp bending makes in the thermo-electric condition of german-silver wire.

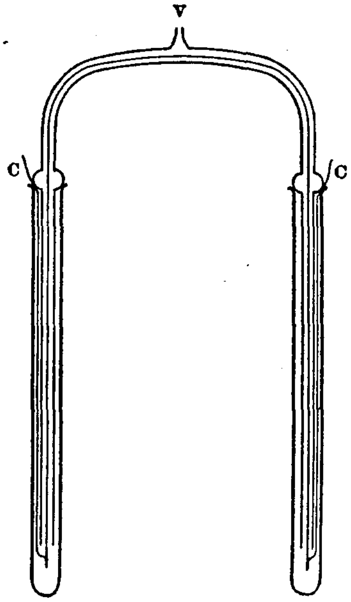

FIGURE 3.

On the Effect of bending German-Silver Wire.

As we had used, in the research on the Thomson effect in iron, spirals of german-silver wire wound on a cylinder 1 cm. in diameter, assuming that the bending produced no important change in the thermo-electric performance of the wire, investigation of the effect of this particular curvature seemed now especially appropriate, even if somewhat belated.

Four pieces, each 150 cm. long, of just such annealed german-silver wire as had been used in making our thermo-electric couples, were taken, and each was mounted on a glass tube about 1 cm. in diameter and 16 cm. long in the manner indicated by Figure 4. C and C are copper wires lashed to the cork K_1 and soldered to the german-silver wire, which, beginning at A, enters the tube through a hole in the side and runs straight to the farther end, where it emerges under the cork K_2 to follow a spiral course to B. The distance between turns of the spiral is about 0.3 cm.

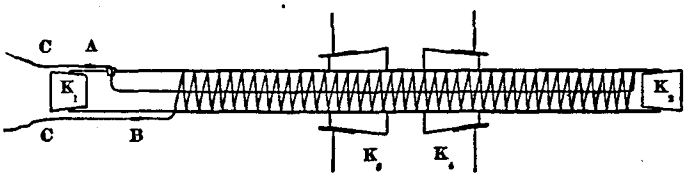

FIGURE 4.

Each tube thus prepared was made to connect two metal vessels containing a thick oil, each end extending through a tight-fitting cork, K_3 or K_4, about 5 cm. into the oil. One body of oil was kept at or very near the temperature of the room; the other was heated to 15 or 20 degrees above this temperature.

The two junctions with the copper being near together in one body of oil were very nearly at the same temperature. The arrangement really gave, then, a thermo-electric couple consisting of straight german-silver wire combined with spirally wound german-silver wire in one continuous piece, the junctions of this couple being in the two vessels of oil with a difference of temperature of some 15 or 20 degrees.

With this condition of things, tests of the four couples, which we may call S_1, S_2, S_3, and S_4, respectively, were made on two days, October 6 and 7. On each day S_1 and S_2 gave deflections in one direction and S_3 and S_4 gave deflections in the opposite direction. Dividing by 4 the algebraic sum of these deflections and comparing them with the effect obtained with a german-silver and copper couple having a like interval of temperature, we found that, on the average, the spiral-straight german-silver couples had a thermo-electric force about $\frac{1}{500}$ as great[2] as that of the german-silver and copper couple, the spiral wire being, apparently, a little farther removed, in thermo-electric quality, from copper than the straight german-silver is.

This seems to show that moderately sharp bending (with a radius of 0.5 cm.) is not very injurious in the ordinary thermo-electric use of german-silver wire. It is to be noted, however, that, if a german-silver and copper couple is used to measure some very small interval of temperature, 0°.1, for example, and if in some bent part of the german silver there is a very much greater interval of temperature, 15°, let us say, such a difference of quality in this bent part as we have inferred from our experiments on the spirals would make an error of 10 per cent in the estimation of the 0°.1 interval. But if in such a case it is practicable so to vary the conditions of the experiment as to reverse the small temperature-interval to be measured, without at the same time changing materially the gradient of temperature in the german-silver wire, error caused by the non-homogeneity of this wire will be practically eliminated from the result of the combined observations.

It seems unlikely that the bends in the upper parts of the german-silver wires of the couples A_2, B_2, C_2, D_2, or A_3, B_3, C_3, and D_3, had any important effect in the performance of these couples.

Thermo-electric Heterogeneity in Straight German-Silver Wire.

We now return to the question whether in a straight piece of german-silver wire there is any discoverable thermo-electric heterogeneity.

After some tentative experimenting the arrangement indicated, as seen from above, by Figure 5 was adopted: G G is the german-silver wire, held nearly straight but with no great tension by the screw-heads S and S ; C and C are copper wires soldered to the german-silver, the junctions being immersed in glass vessels containing oil at the temperature of the room; M M is a meter-rod.

The copper wires being connected with the terminals of a galvanometer, the sharp edge of a piece of ice was placed in contact with the under side of the german-silver wire, as in Figure 6, opposite the 25 cm.

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[2] This electromotive force is too small to be with certainty attributed to the bending.

mark of the meter-rod. The result was usually a small deflection of the galvanometer needle, which deflection remained fairly constant until the ice was withdrawn, when it soon disappeared. The ice was then placed opposite the 26 cm. mark and the deflection was noted. In this way progress was made, by 1 cm. stages, until the 75 cm. mark was reached. Then a return set of observations was made, beginning at the 75 cm. mark and ending at the 25 cm. mark, with 1 cm. stages.

For any one point the return observation usually showed a deflection of the same sign and the same order of magnitude as the deflection shown by the forward observation. The mean of the two deflections was taken for each point examined, and the mean deflections, or quantities proportional to them, were plotted on co-ordinate paper, distances along the wire being measured on a horizontal line and the deflections being represented by vertical distances above or below this horizontal line.

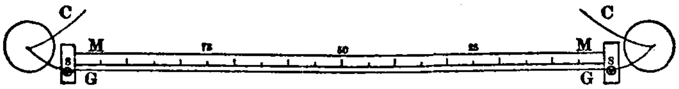

Figure 5.

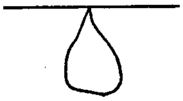

Figure 6.

Through the points thus placed a curve was drawn free hand, which indicates roughly the variations of thermo-electric condition revealed by progressive observations along the wire. C_1 in Figure 7 is the figure thus obtained with an unannealed specimen, which we will call No. 1, of wire from the same piece which has furnished the german-silver of our couples. The maximum height of this curve, which occurs over the 12 cm. point, corresponding to the 37 cm. point of the wire as shown in Figure 5, indicates an electromotive force of 16×10^{-8} volt, approximately, as a result of touching the wire with ice, when the room temperature was about 18° C.

The curve in question crosses the median line, of no electromotive force, 22 times in the 50 cm. of wire investigated. There was no obvious reason for these alternations of thermo-electric effect, or indeed for any thermo-electric effect. The wire, in the part tested, was free from any serious looking kinks or other imperfections.

After the data for curve C_1 had been obtained the wire was annealed in the usual way, without being released from the screws S and S, Figure 5, which held it upon the experimental rack. As the annealing appeared to leave the wire a little slack after cooling, the left-hand S was loosened and the wire was drawn a trifle till it became moderately taut once more; but later, and before any more observations were made on it, the left-hand S being loosened again, the wire was slackened to

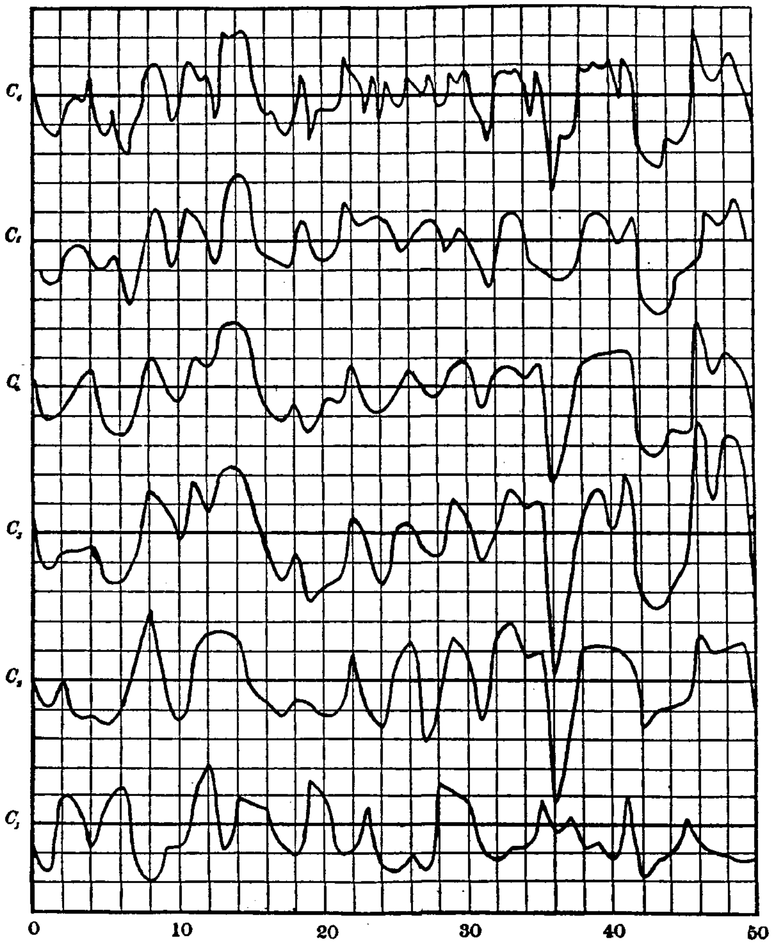

FIGURE 7.

the same state, as nearly as we could tell, in which the annealing had left it. These details are mentioned for the reason that it was important to test after the annealing the same points, as nearly as might be, which had been tested before the annealing; for sometimes a movement of one or two millimeters along the wire made a serious change in the effect observed.

After being annealed the wire was tested again, just as it had been tested before. Curve C_2 of Figure 7 shows the result. This curve is very different from C_1, having a smaller number of changes of sign, 17 (or at most 19) instead of 22, and a greater vertical range, the greatest electromotive force indicated by C_2 being aproximately 30×10^{-8} volt, as against the 16×10^{-8} volt which is the maximum of C_1.

The question arose whether the effects observed could be due in any considerable measure to the surface condition of the wire. Accordingly we now rubbed it with fine emery paper, thus removing at least a large part of the black coating left by the annealing and making the surface comparatively bright once more. After this treatment we tested the wire again, just as before, with the result shown by C_3. Comparison of curves C_2 and C_3 shows them to be very much alike, the differences of detail being, perhaps, no greater than might be expected from two successive tests made without change of condition of the wire.

The next question considered is whether the effects observed are due to slight local bends in the wire, such bends being rather difficult to avoid. It is true that previous experiments, already described, made with spiral wire in combination with straight seemed to give a negative answer to this question, but it was easy and interesting to come at the matter in another way. Accordingly, after marking the 25 cm., 50 cm., and 75 cm. points of the wire with knots of thread, we stretched it until the 50 cm. length which we had especially studied became permanently 55 cm. long, thus taking out any small bends which lay in this part of the wire before the stretching. Then we readjusted the wire on the rack in such a way as to bring opposite the 25 cm. mark of the meter-rod the same point of the wire which had previously been there and to bring opposite the 80 cm. mark of the rod the same point of the wire which had previously been opposite the 75 cm. mark. Then we made a test as before, touching the ice first at the 25 cm. point, then at the 26.1 cm. point, then at the 27.2 cm. point, and so on, by stages of 1.1 cm., to the 80 cm. point, thus trying to get at the same points in the wire which had been used before the stretching. It is to be noted, however, that a wire stretched through the whole of

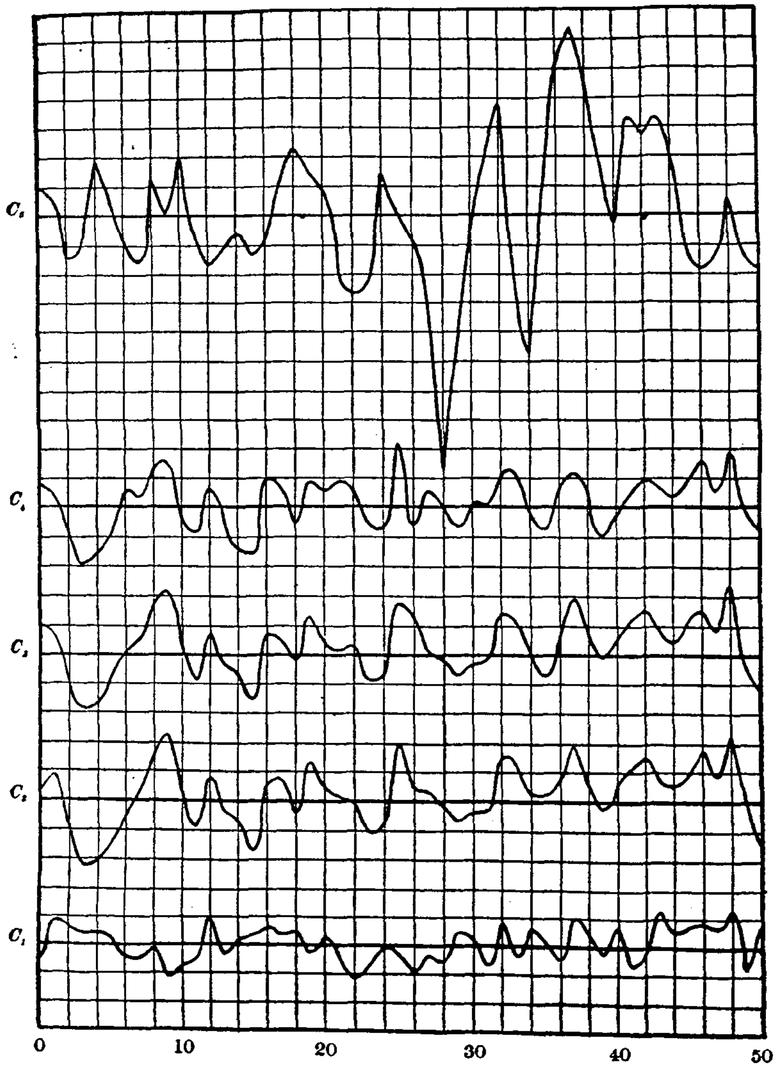

FIGURE 8.

its length at once is not necessarily stretched equally at all parts; for example, the point which had been the 50 cm. point before the stretching might not lie, and in our observations usually did not lie, just half-way between the points which had been the 25 cm. and the 75 cm.

points respectively. Accordingly the points touched by the ice after the stretching were probably not in all cases just the same as the points touched before the stretching. Nevertheless, it will be seen that the curve C_4 of Figure 7, which corresponds to the observations after stretching, agrees very closely in the main with the curve C_3 obtained before the stretching.

The next curve, C_5, is made from data obtained by touching points 25.5, 26.6, 27.7, etc.; that is, points intermediate between those which gave C_4. The differences between these two curves are such as might have been expected.

C_6 is a composite of C_4 and C_5.

So much for the curves of Figure 7, representative of the behavior of wire No. 1. The curves of Figure 8 show the behavior of wire No. 2 taken from the same piece as No. 1. C_1 in Figure 8 is for the stat before annealing; it shows a somewhat smaller range of electromotive force than the corresponding curve of Figure 7, with about the same number of changes of sign. C_2 is for the state just after annealing. As with wire No. 1, annealing has increased the range of electromotive force and diminished the number of changes of sign. C_3 is the curve obtained after brightening the surface of the annealed wire. It differs very little indeed from C_2. C_4 is from the data obtained after permanently stretching the wire 10 per cent, a 50 cm. piece becoming 55 cm., as with No. 1. Here, again, the effect of stretching is very slight. Finally, C_5 shows what was found after reannealing the stretched wire. This annealing, like those previously mentioned, has profoundly changed the character of the representative curve; and here the increase in range of electromotive force has been very great. The maximum electromotive force indicated by C_5 is about 65×10^{-8} volt.

Such thermo-electric heterogeneity as these tests reveal would be found in a wire made by joining together at their ends pieces of german-silver wire from 1 cm. to 6 cm. long, but averaging about 2.5 cm., these pieces varying through some little range of chemical composition or of hardness and each piece having within itself minor differences of composition or of hardness.

Direct experiment with a german-silver and copper couple, similar to those already described in diameter and condition of wire, showed that, when one junction of such a couple is touched with a sharp edge of ice, the temperature of the room being near 18° C., the thermo-electromotive force developed is about 284×10^{-6} volt, about 450 times the maximum indicated by the curve C_5 in Figure 8. As the room temperature when the data for this curve were taken may have been as high as 20°, we will take 500 as the ratio of the electromotive force of a (copper)-(german-silver) couple to that of the most effective (german-silver)-(german-silver) couple which could be made from contiguous parts of the wire which gave the curve C_5, the temperatures to be the same in both couples.

When one junction of this (copper)-(german-silver) couple was in contact with an ice surface about 1 cm. wide, so that contact between metal and ice extended about 0.5 cm. in each direction from the junction, the electromotive force developed was about 3 per cent greater than when a sharp edge only of ice was used, which may be taken as indicating that the sharp edge cools the junction to the neighborhood of 0°.5.

When the german-silver wire was touched by a sharp edge of ice at a distance of 1 cm. from a junction, the electromotive force was such as to indicate that the junction itself was thereby cooled about 3 degrees. Of course the junction, having the highly conducting copper on one side, was cooled somewhat less in this case than a similarly placed point in a continuous german-silver wire would have been. When the copper wire was touched by the edge of ice at a point distant 1 cm. from the junction, the junction was apparently cooled about 12 degrees.

It appears, then, that in our experiments along a straight piece of german-silver wire, exposed to the air of a room at 18° C., the point in contact with the sharp edge of ice was near the temperature 0°.5, while the points a centimeter distant from the point of contact were, perhaps, near the temperature 13°, the mean temperature gradient for the first centimeter in each direction from the point of contact being, therefore, let us say, 12°.

We may, for present purposes, safely regard the temperature change caused by the edge of ice as insensible at 2 cm. from the point of contact. The thermo-electromotive force caused by the inequality of temperature due to the ice is, therefore, dependent on the difference of thermo-electric quality of two pieces of wire, each 2 cm. long, which extend in opposite directions from the point of contact with the ice, and in each of which there is a temperature range of about 18°. Let us call these pieces section l and section r, respectively. The thermo-electromotive force under discussion is equal to the change of electromotive force which would result in one of our ordinary (copper)-(german-silver) couples, if we were to cut out a piece of german-silver having the quality of section l and substitute for it a piece having the quality of section r, the temperatures at the two points of cutting and joining being, approximately, 0° C. and 18° C., respectively.

When we are testing our couples with junction temperatures of, let us say, 140° and 180°, respectively, the german-silver wire between the vapor pots being freely exposed to the air or contained in glass tubes freely exposed to the air, we must have a very steep gradient of temperature in those parts of the wire which lie near the entrance to the pots. If near one pot there happens to be a section of german-silver wire 3 or 4 centimeters long having within itself a range of, let us say, 80°, and if the thermo-electric quality of this section differs from the mean thermo-electric quality of the wire one half as much as certain contiguous sections differ in the wire which gave the data for Figure 8, the electromotive force of the circuit as a whole, when the difference of temperature between the (copper)-(german-silver) junctions is 40°, will be about $\frac{1}{5}$ per cent greater or less than it would be if the given section were of average quality. If this differential effect assists the current when the neighboring (copper)-(german-silver) junction is the hotter one, it will oppose the current when this junction is the cooler one, thus making the difference between the "parallel" arrangement current and the "crossed" arrangement current $\frac{2}{5}$ per cent. If there is, as there may happen to be, near the other pot an equally eccentric section of wire, its eccentricity being of the opposite sign, we may get a difference between "parallel" and "crossed" amounting to $\frac{4}{5}$ per cent.

This goes some distance toward accounting for such difference (P — C) as we have observed, but it goes hardly far enough. The mean of the arithmetical values of (P — C) with the temperature interval 140° to 180° is about 1.3 per cent for the couples A_2, B_2, C_2, D_2, and, if we confine ourselves to observations thus far mentioned, about 3 per cent for the couples A_3, B_3, C_3, D_3. With couple C_3 on September 26 (P — C) was more than 5 per cent of the mean effect. It seems doubtful whether so great a discrepancy can reasonably be accounted for by such heterogeneity as exists in a freshly annealed wire; but the results of certain later calibration tests, now to be described, tend strongly to confirm the opinion that the (P — C) value is due in some way to the condition of the german-silver wire between the pots.

Further Calibration Tests of Couples A_3, B_3, C_3, and D_3.

We made on October 28 another calibration test with couples A_3, B_3, C_3, and D_3, in which we tried to keep the upper parts of the glass tubes containing the german-silver wires in a state of temperature intermediate between the temperatures of the two pots, about 100° and 218°, respectively. With this purpose we closed in the air space about these tubes with shields of asbestos and cotton, thus leaving it to be heated from the tops of the pots. The temperature attained in the middle of this air space was perhaps 100°, not so high as we wished it to be, and we were far from getting a uniform temperature gradient in the wires leading from one pot to the other. At any rate, the performance of the couples in this trial was very little if any more harmonious than it had been on September 29, when no care had been taken to keep the parts in question warm. On the other hand, the mean electromotive force found on October 28 was about $\frac{2}{3}$ per cent less than it had usually been with the same interval of terminal temperatures.

Having now come to the conclusion that the arched[3] glass tubes, containing the german-silver wires between the pots, were an obstacle to the attainment of the temperature condition desired in this part of the apparatus, we now cut out these connecting parts of the glass tubes, thereby returning to an arrangement very like that shown in Figure 1. Placing the pots as near together as they could well be, we surrounded their tops with an elliptical wall of asbestos board, within which we placed loose asbestos fibre. Within this bed of asbestos ran the german-silver wires from pot to pot, each wire being covered to a depth of some centimeters. As the space from pot to pot was shorter for couples B_3 and C_3 than for A_3 and D_3, the junctions of B_3 and C_3 were let down into the pots two or three centimeters farther than those of the other couples.

With this arrangement, calibration tests were made on December 16 and 20 with a new interval, 180° to 218°, approximately. The high mean temperature of the junctions and the comparatively small interval of temperature would naturally have given still larger values of (P — C) than we had commonly found, even with the 140° to 180° interval, if we had not taken care to shield and warm the middle parts of the german-silver wires. More care in this regard was taken on December 20 than on December 16. The term (P — C) is not strictly applicable to the tests of these two days; for, both pots containing naphthalin, the reversal of conditions was effected not by changing from "parallel" to "crossed" or *vice versa*, but by making first the north pot and then the south pot the hotter, or *vice versa*. We shall therefore use the term (N — S) instead of (P — C). The summaries of galvanometer deflections for the two days are as follows:

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[3] We should probably have done well to retain these connecting parts of the tubes, reducing the arch as much as possible so as to bring the wires down near the tops of the pots.

	A_3 (N − S)	B_3 (N − S)	C_3 (N − S)	D_3 (N − S)
Dec. 16	$21.20_{\overset{+}{(0.24)}}$	$21.03_{\overset{+}{(0.45)}}$	$20.89_{\overset{+}{(0.29)}}$	$20.97_{\overset{-}{(0.16)}}$
" 20	$22.10_{\overset{+}{(0.05)}}$	$21.93_{\overset{+}{(0.20)}}$	$21.75_{\overset{+}{(0.26)}}$	$21.83_{\overset{-}{(0.15)}}$

The mean of the numerical values of (N − S) is 0.29, about 1.4 per cent of the deflection, for December 16, and 0.17, about 0.8 per cent of the deflection, for December 20.

On December 22 and 23 calibration tests were made with the temperature interval 100° to 140°, approximately. Both of the pots used on these days contained water, and the reversal of temperature conditions was effected, as on December 16 and 20, by having first one and then the other of the pots the hotter. The usual summary is as follows:

	A_3 (N − S)	B_3 (N − S)	C_3 (N − S)	D_3 (N − S)
Dec. 22	$19.76_{(0.00)}$	$19.58_{\overset{-}{(0.15)}}$	$19.36_{\overset{-}{(0.11)}}$	$19.44_{\overset{-}{(0.16)}}$
" 23	$19.78_{\overset{-}{(0.07)}}$	$19.60_{\overset{-}{(0.20)}}$	$19.39_{\overset{-}{(0.19)}}$	$19.50_{\overset{-}{(0.21)}}$

The mean of the numerical values of (N − S) is 0.105, about 0.5 per cent of the deflection, for December 22; and 0.17, about 0.9 per cent of the deflection, for December 23. Nearly all the (N − S) values of these two days have the minus sign, whereas on December 16 and 20 the plus sign prevailed. But the fact is that on the two earlier days the "crossed" arrangement existed, whereas on the two later days the "parallel" arrangement was used.

Finally, on December 28, a test was made with the interval 140° to 180°, appproximately, one pot now containing water and the other naphthalin, as in the early experiments. The result was:

A_3 (P − C)	B_3 (P − C)	C_3 (P − C)	D_3 (P − C)
$20.30_{\overset{+}{(0.06)}}$	$20.19_{\overset{+}{(0.14)}}$	$19.96_{\overset{+}{(0.22)}}$	$20.01_{\overset{-}{(0.01)}}$

The mean of the numerical values of (P − C) is here 0.11, about 0.5 per cent of the deflection.

The differential percentages, (N − S) or (P − C), found on these last five days of the calibration tests of couples A_3, B_3, C_3, and D_3, have a mean of 0.8 per cent. The temperature interval on each of these days was not far from 40°. The corresponding mean for previous tests of the same couples made with a like interval of temperature is 3 per cent. It is plain, then, that the policy of keeping the middle parts of the german-silver wires warm is fully justified. It is doubtful whether on any day these parts were kept warmer than the top of the cooler of the two pots. It is reasonable to believe that, if an approximately uniform gradient of temperature had been maintained in these parts from one pot to the other, the differential percentage would have been reduced still further.

The Absolute Values of the Electromotive Force of the Couples.

In the following table, (2) indicates that the observations were made on couples A_2, B_2, etc.; (3) indicates that they were made with A_3, B_3, etc. The column Max. ÷ Min. gives the ratio of the highest mean electromotive force shown by any couple to the lowest mean electromotive force shown by any couple on the day considered. Results obtained before July 21 are omitted from this table because of some uncertainty as to the reduction factor of the galvanometer previous to this date. Results obtained after September are separated from those obtained earlier, because in the experiments made after September the policy of keeping the middle parts of the german-silver wires warm was followed, whereas in earlier experiments practically the opposite policy was maintained.

Date.	Set of Couples.	Temp. Interval.	Max. / Min.		Mean e. m. f. (volt) per Degree.	
July 21, 1905	(2)	140°.10 to 180°.36	A ÷ C = 1.004	1.004	2382×10^{-8}	2386
" 22 "	"	140°.14 " 180°.35	" = 1.003		2386 "	
" 24 "	"	140°.10 " 180°.50	D ÷ C = 1.004		2389 "	
Sept. 26 "	(3)	140°.20 " 180°.30	A ÷ C = 1.014	1.014	2375 "	2385
" 28 "	(3)	140°.10 " 180°.20	" = 1.013		2394 "	
		140°.13 " 180°.34			2385 "	
Sept. 22, 1905	(3)	140°.10 to 217°.80	A ÷ C = 1.010		2484×10^{-8}	
" 23 "	"	140°.10 " 217°.90	" = 1.014		2475 "	
" 25 "	"	140°.00 " 217°.90	" = 1.011		2472 "	
		140°.07 " 217°.87	1.012		2477 "	
July 25, 1905	(2)	99°.90 to 217°.90	D ÷ C = 1.004	1.004	2379×10^{-8}	
" 26 "	"	100°.00 " 218°.00	" = 1.004		2380 "	
" 29 "	(3)	100°.00 " 218°.00	A ÷ C = 1.005	1.007	2389 "	
Aug. 3 "	"	100°.00 " 218°.10	" = 1.005		2385 "	
Sept. 29 "	"	100°.10 " 218°.20	" = 1.010		2389 "	
		100°.00 " 218°.04			2385 "	

From this table we get, by reductions which need not be detailed:

Temp. Interval.	Total e. m. f.	Mean e. m. f. per Degree.
140° to 180°	9536 × 10^{-7}	2384 × 10^{-8}
140° " 218°	19320 "	2477 "
100° " 218°	28140 "	2385 "

From these reduced values we get:

α = total e. m. f. from 100° to 140° = 8820 × 10^{-7}
β = " " " 140° " 180° = 9536 "
γ = " " " 180° " 218° = 9784 "
Hence $\alpha + \beta + \gamma = 28140$ "

Taking now the results obtained after September, except those of December 2, which are rejected because of strong evidence that uniform temperature conditions were not maintained, we have

Date.	Set of Couples.	Temp. Interval.	$\frac{\text{Max.}}{\text{Min.}}$	Mean e. m. f. (volt) per Degree.
Oct. 28, 1905	(3)	100°.30 to 218°.60	A ÷ C = 1.011	2375 × 10^{-8}
Dec. 16 "	"	180°.35 " 218°.53	" = 1.015	2543 "
" 20 "	"	179°.77 " 218°.63	" = 1.016	2545 "
" 22 "	"	99°.93 " 140°.07	" = 1.021	2176 "
" 23 "	"	99°.75 " 140°.02	" = 1.020	2178 "
" 28 "	"	140°.25 " 180°.13	" = 1.017	2362 "

From these values we get:

Dec. 22 and 23 α = total e. m. f. from 100° to 140° = 8708 × 10^{-7}
" 28 β = " " " 140° " 180° = 9444 "
" 16 and 20 γ = " " " 180° " 218° = 9660 "
Hence $\alpha + \beta + \gamma = 27812$ "

Oct. 28 $\alpha + \beta + \gamma$ = total e.m.f. from 100° to 218° = 28000

Giving to the value of $\alpha + \beta + \gamma$ found on October 28 one half as much weight as to the value found by combining the December results, and revising the December values of α, β, and γ accordingly, we get:

		Mean e. m. f. per Degree.
α (before Oct. 1) = 8820 × 10^{-7} " (after " ") = 8728 "	8774 × 10^{-7}	2194 × 10^{-8}
β (before " ") = 9536 " " (after " ") = 9466 "	9500 "	2375 "
γ (before " ") = 9784 " " (after " ") = 9683 "	9734 "	2562 "

The values obtained before the policy of heating the middle of german-silver wires was adopted are at every stage somewhat greater, and on the average about 1 per cent greater, than those given by the later experiments. The reason for this is not apparent. It seems best to take the mean value for each stage. The numbers thus obtained indicate a nearly uniform increase of "thermo-electric height," electromotive force for 1° difference of temperature of the junctions, for the interval from 100° to 218°. Perfectly uniform increase would require the values found for the 140° to 180° region to be raised about 1 in 400.

If we take 2194×10^{-8} as the thermo-electric height for 120°, the middle of the α interval, and 2562×10^{-8} as the thermo-electric height for 199°, the middle of the γ region, we get 4.7×10^{-8} as the mean increase per degree from 120° to 199°. This rate of change applied downward from 120° would give 2100×10^{-8} as the thermo-electric height for 100°.

At the end of the paper[4] referred to at the beginning of this article a summary is given of the results of calibration experiments on sets of thermo-electric couples other than those here studied but made from the same materials in the same way. From this summary we get

thermo-electric height at 89°.4 = 2063×10^{-8}
" " " " 17°.4 = 1713 "

From these values we get 4.9×10^{-8} as the rate of increase per degree from 17°.4 to 89°.4. This rate continued gives 2115×10^{-8} as the thermo-electric height at 100°. This differs about 0.7 per cent from the value found above. The value for 199° calculated from the data obtained in the region below 100° would be 2600×10^{-8}, only 1.5 per cent greater than the value found in this paper.

Taking 2100×10^{-8} as the thermo-electric height at 100° and taking the rate of increase per degree rise of temperature as 4.8×10^{-8}, we should get values which would not at any point between 0° and 218° differ as much as 1 per cent from the most probable value which we have for that point.

It has been stated earlier in this paper that in the (2) set of couples the maximum difference of mean electromotive force among the various couples was about 0.3 per cent of the mean electromotive force. The corresponding difference was greater in the (3) set of couples, being, on the average, perhaps as much as 1.5 per cent of the mean electromotive force, and amounting in some cases to 2 per cent. This difference seemed to grow greater in set (3) as time passed, for at the start, in July, it was only about 0.5 per cent. This change may have been due to changes in the method of experimenting or to some more or less damaging experiences which the couples had suffered in some months of existence and many days of use. It is difficult, especially when the german-silver wire is not protected along its middle by tubing, to prevent an occasional accidental pull which may stretch some part permanently a little and thus change its thermo-electric quality. We know that a couple made of hard-drawn german-silver with copper is 3 or 4 per cent more powerful than a couple made of the annealed german-silver wire with copper.

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[4] These Proceedings, May, 1905.

On this point certain experiments made December 6 and 7, 1905, have a bearing. Four pieces of carefully annealed german-silver wire were each stretched to a permanent extension of 10 per cent in one part, and then the stretched part of each was used with the unstretched part of the same wire as a thermo-electric couple. The mean thermo-electric force of the four couples thus formed was found to be about 0.6 per cent of that of a couple made with copper and the annealed german-silver. The stretching, which hardened the wire somewhat, thus bringing it slightly toward the hard-drawn condition, changed its thermo-electric quality slightly toward that of hard-drawn wire. If slightly different annealing currents or slightly different periods of application of such currents can leave different pieces of wire as unlike as the stretched and the unstretched pieces here tested, the observed differences of sensitiveness of the couples A_3, B_3, C_3, and D_3 may be due, in considerable measure, to such inequalities.

If we were now making a new set of couples, we should take more care than we have usually taken heretofore to have all the pieces of german-silver wire heated by the same strength of current and for the same number of seconds. A current of 3 amperes applied for 5 seconds is suitable for german-silver wire such as we have used. This wire is about 0.02 cm. in diameter, being No. 32 of the Brown and Sharpe gauge.

Behavior of Manganin and of Constantin.

The thermo-electric heterogeneity shown in german-silver by some of the tests which have been described in the preceding pages induced us to make somewhat similar tests with manganin and with constantin.

A piece of annealed manganin wire 0.205 cm. in diameter was drawn down without further annealing to a diameter of 0.051 cm. It was then tested, by touching with ice, as the german-silver wire had been tested, first in the hard-drawn condition, then in the annealed condition, then after rubbing bright, then after stretching. The results, in general character and in magnitude of the local electromotive forces developed, were similar to those found with german-silver, though no eccentricity so great as that indicated by certain parts of the curve C_5 in Figure 8 was developed. Annealing increased the local eccentricities. Stretching affected them but little. The maxima and minima appeared to be somewhat less numerous than in the german-silver wires. The curve obtained after brightening differed considerably from that found before brightening, but as six days intervened between the two tests which gave these curves, it is doubtful whether the change is to be attributed to the scouring alone.

A piece of constantin wire was drawn cold from a diameter of 0.051 cm. to a diameter of 0.036 cm., approximately. It was then tested, by touching with ice, in the hard-drawn condition, and later in the annealed condition,. the annealing being done, as in the case of german-silver and of manganin, by heating to incandescence for a few seconds by means of an electric current. The local electromotive forces found were very small, perhaps one fifth as large as those in german-silver and in manganin, and indeed the difficulty of determining their size was so great that the wire was tested in two conditions only. The maxima and minima appeared to be about as frequent as in german-silver. Annealing changed the character of the representative curve, but did not increase its eccentricities.

It might be suspected that the constantin wire, being nearly twice as large in diameter as the german-silver wire, might not be so much affected locally in temperature by contact with the edge of the ice. Experiments made December 26 showed this idea to be without foundation. The edge of ice cooled a copper-constantin junction as effectually as it would cool the smaller (copper)-(german-silver) junction; when placed 1 cm. distant from the junction along the constantin it produced about one eighth part as great an effect as when applied at the junction; at 1.5 cm. from the junction along the constantin the effect was about 3 per cent of that at the junction. On the other hand, when the edge of ice was applied at a distance of 25 cm. from the junction along the copper wire, 0.036 cm. in diameter, its effect was still perceptible, being apparently more than 1 per cent as great as the effect produced by applying the ice directly to the junction. In these tests the wire between the ice and the junction was in free air at the temperature of the room, about 16°C.

Experiment showed that a couple made of copper and unannealed constantin had an electromotive force about 1.5 per cent greater than that of a couple made of copper and annealed constantin, from the same original piece as the unannealed.

The electromotive force of a copper-constantin couple for a given interval of temperature near the temperature of the room is two or three times as great as that of a (copper)-(german-silver) couple under like conditions.

So far, then, as our tests go, it appears that in every particular constantin is a better metal than german-silver to use thermo-electrically with copper, but we have tested only one specimen of constantin.

Our thanks are due to many persons for assistance received in the prosecution of this research, but especially to Mr. John Connors, the head janitor of the Jefferson Physical Laboratory, for much difficult work in the construction of the Crafts heating apparatus, and to Mr. John Coulson, Professor B. O. Peirce's mechanical assistant, for the skilful construction and testing of resistance coils.

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ON HARMONIC FUNCTIONS IN TWO DIMENSIONS.

BY MAXIME BÔCHER.

ON HARMONIC FUNCTIONS IN TWO DIMENSIONS.

By Maxime Bôcher.

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A function $u(x, y)$ is said to be harmonic throughout a certain region T of the x, y-plane if 1) it is analytic there, and 2) it satisfies Laplace's equation. It is well known that the first of these requirements may be replaced by the apparently much less restrictive one that the function u be continuous and have continuous first and second partial derivatives. In fact, it is sufficient to demand even less than the continuity of the second partial derivatives; but some demand beyond the mere existence of these derivatives must be made in order that it should be possible to apply Green's theorem.

It is my object in the present note to show how the theory of harmonic functions in two dimensions may he established without demanding even the existence of the second partial derivatives. This makes it necessary for us to take as our point of departure not Laplace's equation, since this involves the second partial derivatives whose existence we do not wish to demand, but some other characteristic property of harmonic functions. We select for this purpose the fundamental property that the integral of the normal derivative of a harmonic function, extended around a closed curve, is zero, and we thus have as our starting point not a differential equation of the second order, but a differentio-integral equation which involves only first partial derivatives.[1] It is needless to insist on the fact that it is this differentio-integral equation and not Laplace's equation which forms the starting point in most physical applications of harmonic functions.

We will lay down the following

Definition: u (x, y) *is said to be harmonic throughout a region* T *of the* x, y*-plane if it is single valued and continuous there, has continuous first partial derivatives, and, for any circle which lies wholly within* T,[2] *satisfies the equation :*

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[1] We note in passing that a pure integral equation for harmonic functions is given by Poisson's Integral.

[2] That is, all points within the circle or on its circumference are to lie within the region T and not on its boundary.

$$\int \frac{\partial u}{\partial n}\, ds = 0 \tag{1}$$

the integral being extended around the circle, n *denoting the exterior normal and* s *the arc.*

It is clear that this includes all functions which are harmonic according to the ordinary definition. It is not so clear that it does not also include other functions; for since we have not demanded even the existence of the second derivatives of u, we cannot pass from (1) to Laplace's equation by the familiar application of Green's theorem.

THEOREM I. *On the circumferences of any two concentric circles lying wholly in* T *the average values of* u *are the same.*[3]

Call the radii of the circles a and b ($a < b$) and use polar coördinates (r, ϕ) with pole at the centre of the circles. Then by our definition

$$\int_0^{2\pi} \frac{\partial u}{\partial r}\, d\phi = 0 \qquad (a \leqq r \leqq b)$$

$$\therefore \qquad \int_a^b \left[\int_0^{2\pi} \frac{\partial u}{\partial r}\, d\phi \right] dr = 0.$$

Since $\partial u / \partial r$ is a continuous function of (r, ϕ) we may reverse the order of integration. Denoting by u_a and u_b respectively the values of u on the smaller and larger circle, we thus get:

$$\int_0^{2\pi} [u_b - u_a]\, d\phi = 0.$$

from which our theorem follows at once.

By allowing a to approach zero as its limit we get

THEOREM II. *The average value of* u *on the circumference of any circle lying wholly in* T *is the value of* u *at the centre of this circle.*

This theorem we next state in the following generalized form:

THEOREM III. *If* u *is continuous within and on the circumference of a circle* C, *and is harmonic within it, then the average value of* u *on the circumference of* C *is its value at the centre of* C.

For the average value of u on the circumference of a circle a little smaller than C and concentric with it will, on account of the continuity of u, approach the average value of u on the circumference of C as the smaller circle approaches C as its limit. From this fact and Theorem II our theorem follows.

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[3] The proofs here given of this theorem and the next were used by the writer in a paper on Gauss's Third Proof of the Fundamental Theorem of Algebra. Bull. Amer. Math. Soc., May, 1895.

THEOREM IV. *If* u *is harmonic within* T, *and the region* T *is carried over by inversion into a region* T′; *and if we define a new function* u′ *in* T′ *so that it has the same value at every point of* T′ *that* u *has at the corresponding point of* T *then* u′ *is harmonic in* T′.

In the first place, inversion being an analytic transformation, u' is evidently continuous throughout T' and has continuous first partial derivatives there. Now consider any circle C' lying wholly in T', and denote by s' the arc and by n' the exterior normal of this circle. If C is the circle in T of which C' is the inverse and we consider corresponding points on these two circles, we have, since inversion is a conformal transformation, after neglecting infinitesimals of higher order:

$$\frac{ds'}{dn'} = \frac{ds}{dn}.$$

Accordingly

$$\int \frac{\partial u'}{\partial n'}\, ds' = \int \frac{\partial u}{\partial n}\, ds = 0,$$

the first integral being taken around C', the second around C. This completes the proof that u' is harmonic.

THEOREM V. *If* u *is continuous within and on the circumference of a circle* C, *and is harmonic within* C, *then the value of* u *at any point* P *within* C *is given by the integral:*

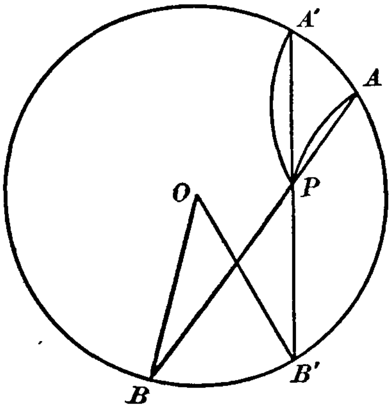
$$(2) \qquad \frac{1}{2\pi}\int_{\theta=0}^{\theta=2\pi} u_\theta\, d\theta,$$

where θ *denotes the angle between a fixed and a variable circular arc which both start from* P *and which both cut* C *at right angles, and* u_θ *denotes the value of* u *at the point where this variable arc cuts* C.

In order to prove this, let us construct the point Q inverse to P with regard to C. If we invert the whole figure with regard to a circle having Q as centre, P goes over into a point P', C into a circle C' having P' as centre, and the fixed and variable arcs referred to in the theorem go over into fixed and variable radii of C' making the variable angle θ with one another. Finally, we define a function u' within and on the circumference of C' so that it has the same value at every point that u has at the corresponding point of C. This function u' will then be continuous within and on the circumference of C', and by theorem IV is harmonic within C′. Accordingly its average on the circumference of C', which is obviously given by (2), is equal by III to its value at P', that is to the value of u at P. Thus our theorem is proved.

Formula (2) is merely an unfamiliar form of Poisson's Integral.[4] This may readily be shown by means of the following lemma, whose extremely simple proof we suppress:[5]

LEMMA; *If from a point* P *within a circle* C *two arcs are drawn orthogonal to* C *and meeting it at* A *and* A′, *the angle between these arcs is measured by the arc* BB′ *of* C *intercepted between the straight lines* AP *and* A′P *produced.*



Applying this to the case in which AA' is the element of arc ds, the angle between the two arcs PA and PA' will be $d\theta$. Denoting the arc BB' by ds_1, and the radius of the circle by R, this lemma tells us that $ds_1 = Rd\theta$. Accordingly (2) reduces to

$$\frac{1}{2\pi R}\int u_{,} \, ds_1, \tag{3}$$

the integral being extended around the circle and $u_{,}$ denoting the same thing as u_θ. This is the form of Poisson's Integral given by Schwarz on p. 360, Vol. 2, of his collected works.

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[4] See a *Note on Poisson's Integral*, by the writer, Bull. Amer. Math. Soc., June, 1898, where a somewhat different treatment is given.

[5] See a paper by the writer on Fourier's Series in the Annals of Mathematics for January, 1906.

This form may be still further changed by means of the proportion $ds_1/\rho_1 = ds/\rho$, where $\rho_1 = PB$, $\rho = PA$. It may thus be written:

$$\frac{1}{2\pi R}\int u_s \frac{\rho_1 \rho}{\rho^2}\, ds.$$

Now, $\rho_1\rho = (R - r)(R + r)$, where r denotes the distance from P to the centre of C. Thus, finally, (2) takes the form:

$$\frac{1}{2\pi R}\int u_s \frac{R^2 - r^2}{\rho^2}\, ds, \tag{4}$$

which is Poisson's original form.

Since $(R^2 - r^2)/\rho^2$ is an analytic function of the coördinates (x, y) of P, the same will be true of the whole integral (4), and therefore $u(x, y)$ is analytic throughout the interior of the circle C. Moreover, $(R^2 - r^2)/\rho^2$ may be seen, by direct differentiation or otherwise, to satisfy Laplace's equation. Hence we infer that the same is true of the function $u(x, y)$ represented by (4).

We have been assuming that u is harmonic merely within C. If, however, it is harmonic throughout any region T, we can now infer at once that it is analytic at any point P of this region, and satisfies Laplace's equation there by merely surrounding P by a small circle and applying the results just established to this circle. Thus we have proved

THEOREM VI. *If* u (x, y) *is harmonic within any region* T, *it is analytic and satisfies Laplace's equation at every point within* T.

We have thus established the identity of the class of functions defined above as harmonic with the class ordinarily so defined.

In conclusion I add the following remarks on the result thus obtained.

1) In the definition of a harmonic function it is obviously not necessary to demand that (1) hold for *all* circles contained in T. All that is essential in the reasoning we have given will apply at once if we start from the following definition:

The function u (x, y) *is said to be harmonic at a point* (x_1, y_1), *if it is possible to surround this point by a region so small that* u *is continuous and has continuous first partial derivatives there, and for every circle contained in this region satisfies equation* (1). *The function* u *is said to be harmonic throughout a region* T *if it is harmonic at every point of this region.*

2) From the point of view of the physicist it may be desirable to demand in the definition *more* than was demanded above, namely, that (1) hold for all regular[6] closed curves lying within T, not merely for all circles. Since it is well known, and readily proved by an application of Green's theorem, that (1) is satisfied for all such curves when u is any harmonic function, it is clear that this will not affect the class of functions covered by the definition. Whether included as part of the definition or proved as a theorem, the fact just stated enables us to see almost intuitively that a harmonic function remains harmonic when the plane is subjected to any conformal transformation, the reasoning being the same as was used in the proof of theorem IV above.

3) The introduction of this general idea of conformal transformation enables us to extend at once our definition of harmonic functions to the case in which T is a region not in the plane but on any analytic curved surface.[7] The definition of a harmonic function on such a curved region T would be identical with the one given above for a plane region, if we understand the partial derivatives there referred to to mean the derivatives along the surface, and if we require that (1) hold for all regular (or even analytic) closed curves on T. If now we transform the region T, or any part of it, conformally onto the plane, we see at once that the function thus defined in the plane is harmonic, and hence the function on the curved surface was analytic there. — Here, as before, we may if we choose require that (1) be fulfilled only for certain classes of closed curves. There seems, in general, no object in doing this. In the case of the sphere, however, the developments given above can be carried through almost word for word if we restrict our attention to circles.

4) Finally, it may be mentioned that this method of approaching the subject is particularly convenient in connection with the theory of conjugate functions. Two functions u and v are said to be conjugate in a plane region T if throughout this region they are single valued, continuous, and have continuous first partial derivatives and satisfy the equations

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}.$$

If we knew that u and v had continuous second partial derivatives, we could infer at once that they are both harmonic. To establish the continuity of these derivatives, recourse is ordinarily had to the theory of functions of a complex variable. Instead of this, we may first deduce from the last written differential equations the relation

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[6] A curve is said to be regular if it is continuous, and is made up of a finite number of arcs each of which has a continuously turning tangent at every point.

[7] Cf. C. Neumann, Math. Ann., **10**, 569 (1876).

$$\frac{\partial u}{\partial \sigma} = \frac{\partial v}{\partial \tau}$$

where σ is any direction and τ the direction which makes a positive right angle with σ. From this we infer that

$$\int \frac{\partial u}{\partial n} ds = \int \frac{\partial v}{\partial s} ds, \qquad \int \frac{\partial v}{\partial n} ds = - \int \frac{\partial u}{\partial s} ds.$$

These integrals will therefore vanish when extended around a closed curve, since u and v are both single valued. From this it follows that u and v are both harmonic.

Harvard University,
Cambridge, Mass.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY
HARVARD UNIVERSITY.

STUDIES ON FLUORITE.

I. *THE FLUORESCENCE OF FLUORITE.*
II. *THE THERMO-LUMINESCENCE OF FLUORITE.*
III. *GASEOUS AND LIQUID INCLUSIONS IN FLUORSPAR.*

BY HARRY W. MORSE.

WITH THREE PLATES.

INVESTIGATIONS ON LIGHT AND HEAT MADE OR PUBLISHED, WHOLLY OR IN PART, WITH APPROPRIATIONS FROM THE RUMFORD FUND.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

STUDIES ON FLUORITE.

By Harry W. Morse.

Presented by John Trowbridge October 11, 1905. Received January 11, 1906.

I. The Fluorescence of Fluorite.

Although the phenomena of fluorescence have been the object of much research for many years the results obtained in the way of general laws must be called very meagre. The law of Stokes, which appears on closer investigation to have many exceptions, is almost the only generalization with direct bearing on the subject. The other general law, that all substances which fluoresce show selective absorption, is of more general bearing, and may perhaps be deduced directly from fundamental thermodynamic principles.

Dividing the subject matter quite arbitrarily into solids, liquids, and gases, it may be said that in gases only a very few cases of fluorescence have been observed. The fluorescence of sodium vapor, first observed by Wiedemann and Schmidt[1] and lately studied with most interesting results by R. W. Wood,[2] offers at least the possibility of a great increase in our general knowledge of the subject. In gases the factors determining luminescence are probably easier and more amenable to experimental control than in the other states of matter.

Among fluorescent liquids there appears a more or less general possibility of systematization. Certain organic groups or radicals seem to carry with them under definite conditions the power of conferring fluorescence on compounds containing them, and the presence or absence of the property of fluorescence may therefore be prophesied with some certainty from a knowledge of the chemical composition of the substance.

With regard to the phenomena of fluorescence in solids we are without any general laws or relations whatever. Those organic substances which show fluorescence in solution often show it also when dried in gelatine, and sometimes even in the pure solid state; but in the latter case the fluorescence color is usually quite different from that of the same substance in solution.

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[1] Wied. Ann., **57**, 447 (1896).

[2] Phil. Mag. [6], **10**, 513 (1905).

Our present methods of investigation seem inadequate. From the mass of fact which has been gathered it seems possible that fluorescence in solids, and especially in inorganic solids, is determined by the presence of small quantities of impurities, but in the uranium salts and the platinocyanides we have substances which are, according to the usual standards, pure, and which are nevertheless capable of exhibiting strong fluorescence, and organic substances which are pure in the same sense often exhibit this property in the strongest degree. The conclusions which may be drawn from the chemical attacks on the question are few and uncertain, and there are as yet no general principles which can be stated.

The only methods of investigation which have been applied to the problem with the exception of chemical analysis and synthesis are those of spectroscopy.[3] In spectroscopy again we have a multitude of facts with only an occasional connection of narrow scope, and the results which may be hoped for are immediately limited by our limited knowledge of the method itself. There is satisfaction in knowing the spectrum of the light emitted by a fluorescing substance under definite conditions of excitation, and such knowledge has of course scientific value, but any extension of our knowledge of fluorescence based on spectroscopic methods can go but very little farther, since we know practically nothing of the mechanism of spectrum production even in the simplest case. The fundamental problem of the appearance of a fluorescence spectrum goes back to the question how any spectrum is produced, and the complete answer to one of these questions includes the answer to the other.

One general fact appears to hold for every case of fluorescence, and that is *selective absorption* by the fluorescing substance. This substance is able to select from the variety of light vibrations which enter it those of certain definite periods, transforming their energy in part into heat, about the period of which we know very little, and in part into light, which is emitted and which determines the spectrum of the fluorescence. This fact of selective absorption brings with it the conclusion that there is something in the substance which has the power of taking up periodic disturbances of definite period, and with their aid setting up within itself disturbances of the same general nature but usually of different period, which result in the emission of light. It would be comparatively easy to picture resonance of such a character that the absorbed and emitted periods should be the same, but there does not appear to be any experimental evidence that such a case exists. Even in sodium vapor, which is nearest this ideal case, the excited fluorescence light has not exactly the same wave-length under all conditions as the exciting light, as may be seen immediately from Professor Wood's photographs. And when we remember that very frequently luminescence spectra of very nearly the same character are given out by a substance when it is excited by light, heat, kathode bombardment, X-rays, friction, or radium, it is evident that a very general property is involved of which excitation by light vibrations is only a phase.

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[3] For a possible variation see Nichols and Merritt, Phys. Rev., **19**, 396 (1904).

It seems probable that neither chemical analysis nor spectroscopic analysis in its present condition can serve as an adequate tool with which to attack the fundamental problems of fluorescence, yet they are almost the only tools available. Burke's investigation on the change of absorption during fluorescence,[4] and Nichols' and Merritt's work on the change of electrical conductivity of solutions while fluorescing,[5] are suggestions of possible variations in the method of attacking these phenomena.

The present paper contains a brief summary of the spectroscopic results obtained during a study of a single substance — fluorite. This substance is remarkable because of the large number of methods of excitation which may be employed with it and because of the brightness of the luminescence light emitted under the influence of most of these exciting sources.

It is the author's intention to carry the study of light emission by fluorite through the entire series of methods of excitation. The present paper contains data on the fluorescence spectra only; that is, on the light emitted by fluorite when the exciting source is light.[6]

Experimental Data.

The crystals examined were all from Weardale, England. Spectroscopic data on the results of the study of these crystals has already been given in full in the Astrophysical Journal for March, 1905. It has seemed advisable to submit a summary of these and of other results obtained since that paper was published to the American Academy, since succeeding results are to be published in the Proceedings, and the connection between the facts of fluorescence and those obtained from other methods of excitation is a very close one.

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[4] Phil. Trans., **191**, 87 (1898).

[5] Phys. Rev., **19**, 396 (1904).

[6] The spectrum of the light emitted being examined *during* excitation.

In Plate 1, the general appearance and something of the detail of the fluorescence spectra from two crystals of fluorite may be seen. These are crystals which have been numbered 4 and 5, and the cuts show the spectra produced when they are excited by the light from the spark between electrodes of various metals. For the tables of measured wave-lengths reference should be made to the paper in the Astrophysical Journal.[7] The facts exhibited by the tables and the plates may be briefly summarized as follows: —

1. The light of the fluorescence excited in a crystal of fluorite by the light of the condensed spark between electrodes of certain metals shows in its spectrum many sharp lines and narrow bands.

2. A single fluorite crystal is able to emit a very great number of these lines and bands, a part only of which appear under excitation by the spark from one metal, another part under excitation by another metal, and so on. The entire fluorescence spectrum of a fluorite crystal must then be considered the sum of all the lines excited by all the different sparks.

3. Certain lines appear to be common to the fluorescence spectra excited by different sparks. This statement holds true to the limit of accuracy of the measurements on the photographs obtained.

4. Certain of these lines and bands are evidently peculiar to the light excited by a single metal. No error of measurement could account for the differences of distribution of the strong lines in the various spectra emitted by the same crystal under excitation by different sparks.

5. The fluorescence spectrum varies from crystal to crystal, not only in minor points, but even in the strongest lines.

6. Metals having strong lines in the ultraviolet part of the spark spectrum excite extended fluorescence spectra.

7. The exciting source for the sharp lines of the photographs is light between wave-lengths λ 3000 and λ 2000, since it is absorbed completely by a thin layer of glass, but not by a considerable thickness of quartz.

8. The strong sharp lines of the fluorescence spectra lie between λ 5700 and λ 6400. No sharp lines whatever have been observed of shorter wave-length than λ 4700.

9. The lines in these fluorescence spectra do not appear to belong to

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[7] Astrophys. Jour., **21**, 83 (1905).

any known substances, and the fact that the fluorescence spectrum changes completely with a change in the exciting source immediately excludes all ordinary substances from consideration.

10. A change of this sort, i. e. a redistribution of sharp lines in a spectrum, corresponding to a change in an exciting source, is something wholly new in spectrum observations.

11. The possibility of optical resonance has been examined with wholly negative results. No relation between sharp spectral lines in the exciting source (of wave-length between λ 3000 and λ 2000) and the strong sharp lines of fluorescence has been found.

12. There is a sharp strong line at about λ 5735 in the fluorescence spectra excited in the same crystal by the spark between the following terminals: — iron, aluminium, cadmium, magnesium, and tin, and the ultraviolet spectra of these metals are so very different as to exclude any relation between a strong line in all of these spectra and a strong line of the fluorescence spectrum. The possibility of optical resonance is made still more improbable by the fact that the spectrum varies sharply from crystal to crystal with the same exciting source.

13. The possibility is suggested that these sharp lines of fluorescence may be connected with an impurity in the crystal. The fluorites giving sharp lines are colored, and the colored parts form layers or strata in which the fluorescence appears stronger than in the other parts of the crystal. These strata were noted by Stokes as being the seat of the "red fluorescence" which he observed and which is here resolved into sharp lines.

14. Whatever the source of fluorescence, it is completely removed or destroyed by heating the crystal to a temperature above 300°.

15. A crystal which has been so heated may be so regenerated by exposure to light that it will give the broad blue band of fluorescence, but the sharp-line portion of the fluorescence light is not regenerated.

16. Under the exciting influence of sunlight the fluorescence spectrum of these crystals contains only the broad diffuse band from about λ 4800 to about λ 4000, without maxima or minima.

The Absorption of Fluorite.

The absorption and emission of fluorite of various colors have been examined in the visible part of the spectrum by Nichols and Merritt.[8] Their results show that as far as their experiments reach the absorption is slight, and that there are no sharp bands in it. Remarkable is the fact that fluorite, which shows no marked absorption out as far as λ 1200, should be strongly phosphorescent. This is the case with some very pure white plates which are used by Dr. Theo. Lyman in his work on the photography of very short wave-lengths. They show no measurable absorption of a selective sort, and permit the passage of light evenly as far as λ 1200, while they are more strongly phosphorescent under the influence of the spark between metallic terminals than many colored fluorites.

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[8] Phys. Rev., **19**, 18 (1904).

Photographs of my own which extend as far as λ 2100 show no marked absorption and no selective absorption whatever. These facts simply point to the flatness of the absorption curve for this substance, and they are all the more remarkable when we consider the great sharpness and intensity of the fluorescence lines and bands from these same fluorites.

Plate 2 shows three photographs enlarged from small plates like those in Plate 1, and they show very clearly that many of the lines in the fluorescence spectra are equally as sharp as the lines in the comparison spark spectra. Some of these lines occur in groups as doublets and triplets, having all of the characteristics as regards appearance and sharpness which characterize the spectra of incandescent metals and gases.

Figure 1 shows a portion of the fluorescence spectrum of a crystal of fluorite excited by the spark between magnesium terminals. It is especially characterized by three strong lines, one of them very strong indeed.

Figure 2 shows the fluorescence spectrum of the same crystal under the exciting influence of the spark between iron terminals. Beside the strong groups in the yellow, orange, and red, there may be seen in this photograph a large number of lines which are partially obscured by the broad band. That these are not lines which have entered the spectrograph from the light of the spark is proven by the fact that the very strongest spark lines are missing from the fluorescence spectrum. In fact, every line which can be seen in this spectrum is a true line of fluorescence light.

Figure 3 shows the fluorescence spectrum of the same crystal under the exciting influence of the spark between cadmium terminals. A number of lines have in this case been reflected in the crystal through poor adjustment, and these appear in the fluorescence spectrum. The strong triplet λ 5737 to λ 5711 is however made up wholly of fluorescence lines, and the other lines belonging to this spectrum can readily be separated from the spark lines which have crept in. The bands in this fluorescence spectrum have every appearance of being made up of unresolved fine lines just as the ordinary bands in metallic and gaseous spectra are.

These lines of fluorescence have, in fact, all the characteristics of sharp spectral lines, comparable with those of metals.

This result is so important and so wholly at variance with the existing ideas about fluorescence that the whole matter deserves minute investigation, and further study is already under way with the assistance of a grant from the Rumford Fund of the American Academy.

II. The Thermo-luminescence of Fluorite.

The more obvious facts about the thermo-luminescence of substances have been known for a very long time. That many substances emit light on being heated to a temperature far below that of incandescence has been known since the time of Boyle, and some crystals having this property were sent to the French Academy of Sciences in 1724. These were evidently fluorspar crystals, and this substance forms the basis of most of the older experiments on the subject, although long lists of thermo-luminescent substances are to be found in the researches of Wedgwood,[9] Brewster,[10] and Pearsall.[11] Later researches of importance have been carried out by Becquerel,[12] Le Bon,[13] and Wiedemann and Schmidt.[14]

General Facts.

The general facts concerning the production of a thermo-luminescence may be briefly summarized as follows: —

1. Certain minerals (fluorite, diamond, leucophane, apatite, scheelite, etc.) and many artificially prepared sulphides of the alkaline earth metals, as well as many salts, emit light when heated to a temperature below that of incandescence.

2. These substances appear without exception to exhibit fluorescence or phosphorescence, or both, under excitation by light, but many of the most brilliantly thermo-luminescent substances are not at all brilliant in fluorescence or phosphorescence, and many brilliantly fluorescent or phosphorescent substances exhibit no measurable thermo-luminescence.

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[9] Phil. Trans., **82**, 28 (1792). Refers to still older literature.

[10] Edinb. Phil. Journal, **1**, 383 (1819).

[11] Journ. of Royal Institution, **1**, 77 and 267 (1830).

[12] C. r., **112**, 557 (1891).

[13] Revue Scientif., **14**, 289 and 327 (1900).

[14] Wied. Ann., **56**, 201 (1895).

3. The time during which the thermo-luminescence persists appears to be a quite definite function of the temperature of heating. Some phosphorescent sulphides may be exhausted of thermo-luminescence at a high temperature in a few seconds, while the luminescence of a crystal of fluorspar may be either exhausted in a short time (a minute or two) at a red heat or may persist for hours or days with a nearly constant intensity if the temperature be kept at 100°. Measurements of the relation between temperature, time of heating, and intensity of the emitted light are now being made.

4. The temperature at which light begins to be emitted by a substance may be a very low one. Some specimens of chlorophane and certain diamonds (especially Brazilian ones) are so sensitive that they show visible light plainly when warmed by the hand, and many other substances show the effect plainly at temperatures below 200°.

5. In many cases the store of light contained in the substance appears not to become exhausted by heating to a temperature below a certain definite maximum, no matter how long the heating is continued. For after all emission has ceased at the lower temperature, more may be produced by raising the temperature to a higher point, and when the light supply has been exhausted at this higher temperature more light is emitted when the temperature is raised still higher. There appears, however, to be a certain maximum temperature at which all of the store of light in the body is exhausted, and further increase in temperature beyond that point does not result in the production of further luminescence.[15]

6. On standing in the dark, the power to emit light on being heated is not regenerated. This is clearly shown by heating a crystal of fluorite to a definite temperature below the maximum until no further luminescence is visible. If the crystal is now cooled and allowed to remain in the dark, and then brought back to the same temperature as before, no light is emitted. If it is heated to a higher temperature, the luminescence begins again.

7. With respect to the regeneration of the thermo-luminescence by light, various substances show differences so great as to make it certain that there is more than one kind of thermo-luminescence. The phosphorescent sulphides are easily regenerated by a short exposure to light, and this effect would naturally be expected from the method of their prepara-

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[15] Le Bon, Revue Scientif., **14**, 289 and 327 (1900); Becquerel, C. r., **112**, 557 (1891).

tion. The thermo-luminescence of substances of this class depends on something which is not destroyed by the action of heat to such an extent or in such a way that it cannot be reformed under the exciting influence of light. But the thermo-luminescence of fluorite is of a wholly different kind. When the store of light has once been exhausted, it may be in a slight degree recovered under the influence of light, but the light produced by a subsequent heating is not the same as the original thermo-luminescence light. This point will be brought out more clearly in the description of my own experiments.

8. With respect to the regeneration of thermo-luminescent power under the influence of the electric spark, these same differences in the two classes of substances are evident. The alkaline earth sulphides are easily regenerated, probably by the light of the spark rather than by any special electric action. Some fluorites, especially those which are usually called *chlorophanes*, which give a strong green thermo-luminescent light, showing a spectrum which is described in a later part of this paper, cannot be so regenerated that the original light is emitted. The power of emitting light is indeed restored under the excitation of the electric spark, but the light emitted is never the same as that produced during the first heating.[16]

9. Many substances change during heating, so that the light, while in some cases remaining apparently nearly constant in intensity, changes in color during the experiment. Others show a great change in intensity, together with changes in the color of the emitted light. It will be seen later that the spectrum of the light from certain fluorites consists of two distinct parts, one of which persists after the other has practically disappeared; and this fact goes one step toward an explanation of the changes in color during heating.

Experimental Data.

The present paper contains data on the spectra of thermo-luminescence of two distinct varieties of fluorite, — one a "chlorophane" from Amelia Court-House, Va., and the other a common, clear, colorless (or very slightly greenish) specimen from Westmoreland, N. H. Both are slightly fluorescent under excitation by ultraviolet light, the fluorescence being of the usual lavender color common apparently to all fluorites under these conditions, and both are phosphorescent, showing this lavender-colored light for some time after removal from the exciting source. Neither is at all brilliantly fluorescent or phosphorescent under the conditions to which they have been subjected, but both are very strongly thermo-luminescent, the chlorophane emitting a very strong bright green light, easily visible even in a well-lighted room, and the other fluorite emitting a lavender or purple light, not very different to the eye from the light emitted by the same substance in fluorescence and phosphorescence.

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[16] Pearsall, Journ. of Royal Institution, **1**, 77 and 269 (1830).

Spectroscopic examination of the light from the chlorophane shows what appears to be a continuous spectrum with a maximum in the green-yellow. Carefully rested eyes enable some other sharper maxima to be seen, but little detail can be made out.

The light from the other fluorite shows several broad bands of a rather diffuse nature, and superimposed over these some sharper lines can be made out. The broad bands of the two spectra are the source of most of the emitted light, and the sharp lines are much obscured by their presence.

The spectrum of fluorites much like the New Hampshire variety has been described by a few observers. The oldest reference found was in a brief note in Poggendorf's Annalen, "from a letter from Herr Kindt."[17] In this note the broad bands of the thermo-luminescence spectrum are mentioned, and the remark is made that the bands are as strong as the absorption bands in the spectrum of the didymium salts. Later references to the same spectrum are to be found in the researches of Hagenbach[18] and Liveing,[19] both of whom observed the broad bands and give their approximate wave-lengths. No reference to photography or more minute examination of the question has been found.

There are at least three distinct stages in the thermo-luminescence of the chlorophane. At a low temperature (50° to 100°) the light is bright green and strong. After the light has begun to grow dim and the temperature is increased and maintained for some time, a yellowish color of less intensity and short duration takes its place, to be in turn replaced by a weak lavender luminescence at a still higher temperature. This final color is very much like that of the fluorescence under excitation by light.

In the case of the other specimen chosen for examination (the colorless crystal from New Hampshire), no such marked differences in the color emitted at different stages of heating were found, but it is evident that the light produced at higher temperatures, toward the end of the luminescence, is of a deeper purple color than that emitted at lower temperatures. In the research of Pearsall[20] a long list of the various colors emitted by different fluorites at various temperatures and times of heating is given. In this same research the matter of the regeneration by the electric spark is taken up fully, but of course without any use of spectroscopic methods.

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[17] Pogg. Ann., **131**, 160 (1867).

[18] Archiv. de Geneve [2], **60**, 297 (1877).

[19] Proc. Camb. Soc., **3**, 96 (1877).

As has been stated, the chlorophane begins to glow at a low temperature. It reaches its maximum at a point not much above 350°, while the other fluorite requires a higher temperature to start the luminescence, and the temperature must be raised to a slightly higher point to completely exhaust the power of luminescence.

The Photography of the Thermo-luminescent Spectra.

Experimentally, the photography of spectra of luminescences of the intensity of these offers no difficulty whatever. Patience is necessary, since a large amount of material must be allowed to luminesce in front of the slit of the spectroscope, and the time during which a single crystal of fluorite glows brightly is rather short. Exposures of about four hours were required to yield measurable plates. Longer exposures could of course have been made, but the advantage gained would be slight, since the continuous background of diffuse light is so strong that the sharp lines are not much more clearly differentiated than with the shorter exposures. A great advantage would be gained by the use of greater dispersion, as the sharp lines could in this way be brought out while the continuous background remained weak. For this preliminary study, however, the spectrum as a whole is of interest, and the dispersion employed is well suited for the purpose. The spectrograph employed was the wide-angle one which had already done good service in the examination of the fluorescence spectra of these same fluorites.[21]

After discarding mechanical arrangements of the nature of hot revolving plates and dishes, the simplest possible method of exposure was employed. The material was heated in a test tube out of range of the slit until it began to glow. It was then held in front of the slit by hand until the glow from the first heating had begun to grow weak, when it was reheated and brought back in front of the slit, and so on until the luminescence was exhausted. Fresh material was then put through the same treatment, and the process continued throughout the time of the exposure. Later, a small electric furnace was used in the same way; the only advantage which it has over the test tube being the better control of the temperature, and this is in part offset by the difficulty of keeping the brightest luminescence directly in front of the slit. The heated crystals decrepitate, and break up into smaller fragments, and occasionally a heavier explosion than usual blows everything out of the furnace, or at least disturbs the position of the main mass of luminescent material. The test tube, held in the hand, can be turned so that the brightest part is always toward the slit, and in general the cruder method is at least as easy in the end.

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[20] Journ. of Royal Institution, **1**, 267 (1830).

[21] Astrophys. Journ., **21**, 83 (1905).

Spectroscopic Results.

TABLE I.

CHLOROPHANE FROM AMELIA COURT-HOUSE, VA.

Continuous spectrum from violet to red, with maximum in the green-yellow both visually and in the photographs. Superimposed over this continuous band the following lines and narrow bands:

4125	2 diffuse.	4415	5 quite sharp.	5375	5 sharp.
4145	2 "	4457	2 diffuse.	5400	diffuse.
4315	1 "	4525	1 "	5435	"
4325	2 sharp.	4580	5 sharp.	5500	"
4335	1 diffuse.	4630	1 diffuse.	5600	"
4347	10 sharp.	4700	2 sharp.	5700	"
4365	4 diffuse.	5175	diffuse.	5893	sharp.

TABLE II.

Clear, colorless fluorite from Westmoreland, N. H. Underlying diffuse band from green to violet, rather sharply bounded at green end, but shading off in violet. Superimposed over this, and in the yellow and red, the following lines and bands:

4108	1 sharp.	4622	1 diffuse.
4130	2 diffuse.	4658	4 sharp.
4150	2 "	4690	2 "
4192	1 "	4703	2 "
4230	2 "	4715	1 diffuse.
4250	2 "	4770	5 nearly sharp.
4298	1 "	4800	2 "
4312	1 "	4815	3 "
4330	2 "	4840	7 diffuse.

TABLE II. — *continued.*

4347	10	sharp.	4892	6	nearly sharp.
4365	5	max. in diffuse band.	4903	5	“
4370	2		5200 to 5265		broad band.
4382	2	sharp, in diffuse band.			
4398	1				
4415	5	not quite sharp.	5375	9	sharp.
4450	1	diffuse.	5435	4	“
4462	2	“	5520	2	“
4475	1	“	5720	1	diffuse.
4502	2	“	5800	1	“
4540	2	“	5893	8	(sodium lines unresolved ?)
4580	4	sharp.			

TABLE III.

LINES COMMON TO BOTH SPECTRA.

Westmoreland.	Chlorophane.	Mean.	Intensity.	Maximum Error.
4130	4125	4127	2	
4150	4145	4147	2	
4312	4315	4313	1	
4330	4325 4335	4330	2	
4347	4347	4347	10	±2 A. E.
4365	4365	4365	5	±2 A. E.
4415	4415	4415	5	
4462	4457	4460	2	
4580	4580	4580	4	
4625	4630	4627	1	
4690 4703	4700	4700	2	
5375	5375	5375	9	±3 A. E.
5435	5435	5435	4	±4 A. E.
5893	5893 (?)	5893	8	±2 A. E.

In all of the photographs taken magnesium was used as comparison metal, and lines in the luminescence spectrum which fall near sharp magnesium lines can therefore be measured with considerable accuracy. It is on these lines that the values in the column headed "Maximum Error" are based. For example, the line at λ 4347, a sharp strong line in the thermo-luminescence spectrum of each of these fluorites, lies near the λ 4352 line of the magnesium spark spectrum, and the error of measurement on several plates is not greater than the value given. The line at λ 5893 coincides exactly with the unresolved sodium lines in the comparison spectrum.

Plate 3 is a reproduction of a photograph of the thermo-luminescence spectrum of the Westmoreland fluorite, with the comparison spectrum of the magnesium spark beside it. The magnesium spectrum is much over-exposed in this photograph, and the exposure for the luminescence spectrum has been perhaps somewhat too short. A longer exposure brings out the diffuse background rapidly, and the lines are not made easier to measure. The chlorophane spectrum has so much of the continuous background as to make it difficult of reproduction, and doubtless many weaker sharp lines escaped measurement from this same cause.

Conclusions.

It would appear proven that the sharp-line portion of the spectrum from the two fluorites is the same. All of the strong lines agree in wave-length and in relative intensity within the limit of accuracy of the measurements. It should be remarked that "intensity" measurements of lines nearly hidden in the diffuse bands are of little significance, but they have been graded on a scale of 1 to 10 for the purpose of a qualitative comparison.

The diffuse bands of the two spectra are, however, entirely different, and the color of the thermo-luminescence light is determined by the preponderance of one or the other series of diffuse bands, the sharp-line portion being in either case comparatively weak.

The change of color of the luminescence light during the heating can now be explained. It is due to the persistence of one of the sources of luminescence — the one causing the appearance of the sharp lines or the one causing the appearance of the diffuse bands — after the other source has ceased to be active. In the Westmoreland fluorite the sharp lines appear to weaken and disappear before the broad bands, and the same seems to be true of the chlorophane, although the question is more difficult of decision in the latter case. A spectro-photometric study of the rate of decay of the lines and bands is already under way.

All attempts to place the sharp lines to the credit of any known elements have been so far unsuccessful. There are of course many coincidences with lines of well-known substances, especially since the dispersion and the corresponding accuracy of measurement are so small; but the other strong lines of the substance are absent in every case, and one would demand the coincidence of an entire series of lines of an element before admitting that a metal or a gas could show a line spectrum under the conditions of the experiments. The very exact coincidence of the line at λ 5893 with the mean of two sodium lines suggests the possibility that this may be that pair unresolved; but it would seem that resolution into the two constituent lines is the very least that could be demanded before deciding that the sodium pair do actually appear under these circumstances, and any decision on this point should be reserved until this resolution has been accomplished.

There can be no doubt that these thermo-luminescence spectra are composites, and that the sources, whatever they may be, of the parts of the composite, are distinct. One of these sources gives rise to the diffuse band part of the spectrum in each case, and this part is like the usually observed spectra of fluorescence and phosphorescence. It is evidently different in the two fluorites. The other source is capable of giving a spectrum in most respects similar to the sharp-line spectra usually associated with incandescent gases. This source is *common to the two fluorites.*

It seems imperative that as minute an examination as possible of the impurities present in fluorites should be made, to include not only a chemical analysis but also an examination of the gaseous and liquid inclusions in the mineral.

A generous grant from the Rumford Fund of the American Academy of Arts and Sciences has been of much aid in this work, and a continuance of the investigation in several directions is already under way.

III. GASEOUS AND LIQUID INCLUSIONS IN FLUORSPAR.

Many facts connected with the various phenomena of luminescence in the mineral fluorspar point to the possible influence of impurities of a gaseous or liquid nature, which are usually present as inclusions, and which are driven out or decomposed when the mineral is heated. There is a very general impression that these impurities exercise a determining influence in the emission of light by the substance,[22] and especial weight has often been laid on the analogy with the necessary presence of small quantities of impurities (usually copper, bismuth, or manganese, and a salt of sodium or potassium) in order that phosphorescence shall be exhibited by the sulphides of calcium, barium, and strontium. Since most colored minerals lose their color, their power of fluorescing and phosphorescing, and their content of included gases at about the same temperature and after about the same duration of heating, a direct connection between these three properties has often been suggested, and a relation of this kind is usually accepted as a probability by every one who studies the phenomena in question.

Spectroscopic investigations on the fluorescence and thermo-luminescence of the mineral fluorspar have yielded many facts of interest, and several new and wholly unexpected discoveries.[23] It seems necessary to examine the included gases and liquids and the other impurities in the naturally occurring mineral as minutely as possible, with especial reference to some chemical factor which might exert an influence on the luminescence phenomena.

The Color of Minerals.

The mineral fluorite is remarkable for the variety of colors which it may exhibit. It occurs in fact in nearly every color of the entire spectrum, with the possible exception of bright red, and not only do fluorites from different localities show this difference in color, but even the most homogeneous and perfect crystal often shows a series of layers of different colors sharply separated from each other, and usually lying in planes parallel to the natural faces of the crystal. The same crystal often shows as many as four or five sharp differences in color, and the colored strata often repeat in sets in which the colors take different sequences until the number of visible separate layers reaches twenty or more. Sir David Brewster[24] was the first to call attention to this structure of fluorite, and even at that early date he suspected a relation between these strata and the remarkable phenomena of fluorescence which he was investigating.

The general question of the cause of color in minerals has been made the subject of a great deal of investigation, without any satisfactory or final answer having been reached. Two diametrically opposite opinions still exist, — one, that the color of many minerals, fluorite among them, is due wholly to the presence of organic coloring materials,[25] the other, that there is no sufficient proof for this opinion.[26] The method of study in all of these researches was the one which suggests itself immediately, — heating the mineral until the organic substances present are driven off or decomposed, and collecting the gases and the liquid distillate when any are produced.

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[22] See Becquerel, C. r., **112**, 557 (1891).

[23] See the two preceding divisions of this paper.

[24] Edinb. Phil. Trans., **16**, [2], 111 (1812) ; Edinb. Jour. Sci., **6**, 115 (1821).

Wyrouboff[27] was the first to make a categorical statement in the matter. He heated minerals in a current of oxygen and determined the resulting carbon dioxide and water, finding from his experiments that the amount of carbon and hydrogen found in the analyses was nearly proportional to the intensity of the color of the mineral. These experiments were especially on fluorspars from different localities and of different colors, and his decision was that the colors were *organic*. Foerster[28] repeated these experiments on a number of fluorites and showed that the quantities of hydrogen and carbon produced are not by any means proportional to the intensity of color in the mineral, clear white specimens often showing as large an amount of organic impurity as the most deeply colored. My own analyses confirm these results of Foerster's in every way, but it does not by any means follow from these results that the coloring substances are not organic. The only conclusion which seems to me justified is that the other crystals contain organic substances which are not colored. At a much later date Weinschenck[29] made an exhaustive series of analyses to determine the same question, arriving at the conclusion opposite to that of Wyrouboff, and a little later Kraatz and Wöhler[30] arrived at the opposite result from an equally careful series of analyses. The question as to the cause of the color of fluorspar still remains in this unsettled state.

The Odor of Fluorspar.

Another peculiarity of fluorspar from many sources, and one which might perhaps be expected to throw some light on the source of its luminescences, is the very strong odor which is produced when the mineral is heated or ground to powder in a mortar. Schafhäutl [31] showed that chlorine monoxide is produced when the famous fluorite from Wölsendorf is heated. Schönbein, Meissner, Schrötter,[32] and others were of the opinion that the odor produced on heating is due to ozone. Becquerel and Moissan [33] found traces of free fluorine in the gases distilled from some fluorites, and ascribe the odor to this substance. The great majority of all the analyses of the mineral show the presence of small but appreciable quantities of chlorine.

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[25] Wyrouboff, Bull. Soc. Chim., **1866**, 334; Kraatz and Wöhler, see Ztsch. für Min. u. Cryst., **33**, 632 (1900).

[26] Weinscheuck, Ztsch. anorg. Chem., **12**, 375 (1896); Foerster, Pogg. Ann., **143**, 173 (1879).

[27] Bull. Soc. Chim., **1866**, 334.

[28] Pogg. Ann., **143**, 173 (1879).

[29] Ztsch. anorg. Chem., **12**, 375 (1896).

[30] See Ztsch. für Min. u. Cryst., **33**, 632 (1900).

The odor of all the samples of fluorite which I have examined by heating is what would be called distinctly "empyreumatic" — closely resembling the odor arising from the distillation of wood, and certainly of organic origin. None of the many samples which have been at my disposal have shown any very marked odor on attrition, but a few give an "earthy" odor, not very characteristic, but suggestive of organic origin.

Inorganic Impurities.

The most general of these appear to be chlorine, iron, and phosphoric acid, neither of which is a perfectly general impurity, and neither of which is usually supposed to exercise any influence on luminescences. Not long ago Humphreys [34] showed that yttrium is an almost universal impurity in fluorite, using the delicate spectroscopic test for the presence of this strange element. For the present, attention has been confined to those substances which can be volatilized or decomposed by a low red heat, and yttrium compounds would not be included, though the presence of this substance is certainly suggestive.

Gaseous and Liquid Inclusions in Minerals.

These have been the subject of much investigation since the discovery of carbon dioxide in liquid form in crystal cavities by Brewster.[35] While carrying out this investigation he found that air and water are present in certain fluorites, and since that time carbon dioxide, hydrogen, nitrogen, carbon monoxide, oxygen, and hydrocarbons have all been reported as existing in the form of inclusions in this mineral. Tilden [36] and Gautier [37] examined the gases produced by heating many rocks and minerals, with especial attention to the probable conditions of temperature, pressure, and concentration of water vapor which existed on the earth at the time the minerals were formed, and they suggest reactions between water vapor, carbon dioxide, and the ferrous compounds of the rocks and minerals to explain the appearance of the gases evolved. Ramsay and Travers[38] also examined a number of minerals with the same object in view.

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[31] Liebig's Ann., **46**, 344 (1843).

[32] Pogg. Ann., **111**, 561 (1860).

[33] C. r., **111**, 669 (1890).

[34] Phys. Rev., **19**, 300 (1904).

[35] Edin. Trans., **10**, 1 (1826).

[36] Roy. Soc. Proc., **60**, 453 (1896).

[37] Bull. Soc. Chim., **25**, 402 (1901).

It is evident that two important questions are involved, — whether or not the permanent gases can remain as such as inclusions in minerals, and whether any conclusions as to the conditions existing on the earth at the time the minerals were formed can be drawn from the inclusions present in them. Travers[39] shows that it is doubtful whether carbon monoxide, nitrogen, oxygen, and hydrogen can remain for a long time included in cavities in minerals, and suggests that they may be formed during the examination of the minerals as a result of the methods of analysis. He shows that many minerals which yield both carbon monoxide and carbon dioxide on being heated give only carbon dioxide when the same mineral is decomposed by the action of an acid.

Direct evidence on this point may be found in the analyses of Hüttner,[40] who heated a number of minerals to 800°–850°, driving out air by means of carbon dioxide and carrying the products of reaction along into the receiving apparatus by means of the same gas. Hüttner expresses the opinion that the hydrogen which he finds in all of his analyses is formed by the reaction suggested by Gautier, i. e., by the action of the ferrous and manganous compounds of the mineral with the water vapor which is always present, and he further offers the opinion that the carbon monoxide found results not merely from the reaction between carbon dioxide and the ferrous compounds, as suggested by Gautier (l. c.), but rather from the direct reduction of carbon dioxide by hydrogen resulting from the above reaction. Travers's conclusion that no carbon monoxide is produced when the mineral is decomposed by the action of an acid is confirmed by Hüttner's results obtained by the same method.

In the special case of fluorite no extended series of analyses of the included gases has been made. The very great difficulties involved in the decomposition of this mineral by an acid and collecting the gases evolved practically excludes this means of deciding whether the gases found exist as such in the crystals or are formed during the progress of the analysis at the temperature necessary for the setting free of the included substances. There are, however, a number of points in previous researches which fall within our general knowledge of chemical equilibrium, and it is the purpose of the present paper to discuss these and to add some new experimental evidence in the matter.

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[38] Roy. Soc. Proc., **60**, 442 (1896).

[39] Ibid., **64**, 130 (1898).

[40] Ztsch. anorg. Chem., **43**, 8 (1905).

The Equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$.

The fundamental conclusion which Hüttner draws from his results is that the carbon monoxide which he finds is formed at the temperature of the experiment by the action of hydrogen on carbon dioxide. Now the results of all the investigations on the water-gas equilibrium show that the equilibrium constant

$$K = \frac{C_{CO_2} \cdot C_{H_2}}{C_{CO} \cdot C_{H_2O}}$$

is about unity at 800°; that is to say, at this temperature carbon monoxide and hydrogen are equally strong reducing agents and carbon dioxide and water vapor are equally strong oxidizing agents. At lower temperatures carbon monoxide is the stronger reducing agent, while at higher temperatures hydrogen is stronger.[41]

The results of experiments on this equilibrium have shown that under ordinary conditions, where the gases are simply mixed together in a vessel, the reaction

$$CO_2 + H_2 \rightarrow CO + H_2O$$

has not a measurable velocity at 900°. Hoitsema[42] used the most sensitive tests for carbon monoxide, but could find no trace of this gas even after hours of heating of the reacting mixture at 900°. Hahn[43] found the presence of a great surface of platinum necessary to cause the reaction to reach equilibrium even in the space of many hours. It is of course possible that the minerals themselves act as catalyzers for the reaction, but even if equilibrium were completely reached during the progress of the experiment the relative amounts of hydrogen, carbon dioxide, carbon monoxide, and water vapor found in my experiments cannot possibly be represented by the reaction which Hüttner assumes to be fundamental.

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[41] Hahn, Ztsch. phys. Chem., **44**, 541 (1903).

[42] Ztsch. phys. Chem., **25**, 686 (1898).

[43] Ibid., **44**, 541 (1903).

Experimental Arrangements.

The present series of experiments consisted only in heating fluorite in a vacuum and making a complete analysis of the gases evolved at various temperatures, varied from about 250° to 700°. Samples of gas were pumped out at various stages of the heating and the gases corresponding to the reaction (whatever it may be) at various temperatures for the same specimen of fluorite were separately analyzed. For the lower temperatures the mineral was heated in a glass or porcelain retort or a glass or porcelain tube in an air bath. For the higher temperatures the fluorite was heated in a porcelain tube in a combustion furnace, and the temperatures are therefore only rough approximations. The retort or tube was connected with a cooled receiver in which the liquid distillate condensed, and beyond the receiver with a Toepler vacuum pump of the form usual for the collection of gases. The gases were collected over mercury at the delivery end of the pump, and passed into the usual apparatus for gas analysis wholly through sealed glass connections. For the determination of the rare gases a second Toepler pump was connected with the first through a series of combustion furnaces containing reagents necessary for the removal of nitrogen. The purified residue was finally passed through the second pump into a Geissler tube for spectroscopic examination. In order to obtain a sufficient quantity of gas for an accurate test for helium, argon, etc., a large quantity of fluorite (10 kilos) was heated in an iron retort and the total residue of the whole volume of gas obtained (about 7 liters) was sent through for the final spectroscopic examination.

The usual weight of fluorite taken for an analysis was 250 gms. and the amount of gas obtained from this amount of the mineral varied greatly. Some of the colored and not very transparent Weardale fluorites yielded as high as 500 c.c. of gas for this amount of material, while other clearer specimens from the same locality gave as low as 50 c.c. Some American fluorites having a deep green color gave only about 20 c.c. of gas, even when they were heated to 700°.

The vacuum reached before heating began was usually about 1 mm. At ordinary temperatures no diminution in the vacuum, such as would be caused by the escape of gas from cavities, was observable after 24 hours.

Analyses of the Gases from Fluorites.

The following are characteristic sets of analyses from a few of the twenty samples analyzed. The results obtained on different samples of the same origin are all of the same order, the percentages varying only within the limit of error of the analytical methods employed.

1. Weardale (England) fluorite. Purple in color. Not very transparent. Contains many inclusions visible under the microscope. Amount taken for analysis, 250 gms. Gas evolved, 320 c.c. Temperature of heating, 280°. Time of heating, four hours. Water produced, 2.4 c.c. 85.3 c.c. taken for analysis.

Composition:

Carbon dioxide	46.7 %
Oxygen	2.9 "
Carbon monoxide	16.7 "
Hydrogen	18.6 "
Nitrogen (remainder)	15.2 "
	100.1 "

2. Same sample. 250 gms. Gas evolved, 280 c.c. Temperature, low red heat (about 650°). Time of heating, two hours. Water, 3 c.c.

Composition:

Carbon dioxide	42.9 %
Oxygen	2.0 "
Carbon monoxide	21.7 "
Hydrogen	16.4 "
Nitrogen (remainder)	16.4 "
	99.4 "

3. Perfectly clear Weardale fluorite crystal. Contains only very minute inclusions, and only a few of these. Deep purple color, slightly brownish. 200 gms. Gas evolved, 68.5 c.c. Temperature, 340°. Time of heating, five hours. Water, about 1 c.c.

Composition:

Carbon dioxide	65.3 %
Oxygen	0.8 "
Carbon monoxide	2.0 "
Hydrogen	26.4 "
Nitrogen (remainder)	4.7 "
	99.2 "

4. Sample from same specimen as 1 and 2, but taken at low red heat and consisting only of the last portions of the gas which came off.

Composition:

Carbon dioxide	64.7 %
Oxygen	0.7 "
Carbon monoxide	12.8 "
Hydrogen	14.8 "
Nitrogen (remainder)	8.0 "
	101.0 "

5. Average analyses of six samples of Weardale fluorites. These were not clear and contained many inclusions. The large amount of carbon dioxide is probably due in part to adhering small particles of calcite, although great care was taken in separating the visible calcite as thoroughly as possible.

(a) Temperature, 28° (air bath)

Carbon dioxide	50 %
Oxygen	2 "
Carbon monoxide	6 "
Hydrogen	14 "
Nitrogen	28 "

(b) At low red heat (combustion furnace)

Carbon dioxide	50 %
Oxygen	1 "
Carbon monoxide	5 "
Hydrogen	16 "
Nitrogen	28 "

6. (a) Clear deep green fluorite from Westmoreland, N. H. No inclusions visible with the microscope. 250 gms. Gas evolved, 18.9 c.c. Low red heat (combustion furnace). Time of heating, four hours.

Composition:

Carbon dioxide	10.0 %
Oxygen	6.3 "
Carbon monoxide	6.9 "
Hydrogen	51.7 "
Nitrogen (remainder)	24.6 "
	99.5 "

(b) Same. Temperature, 260°. Gas evolved, 13.2 c.c.

Composition:

Carbon dioxide	9.2 %
Oxygen	4.7 "
Carbon monoxide	6.6 "
Hydrogen	51.9 "
Nitrogen (remainder)	29.0 "
	101.4 "

The analytical methods were as follows: — Carbon dioxide by absorption in caustic potash; oxygen by alkaline pyrogallate; carbon monoxide by copper ammonium chloride, and from the explosion results; hydrogen by explosion with pure oxygen; nitrogen as remainder, subject to check of more minute analyses of the rare gases. Unsaturated hydrocarbons were tested for by bromine water. Hydrocarbons of the marsh-gas series would have shown themselves in the combustion results if present.

The test for the gases of the helium group was made on the whole amount of gas from about 10 kilos of fluorite. This volume of gas (about 7 liters) was freed from carbon dioxide by collecting it over caustic soda solution, and then put through the regular purification process as given by Travers[44], finally passing over metallic calcium in a series of combustion furnaces. The residue was then subjected to the action of the spark for some time, and the very small quantity of gas remaining was sent on into the spectrum tube through the second pump, which had previously been used to obtain a vacuum in the Geissler tube. The volume of gas remaining after purification was somewhat less than 2 c.c., and this showed only lines of nitrogen and hydrogen, without any indication of the gases of the helium group. A further purification of this residue resulted in its practical disappearance. It was expected that argon would be present, but no lines of this element were observed. A careful test for helium in fluorite has been made by J. Thomsen with negative results.[45]

The Liquid Distillate.

This consists practically entirely of water, although it is brown in color in every case and has a very strong "empyreumatic" odor. It boils at 100° exactly, and gives nothing but water on fractionation.

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[44] The Experimental Study of Gases (1901).

[45] Ztsch. für phys. Chem., **25**, 114 (1898).

Discussion of Results.

As will be seen immediately from the analyses and the corresponding temperatures, there can be no question of the formation of the carbon monoxide in the way which has been suggested by Hüttner. These temperatures are all so low that even if equilibrium had been reached at the temperature of the experiment, which is not at all probable, nothing like the required concentrations of carbon monoxide could result. In fact, in some of the analyses the carbon monoxide equals in concentration the hydrogen, and the relations of the gases are such as would correspond to equilibrium at a temperature above 1000°. Nor can any concentration of the water vapor, which is present in concentrations of the same order as those of the other substances, explain any such results. It seems equally improbable that the carbon monoxide should have been formed from carbon dioxide at any time in the history of the mineral, since the presence of these volatile coloring matters proves that the temperature of the mineral has never been higher than 300°, at which point the color is completely discharged.

There can be no doubt whatever as to the existence of organic substances in these fluorites. The brown substance which comes over with the water in the distillate can be extracted with ether, and is evidently a product of destructive distillation. I see no reason whatever to question the statement that the color of the fluorites is due to organic coloring substances. It is perfectly true that there is no relation between the intensity of color in a crystal of this mineral and the amount of carbon and hydrogen found in the analyses. I have examined some perfectly clear white samples which gave a great deal of gas per weight of substance. But it seems to be just as reasonable that certain of these organic substances should be *colorless* as that they should be anything else, and in the light of this idea all of the conflicting conclusions drawn from the analyses of the past forty years can be easily understood and reconciled.

The Cavities in Fluorite.

Microscopically examined, all sections of these fluorites, with the exception of some very homogeneous and clear ones, show many small cavities, each containing liquid and gas. In some cases the distribution of these cavities follows the colored stratified structure of the crystal, but in others the distribution appears to be wholly irregular. The number of these cavities appears to bear no relation whatever to the intensity of the color, for some of the whitest and clearest crystals have the largest number of cavities, and some deeply colored green and purple ones are almost wholly free from them. The change in appearance of some of these deeper colored crystals, on being heated, is a very marked one: the colored strata become milky and opaque, and lose their color completely at a temperature not far above 300°. Those parts of the crystal which were previously uncolored remain perfectly clear and show not even a trace of milkiness. Other crystals, especially those o yellow or green coloring, are comparatively without stratification, and so far as the experiments have shown, these are the ones which yield but little gas on being heated.

Conclusion.

Although many facts brought out during the study of the fluorescence and thermo-luminescence of fluorite appear to point toward impurities which might determine the luminescence, the present examination of the gaseous products obtained by heating these same fluorites *in vacuo* reveals nothing out of the ordinary among the constituents of the mineral. The analyses have only confirmed what was already known; that is, that practically all fluorites contain organic materials. The old question as to the cause of the color is of course not definitely settled by these experiments, which show only that color, gas content, and fluorescence disappear at the same temperature, and that the gases produced while the color is disappearing are such as would be obtained from an organic distillation of a destructive sort. Taking into account the accurate work of other observers, the necessary conclusion seems to be that the hydrogen, carbon dioxide, and carbon monoxide which are produced when fluorspar is heated are the result of the decomposition of some organic coloring substance which is present in the mineral. Oxygen and nitrogen are in no case present in these analyses in the proportions in which they exist in air, and this suggests that the greater part of the oxygen has been used up to form carbon dioxide and water, whether during the progress of the analyses or not cannot be directly decided.

No. proof whatever that these organic substances have anything to do with the fluorescence and thermo-luminescence of these crystals has been found. In fact, the regeneration of a part of the thermo-luminescence power of chlorophane, described in the preceding section of this paper, seems to indicate an inorganic source for at least a part of the emitted light.

The results of this investigation may be considered wholly negative, in so far as the explanation of the facts of luminescence in the mineral fluorite is concerned. It has, however, seemed a very necessary step in the series of investigations which is now being carried on with the generous aid of the Rumford Fund. It permits attention to be turned toward other questions with a sense of at least having looked in this direction for the mysterious source of the luminescence phenomena in this substance, fluorite.

Jefferson Physical Laboratory,
Harvard University,
Dec. 27, 1905.

EXPLANATION OF PLATE 1.

FIG. 1. Fluorite crystal No. 5. Magnesium spark as exciting source.

FIG. 2. Same crystal and spark, with inductance in the secondary discharge circuit.

FIG. 3. Same crystal. Iron spark.

FIG. 4. Same as No. 3, with inductance in secondary circuit.

FIG. 5. Fluorite crystal No. 4. Iron spark.

FIG. 6. Fluorite crystal No. 5. Cadmium spark.

FIG. 7. Fluorite crystal No. 4. Cadmium spark.

FIG. 8. Fluorite crystal No. 5. Aluminium spark.

FIG. 9. Same, with inductance in secondary circuit.

FIG. 10. Fluorite crystal No. 4. Aluminium spark.

FIG. 11. Fluorite crystal No. 4. Zinc spark.

FIG. 12. Fluorite crystal No. 5. Arc between carbon terminals as exciting source.

In each photograph the exciting spark forms the two outer spectra and the fluorescence spectrum is in the middle.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

THE ELECTROSTATIC FIELD SURROUNDING TWO SPECIAL COLUMNAR ELEMENTS.

By P. W. Bridgman.

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THE ELECTROSTATIC FIELD SURROUNDING TWO SPECIAL COLUMNAR ELEMENTS.

BY P. W. BRIDGMAN.

Presented by B. O. Peirce November 8, 1905. Received January 12, 1906.

MAXWELL, in his Electricity and Magnetism, gives diagrams showing the equipotential lines and lines of force surrounding certain two dimensional distributions of electrostatic charge. He remarks on the value of these diagrams in enabling us to form a rough idea of what will happen when we have a charged conductor of approximately the same shape as any of the equipotential lines of the diagram. In this paper, diagrams are presented of a few simple cases not given by Maxwell. Before giving a description of the diagrams themselves, a short account will be given of the method by which they were drawn.

It is well known that the potential due to a two dimensional distribution of electricity is continuous, and in empty space satisfies Laplace's equation,

$$\nabla^2 \phi \equiv \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0.$$

Within the body of the charge $\nabla^2\phi = 4\pi\rho$, and at surface charges the normal derivative jumps by $4\eta\sigma$. Conversely, if we are given a continuous ϕ, then a distribution is uniquely determined satisfying the relations above. Either the charge determines the potential (except for an additive constant) or the potential determines the distribution.

In 1861 Newmann discussed the equation $\nabla^2\phi = 0$. It appears that this is the necessary and sufficient condition that ϕ have a conjugate function ψ. This new function ψ also satisfies Laplace's equation, and its level lines are orthogonal to the lines of constant ϕ; that is, the lines of constant ψ are the lines of force of the distribution of which ϕ is the potential. It is evident then that if we are given two continuous conjugate functions, we may regard the level curves of either one of them as the equipotential lines, and those of the other as the lines of force of a distribution determined by the equations above. In general, the distribution will depend upon which function is taken as the potential function.

It is a familiar fact that conjugate functions are additive; that is, if ϕ_1 ψ_1 and ϕ_2 ψ_2 are two sets of conjugates, then $\phi_1 + \phi_2$ is conjugate to $\psi_1 + \psi_2$. But the potential function is also additive. Hence if we know the potential and the force functions of two distributions separately, we may find the potential and force functions of the combination by adding the functions of the distributions separately. This addition may be performed very simply by a graphical method described by Maxwell, provided that we know the shape of the curves of constant ϕ_1 and ϕ_2 separately. We draw the set of curves $\phi_1 = C_1$ for a set of values of C_1 differing by some constant, and the corresponding set $\phi_2 = C_2$, for successive values of C_2 differing by the same constant. We have thus divided the plane into a number of small curvilinear parallelograms, at every vertex of which we know the new potential $\phi_1 + \phi_2$. An equipotential line of $\phi_1 + \phi_2$ is obtained by drawing a

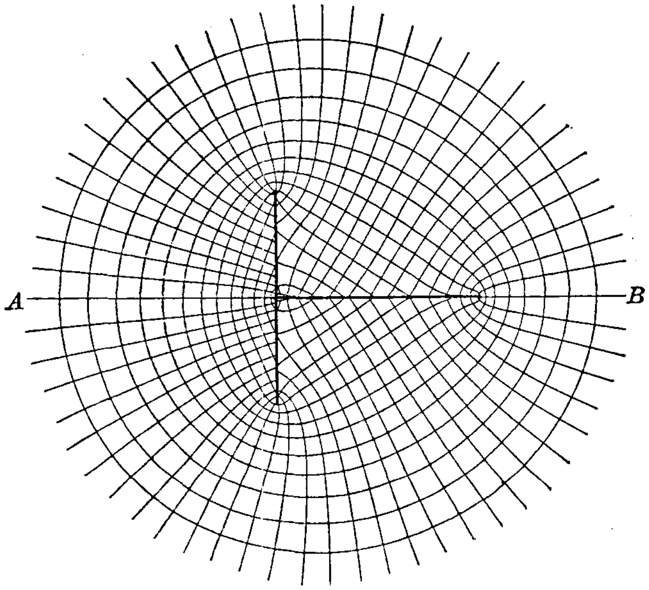

FIGURE 1.

smooth curve through vertices at which $\phi_1 + \phi_2$ has the same value. This line will have the general direction of one of the diagonals in each parallelogram. The line of constant $\phi_1 - \phi_2$ is obtained from the same figure by following along the other diagonal. By an exactly similar method we find the lines of force $\psi_1 + \psi_2$ of the new combination. This is the method by which the curves of this paper were drawn. The diagrams represent the field surrounding two elements, the field of each of which singly is known.

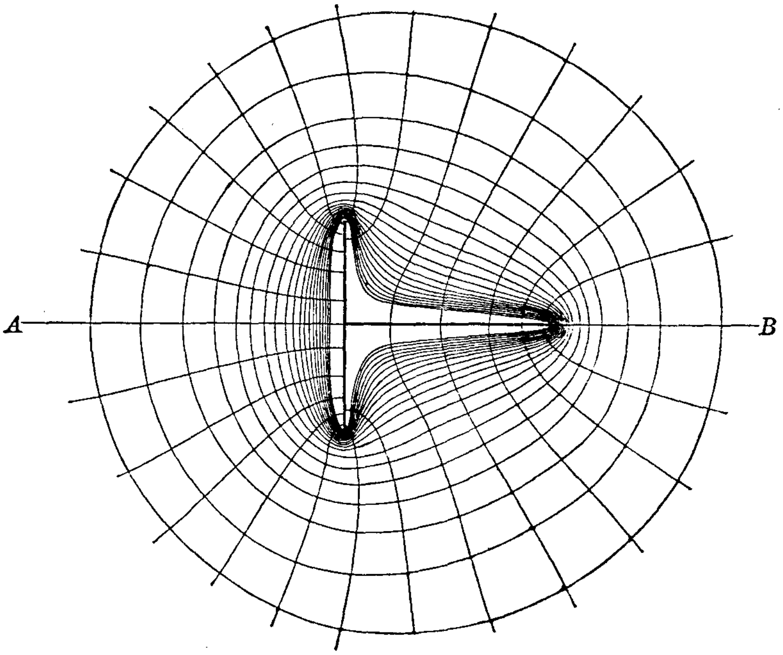

FIGURE 2.

The simple functions that were combined are:

$$\left.\begin{aligned} x &= \cosh\phi\cos\psi \\ y &= \sinh\phi\sin\psi \end{aligned}\right\} \quad \text{I}$$

x and y are conjugate functions of ϕ and ψ, and hence conversely ϕ and ψ are conjugate functions of x and y. Therefore both ϕ and ψ satisfy Laplace's equation, and if they are continuous, may be taken as potential functions. Eliminating ψ and ϕ successively between the equations above gives:

$$\left.\begin{array}{r}\dfrac{x^2}{\cosh^2\phi}+\dfrac{y^2}{\sinh^2\phi}=1\\[2ex]\dfrac{x^2}{\cos^2\psi}-\dfrac{y^2}{\sin^2\psi}=1\end{array}\right\}$$

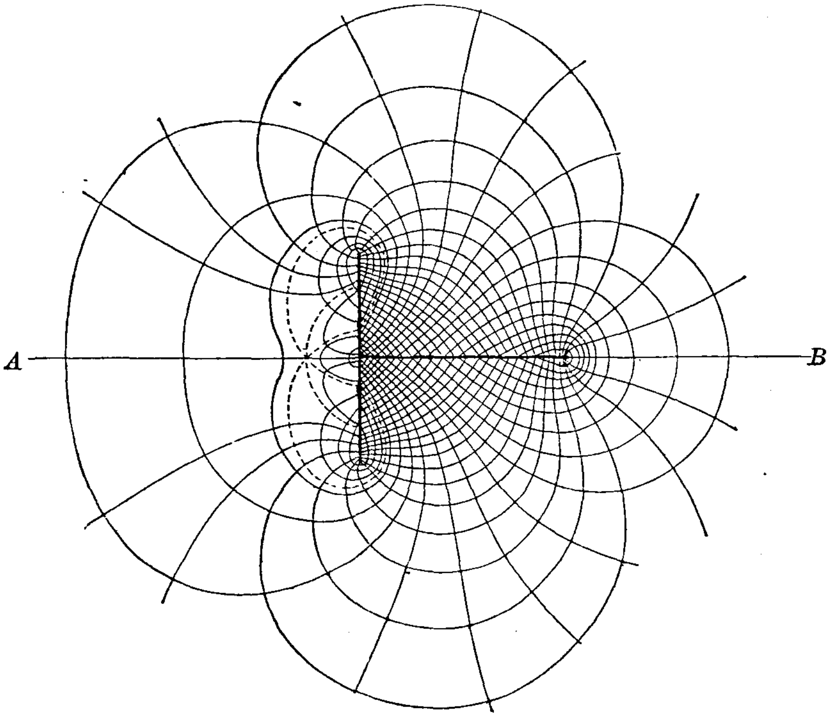

Figure 3.

ϕ defines a family of confocal ellipses, one of which reduces to the straight line joining (1, 0) with (—1, 0). ψ defines a family of hyperbolas with the same foci as the family of ellipses. One of the hyperbolas is the X axis with the segment joining (1, 0) to (—1, 0) omitted. ϕ and ψ define then a system of orthogonal curvilinear co-ordinates. Now if we examine the original equations I, we shall see that x and y do not uniquely define ϕ and ψ. In the first quadrant, for example, ϕ and ψ may be either both positive or both negative. This ambiguity enables us so to assign values that either ϕ or ψ shall be continuous, and hence a potential function.

If ϕ is the potential, we give it the same value at every point on one of the ellipses. ϕ varies from 0 on the straight line joining the foci to infinity at infinity. The first derivatives are continuous everywhere except on the line, where the amount of discontinuity shows that there is a surface charge of

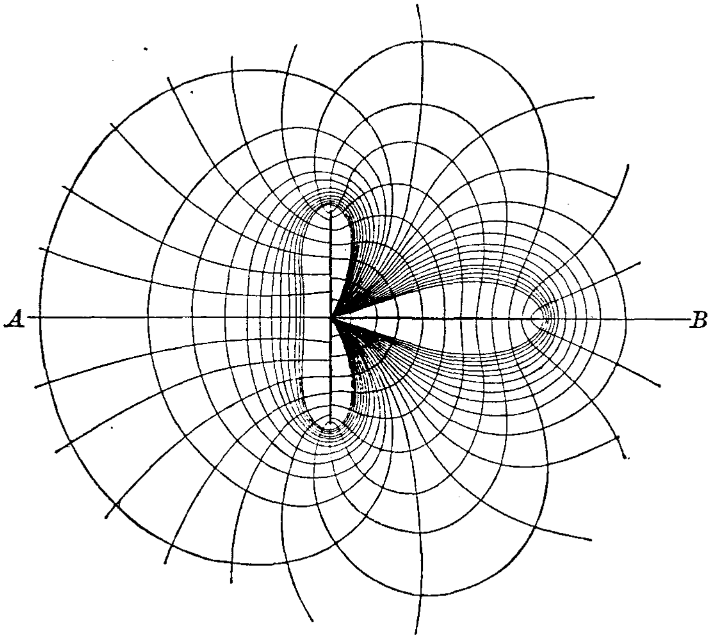

FIGURE 4.

$$\sigma = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{1 - x^2}},$$

counting the charge on both sides of the line. This gives a total charge all along the line of $\frac{1}{2}$. The values of ψ are now determined. Everywhere on the same branch of a hyperbola it has the same numerical value, but suffers an abrupt change of sign on passing through the vertex, being positive above and negative below the X axis. On the two different branches of the same hyperbola ψ has supplementary values.

We have therefore found the potential and lines of force due to a conducting board infinitely long, two units wide, and charged with half a unit of electricity per unit of length. Because of the shape of the equipotential lines, we may call this the elliptic element.

Now to make ψ the potential, it must be made continuous. This is done very simply by keeping the same distribution of values as before in the first and second quadrants, while changing the sign in the third and fourth. ψ now has the same value all along one branch of a hyperbola and the supplementary value on the other branch. It increases in value from zero on the positive piece of the X axis to π on the negative part. The discontinuity shows that there is a surface charge on the interrupted X axis of

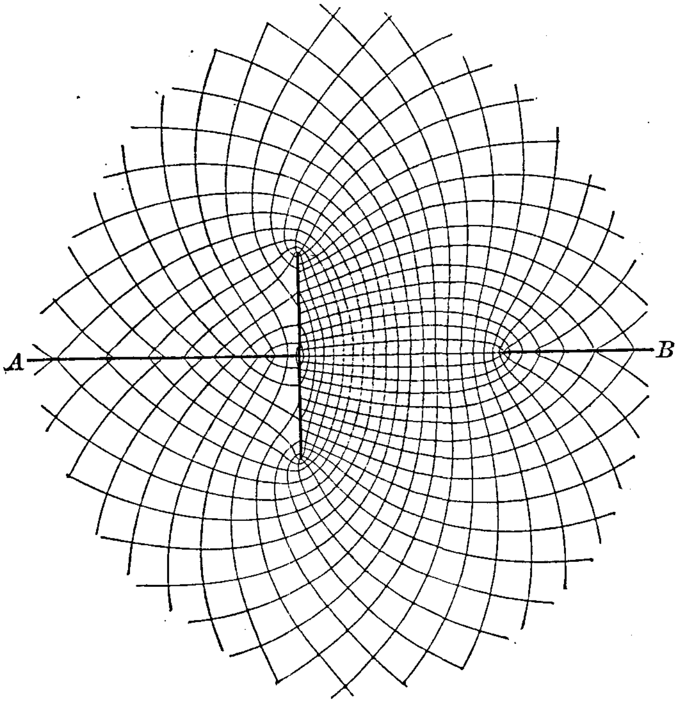

FIGURE 5.

$$\frac{\pm 1}{2\,\pi\,\sqrt{x^2 - 1}},$$

being positive to the right, negative to the left. ϕ has now become discontinuous, it has the negative of its old values below the X axis. We have now found the potential and force functions due to an infinite conducting plane, with a strip two units wide removed, one half charged positively and the other negatively to the same density. This may be called the hyperbolic element, since the equipotential surfaces are hyperbolic cylinders.

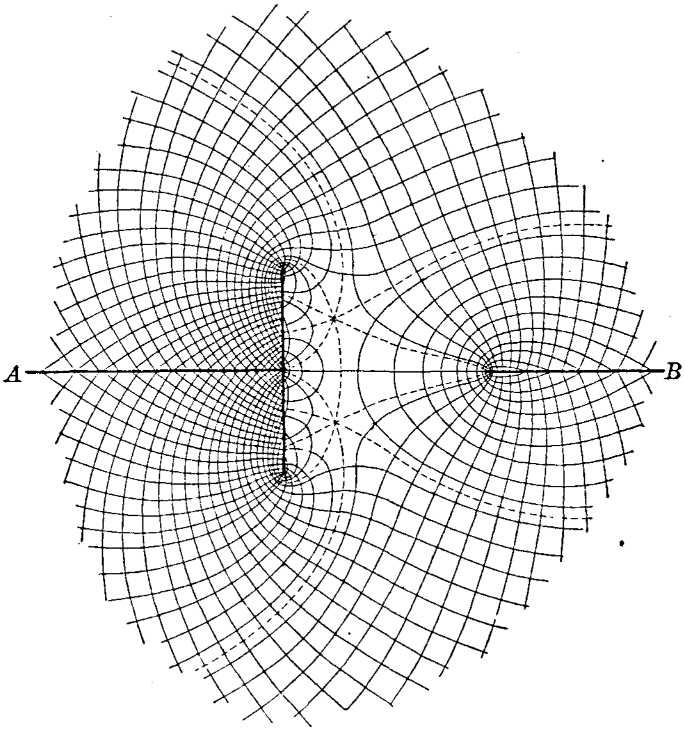

FIGURE 6.

The figures illustrate all possible combinations of two elliptic or hyperbolic elements, the centres being unit distance apart and the direction of one perpendicular to that of the other.

Figure 1 shows two elliptic elements charged with electricity of the same sign. It is to be noticed that the elements are no longer equipotential; that is, they can no longer be regarded as conducting. We must think of the charges as rigidly attached to the boards in the position that they would naturally assume when free from external forces. It should be noted how, near the point of contact, one element dominates the other, a line of force springing across from one to the other, although the charges are of the same sign. This is because the surface density is infinite at the free end of an element.

Figure 2 is not a combination of any of the elements already described. The heavy lines represent rods charged uniformly with electricity of the same sign, and the figure is a cross section through the plane of the rods of a three-dimensional family of equipotential surfaces. The equipotential surfaces of one of the rods singly are confocal ellipsoids of revolution, and therefore a plane section of these surfaces through the rod is a family of confocal ellipses of exactly the same shape as those of the elliptic element. The potential varies from ellipse to ellipse in a different manner, however; for the elliptic element it remains finite, while it becomes infinite on approaching the rod. The figure was drawn to show how

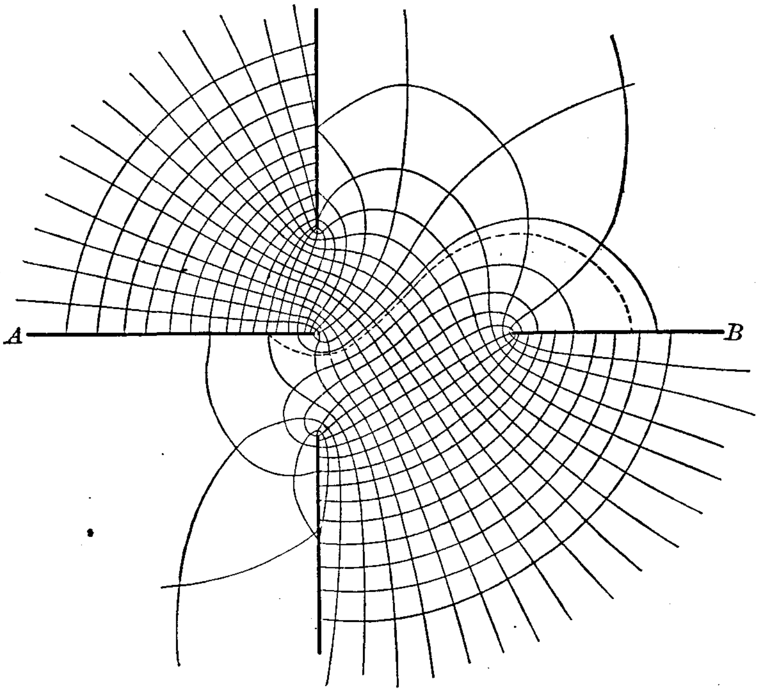

FIGURE 7.

changing the parameters of two families of curves, while keeping their shape constant, may change the shape of the family obtained by adding the parameters. At distances remote from the charges, the shape of the curves of Figures 1 and 2 are the same, but on approaching the rods the equipotential surfaces become infinitely close together, but never cut the rods. It was impossible to draw this, and the space about the rods was left vacant. The lines of force were drawn in free-hand. In three dimensions there is no general additive force function corresponding to that in two dimensions.

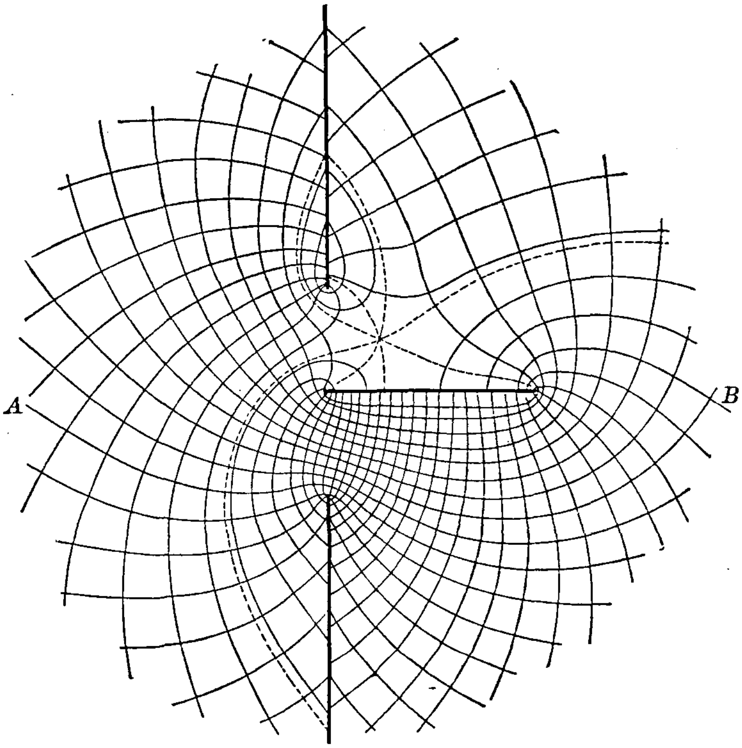

FIGURE 8.

Figure 3 represents the same combination of elements as Figure 1, except that now the signs of the charges are opposite. At the point, distant half a unit from the origin, through which the dotted lines pass, the potential has a stationary point. The lines of force point radially at this point from four perpendicular directions; at the point itself, the magnitude of the force has sunk to zero. On the X axis, to the right of this point, the force is in the direction BA; to the left it has the direction AB.

Figure 4 is the same as Figure 2, except that the charges are of opposite signs. The line of zero potential has the same shape in both Figures 3 and 4. A curious survival of the tendency to the stationary point is seen as we come up toward the origin from A. The equipotential lines are drawn in toward the origin, changing from concave to convex. As in Figure 2, the lines of force were drawn free-hand.

Figure 5 shows an elliptic and hyperbolic element, the parts in contact being charged with the same sign. As in Figure 1, the overpowering of one element by the other at the origin is to be remarked, the lines of force springing from one charge to another of the same sign.

Figure 6 is the same as Figure 5, except now the parts in contact are charged oppositely. Here there are two stationary points at $(\frac{1}{2}, \frac{1}{2}) \cdot (\frac{1}{2}, -\frac{1}{2})$, taking AB as the X axis and the other element as the Y axis.

Figures 5 and 6 represent the two symmetrical combinations of elliptic and hyperbolic elements. Figures 7 and 8 show the two remaining unsymmetric combinations. It will appear that these figures contain nothing new; the separate quadrants have already appeared in one of the other figures, but equipotential lines have now become lines of force, and *vice versa*. Along the elements separating one quadrant from another, discontinuities arise, which are explained by the previous consideration of the discontinuities in ϕ and ψ when regarded as force functions. In what follows we shall abbreviate first, second, third, and fourth quadrants by I, II, III, and IV.

Figure 7 shows two hyperbolic elements. Except for the interchange of lines of force and equipotential lines, I is the I of Figure 3, II the II of Figure 1, III the III of Figure 3, and IV the IV of Figure 1 again. The half stationary point is to be noticed, the transition from Figure 1 to Figure 3 coming across the line dividing this point.

Figure 8 shows a hyperbolic and elliptic element. I is the I of Figure 6, II of Figure 5, III of Figure 6, IV of Figure 5. The potential here has only one stationary point, the halving of the number in Figure 6 corresponding to the halving of the single point of Figure 3.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

ELECTRIC DISCHARGES.

I. *SLOW MOVING ELECTRICAL LUMINOUS EFFECTS.*

II. *MAGNETIC FIELD AND ELECTRIC DISCHARGES.*

By John Trowbridge.

With Four Plates.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

ELECTRIC DISCHARGES.

BY JOHN TROWBRIDGE.

Presented October 11, 1905. Received January 11, 1906.

I. SLOW MOVING ELECTRICAL LUMINOUS EFFECTS.

OUR knowledge of electrical manifestations is based upon the comparatively feeble effects which we can produce in laboratories; and even with the best appliances which modern applications of electricity afford we are far within the limit of lightning discharges.

It is unsafe, therefore, to assume that the testimony of many observers in regard to ball lightning and slow moving discharges, observed by them in electric storms, has its origin in disordered nerves. The following experiments lead me to believe that a slow ionization of rarefied air might be caused by currents of electricity of great strength following lanes of such rarefied air in thunder-storms.

The most notable experiments upon slow moving electrical luminous effects are those of Rhigi.[1] He charged condensers of large capacity by means of an electrical machine; discharged them through a large liquid resistance, and observed globular or elongated spindle-shaped discharges which moved slowly along the tubes of rarefied gases. He gives reasons for believing that such discharges are not striae, but that they belong to a particular kind of discharge.

It is to be noted that he interposed a spark gap in the discharge circuit.

I have repeated Rhigi's results, using instead of discharges from large condensers the current from a storage battery of 10,000 cells without the interposition of a spark gap, and I obtain slow moving luminous effects such as are represented in the illustrations of his memoir. These luminous masses are shot out from the anode and move toward the cathode with increasing strength of current, and retreat to the anode with decreasing strength of current. A striking fact, however, is this: The movement of the luminous masses is toward the negative pole when the pressure in the tube is suddenly increased by a small amount. This movement, however, is dependent upon the strength of the current flowing through the tube, and cannot be perceived with feeble currents. I have employed currents from 10 milliamperes to 40 milliamperes. The number of luminous masses increases with the strength of current. The method of experimenting was as follows.

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[1] Nuove Esperienze sulle Scintille Electriche, Constituite Da Masse Luminose che si Muovono Lentamente. R. Accademia, delle Scienze dell Instituo di Bologna, 19 Maggio, 1895.

The current was led to a reservoir of water. At the base of the reservoir the water flowed out through a long glass tube of 1 cm. internal diameter. A wire introduced in this tube could be pushed in or out, thus modifying the resistance of the circuit. This high resistance was found sufficiently exact for the character of the experimentation. A milliamperemeter (Siemens and Halske) was introduced in the circuit of this water resistance and the exhausted tube while the water was kept running. The indications of the milliamperemeter were perfectly steady. Other forms of high resistance which I have tried heat under the powerful currents necessary for the production of the movements of the luminous masses. Running water gave, as far as the indications of the milliamperemeter was concerned, a perfectly steady resistance; when a telephone, however, was introduced in the circuit, rumbling crepitations were heard, due probably to electrolysis in the liquid. The exhausted tube also produces a note which is superposed on the rumbling of the liquid resistance. This latter note is especially noticeable in comparatively large tubes. I experimented first with a tube 60 cm. long, 8 cm. wide, provided with circular flat electrodes 3 cm. in diameter. The striae were in constant movement, producing a loud hum in the telephone introduced in the circuit. It was noticeable that the striae were straight; not curved toward the positive electrode, except near the sides of the tube, where a curvature like that of a liquid meniscus was formed.

The tubes were maintained in connection with a mercury pump, by means of which the pressure in the tubes could be changed to any desirable extent.

The slow movement of the luminous masses was best shown in tubes of 4 cm. internal diameter. Figure 1 is a photograph of a typical form of such slow moving luminous effects. It will be noticed that the ends of the mass are curved as if the centre of the mass constituted a positive pole; the intervals between the masses acting like cathode spaces, the number of luminous masses increases with the strength of the current passing through the tube. They seem to be shot slowly out of the positive pole, and in their movement constitute a very beautiful experiment. The accompanying table represents this increase of the masses with increase of current.

Current in Milliamperes.	No. of Masses.
5	1
9	2
11	3
12	4
15	5
20	Continuous discharge from one terminal to the other.
40	

It would seem that the chain of positive and negative intervals with very powerful currents becomes a chain with infinitely small intervals between the positive and negative masses. Perhaps this is also the mechanism of the electric discharge through solid conductors. The pressure in the tube varied from four centimeters to half a centimeter. With lower pressures the ordinary form of striae was obtained. With narrow tubes the curvature of the striae increased, and the striae were apparently firmly held by the walls of the tube, recalling the phenomenon of the liquid meniscus. If we suppose, according to Townshend's hypothesis, that the striae indicate the collision of positive and negative ions, this collision would probably be modified by changes of velocity of collision near the walls of the tube in much the same way that the molecular actions at the surface of a fluid are modified by the attracting forces of the containing tube.

I believe that the slow movement observed by Rhigi is an ionization change produced by a change of pressure resulting from the heat of discharge of a condenser, combined with a change in electric potential. It has nothing to do with the velocity of the electric impulses, and cannot be called a slow movement of electricity. It seems probable that globe or ball lightning, and other apparently slow movements of luminous effects observed in thunder-storms, are of the nature of the ionization effects described in this paper.

The ionization theory of Townshend to my mind is the best explanation of these slow moving luminous effects. One can conceive of a non-luminous condition of ionization pervading the space between the poles of a battery in a wide tube at a comparatively high pressure of the containing gas, perhaps 3 centimeters. The positive carriers are not restrained or held back by the swifter moving negative carriers, which cannot manifest their energy in a limited free path. On a sudden increase of pressure luminous clouds emerge from the anode. This phenomenon seems to indicate a greater proportional falling off in the energy of the negative carriers and a manifestation of superior energy of the positive carriers. The cloud of collision between the two moves slowly to the negative pole, the conductivity of the gas changing under the difference of potential to which the tube is subjected. Two luminous clouds can be formed by a suitable change of pressure in the tube; a change of pressure easily produced by condenser discharges in a side tube. These clouds, after the cessation of the condenser discharges, travel slowly to the negative pole in the tube, and coalesce before they disappear in this pole. We seem to have here a picture of the relative energy of the positive and negative carriers with the war of collision between them evidenced by the slow moving luminous clouds. The forms of these slow moving luminous effects are perfectly represented by the illustrations in Professor Rhigi's memoir. It is noteworthy that such slow moving electrical clouds can be produced by a steady discharge from a storage battery of ten thousand cells as well as with a powerful discharge from a large condenser through a water resistance, as in the case of Rhigi's experiments.

The form of the electrodes is of considerable importance. If the positive terminal is an aluminum plate of comparatively large diameter, four centimeters, and the negative terminal a point, the luminous masses retreat to the positive pole instead of moving to the negative pole on an increase of pressure. The surface density on the positive plate does not seem sufficient to force a volume ionization through the increase in the volume of the gas.

On the contrary, when the positive terminal is a point and the negative terminal the plane, an increase of pressure, with strong currents, causes a slow movement of the luminous masses to the negative pole.

A constriction in the glass tube results, in the formation in the constriction, of a fixed luminous mass which acts as a cathode. The position of a side entrance to the tube leading to the pump does not seem to have any influence on the position of the luminous masses. Perhaps the most interesting case of slowly moving electric luminous discharges is that of the very slow retreat of the positive column when the anode is made the pole of a strong electro-magnet. This retreat, while the magnetic field remains constant, occupies several minutes and seems to indicate a decrease in the conductivity of the gas due perhaps to an increase of temperature caused primarily by an increased admittance due to the magnetic field.

II. Magnetic Field and Electric Discharges.

I wish to acknowledge my indebtedness to previous workers in the field which I enter, especially to Lehmann (Drude's Ann., No. 7, p. 1, 1902); and since Professor J. J. Thomson has given a valuable résumé of papers on this subject in his treatise entitled "Conduction of Electricity through Gases, 1903," I refrain from repeating a bibliography here.

The production of the phenomena described in this paper depends primarily on strong, steady currents, high voltage, and powerful magnetic fields. The effects were excited in cylindrical glass tubes of large diameter; these tubes were always connected to the pump. The current was regulated by a running water resistance and the amperemeter and electrometer were suitably connected.

The cathode consisted of a rod of soft iron 16 cm. in length, 2 cm. in diameter. The rod was sealed into the tube by means of a platinum wire which, in turn, was soldered to the rod of iron. This rod formed the core of an electro-magnet. A similar rod of iron with a coil of wire forming another equal electro-magnet could be placed in any position outside the tube. Thus the magnetic lines of force could be formed along the line of flow of the current in the tube or at right angles to this flow. It was realized early in the course of the investigation that the size of the tube, the character of the electrodes, shape, and material, together with the strength of the current, electromotive force, and strength of magnetic field, could modify the phenomena in a perplexing way. Each trial seemed to bring out a new phenomenon. I will select, however, those phenomena which could be definitely repeated under known conditions, and since mere descriptions without photographs of such phenomena are not particularly clear or convincing, I have made a number of photographs of certain typical phenomena.

The mathematical theory of the effect of a magnetic field on electric discharges in rarefied gases assumes, first, that if the viscosity of the gas causes the velocity of the ion to be proportional to the force arising from the electric intensity, the motion of the ion depends on the product of the electric intensity and the strength of the field. According to the value of this product we can have motion along the line of the electric intensity, motion at right angles to the lines of magnetic force, or motion along these latter lines. In a vacuum, where the only force acting is that of the magnetic field, there is a strong tendency in the ion to follow the lines of magnetic force. This tendency was early shown by the experiments of Plucker; it is commented upon at length in Professor J. J. Thomson's work upon electric discharges in gases, and the important paper by Lehmann gives many experimental proofs of it.

One feels strongly in the presence of the phenomena that the mathematical theory is very inadequate; and the conviction has arisen in my mind that no theory based merely upon the impact and collision of particles moving with different speeds can explain the characteristic phenomena which I shall describe. A mathematical theory which shall embrace the entire range of such phenomena must draw upon the known laws of electrodynamics, those of the kinetic theory of gases, and above all those of hydrodynamics.

I shall first describe the experience with a tube of rarefied air 36 cm. long, 4 cm. in diameter. The electro-magnets were excited by a current of twenty-five amperes. The current through the discharge tube varied from 10 milliamperes to 25 milliamperes. The difference of potential between the terminals of the tube varied from 3000 volts to 6000 volts.

1. The lines of magnetic force being along the line of discharge, the cathode forming the magnetic pole, the striations are drawn into or toward the cathode; the Faraday space can be completely annihilated, new striations being formed, and the increase being proportional to the current and the field.

2. In this experiment the tube was 15 cm. long, 4 cm. wide. The cathode was the core of the electro-magnet. This core was pointed at the end. The anode was a pointed aluminum rod. The distance between the terminals was 12 cm. At a high pressure, about 1 cm., when the only light in the tube consisted of a slight glow at the terminals, the excitation of the magnetic field drew a luminous discharge from the anode up to a Faraday space, forming at the end of the discharge a very sharp line of demarcation, with one or two striae. This luminous discharge came up slowly, and retreated very slowly, after a minute or two, into the anode. The pressure was far below the ordinary stratification stage without the influence of magnetism.

3. At a lower pressure, 2 mm., a series of striae emerged from the anode with a bulge at their centre (Figure 2).

At still lower pressures ring-formed striae issued from the anode and travelled slowly toward the cathode. These rings seemed to be formed primarily from the gases which came out of the electrode. In all cases the bulge at the centre of the striae disappeared when the water resistance was greatly increased or a spark gap was inserted in the exterior circuit. The diminution in the density of the positive carriers in the central line of discharge seemed to bring out the diverging effect of the magnetic field on the cathode ions shown by the luminous effect of the collisions in a ring form. This ring form is more strongly developed near the cathode.

Figure 3 is a photograph of two of these rings taken in perspective. The walls of the tube distort the appearance, but the ring character of the striae, I believe, is shown.

The table on page 636 shows the effect of the magnetic field with varying pressure and varying currents. The lines of magnetic force are, in every case, directed along the line of discharge.

In all cases there was an increase in the cathode light when the magnetic field was excited. If the increase or diminution of the light in the tube due to the magnetic field was plotted as an ordinate and the pressure as the abscissa of a curve, it was seen that at 1 cm. pressure the light rose to a maximum with a current of 6 milliamperes, then fell very slowly with increasing current. At 3 mm. pressure light increased with comparatively weak currents. At 1 mm. pressure light decreased, the ordinate representing the strength of light becoming negative, the curve crossing the axis of X. There seemed to be several critical points.

4. The tube in this experiment was 36 cm. long, 4 cm. wide. The rod of iron, forming one terminal, was provided at its end with an aluminum disc diameter. The coil described in Experiment 1 formed with the rod of iron the electro-magnet. When the aluminum disc formed the cathode, at pressures below the stratification point, between 2 cm. and .5 mm., the violet light on the cathode enlarged from a spot, so as to cover apparently uniformly the entire rim of the disc, while luminous streams emerged from the edges of the disc. The effect of the magnetic field was to drive the discharge away from the centre of the magnetic field.

5. When the disc was made the anode, the effect of the field was just the opposite. The discharge was drawn to the centre of the field.

6. A revolving mirror was so arranged that an image of the aluminum disc was thrown into a camera, the tube was covered with black paper so that the light only of the cathode was reflected into the camera. The photograph (Figure 4) shows a unipolar rotation. This increased in

velocity of rotation as the pressure of air in the tube was diminished. The effect of the magnetic field is to separate the discharge from the cathode into two discharges, one a violet glow, the other a plume-like orange light, which seems to extend across both cathode dark space and Faraday space. Its presence is shown by the flaring light on the spires of the helix of the photograph. The effect of the field at the anode also is to separate the discharge into a plume-like rose discharge and a smaller plume-like violet discharge. These discharges also rotate about the pole.

Current in Milliamperes.	No. of Striae without Magnetic Field.	No. of Striae with Magnetic Field.
1 Cm. Pressure.		
6	0	4 striae. Slow advance to cathode.
18	0	2 striae. Very slow retreat to anode.
20	0	No striae. Slow retreat to anode.
3 Mm. Pressure.		
9	4	5
17	3	2
20	0	0
1 Mm. Pressure.		
20	0	No striae. Instantaneous retreat of glow at the anode.
.5 Mm. Pressure.		
6	0	3
10	0	3
20	2	5
.25 Mm. Pressure.		
20	0	Continuous discharge from anode to cathode.

7. The electro-magnet was placed with its core at right angles to the line of discharge at the anode. The discharge was thrown up or down,

the direction being dependent naturally upon the direction of the current and the polarity of the field. The space around the anode was cleared of the striae, which existed before the field was excited; but the striations just beyond the most intense portion of the field were broken up into smaller striae.

8. The electro-magnet was placed at right angles to the cathode. The discharge was thrown up or down according to known electro-magnetic laws, but in addition striae were formed at a stage of pressure far below the ordinary stratification stage, at pressures of 2 cm. to .5 mm.

9. The electro-magnet was placed at right angles to the discharge at a distance from either electrode, in the centre of the tube (Figure 5). What may be called the magnetic striae appeared on the diverted path of discharge. These striae only appear with strong currents at comparatively high pressures, 2 cm. to .5 mm.

10. The ordinary form of X-ray bulb does not lend itself easily to the application of the magnetic field, either at the anode or the cathode; and even in the special form of bulb which I have used in this investigation it was not possible to develop magnetic lines of force over the entire surface of the anode or the cathode. The cores of the electro-magnets were hollow in order to allow of the approach of the coil of the electro-magnet to the terminals of the bulb, the glass seals of these terminals thus projecting into the hollow iron cores; only a circular area, therefore, on the cathode or anode forms the effective magnetic field. It seems probable that the best results would be obtained by inclosing the iron cores entirely inside the bulb, platinizing the end of the iron core forming the anode, placing the aluminum mirror forming the cathode directly in the end of an iron core and nickel plating both iron cores to prevent the constant escape of gases from such large surfaces of iron. I had a bulb constructed of this general description, but found it impossible to exhaust it to the X-ray stage on account of the escape of gases from the iron; the iron was not nickel-plated, however. The bulb was exhausted while it was strongly heated in an oven.

Figure 6 is a photograph of the discharge in the X-ray bulb somewhat before the X-ray stage; a dark space surrounds the cathode.

Figure 7 is a photograph of the same tube when the magnetic field is applied to the anode. This cone of rays is solid until just before the X-ray stage; then it becomes hollow; and at the X-ray stage it appears only at the instant the magnetic field is excited as a violet light, and then becomes indistinguishable in the fluorescence of the bulb. A violet brush, however, persists on the surface of the anode at the X-ray stage.

When the vacuum was increased to so high a degree that the bulb could not be excited by a coil giving a six-inch spark, the application of the magnetic field immediately resulted in the production of the rays.

At the X-ray stage, when the cathode formed the end of a powerful electro-magnet and the lines of magnetic force were directed along the line of discharge, a violet light in the form of a ring appeared on the back of the cathode toward the magnet, while the X-rays were greatly increased in brilliancy together with the general fluorescence of the tube.

The increase in the X-rays, however, only appeared in a tube highly exhausted and with the employment of a coil giving a twenty-four inch spark. The coil was also provided with large condensers. The increase in brilliancy of the X-rays viewed on a fluorescent screen was fully one hundred per cent, and this increase suggests the possibility of a practical use of the magnetic field in connection with the production of X-rays. One can modify the state of the tube by modifying the magnetic field. It may be also that by studying the effect of the field we can identify the particles which bombard the anticathode.

It was difficult to measure the strength of the magnetic field inside the exhausted bulbs. The size of the electro-magnet was as follows: Diameter of core, 2.5 cm.; 15 layers of no. 12 copper wire, 37 turns to a layer; length of electro-magnet, 9.5 cm.; distance of end of hollow iron core from anode or cathode, 2 cm. Exciting current in electro-magnet, 15 to 25 amperes.

When the cathode mirror of an ordinary X-ray bulb is made the anode instead of the cathode, the current passing in the usually unfavorable way for the production of the rays, and at the same time is also the pole of a powerful electro-magnet, the bulb gives out X-rays in great abundance. This is not the case when the magnet is not excited. The magnetic field, therefore, causes the anode to produce X-rays, probably by an increased energy of bombardment of the platinum focal plane by the positive ions. I believe that if one studies the effect of the magnetic field on the anode, in the case of the stratifications showing an effect of increased pressure at their centres (Figure 2), the production of ring stratifications, one must be convinced that the effect of the magnetic field is to increase the free path of the positive ion and to permit it to produce the X-rays by a bombardment similar to that of the negative ion. There is another supposition, namely, that the magnetic field produces rays similar to the cathode rays; certainly one can separate the anode discharge into two by means of the magnetic field.

Figure 8 is a photograph of the X-ray bulb when the cathode mirror has been made the magnetic pole. Without the excitement of the magnetic field the bulb could not be made to give the X-rays even with a coil producing a six-inch spark, the vacuum having increased greatly during the previous use of the bulb. When the field was excited, however, a brilliant fluorescence was produced without any appearance of X-rays. The negative ions apparently did not reach the anticathode, but instead formed fluorescent rings around the cathode. The cathode is thus made part of a hollow hemisphere of, in this case, orange light. When, however, the bulb was excited by a coil giving a twenty-inch spark with Leyden jars, the bulb gave very brilliant X-rays at the moment of exciting the magnetic field. In the case of the use of the six inch spark coil there were no Leyden jars in the circuit.

Besides the scientific side of the manifestations of the effect of the magnetic field on discharges in high vacua, there seems to be a practical use of the electro-magnetic field in connection with the regulation of the discharge in X-ray bulbs. At present, when the vacuum has risen so high that the bulb cannot be excited, one is forced to apply heat to various regulators in order to drive out gases to increase the conduction. All regulators hitherto used are uncertain and dangerous to the life of the bulb in their application. I believe that a magnetic regulator applied to the anode would be of great service in hospital plants where a suitable electrical equipment can be had. The magnetic regulator is entirely safe and is constant in its action. It also enables one to pass readily from the production of hard rays to that of soft rays by the modification of the strength of the magnetic field, a modification difficult to accomplish without the application of the magnetic field.

In Experiment 1 can we not say that the revolving factor in the case of the negative ion is greater than its progression factor? The energy, therefore, along the line of discharge is less to oppose the energy of the positive ion, and the negative ion is therefore diverted from the line of discharge, thus allowing the positive column to advance to the cathode.

In Experiment 5 the revolving factor is less than the progression factor, the discharge is ejected from the centre of the magnetic field. This is independent of the name of the magnetic pole.

In Experiment 6 the revolution of the cathode light indicates this superior revolution factor in the case of the negative particle. This is independent of the name of the magnetic pole.

In Experiment 10 we see the positive particles driving back the negative particles so as to diminish the dark cathode space.

In Experiment 3 the formation of the rings indicates a superior pressure of the positive particles shot out from the positive pointed terminal against a diminished pressure of the discharge from the plate terminal of the cathode. This manner of formation is analogous to that of smoke rings.

In Experiment 8 the revolving factor is greater than the progression factor for both the positive and the negative ions, and the effect of the field is to retard the discharge. The field seems to oppose an obstacle like that of a rock in a running stream producing ripples.

The theory of positive and negative ions moving with different velocities and colliding according to Townshend's Hypothesis, — an hypothesis which is illuminating in regard to electric discharges in general, — does not throw much light upon the phenomenon of striae. It does not seem possible to form striae by the indiscriminate projection of colliding particles. Such striae, or stratifications, are analogous to standing waves in a liquid, and if we suppose electric pulsations issuing both from the cathode and the anode we can perhaps conceive of the method of formation of the stratifications. Even with the employment of the steady current from a storage battery a high note is heard in a telephone intercalated in the circuit which is not due to crepitations in the liquid resistance employed.

The unipolar rotation described in Experiment 6 leads my mind to connect the phenomenon of coronal streamers seen at the poles of the sun in an eclipse with the effect of a magnetic field on possible electrical discharges between the equatorial regions of the sun and the poles of the sun. If we suppose that a difference of electrical potential can arise between the swiftly moving strata of gases or from the eruptions which take place mainly along the equatorial belt and the polar regions, the magnetic poles of the sun would undoubtedly tend to cause the resulting electric discharges to revolve about the pole. On account of the vast circumferential area about the poles a number of discharges could occur at different points around the pole and each discharge would revolve under the effect of the pole. In observing the effect of a strong magnetic pole on plate terminals in wide tubes of rarefied air at comparatively high pressure of air under conditions of high electromotive force and great current density, one can observe phenomena of rotation which cannot be photographed, yet which present to the eye a strong analogy to the appearance of coronal streamers.

I have arranged a number of collections of bristles on a disc, which was then set in rapid rotation. Figure 9 is a photograph of the appearance of such revolving streamers which fairly represent what may be seen at the terminal of a discharge tube in a magnetic field, and call to mind the coronal streamers. The magnetic phenomenon described in 8 may also have a bearing upon the coronal streamers at the poles of the sun. When the lines of magnetic force are at right angles or transverse to the direction of the electric discharge, at comparatively high pressures, one to two centimeters with currents from 5 to 20 centimeters, 3,000 to 8,000 volts in wide tubes, streamers radiate from the position of the magnetic pole. It will be noted that these striae make their appearance at a much higher pressure than that of the usual striae in rarefied gases.

Thus electric discharges around or toward the poles of the sun transverse to the lines of magnetic poles of the sun could be separated into streamers. What conclusions can we draw from such varied phenomena? A short consideration of the elementary mathematical treatment given by J. J. Thomson in his treatise on "Discharge of Electricity through Gases" will help us, I think, to analyze the phenomena. Professor Thomson discusses the case of one ion moving through a gas, the viscosity of the gas causing the velocity of the ion to be proportional to the force acting upon it. Since there is a mechanical force acting upon the ion perpendicular both to the electric force and the direction of the magnetic force, we have

$$\left.\begin{matrix}X\\Y\\Z\end{matrix}\right\}\text{components of electric force,}\quad \left.\begin{matrix}\alpha\\\beta\\\gamma\end{matrix}\right\}\text{components of magnetic force,}\quad \left.\begin{matrix}u\\v\\w\end{matrix}\right\}\text{components of magnetic force.}$$

Components of mechanical force X_ϵ, Y_ϵ, Z_ϵ, ϵ being the charge on the ion, R the velocity of the ion under unit electric intensity. We then have

$$\epsilon\,(\beta\,w - \gamma\,v),$$
$$\epsilon\,(\gamma\,u - \alpha\,w),$$
$$\epsilon\,(\alpha\,v - \beta\,u).$$

$$u = R\,(X + \beta\,w - \gamma\,v),$$
$$v = R\,(Y + \gamma\,u - \alpha\,w),$$
$$w = R\,(Z + \alpha\,v - \beta\,u).$$

$$u = \frac{R\,X + R^2\,(\gamma\,Y - \beta\,Z) + R^3\,\alpha\,(\alpha\,X + \beta\,Y + \gamma\,Z)}{1 + R^2\,(\alpha^2 + \beta^2 + \gamma^2)},$$

$$v = \frac{R\,Y + R^2\,(\alpha\,Z - \gamma\,X) + R^3\,\beta\,(\alpha\,X + \beta\,Y + \gamma\,Z)}{1 + R^2\,(\alpha^2 + \beta^2 + \gamma^2)},$$

$$w = \frac{R\,Z + R^2\,(\beta\,X + \alpha\,Y) + R^3\,\gamma\,(\alpha\,X + \beta\,Y + \gamma\,Z)}{1 + R^2\,(\alpha^2 + \beta^2 + \gamma^2)}.$$

Since it is supposed that there is not only a negative particle ϵ shot out from the negative terminal, but also a positive particle ϵ' shot out from the positive terminal, we have similar expressions for u_1, v_1, w_1, for the velocity of the positive particle.

Professor Thomson analyzes thus the equations for u, v, w. The first term in the numerator represents a velocity parallel and proportional to the electric force; the second term, a velocity at right angles both to the electric and magnetic forces. The third term represents a velocity parallel to the magnetic force. The relative importance of these terms depends upon the value of $R\ H$. If this quantity is small the first term is the most important, and the ion moves parallel to the electric force; if, on the other hand, $R\ H$ is large, the last term is the most important and the ion moves parallel to the magnetic force. As R varies inversely as the pressure of the gas, it might, at very low pressures, be possible to make $R\,H$ large enough and thus make the ions travel along the lines of magnetic force. The same reasoning can doubtless be applied to the movement of the positive particle, and we then have a consideration of the relative rotation effect under the influence of the magnetic field on the two particles, the negative and the positive, and the relative progression effect along the lines of magnetic force. In the discussion also we must bear in mind the fruitful hypothesis of Townshend in regard to the collision of the ions.

It may be that the difference in magnetic properties of oxygen and hydrogen contained in the rarefied air also plays an important part in the phenomena which are presented in this paper. I hope to present later the results of a study of the behavior of oxygen and other gases submitted to electric discharges in a magnetic field. The conclusions I draw from these experiments are as follows: —

1. Strong, steady currents and high differences of potential introduce us to phenomena which are concealed by inadequate currents, low potential differences, and weak magnetic fields.

2. At definite pressures, where the free path of the molecule is short, the cathode violet light is repelled to the edge of the circular disc forming the cathode by a magnetic field, the lines of which are directed along the line of electric discharge, and revolves with a speed dependent upon the pressure of rarefied air. We have, in this case, a distinct unipolar rotation. Besides the violet light, there is also a rotation of an orange-colored plume-like discharge.

3. When the anode is made the core of an electro-magnet at high pressures, the discharge is separated into two; one a violet discharge,

the other a rosy red discharge. These discharges are brought to the centre of the disc constituting the end of the anode, instead of being repelled to the edge of the disc, as in the case of the cathode (see 2 above). The effect in both 2 and 3 is independent of the name of the pole.

4. When the cathode light suddenly diminishes, at a definite pressure caused by the sudden introduction of air, ring-shaped striae issue from the anode and travel toward the cathode.

If a spark gap is introduced in the exterior current circuit, the cathode light diminishes and ring-shaped striae issue from the anode. The energy of projection of the negative particle apparently diminishing quicker than the larger positive particle, the stream of the latter issuing at a greater density from a pointed electrode, can form a luminous ring analogous to vortex rings.

5. The suppositions that under certain conditions of free path the negative ion is diverted more readily from the line of discharge under the influence of the magnetic field than the positive ion; or, in other words, has a greater energy of movement around the lines of magnetic force than of energy along the line of discharge, and that the reverse holds for the positive ion seemed to be borne out by experiment.

6. The formations of what I have termed magnetic striae, when the magnetic field is at right angles to the line of discharge, seems to indicate that the revolving factor of both negative and positive ions being greater than their energy along the path of discharge, the discharge is hampered, as if in a flow of a liquid an obstacle should be interposed, thus forming ripples.

7. If we suppose pulsations issuing at different rates from the two terminals of the discharge tube we can by hydrodynamical analogies get an insight into the formation of stratifications.

8. When the cathode forms the end of a powerful electro-magnet under suitable conditions the output of X-rays is greatly increased.

9. A new form of discharge is seen in case 8 on the back of the cathode. If these are canal rays, they are formed without a canal.

10. The fluorescent light on the walls of the X-ray bulb revolve through a comparatively large angle about the pole.

11. When the anode in an X-ray bulb is also the end of a powerful electro-magnet the application of the magnetic field results in the production of X-rays from a bulb which cannot be excited without the application of heat.

12. When the anode forms the magnetic pole a violet brush-like light

appears on the anode, on the side away from the electro-magnet, while the fluorescent light is forced somewhat beyond the cathode.

13. The use of Leyden jars in the case of low potential coils greatly modifies the effect of the application of the magnetic field, while with coils giving sparks of over 20 cm. with comparatively large Leyden jars in circuit, the application of the magnetic field to either anode or cathode results in greatly increased production of X-rays.

14. The application of a strong magnetic field at the anode, with lines of force along the line of electric discharge, forms a safe and useful method of regulation of X-ray bulbs.

15. The application of the magnetic field at a suitably placed anode constitutes a magnetic rectifier for alternating currents.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

DISPERSION IN ELECTRIC DOUBLE REFRACTION.

BY HOWARD LANE BLACKWELL.

WITH A PLATE.

INVESTIGATIONS ON LIGHT AND HEAT MADE OR PUBLISHED, WHOLLY OR IN PART, WITH APPROPRIATIONS FROM THE RUMFORD FUND.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

DISPERSION IN ELECTRIC DOUBLE REFRACTION.

By Howard Lane Blackwell.

Presented by John Trowbridge, April 8, 1905. Received December 31, 1905.

The object in the following paper is to present a study of dispersion in the electric double refraction of carbon bisulphide; for though as many as fourteen observers have experimented with electric double refraction since its discovery thirty years ago, even the existence of dispersion in the phenomenon has been unnoticed by all except two. Only one of them made any suggestion as to what function of the wave-length represents the double refraction, and the function which he suggested turns out not to hold even in the visible spectrum. The departure from it is important both in the direction of electro-magnetic waves and towards the ultra-violet.

When a condenser is charged, the dielectric becomes doubly refracting. Under the influence of the electric field it acquires the optical property of a uniaxial crystal. Electric double refraction has been induced in glass, resin, and other transparent solids, and in over one hundred insulating liquids. Even conducting liquids, — water, nitrobenzol, and others, — are capable of exhibiting this effect, though only for an instant. Solid insulators, like glass, require a number of seconds after the field is established to acquire the new property most strongly. In liquids, however, the change is sensibly instantaneous. Moreover, just as natural uniaxial crystals fall into two classes, positive and negative, so substances rendered doubly refracting by the electric field behave some of them like positive, some like negative crystals. In other words, the velocity of the extraordinary wave is increased over that of the ordinary in some substances and decreased in others. Carbon bisulphide becomes positively uniaxial and exhibits electric double refraction more strongly than almost any other liquid tried.

The first investigation in this general field was an unsuccessful attempt by Faraday in 1834 to discover some effect of electrostatic strain on the propagation of light.[1] The first successful experiment was the discovery of the effect by Kerr in 1875, and in an admirable series of papers extending over nearly twenty years, he continued to contribute to the advance in this new field.[2] Other investigators, following Kerr, are: Gordon,[3] Mackenzie,[4] Roentgen,[5] Grunmach,[6] Blondlot,[7] Brongersma,[8] Kundt,[9] Quincke,[10] Lemoine,[11] Abraham,[12] Schmidt,[13] and Elmén.[14]

Even after Kerr's discovery, Mackenzie, Grunmach, and Gordon in his first attempt, failed to observe electric double refraction. Gordon in his second attempt, and Brongersma confirmed Kerr's discovery. Roentgen added new liquids to Kerr's list, and studied the electric double refraction of quartz. Kundt observed quartz only.

The first quantitative observations were made by Kerr, who showed (with white light, but with great care) that the double refraction of carbon bisulphide varies as the square of the strength of electric field. In 1883 Quincke undertook to determine the coefficient of proportionality between double refraction and field-strength. He worked partly with white light and partly with light from a red color screen, but did not mention dispersion, and his results do not show it. Lemoine in 1896 redetermined this coefficient, which Quincke called "B," for carbon bisulphide, with a red color screen, but gave no data and few details to show the accuracy of his result. Elmén, working with Brace, in 1904, observed "B" in very weak electric fields. None of them noted dispersion.

It was left for Kerr in 1892 to show the existence of dispersion in the phenomenon he had discovered and studied so successfully. He did not multiply observations on his latest discovery, however, but inferred from what seem scanty data, that in carbon bisulphide the double refraction " varies exactly, or very nearly " inversely as the square-root of the wave-length. The only other observer who noticed dispersion is Schmidt. In 1901 he published observations of the amount of electric double refraction in a number of organic liquids compared with that in carbon bisulphide as standard. His observations were made with blue and with red light obtained with color screens (light of wave-lengths "$490\mu\mu$" and "$680\ \mu\mu$"), and with what he called the "blue-violet transition tint ($\lambda = 580\ \mu\mu$)," obtained apparently by rotating the Nicols. He expressed the electric double refraction of the substance under investigation in terms of the effect in carbon bisulphide. It thus does not appear whether his data indicate dispersion in the liquid under observation, or in the carbon bisulphide used for comparison, but merely that in one or other, or both, there is dispersion. The reduction of his observations to absolute value would depend on the determinations of "B" in carbon bisulphide made by Quincke and by Lemoine, neither of whom noted the variation of "B" with variation of wave-length.

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[1] Experimental Researches in Electricity, Series VIII, §§ 951–955 (April, 1834).

[2] Phil. Mag., (4) **50**, 337, 446 (1875); (5) **8**, 85, 229 (1879); **9**, 157 (1880); **13**, 153, 248 (1882); **20**, 363 (1885); **37**, 380 (1894); **38**, 144 (1894); Rep. Brit. Ass'n, Edinb., 157 (1892).

[3] Phil. Mag., (5) **2**, 203 (1876); Treat. on Elec. & Mag., **11**, 247 (1880).

[4] Wied. Ann., **2**, 356 (1877).

[5] Wied. Ann., **10**, 77 (1880); **18**, 213 (1882); **19**, 319 (1882).

[6] Wied. Ann., **14**, 110 (1881).

[7] Jour. de Phys., (2) **1**, 364 (1882); Compt. rend., **106**, 349 (1888).

[8] Wied. Ann., **16**, 222 (1882); Ann. d. Phys., **7**, 708 (1883).

[9] Wied. Ann., **18**, 228 (1882).

[10] Wied. Ann., **19**, 729 (1883).

[11] Compt. Rend., **122**, 835 (1896).

[12] Jour. de Phys., (3) **9**, 265 (1900).

[13] Ann. d. Phys., (4) **7**, 142 (1901).

[14] Phys. Rev., **20**, 54 (1905).

In short, Kerr's measurements on the variation of electric double refraction with variation in wave-length furnish the only suggestion thus far as to the law of dispersion.

NEW RESULTS.

In the eight experiments whose results are given below, the temperature remained constant near 11° in five, and near 24° in the other three. The field-strength during each of the experiments was also very nearly constant, and differed little from experiment to experiment. The results are given for a potential gradient of 56,000 volts per centimeter. The observed double refraction, or separation δ, is proportional to the difference between the ordinary and electrically produced extraordinary indices of the liquid, $(\mu_e - \mu_\omega)$. This quantity is the separation in centimeters introduced between the mutually perpendicular components of a wave of plane-polarized light in traversing one centimeter of the substance in question, — in this case carbon bisulphide, — at a certain temperature, in a field of certain strength. The double refraction of natural uniaxial crystals is usually similarly expressed as the difference between the two indices. Accordingly, the present observations are expressed in terms of this characteristic difference. Its values are given in Table I, and plotted in Figure 1, in per cent of its value at 24°.0 at wave-length 5890. This value, the factor reducing the values of Table I to absolute index of refraction, is found in these experiments to be about $7.04 \cdot 10^{-7}$.

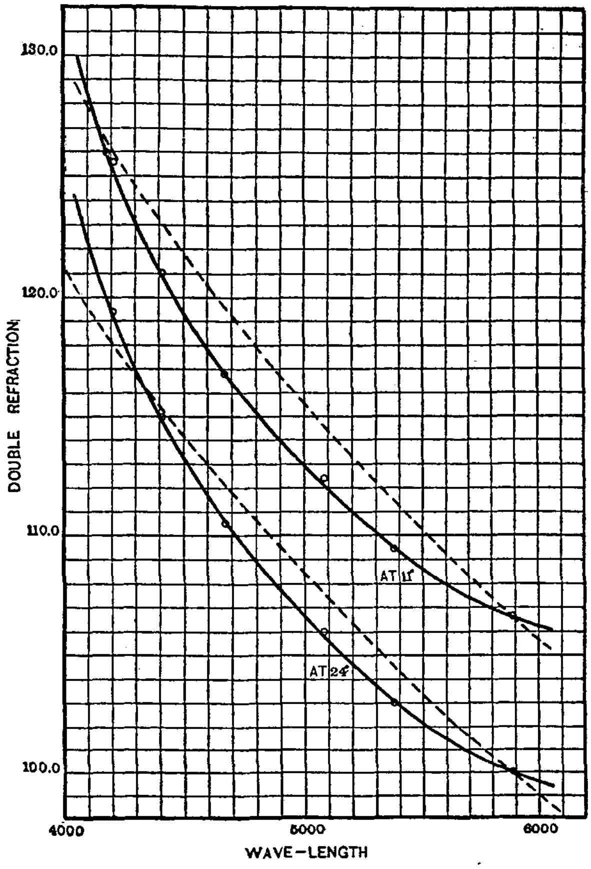

FIGURE 1. ELECTRIC DOUBLE REFRACTION OF CARBON BISULPHIDE.

The heavy curves show the double refraction observed with carbon bisulphide at 11° and at 24°. The dotted curves show how the double refraction was previously supposed to vary,—inversely as the square-root of wave-length. The departure of each observed curve from the dotted one is important, for the dotted curve is an equilateral hyperbola approaching the axes of zero wave-length and zero double refraction as asymptotes, whilst the observed curve rises much more sharply towards the edge of strong absorption in the ultra-violet, and falls much more slowly than the hyperbola towards the red.

The wave-length is expressed in 10^{-10} meters; the double refraction is expressed in per cent of the value of $(\mu_\epsilon - \mu_\omega)$ at 24°.0 with sodium light, — equal to about $7.04 \cdot 10^{-7}$; the field-strength giving these values is 56,000 volts per cm.

TABLE I.

ELECTRIC DOUBLE REFRACTION OF CARBON BISULPHIDE

Experiment.	Conditions under which observations were made:		Double refraction, reduced to 24°.0 and 11°.0 as standard temperatures, and 8990 volts (= 56,000 volts per cm.) the value at λ = 5890 being taken as 100 at each standard temperature:							
No.	Volts.	Temp. of CS_2.	Standard Temp.	λ 4180	λ 4216	λ 4415	λ 4677	λ 5085	λ 5378	λ 5890
1	9000 ?	24°.3	24°.0		120.2	115.5	111.2	106.1	103.2	100.0
4	8980	25°.9	"	118.9		114.1	110.9	105.9	103.1	"
8	8890	22°.5	"		118.5	114.4	109.5	105.9	102.7	"
Mean (plotted in Fig. 1).			24°.0	(118.9)	119.4	115.0	110.5	106.0	103.0	100.0
2	8800 ?	9°.2	11°.0	119.2	118.0	112.9	109.2	105.4		100.0
3	8885	10°.9	"	118.9		114.3	110.9	106.9	103.7	"
5	9105	11°.9	"	116.1		112.2	108.9	104.9	101.9	"
6	9150	11°.0	"	117.9	117.3	113.8	109.3	105.3	102.5	"
7	8990	10°.4	"	118.9	118.2	114.1	110.2	105.7	103.0	"
Mean			11°.0	118.2	117.8	113.5	109.7	105.6	102.8	100.0
Mean at 11°.0 expressed in per cent of value taken as 100 at 24°.0 and λ = 5890 (plotted in Fig. 1).				126.0	125.6	121.0	116.8	112.4	109.4	106.6

Absolute values are obtained in terms of $(\mu_\epsilon - \mu_\omega)$ by multiplying the above values by $7.04 \cdot 10^{-7}$, the observed absolute value of the double refraction at 24°.0 and λ = 5890.

Observations were terminated in the violet by the absorption of the liquid. The sharpness with which absorption begins is shown in the photograph of the iron spectrum taken through 96 cm. of carbon bisulphide. (Plate, Figure A.) At wave-length 4180 the spectrum was still bright; at 4144 very faint. Within 50 Ångström units change of wave-length the light was practically cut off. In the following observations on dispersion the light passed through 109 cm. of liquid, and measurements of the photographs could not be made on account of the absorption beyond wave-length 4180, a point near the limit of the violet.

As this sharply bounded region of total absorption is approached, the dispersion varies rapidly. According to the law suggested by Kerr, the double refraction should increase as the inverse square-root of the wave-length. This curve is an equilateral hyperbola whose axes are the axes of co-ordinates. It is shown dotted in Figure 1. Each hyperbola was calculated by starting with the observed value of the double refraction for the given temperature at wave-length 5890. But the curve of double refraction observed in the present experiments is much steeper than the hyperbola suggested by Kerr, as the region of total absorption is approached. This raises the question whether *the approach to an absorption band* may not be the controlling factor rather than the change of wave-length.

The possible effect of an absorption band is briefly considered mathematically by Néculcèa,[15] in his summary of the mathematical papers on the general phenomenon of electric double refraction by Pockels,[16] and Voigt.[17]

METHOD.

A means of measuring double refraction, and at the same time of taking into account the dispersion, is furnished by Babinet's compensator.

When this is viewed by plane-polarized light and an analyzing Nicol, both properly oriented, a symmetrical system of straight dark fringes appears, of which the central one is achromatic. If a doubly refracting body, also properly oriented, be introduced between the compensator and the source of light, the fringes are shifted through a distance x, proportional to the double refraction. By analyzing the dark central fringe with a spectroscope whose slit cuts the fringe at right angles, a spectrum is formed traversed by a dark band, and any variation in x with variation in color is made evident. If the dispersion of double refraction in the material under observation be the same as that of the compensator, namely of quartz, the straight dark band in the spectrum will shift parallel to itself. If the dispersion be different from that of quartz, the shifted band will be a curve plotting the difference. By the use of a camera and orthochromatic plates in place of the usual observing telescope permanent records may be obtained. This is an advantage, because measurements of the negatives can be made at leisure more accurately than visual observations under the pressure of experiment.

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[15] Le Phénomène de Kerr, p. 60. (Par E. Néculcèa; C. Naud, Éditeur; Paris, 1902.)

[16] Gött. Nachr. Math. Phys. Klasse, **2**, 102 (1896).

[17] Wied. Ann., **69**, 297 (1899); Ann. d. Phys. (Jan., 1901).

APPARATUS.

Accurate measurement of double refraction in a strong electric field requires above all a source of constant electromotive force. The Jefferson Physical Laboratory is especially well fitted for such work in having a storage battery of twenty thousand cells.

Electrodes. — The success of the method depends upon the constancy and uniformity of the electric field. Plane, parallel electrodes are necessary to secure uniformity, and the distance between them must be everywhere the same, and accurately known. In order to reduce the voltage required to produce a given strength of field, the distance, "*a*," between the bars was made only 0.1607 cm. Since the field-strength is the difference of potential divided by this distance, and enters the expression for the double refraction as the square, the percentage error of "*a*" is doubled in the result. To keep the error in distance between electrodes negligible, the electrodes were made of silver deposited in a thin bright film on two heavy glass bars whose faces were prepared at the optical works of the Alvan Clark & Sons Corporation. The Newton's fringes formed when a plane test-plate was placed anywhere on these surfaces before they were silvered showed them to be far more nearly plane than

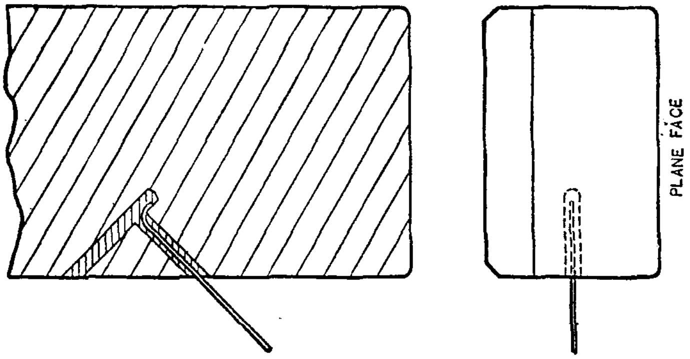

FIGURE 2. Section and end view, actual size, of silvered glass bar used as electrode, showing method of connecting to silver film. The film extends from the plane face around the corners and up the sides higher than the wire.

necessary to equal the accuracy obtainable with the optical parts of the apparatus.

The bars are rectangular, 2.3 cm. thick, with faces 3.6 cm. wide by 96.0 cm. long. The edges and corners are smoothly rounded to a radius of something over one millimeter so that the plane silver films on the faces do not end in edges which might determine spark discharge, but round back on the sides of the bars. Solid electrical connection with the films is made through two pairs of small intersecting holes drilled in each bar on opposite sides near one end (Figure 2). A silver wire is clinched in one hole of each pair and both holes are then filled with the spongy gold used in dentistry firmly tamped in. When the bar is silvered the film comes down perfectly solid and continuous over the glass and the gold, and even if the film should break away from the wire where the latter enters the bar, still through the other filled hole the electrical connection would remain perfect. Reflection from the silver film was always improved by careful dry polishing with settled rouge on washed chamois. Minute scratches are produced, but the gain in reflected light when the bars are in use close together, face to face, is important.

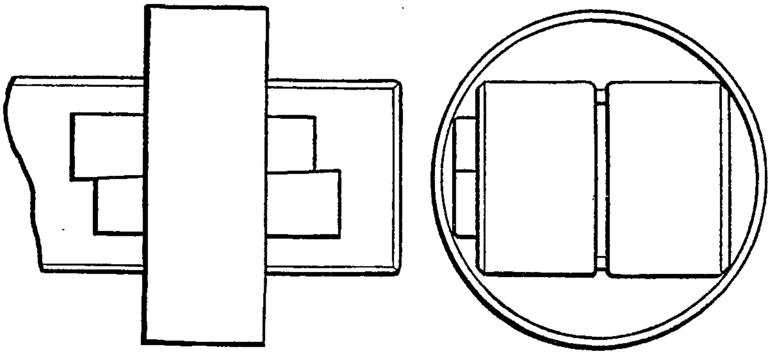

FIGURE 3. Silvered glass bars wedged face to face. Side view and end view, showing ring, wedges, and little quartz pieces separating bars.

To hold the bars rigidly at a constant accurately known distance apart small flat pieces of quartz are placed between them close to each edge about every eight centimeters. These pieces of quartz were all cut from the same plate which had been ground and polished at the Clarks' more closely plane and uniform in thickness than a tenth of one per cent of the total thickness which is 1.607 mm. Similar spacer pieces of glass were previously used, but their insulating power seemed in time to fail. The bars are finally firmly wedged together in glass rings just large enough to slip over them (Figure 3). Thus assembled, they are shown in the Plate, Figure B.

Tube. — The construction of the tube to hold these bars immersed in the liquid to be observed was a difficult problem. The tube must admit insulated leads to the bars from a source of high voltage, must be closed liquid tight by glass end-plates, must be connected by glass tubes with a supply of pure, dry carbon bisulphide, and must be chemically inert and capable of being made chemically clean. It must be three inches in diameter and nearly four feet long (7 × 110 cm.).

Glass tubes were tried first. Four in succession cracked spontaneously. Apparently such large tubes are under great internal strain, and when this is released in grinding the ends, the glass gradually yields. Sometimes a night and sometimes a week elapsed before the crack came, but fortunately it came before any tube was in place and filled with carbon bisulphide.

Glass is too dangerous to rely on, and special enamelled iron tubes were tried. On opposite sides, as near as possible to each end, insulating "lava" bushings about 2.5 cm. in diameter were screwed in before enamelling. The ends of the tubes were flanged, and had to be machined to a smoothly convex surface in order to retain a good continuous coating of enamel. Four attempts were made before a satisfactory tube was secured.

Closing Tube. — The necessity of having enamel on the ends of the tube is due to the method of closing it. In order to avoid the use of washers, or cement, which might be more or less soluble in carbon bisulphide, a fine fit alone is relied upon. Glass discs for the ends were cut out of "crystal plate," about 1.5 mm. thick, only those being used which when finished seemed free from double refraction. "Crystal plate" is ground and polished in manufacture, and consequently free from small irregularities in the surface. This was shown with an optically flat test-plate by the smoothness of the Newton's fringes in the air film between plate and disc. The enamelled ends of the tube were ground, at the Clarks', optically flat and nearly parallel. The tube was then closed by simply pressing the glass plates against the flat ends (Figure 4). Thin rubber rings were inserted between the glass and the rigid cast-iron rings as a precaution against breakage.

When all is in place, a continuous rubber ring, or gasket, previously slipped over the tube, is wired down on tube and ring, and mercury is poured into the annular space between, through a small hole in the iron ring. This makes the ends air tight, so that the tube can be exhausted and the liquid drawn in without any moist or dusty outside air.

Connections. — Ground joints in the enamelled lava bushings admit the side-tubes connecting with the supply of carbon bisulphide and with a filter-pump. A three-way glass stopcock permits the exhaust to be closed and air to be admitted through drying tubes of phosphoric pentoxide when the liquid used in the observing tube is to be run back into its reservoir. The battery connections are made through platinum wires sealed into glass stoppers ground to fit the other lava bushings, and all these connections are also mercury-sealed.

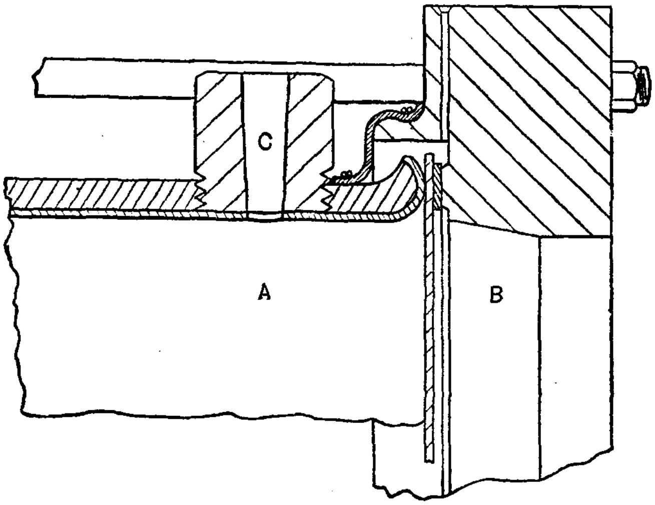

FIGURE 4. SECTION OF THE TUBE SHOWING METHOD OF CLOSING ENDS.

A, Tube in longitudinal section, with glass plate against right-hand end.

B, Cast-iron ring bolted against end, pressing rubber ring, shown in section, against glass plate. Notice small vertical duct in ring through which mercury is poured in to fill annular space enclosed around edge of glass plate by rubber ring shown in section as wired down to flange of iron ring and to tube.

C, "Lava" bushing, hollow, for side tube admitting liquid and electrical connection to electrodes in tube.

Advantage is taken of two of these side outlets to insert permanently, at the upper and lower ends of the tube, two small mercury thermometers which can be read through the glass side-tubes.

OPTICAL PARTS.

The principal parts of the illuminating system are an arc lamp, short focus objective, and Nicol prism. The arc lamp is entirely enclosed in a sheet-iron case into which cool air from another room is forced by a fan. This precaution is taken against fire, so that if the carbon bisulphide should by any chance leak, the heavy vapor could not creep into the lantern and explode. The arc-circuit switch is opened under oil. Since the vapor of carbon bisulphide ignites very much below red heat a water cell is placed in front of the condensing lens of the lantern, so that the slit which is at the focus cannot become hot enough to ignite the inflammable vapor in case of a leak. A half-inch microscope objective sends a bright, thin, very little divergent beam from the slit through the polarizing Nicol and up the tube.

The Babinet's compensator is fastened to the face of the spectroscope slit, behind which, in the collimating tube, is the analyzing Nicol. A direct-vision prism-train receives the light, and a camera with long focus achromatic objective takes the place of the observing telescope.

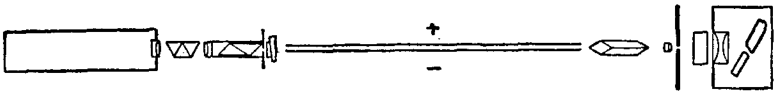

FIGURE 5. ARRANGEMENT OF APPARATUS.

A half-inch objective is used to send a strong beam of polarized light through the carbon bisulphide.

The tube containing the electrodes and liquid is mounted sloping, in order that air bubbles may not lodge under the spacer pins between the bars during filling and cause spark discharge later. The arrangement of apparatus is shown in Figure 5.

The amount of double refraction due to ether drift, if there is such an effect, is less than $10^{-8}\,\mu_\omega$ according to Lord Rayleigh,[18] and the values of the electric double refraction obtained in the present experiments are not less than $70 \cdot 10^{-8}\mu_\omega = (\mu_\epsilon - \mu_\omega)$, so that they cannot be affected by more than 1.5 per cent. But if the effect of ether drift should turn out not to be negligible the position of the tube should be known. It is nearly in the magnetic meridian, and slopes down to the north about 23° from horizontal.

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[18] Phil. Mag., **4**, 678–683 (Dec., 1902).

Electrical Apparatus.

In determining the strength of the electric field, since the double refraction varies as the *square* of the field-strength, it is necessary to measure the difference of potential with a percentage error less than *one half* that of the optical measurements. With differences of potential as high as ten thousand volts and higher, the readings of electrometers depend to an uncertain extent on the condition of the air, especially on the humidity, and it seemed preferable to use a high-resistance potentiometer and Clarks' standard cell.

Potentiometer. — To extend the range of the Crompton potentiometer to fifteen thousand volts, and still permit readings to an accuracy of one tenth of one per cent, required one million ohms of extension coils with a negligible temperature coefficient and a carrying capacity of fifteen milliamperes continuously.

Special Resistance. — The resistance was therefore built up of ninety coils, each of ten thousand ohms, wound with a single strand of number 37 (B. & S.) double silk-covered, paraffined special resistance wire, on seasoned maple spools. Single winding secures the least possible difference of potential between adjacent turns, and successive layers are separated by paraffined paper. The spools before winding were tested, and none found to have less resistance between opposite sides of the core than 100 megohms. The wound spools were gradually heated in an electric oven to 140° C., and maintained so for about three hours, when they were immediately immersed in melted paraffine also at 140°, and kept hot for three hours more. The spools finally ceased to give off air bubbles, and they were left in the bath while it was gradually allowed to solidify, so that as the cooling paraffine in the spools shrank, air might not penetrate. The bath was then barely melted and the spools withdrawn and dipped once more. In this way dry coils, well insulated and well protected from atmospheric influences, were obtained.

They were mounted in groups of nine on bright hard-rubber and the terminals of each coil were soldered without the use of acid to brass binding posts on hard-rubber strips on opposite sides of the ventilated containing boxes.

The resistance of each coil thus mounted can be measured with a standard bridge. The resistance of the sum can be taken as the sum of these resistances, since the coils and their terminals are so highly insulated from one another.

But since the fall of potential may reach as much as one hundred and fifty volts per coil in use, and the resistance was measured with only six volts, it was an open question whether the increased leakage at the higher voltage between such an enormous number of adjacent turns would be negligible. To settle this question, a d'Arsonval galvanometer was calibrated with a known resistance box and standard cell to read current. The current through each coil of a set of nine was then measured with six volts per coil and with one hundred and fifty. Within the limit of error of the observations, one tenth of one per cent, the current proved proportional to the voltage.

Static Voltmeter.—With this resistance and potentiometer satisfactory measurements of potential up to fifteen thousand volts were obtainable, but another difficulty presented itself. The electromotive force of the battery with the potentiometer connected falls perceptibly in a few seconds, even with the small discharge rate of fifteen milliamperes. An auxiliary static voltmeter calibrated immediately before and after using is necessary. A very simple device has proved exceedingly sensitive. Between two parallel plates of brass about fifteen centimeters square a silvered glass disc is hung, four or five centimeters in diameter, firmly cemented to a small, straight, steel clock spring, thus probably securing nearly perfect elasticity in the moving parts within the limits of deformation. The whole is enclosed in wood and glass. The spring and one plate are connected to one terminal and the other plate to the other terminal of the high voltage line, and deflections can be read by a telescope and micrometer eyepiece to one tenth of one per cent. Moreover, successive calibrations agree to this degree of accuracy.

PURIFICATION OF CS_2.

Though small quantities of impurities often do not preceptibly influence the behavior of a substance, it is more satisfactory to work with a pure substance, or definite mixture. Pure carbon bisulphide can be ordered, but is said to deteriorate in a short time by the formation of more complex sulphides.[19] Kerr, Quincke, and Schmidt all lay stress on the need of freeing the carbon bisulphide from solid particles in order to prevent sparks. It is possible also that traces of dissolved water lower the obtainable difference of potential. Accordingly the following method of purifying and drying CS_2 was used.

(1) To commercial carbon bisulphide add quicklime, stand a day, digest four hours (with return condenser), distil.

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[19] Berthellot, A. Ch., (7), **14**, 155; **19**, 150.

(2) Stir with plenty of Hg, repeat till fresh Hg is not tarnished. Decant.

(3) Stir with strong aqueous solution of $HgCl_2$ till no more precipitate, remove $HgCl_2$ (separating funnel), keep in dark.

(4) Dry over fused $CaCl_2$ at least a day. Stir. Keep cool and dark.

(5) Decant, add P_2O_5 (resublimed if necessary to remove P), digest for some hours (with return condenser), distil.

In the last step the distillate before condensation was passed through about four feet (120 cm.) of drying tubes filled with phosphoric pentoxide and glass wool, and kept at about 50° C. It was then washed free from any possible particles of pentoxide by bubbling up through some ten inches (25 cm.) of chemically clean glass beads immersed in a portion of the liquid earlier distilled. The pure dry gas was finally condensed and run into the receiving bottle. The bottle had already been connected to the observing tube so that the liquid could be used without exposure to the undried and more or less dusty air of the room. All joints were of ground glass mercury-sealed. The temperature of the vapor during distillation did not vary more than 0°.02 till near the close, when the vapor began to superheat on the warmer walls of the distilling bottle. Carbon bisulphide thus purified is a beautiful, sparkling, colorless, limpid liquid with a pleasant odor suggesting ether. Probably the most delicate test of its purity is its odor. The bad smell of commercial carbon bisulphide is caused by the impurities.

In the purification of this substance in such large quantities as this, — about twelve liters at a time, — every precaution was taken against fire. No flames were used in the room, and the liquid was distilled from a water-bath warmed by an immersed electric heater. Thus there was no object as hot as boiling water, and no spark, which could possibly be reached by any vapor of carbon bisulphide which might escape.

Such are the method and devices developed step by step by actual experience in solving this threefold problem, — optical, electrical, and chemical.

PROCEDURE.

In these measurements the carbon bisulphide was purified as described, but had stood in glass bottles for several months, though in the dark, hermetically sealed, and dry. After the measurements it was examined, and its freedom from disagreeable odor made it seem probable that deterioration had not taken place.

Each observation consisted essentially of three photographs of the spectrum taken on successive strips of the same plate. With everything in adjustment, but with no electric field, the zero band was taken; with the same adjustments the voltage band was taken, with the electric field; and the reference spectrum was secured by temporarily inserting, immediately in front of the compensator, a sodium flame, and then a cadmium spark. The slit was 0.3 mm. broad, but even with such a broad slit the voltage and zero bands each had to be exposed for at least forty minutes. With this breadth of slit, however, the position of any wave-length could be determined with sufficient accuracy, as it affects the results only as the square-root approximately. A typical photograph is shown in the Plate, Figure C. During the exposure of the voltage band, readings of the potential were taken nearly every minute. Immediately before and after taking a voltage band the static voltmeter was calibrated with the potentiometer and one of three concordant Clark's standard cells. In the worst case the calibration curves differed by four tenths of one per cent, and generally they coincided.

But the measurement of the negatives proved slower and more troublesome than securing them. Each negative was mounted in front of ground glass illuminated by transmitted light, and was measured with a telescope sliding on a vertical scale. In the focal plane of the eyepiece two spider lines crossed at an angle of 60°, and the intersection of the cross-hairs was brought alternately from above and from below to what appeared to be the middle of the band. At least four settings were made on every band and two settings on the edge of the strip. In this way the height of each voltage and zero band above the edge of its strip was found at six and sometimes seven places of known wave-length varying from 5890 to 4180 Ångström units, — the wave-lengths being known from the cadmium reference spectrum and the strong carbon bands in the extreme violet. The negative was then inverted, each measurement repeated, and the mean taken to eliminate possible partiality for settings to the upper or lower side of the field of view. How accurately these settings ran may be seen from the following measurements of one plate, — the worst (Tables II and III). How accurately they *can* be made is shown in Table IV, — perhaps the best.

In spite of these precautions, reduction and comparison of the measurements of eight voltage bands show small discrepancies in the total shift, x, probably due, at least in part, to inaccuracies in the vertical ways on which the telescope was moved. Discrepancies of 0.2 mm., the largest found, might be caused by a very small tilt in the telescope,

TABLE II.

One Set of Measurements in Detail.

Negative, Erect, of Experiment 8. Temp. of CS_2, 22°.5.

Object viewed.	Telescope Scale Reading.						Volts.	Compensator Screw.
	λ 4216	λ 4415	λ 4677	λ 5085	λ 5378	λ 5890		
	cm.	cm.	cm.	cm.	cm.	cm.		Turns.
First zero band	6.286	6.298	6.322	6.312	6.324	6.332		
	300	310	30	30	36	44		
	296	298	18	24	28	26		
	304	304	34	36	44	52		
						32		
						26		
Mean	6.297	6.303	6.326	6.326	6.333	6.335		
Edge of strip	6.040	6.046	6.058	6.062	6.064	6.066		
	40	48	60	66	66	68		
Mean	6.040	6.047	6.059	6.064	6.065	6.067		
Difference, or height of band	0.257	0.256	0.267	0.262	0.268	0.268	0	5.000
Voltage band	4.864	4.782	4.700	4.648	4.576	4.550		
	80	86	08	50	96	62		
	62	78	02	42	86	46		
	80	94	10	58	96	54		
Mean	4.872	4.785	4.705	4.650	4.589	4.553		
Edge of strip	3.802	3.824	3.834	3.848	3.850	3.862		
	04	26	36	50	52	64		
Mean	3.803	3.825	3.835	3.849	3.851	3.863		
Difference	1.069	0.960	0.870	0.801	0.738	0.690	8890	30.000
Second zero band	2.296	2.308	2.324	2.350	2.346	2.348		
	302	22	24	50	46	60		
	294	08	28	40	36	34		
	304	24	30	46	50	50		
Mean	2 299	2.316	2.327	2.347	2.345	2.348		
Edge of strip	1.612	1.618	1.626	1.632	1.628	1.640		
	16	20	30	38	28	40		
Mean	1.614	1.619	1.628	1.635	1.628	1.640		
Difference	0.685	0.697	0.699	0.712	0.717	0.708	0	0.000
	4216	4415	4677	5085	5378	5890	Wave-length checked.	

TABLE III.

TOTAL SHIFT, x_λ.

Summary of Table II and of two similar tables of measurements upon the negative, erect or inverted, of Experiment 8. Temp. of CS_2, 22°.5.

	Height of Zero and Voltage Bands.						Volts.	Compensator Screw.
	λ 4216	λ 4415	λ 4677	λ 5085	λ 5378	λ 5890		
	cm.	cm.	cm.	cm.	cm.	cm.		Turns.
Negative Erect	0.257	0.256	0.267	0.262	0.268	0.268		
" Inverted	0.259	0.262	0.262	0.267	0.269	0.267		
" Erect					0.282	0.273		
(1) Mean height of first zero band	0.258	0.259	0.265	0.265	0.273	0.269	0	5.000
Negative Erect	1.069	0.960	0.870	0.801	0.738	0.690		
" Inverted	1.053	0.995	0.890	0.814	0.769	0.700		
" Erect					0.743	0.686		
(2) Mean height of voltage band	1.061	0.978	0.880	0.808	0.750	0.692	8890	30.000
Negative Erect	0.685	0.697	0.699	0.712	0.717	0.708		
" Inverted	0.697	0.716	0.721	0.719	0.726	0.718		
" Erect					0.709	0.706		
(3) Mean height of second zero band	0.691	0.707	0.710	0.716	0.717	0.711	0	0.000
(2) – (1) = Apparent Shift	0.803	0.719	0.615	0.543	0.473	0.423		
Add for Comp.	2.32	2.32	2.32	2.32	2.32	2.32		25.000
Total x_λ at 8890 v.	3.123	3.039	2.935	2.863	2.793	2.743	8890	
Total x_λ at 8990 v. $= \frac{8990}{8890} \cdot x_\lambda =$	3.192	3.108	3.000	2.928	2.855	2.304	8990	

TABLE IV.

TOTAL SHIFT, x_λ.

Like Table III, this table is a summary of tables similar to II for negative of Experiment 4. Temp. of CS_2, 25°.9.

	Height of Zero and Voltage Bands.						Volts.	Compensator Screw.
	λ 4180	λ 4415	λ 4677	λ 5085	λ 5378	λ 5890		
	cm.	cm.	cm.	cm.	cm.	cm.		Turns.
Negative Erect	1.372	1.364	1.363	1.370	1.364	1.363		
" Inverted	1.367	1.358	1.359	1.365	1.364	1.370		
" Erect	1.372							
(1) Mean height of first zero band	1.370	1.361	1.361	1.368	1.364	1.367	0	−5.000
Negative Erect	1.291	1.171	1.110	1.020	0.957	0.894		
" Inverted	1.268	1.171	1.109	1.012	0.955	0.901		
" Erect	1.265							
(2) Mean height of voltage band	1.275	1.171	1.110	1.016	0.956	0.898	8980	+30.000
Negative Erect	0.879	0.885	0.889	0.878	0.886	0.873		
" Inverted	0.869	0.873	0.880	0.881	0.887	0.875		
" Erect	0.872							
(3) Mean height of second zero band	0.873	0.879	0.885	0.880	0.887	0.874	0	0.000
(2) − (1) = Apparent Shift	−0.095	−0.190	−0.251	−0.352	−0.408	−0.469		
Add for Comp.	3.14	3.14	3.14	3.14	3.14	3.14		35.000
Total x_λ at 8980 v. (Sensibly unchanged at 8990 v).	3.045	2.950	2.889	2.788	2.732	2.671	8980	

which was mounted four feet from the negative (120 cm.). A fixed telescope with micrometer eyepiece is being mounted, and will eliminate this uncertainty in future.

The difference between the positions of the voltage and zero bands gives the apparent shift, — that part of the total amount not taken up by changing the setting of the compensator. The reading of the compensator was reduced to terms of the apparent shift by photographing the zero bands for two settings of the compensator differing by from five to ten turns of the screw, the range being limited by the field of view. Thus it was found that for thirty turns, 2.73 cm. must be added to the apparent shift to get the total shift, x. The values of x obtained from the eight voltage bands thus far observed are recorded in Table V.

TABLE V.

Total Shift, x_λ, at 8990 Volts.

Summary of Tables III, IV, and similar ones, each for one experiment.

Exp.	Temp. of CS$_2$	Total observed shift, x_λ, at temperatures given, reduced to 8990 volts, or 56,000 volts per cm.						
		λ 4180	λ 4216	λ 4415	λ 4677	λ 5085	λ 5378	λ 5890
No.	C	cm.	cm.	cm.	cm.	cm.	cm.	cm.
1	24°.3		2.922	2.831	2.744	2.647	2.589	2.529
4	25°.9	3.045		2.950	2.889	2.788	2.732	2.671
8	22°.5		3.192	3.108	3.000	2.928	2.855	2.804
2	9°.2	3.040	3.010	2.907	2.833	2.760		2.656
3	10°.9	2.992		2.900	2.835	2.760	2.692	2.620
5	11°.9	3.240		3.160	3.087	3.005	2.936	2.907
6	11°.0	3.247	3.232	3.160	3.061	2.980	2.915	2.868
7	10°.4	3.256	3.242	3.158	3.069	2.973	2.917	2.856

Treatment of Data.

Before the true significance of these measurements can be brought out, allowance must be made for dispersion in the natural double refraction of the quartz compensator. When plane-polarized light passes through quartz, the extraordinary wave is retarded over the ordinary by δ cm., an amount equal to the thickness, e cm., of quartz traversed, times the difference between the refractive indices, $(\mu_\epsilon - \mu_\omega)$. In Babinet's compensator, since the optic axis in the second wedge is at right angles to that in the first, the wave retarded in the first wedge is accelerated in the second, and vice versa. Hence

$$\delta = (e - e')\ (\mu_\epsilon - \mu_\omega),$$

With plane-polarized light the dark central fringe occurs where $\delta = 0$, namely, where the thickness of the first wedge, e, is equal to the thickness of the second, e'. This is the zero position. But when the incident light has already undergone double refraction, the central dark fringe will be shifted x cm. from the zero position to that point where the separation δ of the compensator is equal and opposite to the separation produced by the doubly refracting substance under observation. If i be the acute angle of the compensator wedges,

$$(e - e') = 2\,x \tan i$$

and at this point the separation due to the compensator is

$$\delta = 2\,x \tan i \cdot (\mu_\epsilon - \mu_\omega)$$

Since δ, x, and $(\mu_\epsilon - \mu_\omega)$ all vary with the color of the light, and since $(\mu_\epsilon - \mu_\omega)$ here refers to quartz, it is better to write the last equation with the subscripts λ, and Q, thus, (for quartz)

$$\delta_\lambda = 2\,x_\lambda \tan i \cdot (\mu_\epsilon - \mu_\omega)_{(\lambda,\,Q)}$$

In carbon bisulphide, in which $\mu_\epsilon > \mu_\omega$

$$\delta_\lambda = l \cdot (\mu_\epsilon - \mu_\omega)_{(\lambda,\,CS_2)}$$

at a given temperature, and given strength of electric field. In the voltage band the central fringe appears shifted by just the amount x_λ necessary to cause the compensator to counteract the separation induced by the electric field; or, stated algebraically,

$$\delta_\lambda = l \cdot (\mu_\epsilon - \mu_\omega)_{(\lambda,\,CS_2)} = 2\,x_\lambda \tan i \cdot (\mu_\epsilon - \mu_\omega)_{(\lambda,\,Q)} \qquad (1)$$

For sodium light, $\lambda = D$ and (1) becomes

$$\delta_D = l \cdot (\mu_\epsilon - \mu_\omega)_{(D,\,CS_2)} = 2\,x_D \tan i \cdot (\mu_\epsilon - \mu_\omega)_{(D,\,Q)} \qquad (2)$$

By dividing the first of these equations by the second, the double refraction in CS_2 for any wave-length can be expressed in terms of the double refraction for D light, $= (\mu_\omega - \mu_\epsilon)_{(D,\,CS_2)}$, as standard:

$$\frac{\delta_\lambda}{\delta_D} = \left[\frac{(\mu_\epsilon - \mu_\omega)_\lambda}{(\mu_\epsilon - \mu_\omega)_D}\right]_{CS_2} = \frac{x_\lambda}{x_D}\left[\frac{(\mu_\epsilon - \mu_\omega)_\lambda}{(\mu_\epsilon - \mu_\omega)_D}\right]_Q \qquad (3)$$

The values of $\left[\frac{(\mu_\epsilon - \mu_\omega)_\lambda}{(\mu_\epsilon - \mu_\omega)_D}\right]_Q$ are given in Table VI, which was made from data published by Macé de Lépinay in 1892 on the values of $(\mu_\epsilon - \mu_\omega)_{(\lambda,\, Q)}$.[20]

TABLE VI.

Dispersion Factor of Quartz.

The first line is obtained by interpolation from determinations by Macé de Lépinay, Jour. de Phys., (3) 1, 31 (1892).

The second line is calculated from the first.

	λ 4180	λ 4216	λ 4415	λ 4677	λ 5085	λ 5378	λ 5890
$(\mu_\epsilon - \mu_\omega)_{(\lambda, Q)}$	0.00951	950	943	936	926	921	913
$\left[\frac{(\mu_\epsilon - \mu_\omega)_\lambda}{(\mu_\epsilon - \mu_\omega)_D}\right]_Q$	1.042	1.041	1.033	1.025	1.014	1.009	1.000

Differences between different plates, thus far unexplained, are eliminated by using equation (3) in which the *ratio* of the observations to one of their own number, $\frac{x_\lambda}{x_D}$, is formed for each photograph, instead of placing values of x_λ from different plates in comparison, as would be the case in using equation (1). The observations have all been reduced to one strength of electric field, that obtained with 8990 volts, but the actual voltages were very nearly this, so that the corrections resting on the law of squares are small. In this way the values of x_λ given in Table V have been treated by (3) and the resulting values of the double refraction, $(\mu_\epsilon - \mu_\omega)_{\lambda,\, CS_2}$, in terms of $(\mu_\epsilon - \mu_\omega)_{(D,\, CS_2)}$ as one hundred per cent, have been tabulated and plotted near the beginning of this paper (Table I, and Figure 1).

Jefferson Physical Laboratory,
Harvard University.

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[20] Jour. de Phys., (3) **1**, 31 (1892).

EXPLANATION OF PLATE.

Figure A. Iron Spectrum through CS_2, and Direct.

The upper spectrum, taken through 96 cm. of the liquid, shows the rapidly increasing absorption of carbon bisulphide in the extreme violet. The lower spectrum was taken direct.

Figure B. Electrodes. — Silvered glass bars wedged face to face ready to be used as condenser electrodes.

Figure C. Typical Photographic Observation showing Dispersion in the Electric Double Refraction of Carbon Bisulphide.

The dark line throughout each band is the central dark fringe crossing the slit of the spectroscope at right angles and produced by the Babinet compensator.

The amount by which this line is displaced from horizontal in the "Voltage Band" shows the excess of dispersion in the double refraction of carbon bisulphide over that of the quartz compensator.

Each band was taken with a different setting of the compensator screw, and with an exposure of about forty minutes.

The strong bright lines in the extreme violet are carbon lines from the arc.

The reference spectrum was produced by a cadmium spark and sodium flame with exposures of from one to three minutes.

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THE THERMAL CONDUCTIVITY OF LEAD.

By F. L. Bishop.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

THE THERMAL CONDUCTIVITY OF LEAD.

BY F. L. BISHOP.

Presented by F. A. Laws, December 13, 1905. Received January 12, 1906.

THE method of determining the thermal conductivity here employed is the modification of the wall method due to Professor F. A. Laws, and the object of this research is to determine the limits of accuracy of that method.

For a general discussion of the method the reader is referred to the article, "A Method of Determining Thermal Conductivity," by F. A. Laws[1] with F. L. Bishop and P. McJunkin. A part of the apparatus used in this investigation is the same as that used in the previous one.

GENERAL DESCRIPTION OF APPARATUS.

A diagram of the apparatus is shown in Figure 1. The shell A is supported at the centre of a large tank filled with oil and can be maintained at any desired temperature. The nickel wires from the two surfaces of the shell are led to the mercury cups a, b, in the box C. A piece of the original lead L is brought from the shell to one of the calibrating tanks D. To this lead wire is electroplated a nickel wire similar to the wires on the sphere; consequently at T we have a junction exactly similar to the junctions on the shell. The wire is brought to the mercury cup c in the box C. This provides a means of determining with the required degree of accuracy the exact temperature of the outside or inside of the shell.

The wires from the galvanometer G are fastened to the block E, which can be placed in position in the mercury cups a, b, and thus connect the galvanometer with the thermo-electric junction on the shell. The deflection of the galvanometer G was calibrated by connecting the terminals E at M with the fall of potential across the standard ohm R_4, this fall of potential being compared with the Carhart-Clark cell H by means of the resistances R_1 and R_2.

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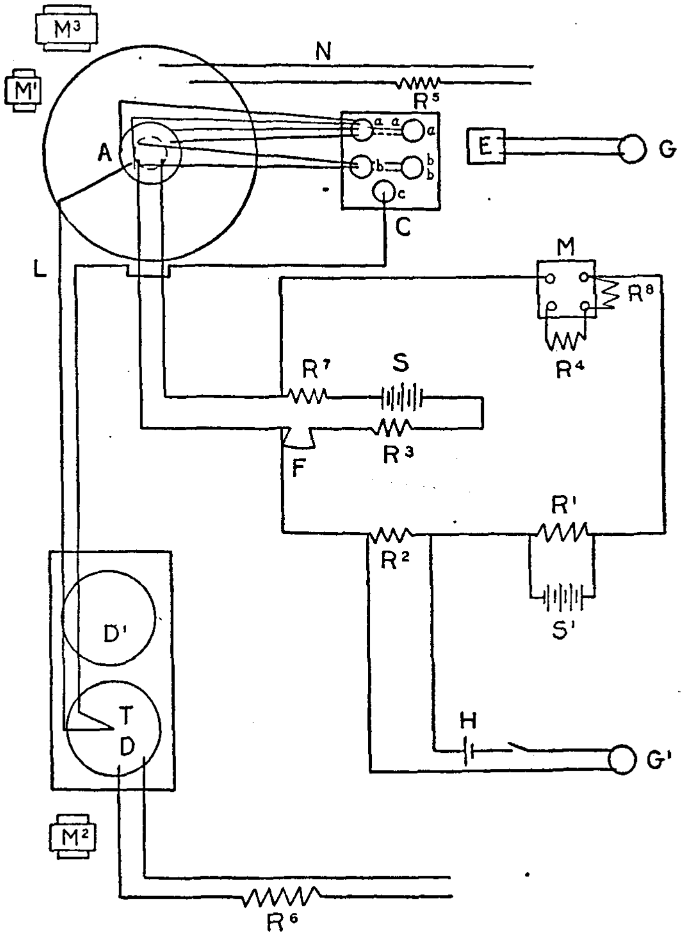
[1] These Proceedings, 1906.



EXPLANATION OF FIGURE 1.

A	Shell.
C	Box containing mercury cups a, b, c surrounded by compressed hair.
D, D_1	Calibrating tanks.
E	Block containing terminals of galvanometer.
F	Ammeter.
G, G_1	Galvanometers.
H	Carhart-Clark cell.
M	Mercury cup containing terminals of standard ohm.
N	220 volt mains.
M_1, M_2, M_3	Electric motors.
R_1, R_2	Resistances in series with fall of potential across heating coil.
R_2	Resistance in series with Carhart-Clark cell.
R_3	0.1 ohm in series with ammeter.
R_4	Standard ohm.
R_5, R_6, R_7	Adjustable resistances.
R_8	Resistance in series with galvanometer to counterbalance resistance of nickel leads.
S, S_1	Storage cells.
T	Nickel-lead junction.

The heat for the inside of the shell is furnished by the storage cells S. This current is measured by the ammeter F, and the fall of potential is balanced against the Carhart-Clark cell H by means of the resistances R_1 and R_2 and the galvanometer G_1. A resistance R_3 of 0.1 ohm connected in series with the ammeter provides a means of calibrating the ammeter in position and during readings. The fall of potential at R_3 was balanced against that of R_1 in series with the storage cells S_1, this being balanced just before and after against the Carhart-Clark cell H.

M_1, M_2, M_3 are electric motors for agitating the oil outside the shell, inside the shell, and around the junction T respectively.

The current from the 220 power mains N furnished heat to the oil outside the shell, while the current from the 110 mains maintained the tank D at constant temperature.

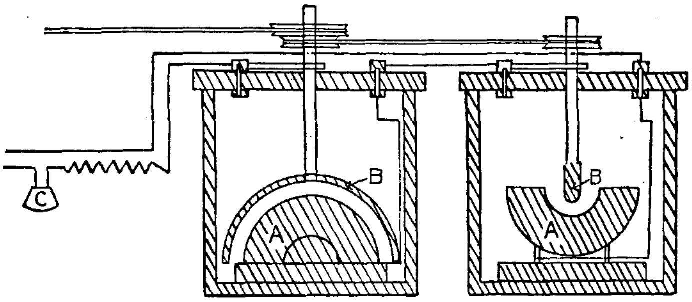


FIGURE 2.

DETAILS OF APPARATUS.

1. *Shell.*

The shell is in the form of a hollow sphere of cast lead, in the casting of which every precaution was taken to have the lead free from blowholes and of uniform quality. It is divided into two hemispheres by a plane passing through a great circle of the sphere. A radial hole was provided for the admission of wires to the heating coil, thermo-electric junctions, and also for the shaft to drive the fans on the interior of the sphere.

The shell was coated by electrolysis with nickel of a uniform thickness of about .7 mm. The apparatus as arranged for this is shown in Figure 2. The solution used was nickel-ammonium sulphate made slightly acid with citric acid.

The nickel wires, six in number, two inside and four outside, were electroplated to the nickel. The two from the inside were brought through a fire-clay plug which would completely insulate them.

2. *Source of Heat.*

The heat is supplied to the interior of the shell by means of an electric heating coil (Figure 3) consisting of German-silver wire. Copper wires were soldered to these, which were taken out of the shell through the fire-clay plug. The oil in the cavity was agitated by two sets of fans soldered to a shaft of steel .7 mm. in diameter. This extended out through the fire-clay plug. A pulley on this shaft was connected by a belt to the small motor M_1. The fans were about 3 cm. in diameter, and so arranged that the oil would circulate towards the centre of the cavity or away, according to the direction of rotation of the fans.

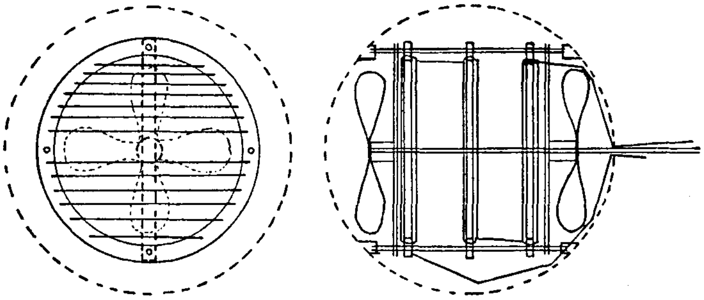


FIGURE 3.

A battery of storage cells supplied the necessary current at a constant pressure.

3. *Constant Temperature Bath.*

For the details of the constant temperature tank see previous article.

DATA.

1. *Constant Data.*

The following are the dimensions of the sphere:

External radius . . .	8.742 cm. ± .00055 cm.
Internal radius . . .	3.695 cm.
Thickness	5.045 cm.

2. *Experimental Data.*

Procedure. — The general method of procedure in taking a set of readings was to adjust the temperature of the large tank to approximately the temperature at which it was desired to take readings. Then the fans in the cavity of the shell were set in rotation and the current from the storage cells turned into the heating coil. The temperature of the outside was adjusted until it remained constant, then the apparatus was allowed to run constant for approximately two hours, tests being made from time to time to determine when it had become constant. The deflection of the galvanometer was read every two minutes. The ammeter reading was maintained constant by an adjustable resistance which would permit of a very small change in the current. The fall of potential across the heating coil was balanced against the Carhart-Clark cell several times during a run, which was usually thirty-two minutes. The variation in the temperature of the oil surrounding the shell was taken at frequent intervals. The galvanometer was immediately calibrated after the readings had been taken.

Readings. — The following set of readings illustrates the constancy of the various quantities. The first column gives the actual readings on the galvanometer, the second gives the difference between each two successive readings, i. e. the double deflection, the third and fourth give the current and the ratio from which to determine the voltage. The column marked "Temperature Tank" gives the variation of the temperature with time of the oil surrounding the shell.

THERMAL CONDUCTIVITY. MARCH 16, 1905.

Readings.	2 D.	Amperes.	Volts.
98.15	. .	4.500	209177
5.80	92.35		9177
98.40	92.60		Ohm at 21°.5 C.
6.00	92.40		C.-C. cell at 21°.6 C.
98.35	92.35		
5.85	92.50		
98.30	92.55		
5.95	92.35		
98.25	92.30		
5.85	92.40		

Mean, 92.433 ± 0.024

Galvanometer Calibration.		Temperature Tank.
98.20	. .	5.203 at 11:03
6.30	91.90	5.202 at 11:08
98.23	91.93	5.200 at 11:16
6.22	92.01	5.199 at 11:28
98.23	92.01	
6.19	92.04	Tank at 92°.4 C.
98.19	92.00	Inside at 101°.7 C.
Mean,	91.982 ± 0.014	Mean at 97°.8 C.

R = 190000 in series with storage cell and ohm.
Ratio to balance Clark cell against storage cell:
209177
9177

THERMAL CONDUCTIVITY. MARCH 18, 1905.

Readings.	2 D.	Amperes.	Volts.
12.65	. .	4.500	209180
91.90	79.25		9180
12.65	79.25		Ohm at 27°.7 C.
92.15	79.50		C.-C. cell at 22°.5 C.
12.65	79.50		
92.10	79.45		
12.65	79.45		
92.00	79.45		
Mean,	79.407 ± .070		

Galvanometer Calibration.		Temperature Tank.
91.10	. .	3.190 at 1 : 44
13.80	77.30	3.189 at 1 : 50
91.10	77.30	3.180 at 1 : 57
13.85	77.25	3.180 at 2 : 11
91.10	77.25	
13.85	77.25	
Mean,	77.270 ± .020	

R = 220000 Tank at 48°.5 C.
Ratio: 209183 Outside at 53°.2 C.
9183 C.-C. cell at 22°.7 C. Mean, 56°.7 C.

MEANS OF DIFFERENT SETS OF READINGS.

Date.	2 D.	Amperes.	Volts.	Temperature.	Thermo E. M. F.
March 16	92.42	4.570	32.82	97°.8	173.20 M. V.
" 18	79.41	4.570	32.75	56°.7	153.01
" 21	63.08	4.570	21.94	102°.5	117.52
" 22	58.22	4.570	22.36	77°.6	111.76
" 23	54.40	4.570	21.01	71°.2	101.07
" 24	65.10	5.079	23.93	58°.0	126.29
April 3	29.95	5.079	13.04	21°.9	60.77
" 4	29.65	5.087	13.02	22°.7	60.40

THERMO-ELECTROMOTIVE FORCE.

The thermo-electromotive force per degree was determined by placing the hemispheres in two adjacent tanks, connecting them with a piece of the original lead, and maintaining these tanks at a constant difference of temperature, this difference of temperature being measured by means of two Beckman thermometers, one of them being a standard. The electromotive force thus produced was measured by noting the double deflection on a galvanometer and then calibrating this galvanometer deflection against a known electromotive force.

Apparatus (Figure 4). — This apparatus consists of two cylindrical sheet iron tanks, each 35 centimeters in diameter and 45 centimeters deep, packed in a wooden box 45 × 50 × 100 centimeters. One hemisphere was suspended on a wooden frame at the centre of each tank. A fan at the bottom of each tank, which could be rotated in either direction, and driven at various speeds by means of an electric motor, served to keep the oil in the tanks in motion.

The frames supporting the hemispheres extended into the tanks about 3 cm., thus preventing the oil from taking up a regular rotary motion. These frames also served to hold the wires of the heating coil, by means of which the temperature of the tanks could be raised. With a bank of lamps and an adjustable resistance in series with the coils, the tanks could be maintained at any desired temperature above that of the room.

The two hemispheres were joined by a piece of the same lead as the spheres. In order to insure perfect electrical contact, a small hole about 0.1 cm. in diameter was drilled into each hemisphere, to a depth of about 0.5 cm., and the end of the lead wire was carefully scraped and forced into this hole with a wooden wedge.

The leads from the nickel were brought to different points on the

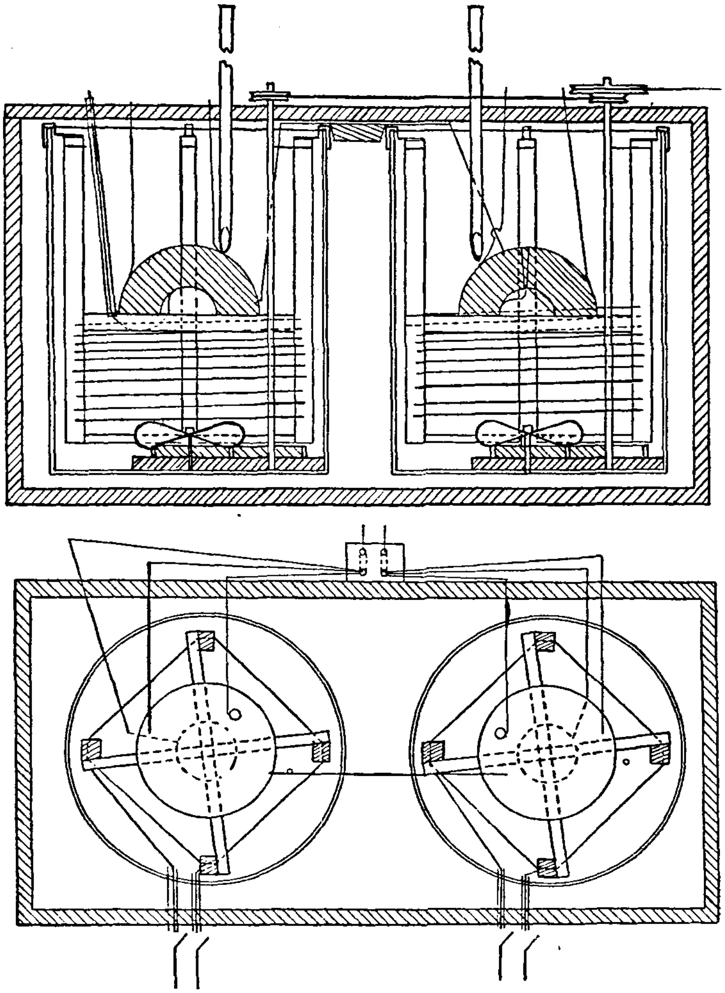


FIGURE 4.

wooden cover of the box which contained the tanks, the leads from the inside being insulated in glass tubes, so that there would be no chance of their touching the sphere at any point except where they were attached.

These leads were brought to two mercury cups which could be connected directly to the galvanometer.

The Beckman thermometers were so placed that they came in contact with the sphere where the nickel wires were attached.

Procedure. — The temperature of the tanks was adjusted until the difference between them was less than 5° C. The standard Beckman was placed in one tank and Beckman No. 2 in the other. The readings were then taken every two minutes, the galvanometer deflection being taken at the same time. After a series of readings the standard thermometer was transferred to the other tank and placed in contact with the sphere as close to Beckman No. 2 as possible. After an interval of sufficient length to insure the two thermometers being at the same temperature, their readings were taken, the tank meanwhile being maintained as nearly as possible at its original temperature. If this tank has not changed in temperature during the comparison of the thermometers, then the difference in the readings of the standard, after correction for calibration, etc., will give the true difference in temperature of the tanks to 0.001° Centigrade. If, however, the reading of Beckman No. 2 has changed, it will be necessary to add or subtract this reading to the difference of the readings of the standard. The true temperature of one of the tanks was taken by means of an ordinary thermometer reading directly to tenths of a degree Centigrade.

READINGS ON THERMO-ELECTROMOTIVE FORCE.

The following set of readings illustrates the method and constancy of the different readings:

THERMO-ELECTROMOTIVE FORCE. FEBRUARY 20, 1905.

All Junctions.		Tank No. 1 Higher at 51°.4.	
Readings.	2 D.	Standard.	Beckman.
79.18	. .	4.289	3.850
49.31	29.87	4.289	3.851
79.15	29.84	4.289	3.853
49.31	29.84	4.289	3.853
79.18	29.87	4.289	3.854
49.33	29.85	4.291	3.854
79.15	29.82	4.292	3.855
49.32	29.83	4.293	3.855
79.18	29.86	4.294	3.858
49.30	29.88	4.294	3.857
	29.852 ± .005	4.291 ± 0.005	3.854 ± .0004

Galvanometer Calibration.	
50.14	. .
79.70	29.56
50.12	29.58
79.71	29.59
50.11	29.60
	29.582 ± .0052

Comparison.	
1.659	3.823
1.628	3.792
1.633	3.797
1.633	3.797
1.638	3.701

R = 75000 at 21°.1
Ohm, at 21°.0
Clark cell, at 21°.1
Ratio: 77430
27430

Temp. Diff., 2°.6504

E. M. F. per degree, 20.58 M. V.
at 50°.0

Means of Readings on the Thermo-Electromotive Force.

Date.	No. of Junction.	2 D.	Diff. of Temp.	Temp.	E. M. F. per degree.
Feb. 7	1 and 4 outside	41.51	4°.318	44°.4	20.56 M. V.
" 7	1 and 3 outside	43.14	4°.366	44°.4	20.57 "
" 20	All	29.85	2°.650	50°.0	20.58 "
" 22	All	35.09	3°.066	76°.5	21.43 "
" 22	1 and 3 outside	32.50	3°.061	76°.5	21.39 "
" 23	All	55.43	4°.814	87.°6	21.81 "

These results are expressed graphically in the following curve (Figure 5).

Sources of Error in Calibrating Thermo-Electromotive Force.

1. *Difference of temperature.* — The difference of temperature was determined by a Beckman thermometer standardized between 0° and 100° by the Reichstanstalt, to an accuracy of $\frac{1}{1000}$° for a difference of temperature not exceeding 5°. All corrections were applied to the standard.

2. *True temperature.* — An error in this reading of 0°.5 would cause an error of less than $\frac{1}{10}$ of 1 per cent in the value of the electromotive force per degree. The error in the thermometer used for this purpose was much less than this.

3. *Resistance of nickel lead wires.* — This error was large, due to the fact that the galvanometer was connected in series with this resistance when finding the deflection, but when the galvanometer was calibrated the deflection was produced by the difference in potential at the termi-

nals of a standard ohm, consequently the resistance in this case in series with the galvanometer was zero. It therefore became necessary to introduce into the galvanometer circuit when calibrating it a resistance equal to the nickel leads. This was done by means of two mercury cups which were connected by a coil of copper wire, thus eliminating the possibility of any additional thermo-electromotive force. The resistance of the nickel leads was determined to a hundredth of an ohm, it being shown that a change of this amount in the galvanometer circuit produced a negligible change in the deflection.

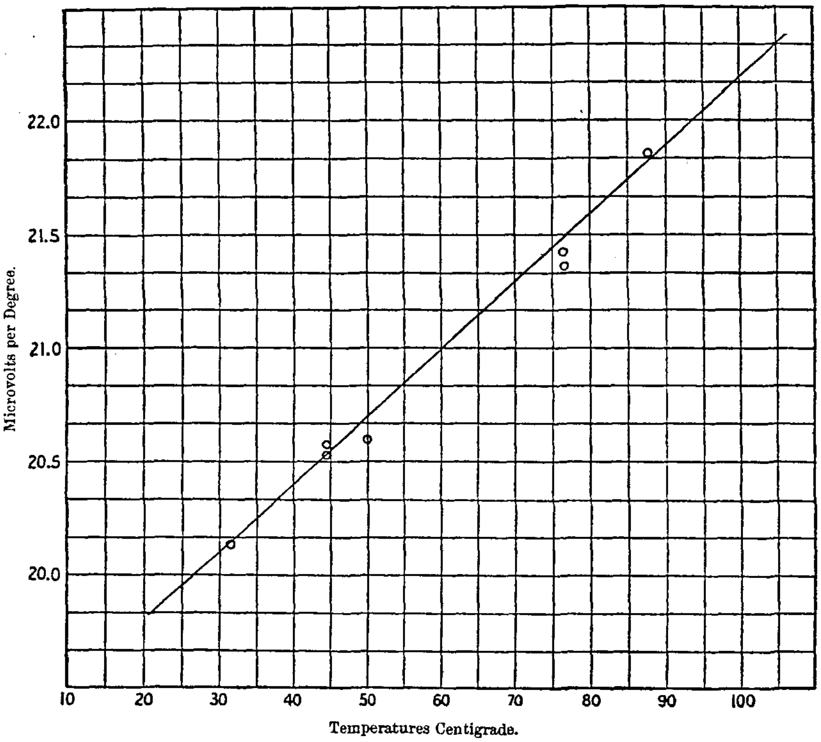


FIGURE 5.
Diagram of Thermo-Electric Force.

4. *Thomson effect.* — The resultant thermo-electromotive force per degree is the sum of the Seebeck and Thomson effects. It is to be noted that the conditions in determining the electromotive force are almost

exactly identical with the conditions under which the electromotive force is to be employed; as, for instance, the nickel wires will extend in each case from a bath of hot oil to the cold air, and the difference of temperature between the oil and the air at any one particular temperature will be practically identical in the two cases. Again, the nickel wires are to terminate in mercury cups, which will in both cases be at the same temperature with respect to the oil. Also the galvanometer will always be connected to the nickel leads in exactly the same way. Again, the terminals of the nickel wires which dip into the mercury were treated alike in each case, — that is, polished with emery cloth, etc.

5. *Thermo-electromotive forces in the galvanometer circuit.* — Reversed readings were taken which would eliminate any electromotive force in the galvanometer circuit. In order that the galvanometer might be as free as possible from any effects produced by changes in temperature, it was placed in a box 30 × 30 × 46 cm., lined with pressed hair 5 cm. thick; a small opening, covered with glass, gave a view of the mirror. A thermometer was placed inside to determine the changes of temperature. When taking a series of readings under the best conditions this thermometer would indicate no change whatever. The room was maintained at a constant temperature to within 2° C., and during the time necessary for a set of readings the variation in temperature was usually much less than 0°.5.

6. *Electromotive force at the contact of the galvanometer terminals in the mercury cups.* — That an electromotive force of considerable magnitude might exist at this point was discovered by connecting the two mercury cups in which the nickel wires terminated by a single nickel wire, then placing an electric light near the mercury cups; they were heated unequally and a marked deflection of the galvanometer was obtained. While both cups were maintained at the same temperature no deflection was noted, but it is possible that the Thomson effect caused by the wires coming from the air into the mercury would be different unless the mercury cups were maintained at approximately the same temperature for different sets of readings, consequently they were surrounded with compressed hair in a wooden box 25 × 25 × 25 cm., and this was maintained at room temperature. When the room temperature differed greatly from the normal, no attempt was made to take readings.

7. *Resistances.* — All resistances were accurate to .1 of 1 per cent or better. All corrections for changes of temperature, etc., were made.

8. *Standard electromotive force.* — The electromotive force used to calibrate the galvanometer was obtained from a number of storage cells,

connected in series through a resistance, and the drop of potential across this resistance used as the source of electromotive force. This was counterbalanced against a Carhart-Clark cell, which had been recently calibrated.

9. *Sensitiveness of the galvanometer used to balance the electromotive force against the Clark cell.* — This was of sufficient sensitiveness to detect a change of 2 ohms in 10,000 ohms in the Carhart-Clark cell circuit.

10. *Insulation.* — The galvanometer leads were carried through air to the stone pier upon which the galvanometer rested. The leads from the heating coil were brought out through the box in glass tubes. The nickel wires were carefully insulated both inside the tanks and out, and were carefully separated even when entering the mercury cups.

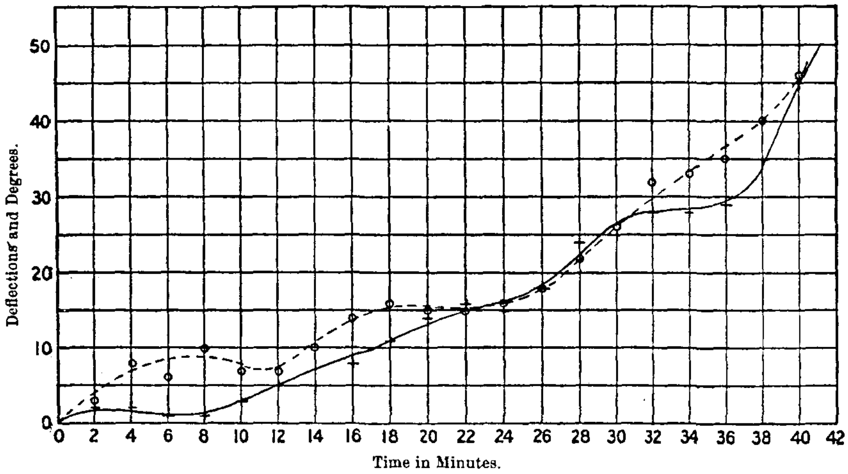


FIGURE 6.

Comparison of variation (*vide* error 11).

11. *Variation in temperature during a reading.* — Since it was impossible to maintain the tanks at an absolutely constant temperature, it was important to know how closely the galvanometer deflections varied with the variations in the readings of the thermometers. In order to show this, a special set of readings was taken with large variations, and the following plot (Figure 6) shows the results. The results for the plot were obtained by taking the difference in temperature, and the gal-

vanometer reading, subtracting each from the first reading, and plotting this difference as ordinates and the time as abscissa. The two curves thus obtained should be approximately the same if the difference in temperature and galvanometer deflection vary at the same rate.

12. *Variation in difference of temperature during a reading.* — That this variation was less than .1 of 1 per cent is shown by the results.

13. *Change of zero in the galvanometer.* — Double deflections were always used, consequently the zero was not needed.

14. *Different junctions give the same electromotive force.* — See Summary of Results.

15. *A variation in the difference of temperature gives the same result for the electromotive force per degree.* — See Summary of Results.

16. *The resultant electromotive force per degree is the same no matter which tank is at the higher temperature.* — See Summary of Results.

17. *The temperature of the oil not the same as that of the spheres.* — The thermometers were placed in the same relative position in both tanks, and since the tanks were constructed the same, and the oil rotated in the same direction, and the difference in temperature being small, the thermometers would both be at the same relative amount above or below the temperature of the spheres. Placing the thermometers in different parts of the tank gave the same difference.

18. *Variation of rate of agitation of oil.* — The rate and direction of rotation of the oil was changed repeatedly, but no effect could be discovered in the results. The direction of rotation was the same in both tanks, but the rate was sometimes the same and sometimes quite different.

For errors in calibrating galvanometer see errors under conductivity.

19. *Error in Beckman thermometer No.* 2. — When comparing the two thermometers, if the tank has changed in temperature, then it becomes necessary to use the difference of two readings of Beckman No. 2, but as this difference never exceeded a few hundredths of a degree it was assumed that there would be no error.

20. *Error in connecting the two hemispheres.* — It was found in the research on iron by this method that a materially different electromotive force was obtained when the position of the iron connecting the hemispheres was changed. A number of experiments were made to clear this point. It was found that the electromotive force was independent of the method of attaching and position of the connecting wire on the hemispheres as long as the electrical contact was good. This was a simple matter with lead.

Sources of Error in Measuring Conductivity.

1. *Purity of lead.* — The lead was obtained as pure as possible in the granulated form. Every precaution was taken in casting to keep it free from impurities. Clean chips from both parts of the shell gave 99.16 per cent pure. Impurities, iron and aluminium. Density: 11.313 gms. ± 0.018 per c.c. at 22° C.

2. *Blow-holes.* — Probably the most difficult correction to determine was that due to blow-holes. Every precaution was taken in the casting to make it free from blow-holes. In the turning, constant and careful watch failed to show the least trace of blow-holes, although a thickness of about 6 mm. was taken from both inside and outside.

The presence of large blow-holes in either half would show in the weight. A special balance was constructed for this purpose, which was sensitive to 50 mmg. under full load. The weight was found in this way to be: —

Top	14,640.98 gms.
Bottom	14,642.66 gms.
Difference . . .	1.68 gms.

This would indicate that the variation in the amount of lead in the two parts is less than .013 per cent. This, together with the close agreement of the volumes as found by measurement, would indicate the probable absence of large blow-holes.

3. *Variation of conductivity due to the two halves not being close together.* — That this is negligible is shown as follows: The surface of the cavity from which heat is conducted equals 172 cm^2. If the two parts are separated by a non-conducting space of .1 mm., the amount of surface in the cavity would be decreased by only .2 cm^2. The surfaces fitted so well that it was practically impossible to slide one over the other. In order to be certain that no oil flowed in or out, the outside junction of the two hemispheres was sealed with shellac.

4. *Conduction through hole carrying shaft and wires.* — This hole at the surface of the cavity was 4.8 mm. in diameter. If this hole was filled with a non-conducting material the error would amount to less than .2 per cent. But the hole was partly filled with copper, nickel, and steel, all of which have a greater conductivity than lead, and partly by fire-clay, which has a smaller conductivity than lead. These would tend to neutralize each other and make this error negligible.

5. *Variation in source of energy.* — The energy was supplied from a battery of storage cells. A rheostat with a very fine adjustment was connected in series, so that any change might be counterbalanced. Adjustment of this rheostat was made every minute if there was any change in the current. In this way the movement of the needle of the ammeter was less than one twentieth of a division.

6. *Energy supplied by agitating fans in the cavity.* — These fans rotated about 800 times per minute. The energy supplied was determined by placing the coil with fans in a calorimeter containing oil, rotating the fans at the required rate, and determining the rate of increase of temperature not exceeding 1° C. Energy was then supplied to the coil until the rate of increase was approximately double.

Three determinations resulted as follows:

0.366 watts
0.344
0.385
Mean, 0.365 ± 0.10.

Since the total energy supplied was about 100 watts, this correction was determined with sufficient accuracy.

7. *Correction on dimensions of shell.* — The shell was measured by a special micrometer caliper made by Brown and Sharpe for this work. This was compared with a standard meter, which was compared by the Société Genevoise with their standard.

8. *Variation in the thickness of the nickel.* — When the nickel was being plated to the lead it was first plated on the outside, frequent measurements being taken to insure its being of a uniform thickness, the electrodes being adjusted when it was necessary to increase the thickness at any particular place.

During the plating of the inside, measurements were taken in the same way, so that the thickness of the nickel probably does not vary over 0.2 mm. However, in the manipulation of the hemispheres small portions of the nickel cracked off, from the outside near the edge of the hemispheres. The amount cracked off in this way is approximately 5 sq. cm., the largest amount in one place being perhaps 2 sq. cm.

9. *Conduction of the heat from the shell at the points where the nickel wires were electroplated.* — In electroplating the wires to the shell the nickel on the shell was filed slightly and a small depression made in which to put the wire. After plating over the wire, the thickness of the

nickel at this point varied little from that at any point. The wires were 0.3 mm. in diameter.

10. *Change in temperature in the oil surrounding the shell.* — It was impossible to maintain this oil at exactly a certain temperature. The temperature was therefore allowed to vary, part of the time increasing and part of the time decreasing, and by the same amount both times. This total change did not exceed a few hundredths of a degree. See Readings.

11. *Variation in the agitation.* — The fan for agitating the oil on the outside of the shell was always driven in the same direction, as it was evident from the motion of the oil that this direction gave much better agitation close to the shell.

On the inside of the shell the fans were usually driven in the same direction, and always at the same speed, but a change in the direction did not appear to produce any change in the deflection. It would appear from a consideration of the construction of the heating coil that a change in the direction of rotation inside the cavity should produce a change in the galvanometer deflection. That it did not may be due to the position selected at which to attach the nickel wires on the inside.

12. *Amount of oil in the cavity.* — The fire-clay plug had two holes by which oil could enter or air leave when the shell was immersed in oil. At first the shell was immersed in oil and the air exhausted from over it by means of a pump, but it was found that this was an unnecessary precaution. A small wedge which could be easily withdrawn was placed between the two hemispheres, thus allowing the air and oil to enter the cavity at the separation of the spheres, and also through the fire-clay plug. The oil was agitated in the cavity and at the same time heated to expel the air. The wedge was withdrawn and the holes in the fire-clay plug sealed with shellac.

13. *Calibration of the ammeter.* — The ammeter read from zero to 5 amperes, the smallest division being 0.05 of an ampere. The resistance of the tenth of an ohm in series with the ammeter was accurate to $\frac{1}{50}$ of 1 per cent. The drop of potential across its terminals could be balanced against the Carhart-Clark cell to an accuracy of 0.1 per cent. Consequently it was possible to determine the current to within 0.1 per cent.

14. *Determination of voltage.* — This was balanced directly against the Carhart-Clark cell, and the galvanometer was sensitive to better than 0.1 of 1 per cent.

SUMMATION OF RESULTS.

Date		Temp.	Diff. T.	C. V.	K
March	16	97.8	7.844	149.97	.05673
"	18	56.8	7.321	149.68	.06067
"	22	77.6	5.198	102.17	.05832
"	23	68.0	4.763	96.03	.05980
"	21	102.5	5.289	100.26	.05645
"	24	58.0	6.034	121.56	.05978
April	3	21.9	3.057	66.24	.06429
"	4	22.7	3.034	66.21	.06475

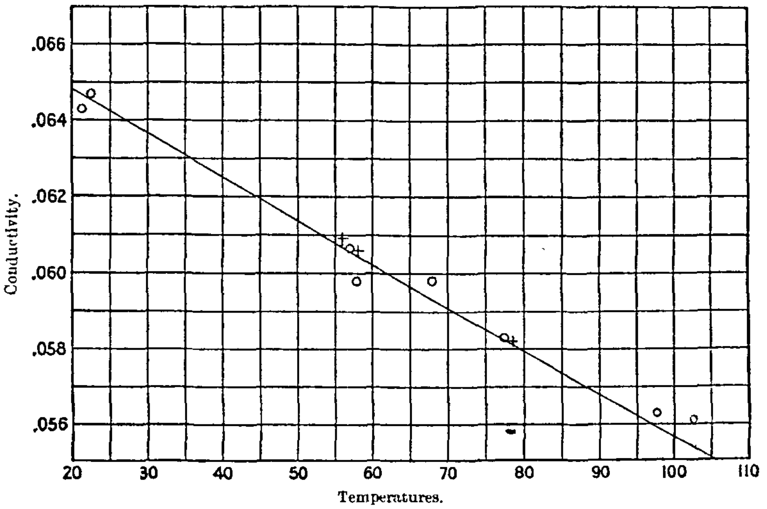


FIGURE 7.

$K_t = 0.0645\ [1 - 0.00159\ (t - 20)]$.

Note: Results marked + are preliminary.

DISCUSSION OF RESULTS.

From the plot of the value of K we find the average deviation of the results to be less than 0.15 per cent, — the maximum variation being a little over 1 per cent. The coefficient $= \frac{K_{100} - K_{20}}{80\, K_{20}}$ as obtained from these results is found to be 0.00159.

A point to be noticed is that the amount of energy supplied to the interior of the shell as represented by the watts was never the same, and varied from 66.21 to 149.97; and again, the difference in temperature varied from 3°.034 C. to 7°.844 C.; consequently we would conclude that the method within these limits is independent of the total energy supplied to the inside, and to the difference in temperature. Three different forms of heating coil being used would indicate that the method was independent of the form of the coil. Another point to be noticed is that four preliminary results, taken when the apparatus was set up in the laboratory at Bradley Institute, agree better than 0.5 per cent with the results obtained with the apparatus used in the Ryerson Laboratory. This would imply that the results here obtained are independent of all the electrical apparatus employed. The results for the thermo-electromotive forces give values and slope of the curve, which are almost identical with the ordinary values given for pure lead and pure nickel. Since the thermo-electromotive force between lead and nickel has been frequently determined and with a high degree of accuracy, we may infer that the results here obtained for the thermo-electromotive force contain no constant error, and as shown by the data are probably accurate to 0.1 per cent. From which we may infer that the error in measuring the electromotive force when determining the thermal conductivity is also accurate to 0.1 per cent.

I wish to express my thanks to the Rumford Committee for financial assistance, and to Professor A. A. Michelson and Dr. R. A. Millikan for many courtesies extended to me at the Ryerson Physical Laboratory.

Physical Laboratory, Bradley Institute,
Nov. 4, 1905.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

THE LONGITUDINAL VIBRATIONS OF A RUBBED STRING.

By Harvey N. Davis.

With Three Plates.

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THE LONGITUDINAL VIBRATIONS OF A RUBBED STRING.

By Harvey N. Davis.

Presented by Wallace C. Sabine, October 11, 1905. Received January 25, 1906.

§ 1. Introductory.
§ 2. Aliquot Cases — Experimental Work.
§ 3. Aliquot Cases — Theory.
§ 4. Rational Cases — Krigar-Menzel's Law.
§ 5. Integral Surfaces.

§ 1. Introductory.

This work was originally undertaken as the first step in an investigation of the effect of vibration on magnetic quality. In the previous study of this subject, although many exact measurements of the magnetic phenomena had been made by a number of writers, in no case had the conditions been such as to admit of an equally exact description of the mechanical phenomena involved. It seemed desirable, therefore, to renew the attack upon the subject, using as the mechanical disturbance a steady state of vibration whose character was known, and whose intensity could be accurately measured and easily controlled. It was found that longitudinal vibrations suitable for this purpose could be maintained in a steel wire by means of a mechanically operated rubbing wheel, and a study of their nature has shown them to be identical in all respects with the familiar transverse vibrations of a violin string, so that the parallelism between longitudinal and transverse vibrations which the mathematics of the subject predicts, finds here a unique and exceedingly detailed experimental verification. This purely mechanical discussion has thus acquired a considerable interest in itself, and has, therefore, been carried far beyond the requirements of the original problem. It is now presented as a separate paper; the magnetic problem will be considered at another time.

In two respects the present paper goes beyond the published work on the violin string. In the first place, the fact that a given state of vibration can be maintained indefinitely and can be reproduced at will has made it possible, for the first time, to measure amplitudes under conditions which are accurately known. This has resulted in a good quantitative verification of the established theory, and especially of the law connecting amplitude with bowing speed. And in the second place, it is hoped that the graphical method which is here presented for solving the differential equation of vibrating strings, subject to the most general boundary conditions which the problem of the rubbed string requires, will prove to be a valuable addition to the theoretical side of the subject. It is possible that this part of the paper will have some interest for the pure mathematician, for it is rather unusual that graphics should be able to give, not simply an approximate, but an exact and rigorous solution of a problem of such complexity. A striking instance of this kind is, of course, Euler's solution of the problem of the plucked string.

As has been indicated, all previous work on rubbed strings has been concerned with transverse vibrations. In this form the problem is fully as old as that of the plucked or of the struck string, but it differs essentially from them in that it is not easy to set up, *à priori*, a satisfactory analytical statement of the action of a bow, as was attempted by the earlier mathematicians.[1] The first results of any importance are due to Thomas Young,[2] who formulated for plucked strings, and less definitely for bowed strings,

YOUNG'S LAW: *No overtone is present which would have a node at the point of excitation.*

But if, he continues, the bowing be, not at a principal node, but near one, "the corresponding harmonic is extremely loud," a fact which has been very prettily verified by Krigar-Menzel,[3] who showed also that its phase changes suddenly by π when the bowing point passes through the node.

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[1] For example, D. Bernoulli, in his "Memoir sur les sons produit par les tuyaux d'orgue," likens the action of a bow to that of a toothed wheel, and J. Antoine [Ann. de chim. et phys., Sér. III, **27**, 191; Pogg. Ann., **81**, 544 (1850)] speaks of "a series of gentle blows." Duhamel [C. R., **3**, 646, (1836); C. R., **9**, 567, (1839); Mem. d. Savants étrang., **8**, 131 (1843)] properly assumes the action to be frictional, but his treatment is not entirely satisfactory. That of F. Lippich [Mittheil. d. deut. math. Ges. in Prag, **1**, 118 (1892)] is much better, but it would scarcely have been possible apart from the experimental work which preceded it.

[2] Phil. Trans. of the Roy. Soc. of London, **90**$_1$, 106 (1800).

[3] "Ueber d. Bewegung gestrich. Saiten," Inaug. Diss., Berlin, 1888. See also J. Ritz, "Zusammensetzung d. Klänge d. Streichinstr.," Inaug. Diss., München, 1883.

A notable advance was made by Helmholtz,[4] who, with the aid of Lissajous's vibration microscope, proved

HELMHOLTZ'S LAW: *When a string is bowed at an aliquot point* [1/k], *the part of the string immediately under the bow moves to and fro with constant velocities, whose ratio is equal to the ratio* [1/(k — 1)] *of the segments into which the string is divided by the point in question.*[5]

He also formulated the following

VELOCITY LAW: *The smaller of these two velocities has the same direction as that of the bow and is equal to it.*

In other words, "This point of the string adheres fast to the bow and partakes of its motion . . . then is torn off and jumps back to its first position . . . till the bow again gets hold of it." From these facts he derived his well known solution,

$$u = A \sum_n \frac{1}{n^2} \sin \frac{n \pi x}{l} \sin \frac{n \pi t}{T},$$

where l is the length of the string, $2T$ is its period, and A is a constant depending on the velocity of the bow. This single expression satisfies the necessary conditions for any aliquot case, and may be called Helmholtz's major solution. In any particular case it must be modified, in accordance with Young's law, by the omission of the kth, $2k$th, etc., terms; that is, by subtracting the expression

$$u' = A \sum_m \frac{1}{k^2 m^2} \sin \frac{k m \pi x}{l} \sin \frac{k m \pi t}{T},$$

which may be called the minor solution for the case in hand. If the point bowed is near one end of the string, as is always true in musical practice, the minor solution is relatively unimportant and may usually be neglected altogether. This work of Helmholtz's solves the problem for any aliquot bowing point completely, and has been ingeniously verified by a number of observers.[6]

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[4] "On the motion of the strings of a violin," Proc. Glasgow Phil. Soc., Dec. 19, 1860; reprinted in Phil. Mag., Ser. 4. **21**, 393 (1861). See also Helmholtz, Sensations of Tone, etc., 3d Eng. ed., 80–88, 384–387; Rayleigh, Theory of Sound, 2d ed., 208 *et seq.*; Donkin, Acoustics, 2d ed., 136–144.

[5] An aliquot point is a point whose distance from one of the fixed ends is $1/k$th of the length of the string, where k is an integer.

Non-aliquot, rational points were considered for the first time by Krigar-Menzel and Raps,[7] who showed that Helmholtz's law must, in the general case, be replaced by

KRIGAR-MENZEL'S LAW: *When a string is bowed at any rational point* p/q *where* p *and* q *are prime to each other, the part of the string immediately under the bow moves to and fro with constant velocities, whose ratio depends only on* q *and is* $1/(q-1)$.

This law, which includes Helmholtz's law as a special case, they verified for values of q up to 10, at which point it seemed to break down. In general, this breaking-point would be determined in some obscure fashion by such circumstances as the pressure and speed of the bowing, the material and size of the strings used, etc. It can easily be proved that in those cases in which Krigar-Menzel's law does hold, this law is enough, with Young's law and Helmholtz's velocity law, to determine the motion of the string uniquely. The theory is therefore complete for all simple rational cases, whether aliquot or not. It must be remembered, however, that it rests wholly on an empiric foundation, and that the foregoing laws, which are that foundation, were enunciated solely for the case of a string vibrating transversely under the action of a violin bow. This paper will show that they are equally applicable to the case of a string vibrating longitudinally under the action of a rough wheel.

The string upon which the following experiments were performed was a steel wire about 0.4 mm. in diameter (density 8.6; Young's modulus, as calculated from the observed frequency, 23,430 kg. wt. per mm^2). It was stretched horizontally, one end passing over a pulley and supporting 10 kgs., rather less than one third the breaking weight. Effective ends for the vibrating portion were obtained by clamping the wire firmly to the faces of two cast-iron blocks, each about 5 kg. in weight. The length of the vibrating portion was 676 cms. During most of the work a node was formed in the middle of the wire by means of a brass knife edge, the pressure on which was that due to a displacement of the wire of about a centimeter from its position of equilibrium. The wire was then rubbed on one side of the bridge and observed on the other, which

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[6] Mach, Pogg. Ann., **134**, 311 (1868); Clem. Neumann, Sitzber. I, and R. Acad. d. Wissenschaften, Wien, Math. Cl., **41**$_2$, 89 (1870); Clifton, cited by Donkin, Acoustics, p. 138. See also Mach, Zeitsch. f. d. phys. u. chem. Unterr., **1**, p. 264 (1888); Fernbach, ibid., **9**, 239 (1896); Kuhfahl, ibid., **10**, 92 (1897).

[7] Wied. Ann., **44**, 623 (1891).

made all parts of a vibrating segment accessible, and furthermore kept the point which was being observed almost entirely free from the lateral motion which imperfections in the shape of the rubbing wheel would otherwise have caused.

The excitor or "bow" of these experiments was a chamois-covered wheel dusted with powdered rosin. In order to give an unbroken rubbing surface the chamois was cut from a skin in the form of a ring and stretched over the edge of the wheel, a method which gave very satisfactory results. During the first part of the work a wooden wheel 15.1 cms. in diameter was driven by a 1/12 H. P., A. C. motor through a system of wooden pulleys and belts which could be so adjusted as to give the peripheral velocity any one of eleven values from 5 to 91 cms. per second. A more satisfactory wheel 4.9 cms. in diameter was subsequently made of brass, to avoid warping, and was driven by the same motor through friction gears of brass and rubber so adjustable as to give a continuous variation in its peripheral velocity from about 3 cms. per second up; it was mounted on the slide rest of a small lathe bed, the two feed screws giving slow motions along and perpendicular to the wire.

These experimental conditions are unusually favorable, as compared with those of previous observers, in three respects. In the first place, as has already been noted, the use of a wheel instead of a bow enables a given state of vibration to be kept up indefinitely and to be reproduced at will, so that comparable observations can be made at different parts of the wire, and the corresponding "velocity of the bow" measured. In the second place, the vibrating segments are much longer than would have been possible for transverse vibrations, so that given points can be more accurately located and the effective width of the "bow" is more nearly negligible. And finally, the system is very nearly "perfectly flexible" because the sharp corners of a transversely vibrating string are replaced by points whose only peculiarity is a change in the law of distribution of density. It is very true that sudden jumps in this law necessarily imply instantaneous changes of finite magnitude in the velocities of the elements of the wire, but, other things being equal, "corners" would not be rounded off so much for longitudinal as for transverse vibrations. On the other hand, the amplitudes obtainable in this way, even with long strings, are much smaller than those of transverse vibrations, and furthermore it is always difficult to perform mechanically so delicate an operation as that of bowing a string, so that the more complicated vibration forms have sometimes been very hard to set up.

§ 2. Aliquot Points — Experimental Work.

The observations to be described in this section were made with the help of glass balls from 0.1 mm. to 0.2 mm. in diameter, which were formed by melting the end of a fine glass fibre in a flame, broken off, and fastened on the wire with a touch of shellac. When illuminated from the side by a Nernst filament behind a small hole in a screen, and observed through a stationary microscope, such a ball, acting as a spherical mirror, gives a bright point image of the source. When the wire is vibrating this point of light becomes a line, and, in the case of the rubbed string, this line is always bisected by the position of the point when the wire is at rest; half its length is, therefore, the amplitude of the vibration at the point in question. If the motion were transverse, the curve obtained by plotting the amplitudes which correspond to a single speed of the rubbing wheel, against distances along the wire, would be one boundary of a region actually swept over by the wire during each vibration, that is, a region which one would see if a white string were vibrating over a black background. In other words, this curve would be the envelope of the various configurations of the wire. In the case of longitudinal vibrations these geometric notions are inapplicable, but it will still be convenient to plot amplitudes as though they were transverse, and to call the resulting curve the envelope of the corresponding vibration form. Such envelopes were obtained for each of the aliquot cases $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, and were found to consist of 2, 3, 4, and 5 straight lines respectively, the corners corresponding to points which divide the wire into 2, 3, 4, and 5 equal parts respectively. The shapes and relative magnitudes of these four envelopes for the same bowing speed are shown in Figure 1, the amplitude of the point bowed increasing directly as the distance of that point from the more distant end of the string,[8] and the other corners being determined by the intersections of two sets of rays

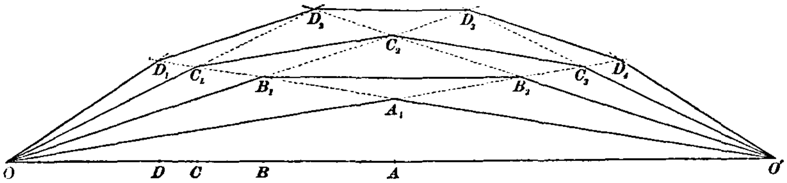

Figure 1. The envelopes for the aliquot cases $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

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[8] In other words, the points A_1, B_1, C_1, and D_1 are collinear.

drawn from the ends of the base line in the manner indicated.[9] In the general case, $1/k$, the envelope would be k straight lines similarly determined. When the speed of the rubbing wheel was varied, all other conditions remaining the same, the amplitude was found to vary directly as the peripheral velocity of the wheel, a fact which was tested for a considerable range of velocities in each of the four cases considered. This fact is an obvious consequence of Helmholtz's two laws.

In all, 331 amplitudes were measured, each observation involving, in general, six settings of a filar micrometer. The results are summarized in Table I. The first column gives the position of the point rubbed, and the second, the position or range of positions of the point observed, in fractions of the length of a vibrating segment; the third gives the peripheral velocity of the rubbing wheel in cms. per second. In the fourth column is given a mean standard amplitude M, obtained from the actually observed amplitudes m, first, by multiplying each of them by an appropriate factor obtained directly from the geometry of Figure 1, giving a

TABLE I.

Point rubbed.	Point observed.	Peripheral Velocity of Rubbing Wheel.	$M \times 10^4$, in cms.	No. of Observations.	Weight.
$\frac{1}{2}$	.091 to .895	22.4	$3.29 \pm .01$	62	6
	.497	5.1 to 90.9	$3.28 \pm .01$	62	6
$\frac{1}{3}$	.091 to .910	30.4	$3.27 \pm .01$	39	4
	3.37	9.9 to 59.3	$3.19 \pm .01$	7	1
$\frac{1}{4}$	.050 to .922	25.7	$3.20 \pm .01$	42	4
	.500 and .750	5.1 to 59.3	$3.24 \pm .02$	40	4
$\frac{1}{5}$	.400 to .919	18.0	$3.34 \pm .01$	67	7
	.400	9.9 to 25.7	$3.24 \pm .01$	12	1

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[9] The distance A A_1 having been arbitrarily assumed, the points B_1, C_1, and D_1 on the line O′ A_1 are determined by the bowing points B, C, and D, and themselves determine the rays through O. The symmetry of the figure determines the corresponding rays through O′.

value for the fundamental ordinate AA_1 of that figure, and second, by dividing this fundamental ordinate by the corresponding peripheral velocity of the rubbing wheel. The resulting M is, therefore, the amplitude in centimeters, of the motion which the middle point of the string would have if the string were rubbed at that middle point with a speed of one centimeter per second. It should be a constant under all circumstances. The smallness of the probable error in the second, fourth, sixth, and eighth lines of the table is a verification of the speed law stated in the last paragraph. Its smallness in the other four cases shows that the shape of each individual envelope is that which Figure 1 predicts. This is shown in another way in Figure 2, in which the observations of these groups are plotted in the manner already mentioned. The envelope in each figure is the best line *of the given shape* that could be drawn. And finally the constancy of M throughout the table shows that the relative magnitude of the envelopes is as described. This verification is not quite so good as the individual probable errors would lead one to expect, because of certain errors, largely in the adjustment of the brass bridge, which are constant throughout any one set of observations, but vary from set to set. The weighted mean of the results in the table is

$$M = .000327 \pm .000001 \text{ cms.}$$

In the case $\frac{1}{2}$ to which this standard ordinate would correspond, the two velocities under the bow would be equal, if Helmholtz's law is applicable in longitudinal cases. If F be the frequency of the string, $4MF$ would be the distance travelled by its middle point in one second, and this should, according to the velocity law, be equal to the corresponding velocity of the bow, that is, to 1 cm. per second. Therefore $F = 1 / 4M$. This formula gives

$$F \text{(calculated)} = 764 \pm 2.$$

A Valentine and Carr set of tonometer forks gave, by the method of beats,

$$F \text{ (observed)} = 762,$$

an excellent quantitative verification of Helmholtz's two laws for longitudinal vibrations.

§ 3. ALIQUOT POINTS — THEORY.

In the preceding section it has been shown that the amplitudes which have been measured in the various aliquot cases considered could all be

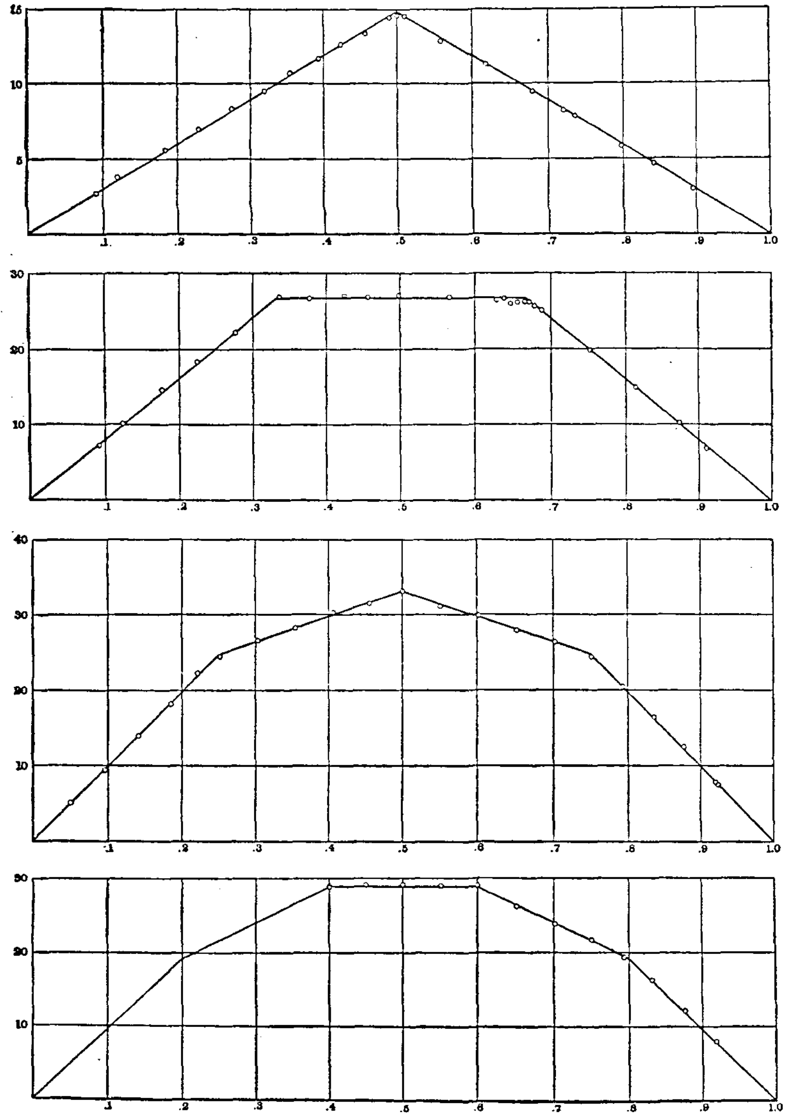

FIGURE 2. Amplitude measurements on a string bowed at points $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of its length from one end. The ordinates are the amplitudes actually observed, in hundredths of a millimeter. The node formed by the brass ridge is at the right.

brought under one simple, but as yet apparently arbitrary, geometric law which has been called Figure 1. In this section it will be proved that this law is a necessary consequence of Young's law, Helmholtz's law, and the velocity law.

The essential characteristics of the geometry of Figure 1 are, first, that the points B_1, C_1, D_1, etc., and B_2, C_3, D_4, etc., lie respectively on the lines $O'A_1$ and OA_1 produced; and second, that the other vertices of the figure are the intersections of two pencils of rays through O and O′ determined by these sets of points. These two statements will be considered in turn.

Let a string of length l be bowed downward at the point $1/k$. According to Helmholtz's law, the motion of the point bowed would be an ascent with a constant velocity f_0, followed by a descent with a constant velocity g_0, and

$$\frac{g_0}{f_0} = \frac{1}{k-1}.$$

Let the amplitude of the point bowed be m_0. Then the time of one oscillation would be

$$\frac{1}{F} = \frac{2\,m_0}{f_0} + \frac{2\,m_0}{g_0} = \frac{2\,m_0}{g_0}\,\frac{k}{k-1},$$

where F is the frequency. Therefore

$$m_0 = \frac{k-1}{k}\,\frac{g_0}{2\,F}.$$

According to the velocity law, g_0 would be equal to the velocity, V, of the bow. Therefore

$$m_0 = \frac{k-1}{k}\,\frac{V}{2\,F} = \frac{V}{2\,Fl}\cdot\frac{k-1}{k}\,l.$$

Now for any of the points, A_1, B_1, C_1, etc., of Figure 1, say C_1, $(k-1)\,l/k$ is the distance O C, and m_0 is the distance CC_1. Therefore, if V is independent of k,

$$\frac{AA_1}{O'A} = \frac{BB_1}{O'B} = \frac{CC_1}{O'C} = \text{etc.},$$

and $O'A_1B_1C_1D_1$, etc., is a straight line. The relative magnitude of the envelopes is, therefore, that which the figure indicates.

There remains to be proved only the fact that the shape of the kth envelope (*a*) as given by Figure 1, and (*b*) as calculated theoretically, is k straight lines inscribed in a parabola.

Take the line A O′ of Figure 1 as an x axis and a line perpendicular to it at A as a y axis, and let the units of length along the two axes be A O′ and A A_1 respectively. Call the vertices of the kth envelope O or K_0, K_1, K_2, . . . K_{k-1}, and K_k or O′. Let the coördinates of any vertex, K_j, be $x(j, k)$ and $y(j, k)$. When $j = 1$ the corresponding vertex lies on the line O′A_1, whose equation is

$$y = - x + 1.$$

Also

$$x(1, k) = -1 + \frac{1}{k} \cdot 2 = \frac{2-k}{k}.$$

Therefore

$$y(1, k) = -x(1, k) + 1 = -\frac{2-k}{k} + 1 = 2\frac{k-1}{k}.$$

Similarly when $j = k - 1$, the vertex lies on the line O A_1, whose equation is

$$y = x + 1.$$

Also

$$x(k-1, k) = -1 + \frac{k-1}{k} \cdot 2 = \frac{k-2}{k}.$$

Therefore

$$y(k-1, k) = x(k-1, k) + 1 = \frac{k-2}{k} + 1 = 2\frac{k-1}{k}.$$

Now the law by which Figure 1 was constructed may be expressed analytically by the statement that the point $[x(j, k), y(j, k)]$ is collinear with the points $[x(j-1, k-1), y(j-1, k-1)]$, and $[-1, 0]$, and also with the points $[x(j, k-1), y(j, k-1)]$, and $[+1, 0]$. It follows, in the first place, that $[x(j, k), y(j, k)]$, $[x(1, k-j+1), y(1, k-j+1)]$, and $[-1, 0]$ are collinear. Therefore

$$\frac{y(j, k)}{y(1, k-j+1)} = \frac{x(j, k) + 1}{x(1, k-j+1) + 1}.$$

Substituting the values found above and simplifying gives

$$(k-j) \cdot x(j, k) - y(j, k) = j - k.$$

Similarly $[x(j, k), y(j, k)]$, $[x(j, j+1), y(j, j+1)]$, and $[1, 0]$ are collinear. The resulting equation is

$$j \cdot x(j, k) + y(j, k) = j.$$

These two equations, when solved for $x\,(j,\,k)$ and $y\,(j,\,k)$ give

$$x\,(j,\,k) = \frac{2j-k}{k},[10]$$

$$y\,(j,\,k) = \frac{2j}{k}\,(k-j).$$

Now the symmetry of Figure 1 shows that the equation of the desired parabola, if one exists, must be of the form

$$x^2 = 2\,p_k\,(a_k - y),$$

where p_k and a_k are undetermined parameters corresponding to the kth envelope. Pass such a parabola through the vertex $[-1,\,0]$ and any other vertex $[x\,(j,\,k), y\,(j,\,k)]$ of this envelope. The equations for determining p_k and a_k are

$$1 = 2\,p_k\,a_k$$

and

$$\frac{(2j-k)^2}{k^2} = 2\,p_k\,[a_k - \frac{2j}{k}\,(k-j)],$$

and these give

$$p_k = \frac{1}{k} \quad \text{and} \quad a_k = \frac{k}{2},$$

each of which is independent of j. Therefore the parabola

$$x^2 = \frac{2}{k}\left(\frac{k}{2} - y\right)$$

passes through all the vertices of the kth envelope, and the first part of the proof is completed.

In the second part of the proof[11] it will be convenient to transfer the origin to the left-hand end of the string. The length of the string is l, as before. Let it be bowed downward at the point $x_0 = l/k$. Helmholtz's solution may be written

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[10] This equation can be put in the form $x\,(j,k) = -1 + \frac{j}{k}\cdot 2$, which proves that the projections of the vertices of the kth envelope are points which divide the length of the string into k equal parts.

[11] This part of the proof follows closely the method suggested in Helmholtz's paper of 1860.

$$y = u - u' = \frac{8H}{\pi^2}\sum_n \frac{1}{n^2}\sin\frac{n\pi x}{l}\sin\frac{n\pi t}{T}$$
$$-\frac{8H}{\pi^2}\sum_m \frac{1}{k^2 m^2}\sin\frac{km\pi x}{l}\sin\frac{km\pi t}{T},$$

where H is a constant. If the first term were the complete solution, the motion of any point x of the string would be an ascent from h units below equilibrium to h units above it, with a constant velocity f, during an interval of length 2τ, followed by a return to the original position, with another velocity g, during an interval of length $2(T-\tau)$, where

$$h = 4H\frac{x}{l}\frac{l-x}{l},$$

$$f = \frac{4H}{T}\frac{l-x}{l},$$

$$g = \frac{4H}{T}\frac{x}{l},$$

$$\tau = T\frac{x}{l}.$$

Suppose any one point to be going up through its position of equilibrium at the time $t = 0$. Then all points would go up through their positions of equilibrium at the times 0, $2T$, $4T$, etc., and would go down through those positions at the times T, $3T$, $5T$, etc.; and each of these times would be for each point the middle of an interval, during which its velocity is constant. The first of the above equations shows the envelope of this major vibration to be a parabola, whose vertex is at the point $(l/2, H)$.[12]

The case in hand is modified by the presence of a term

$$u' = \frac{8H}{\pi^2}\sum_m \frac{1}{k^2m^2}\sin\frac{km\pi x}{l}\sin\frac{km\pi t}{T}$$
$$= \frac{8}{\pi^2}\frac{H}{k^2}\sum_m \frac{1}{m^2}\sin\frac{m\pi x}{l/k}\sin\frac{m\pi t}{T/k}.$$

The motion corresponding to this term alone would be exactly similar to that described above, except that the constants l, T, and H would be replaced by $l' = l/k$, $T' = T/k$, $H' = H/k^2$. That is, at the time $t = 0$, each point in the first kth of the string would be ascending through its position of equilibrium with a velocity

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[12] For a proof of these facts see any of the references mentioned in the note on page 695.

$$f_1' = \frac{4\,H'}{T'}\,\frac{l' - x}{l'} = \frac{4\,H}{T}\left(\frac{1}{k} - \frac{x}{l}\right),$$

and this motion would continue until the time

$$t = \tau_1' = T'\,\frac{x}{l'} = T\,\frac{x}{l} = \tau,$$

the turning point coinciding with that of the major vibration. It would then descend during an interval centred at $t = T/k$ with a velocity

$$g_1' = \frac{4\,H'}{T'}\,\frac{x}{l'} = \frac{4\,H}{T}\,\frac{x}{l},$$

then ascend during an interval centred at $t = 2\,T/k$ with the velocity f_1' as before, and so on indefinitely.

The complete motion, in the actual case, of any point x, $0 < x < l/k$, is, therefore, an ascent from its position of equilibrium, during the interval $0 < t < \tau$, with a velocity

$$f - f_1' = \frac{4\,H}{T}\,\frac{l - x}{l} - \frac{4\,H}{T}\left(\frac{1}{k} - \frac{x}{l}\right) = \frac{4\,H}{T}\,\frac{k-1}{k};$$

followed by a number of descents, the first of them during an interval centred at $t = T/k$, with a velocity

$$g - g_1' = 0,$$

the second during an interval centred at $t = 2\,T/k$, with a velocity

$$g + f_1' = \frac{4\,H}{T}\,\frac{x}{l} + \frac{4\,H}{T}\left(\frac{1}{k} - \frac{x}{l}\right) = \frac{4\,H}{T}\,\frac{1}{k},$$

the third during an interval centred at $t = 3\,T/k$, with a velocity 0 as before, and so on. At the time $t = T$, the point is descending through equilibrium with a velocity which is either $4H/kT$ or 0, according as k is even or odd. The other half of a complete vibration is most easily determined by the fact that both components are odd functions of the time. The whole curve $y = \phi\,(t)$ for any point in the first kth of the string is, therefore, like a flight of steps with one steep ascent and $k - 1$

less steep descents connecting k horizontal stretches. A similar proposition is true for points in the last kth of the string.[13]

The portion of the envelope between $x = 0$ and $x = l/k$ is given by

$$m = \left(\frac{4\,H}{T}\,\frac{k-1}{k}\right)\left(T\,\frac{x}{l}\right) = 4\,H\,\frac{k-1}{k}\,\frac{x}{l},$$

and is a straight line intersecting the parabola

$$h = 4\,\text{H}\,\frac{x}{l}\,\frac{l-x}{l}$$

at the points (0, 0) and $(l/k,\ 4\,H\,(k-1)/k^2)$, which agrees with what was to be proved.

In the second kth of the string, a point under the influence of the minor vibration alone, would, at the time $t = 0$, be descending through equilibrium with a velocity

$$g_2{}' = \frac{4\,H'}{T'}\,\frac{x-l'}{l'} = \frac{4\,H}{T}\left(\frac{x}{l} - \frac{1}{k}\right),$$

and this descent would last until the time

$$t = T' - \tau_2 = T' - T'\,\frac{x-l'}{l'} = T\left(\frac{2}{k} - \frac{x}{l}\right).$$

It would be followed by an ascent during an interval centred at $t = T/k$ with a velocity

$$f_2{}' = \frac{4\,H'}{T'}\,\frac{l' - (x - l')}{l'} = \frac{4\,H}{T}\left(\frac{2}{k} - \frac{x}{l}\right).$$

The whole duration of this interval would be

$$2\,\tau_2 = 2\,T\left(\frac{x}{l} - \frac{1}{k}\right),$$

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[13] This work shows that the figure in Helmholtz's 1860 paper is incorrect. It is true that the differential equation in question has a solution corresponding to the function $y = f(t)$, which his figure represents, and that this solution satisfies the boundary conditions and both of Helmholtz's laws, as can easily be proved by the method of section 5 of this paper. But it does not satisfy Young's law, the 6th, 18th, 30th, etc., partials being present, and it could not, so far as our present knowledge of the violin string goes, be actually set up by bowing a string at the point 1/6.

For some theorems which are very useful in this connection, see Wedmore, Journ. Inst. of El. Engg., **25**, 224 (1896); and Lyle, Phil. Mag., **6**, 549 (1903); ibid., **11**, 25 (1906).

and it would end at the time

$$t = T' + \tau_2 = T\frac{x}{l} = \tau,$$

so that one of the turning points of the minor vibration coincides with that of the major vibration, as before. This is true in all cases.

The complete motion of a point in the second kth of the string is, therefore, (a) two ascents, the first during the interval

$$0 < t < T\left(\frac{2}{k} - \frac{x}{l}\right),$$

with a velocity

$$f + g_2' = \frac{4H}{T}\frac{l-x}{l} + \frac{4H}{T}\left(\frac{x}{l} - \frac{1}{k}\right) = \frac{4H}{T}\frac{k-1}{k};$$

and the second during the interval

$$T\left(\frac{2}{k} - \frac{x}{l}\right) < t < T\frac{x}{l},$$

with a velocity

$$f - f_2' = \frac{4H}{T}\frac{l-x}{l} - \frac{4H}{T}\left(\frac{2}{k} - \frac{x}{l}\right) = \frac{4H}{T}\frac{k-2}{k};$$

followed by (b) a number of descents, the first during an interval centred at $2T/k$, with a velocity

$$g - g_2' = \frac{4H}{T}\frac{x}{l} - \frac{4H}{T}\left(\frac{x}{l} - \frac{1}{k}\right) = \frac{4H}{T}\frac{1}{k};$$

the second during an interval centred at $3\,T/k$, with a velocity

$$g + f_2' = \frac{4H}{T}\frac{x}{l} + \frac{4H}{T}\left(\frac{2}{k} - \frac{x}{l}\right) = \frac{4H}{T}\frac{2}{k};$$

the third during an interval centred at $4T/k$ with the velocity $4H/Tk$ as before, and so on. In no case will the curve $y = \phi\,(t)$ for any point not in the first or last kth of the string have any horizontal portions whatever, but both the ascending and the descending portions will be somewhat "crumpled."

These facts are very prettily illustrated by the appearance under a microscope of the bright lines upon which the observations of the last section were made. When the observed point is in one of the end $k$ths of the wire there is always a very distinct spot of light at each end of the line, and $k-2$ other spots equally spaced between them, corresponding to the k horizontal portions of the vibration curve; but in all other cases the line is of nearly uniform intensity, even at the ends.

The portion of the envelope between $x = l/k$ and $x = 2\,l/k$ is given by

$$m = \frac{4\,H}{T}\,\frac{k-1}{k}\cdot T\left(\frac{2}{k}-\frac{x}{l}\right)+\frac{4\,H}{T}\,\frac{k-2}{k}\cdot 2\,T\left(\frac{x}{l}-\frac{1}{k}\right)$$
$$= 4\,H\left(\frac{2}{k^2}+\frac{k-3}{k}\,\frac{x}{l}\right),$$

and is a straight line intersecting the parabola

$$h = 4\,H\,\frac{x}{l}\,\frac{l-x}{l}$$

at the points $(l/k,\ 4\,H\,(k-1)\,/k^2)$ and $(2\,l/k,\ 8\,H\,(k-2)\,/k^2)$, as was to be proved.

Similar reasoning can be applied to a point in the third kth, or, in general, to any point x, where

$$\frac{j\,l}{k} < x < \frac{(j+1)\,l}{k},\ \text{and}\ j < k;$$

but it is necessary to distinguish between a case in which j is even and one in which it is odd. The resulting formulae are given in Table II. In each case the envelope is a straight line intersecting the above mentioned parabola at the points $[j\ l/k,\ 4\,Hj\ (k-j)/k^2]$ and $[(j+1)\,l/k,$ $4\,H\,(j+1)\,(k-j-1)\ /\ k^2]$, and the second part of the proof is completed.

The geometry of Figure 1, which was obtained experimentally for longitudinal vibrations, is, therefore, a verification of the established theory for the transverse vibrations of a string bowed at an aliquot point.

It is easy to construct from the data of Table II either the curve $y = \phi\,(t)$ for a given x, or the curve $y = \psi\,(x)$ for a given t, in any desired aliquot case. The results for $k = 2$, 3, and 4 are given in Figures 3, 4, and 5. From them one can get a reasonably definite idea of the general case. It is interesting to notice that the vibration of a string bowed in the middle is identical with that of a string plucked in the middle, although under other circumstances the corresponding modes of vibration of a bowed and of a plucked string are wholly different.

TABLE II.

$\frac{jl}{k} \leqq x \leqq \frac{(j+1)l}{k}, j<k.$

	When j is even.	When j is odd.
First ascending velocity	$\frac{4H}{T}\frac{k-(j+1)}{k}$	$\frac{4H}{T}\frac{k-j}{k}$
Second ascending velocity	$\frac{4H}{T}\frac{k-j}{k}$	$\frac{4H}{T}\frac{k-(j+1)}{k}$
Number of ascents	$j+1$	$j+1$
First descending velocity	$\frac{4H}{T}\frac{j}{k}$	$\frac{4H}{T}\frac{j}{k}$
Second descending velocity	$\frac{4H}{T}\frac{j+1}{k}$	$\frac{4H}{T}\frac{j+1}{k}$
Number of descents	$k-j$	$k-j$
Duration of first interval	$T\left[\frac{x}{l}-\frac{j}{k}\right]$	$T\left[\frac{j+1}{k}-\frac{x}{l}\right]$
Duration of second interval	$2T\left[\frac{j+1}{k}-\frac{x}{l}\right]$	$2T\left[\frac{x}{l}-\frac{j}{k}\right]$
Duration of third interval	$2T\left[\frac{x}{l}-\frac{j}{k}\right]$	$2T\left[\frac{j+1}{k}-\frac{x}{l}\right]$
Envelope	$\frac{4H}{k}\left\{\frac{x}{l}[k-2j-1]+\frac{j(j+1)}{k}\right\}$	

Showing the motion of any point, x, of a string bowed at the aliquot point $1/k$. The table describes a half vibration, from equilibrium ascending to equilibrium descending. The ascending velocities alternate until $(j+1)$ ascents are accomplished; then the descending velocities alternate until equilibrium is reached; then the vibration is continued below equilibrium in such a way as to make the displacement an odd function of the time. The last ascent is always with the smaller of the ascending velocities; similarly the first descent is with the smaller of the descending velocities. The table describes $(k+1)$ straight lines, but the first and the last of them are halves of longer stretches in the complete curve which thus consists essentially of $2k$ straight lines. This shows why the duration of the first velocity is given in the table as half that of the third; the other half of the interval is thought of as belonging to the preceding half-vibration.

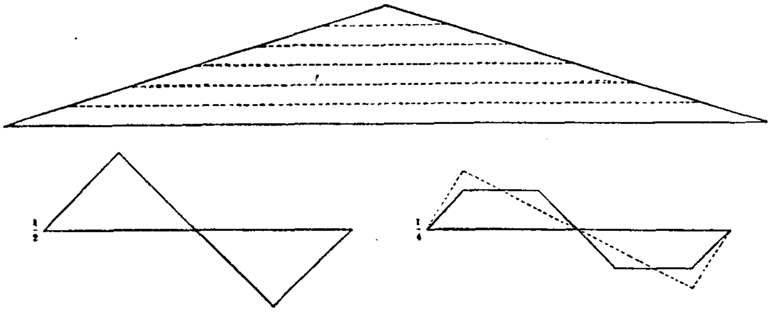

FIGURE 3. The case ½. The upper figure shows the configurations of the string at the times 0, $T/12$, $2T/12$, . . . $6T/12$, $2T$ being the period; the string returns to equilibrium through the same forms; in this, and in the following cases, the configurations during the second half of a complete vibration can be obtained by turning the page upside down. The lower figures show the curves $u = \phi(t)$ for the points ½ and ¼. The dotted lines in this, and in the similar figures to follow, show the motion due to the major vibration alone. It is interesting to notice that this mode of vibration is identical with that of a string plucked at its middle point.

§ 4. RATIONAL POINTS — KRIGAR-MENZEL'S LAW.

The experimental method described in section 2 has two disadvantages. In the first place, it is not easy, especially in a complex case, to discover any vibration form that corresponds to a given envelope; and in the second place, an envelope is sometimes, and perhaps always, not enough to determine its vibration form uniquely. For example, in the case of the struck string, if the initial conditions be taken in the form

$$u = 0 \text{ when } t = 0,$$

$$\frac{\partial u}{\partial t} = 0 \text{ when } t = 0, \quad \text{and} \quad 0 < x < x_0,$$

$$\frac{\partial u}{\partial t} = C \text{ when } t = 0, \quad \text{and} \quad x_0 < x < x_0 + a,$$

$$\frac{\partial u}{\partial t} = 0 \text{ when } t = 0, \quad \text{and} \quad x_0 + a < x < l,$$

it can easily be shown that the envelope depends only on the constants a and C and not at all upon x_0. A second standard method of observation was therefore made use of, and it proved to be more powerful, although much less accurate, than the first. The point of a fine needle was broken off and fastened to the wire with shellac,[14] and a microscope slide with a very light coating of smoke was drawn under it by hand while the wire was vibrating. The resulting curve shows the displacement of the given part of the wire as a function of the time, and it can be proved that a single curve of this kind, together with Young's law, is enough for the complete determination of a vibration form. A number of these curves, corresponding to the three simplest aliquot cases, are reproduced photographically, much enlarged, in Figure 6, and their resemblance to the calculated curves of Figures 3, 4, and 5 is another and a final verification of the conclusions of the last two sections.

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[14] The added weight was that of about half a millimeter of wire.

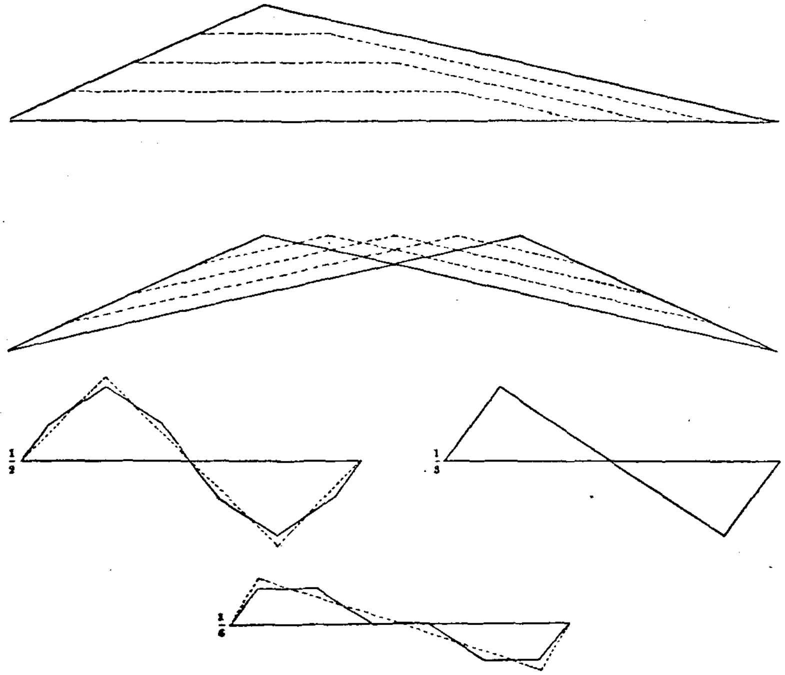


FIGURE 4. The case $\frac{1}{3}$. The upper figures show the configurations of the string at intervals of $T/12$, the first from 0 to $T/3$, and the second from $T/3$ to $2T/3$; the return to equilibrium is through forms symmetric to those of the upper figure. The lower figures show the motion of the points $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$.

This section will describe a number of observations which have been made by this method to test Krigar-Menzel's law for the simpler rational cases. It will be remembered that, according to this law, the motion of the point bowed in the rational case p/q, is an ascent with a constant velocity, followed by a descent with another constant velocity, the ratio of the lesser velocity to the greater being $1/(q-1)$. It follows that the duration of the greater velocity is $1/q$th of a complete period, or, speaking geometrically, that the length of the projection on the t axis of the steeper of the two straight lines which make up a typical segment of the curve $y = \phi\ (t)$ for the point in question, is $1/q$th of the length of that

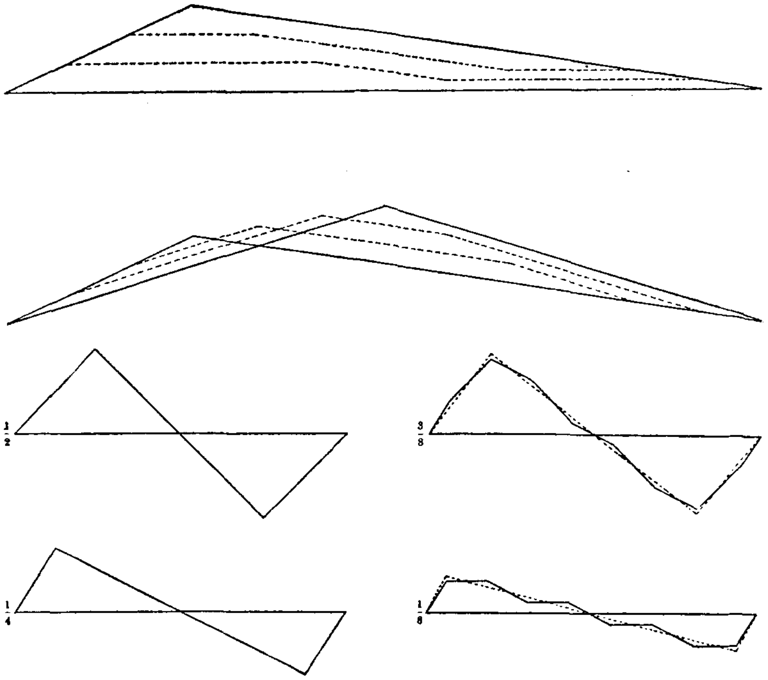
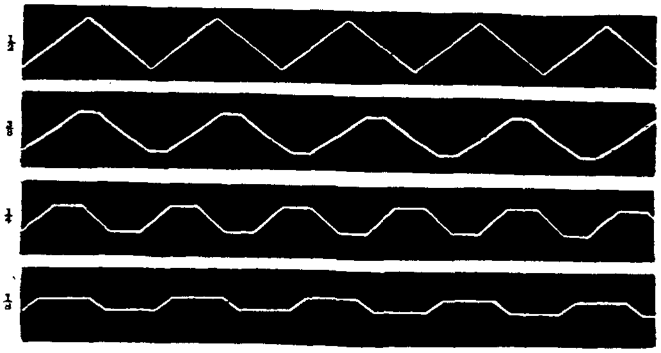


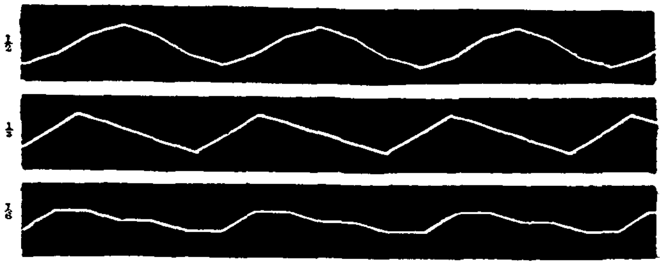
FIGURE 5. The case $\frac{1}{4}$. The upper figures show configurations at intervals of $T/12$, the first from 0 to $T/4$, and the second from $T/4$ to $T/2$; the return to equilibrium is through forms symmetric to those of the ascent. The lower figures show the motion of the points $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{1}{8}$.

segment. A number of measurements of this ratio for the various rational cases up to and including $q = 10$ are presented in Table III.

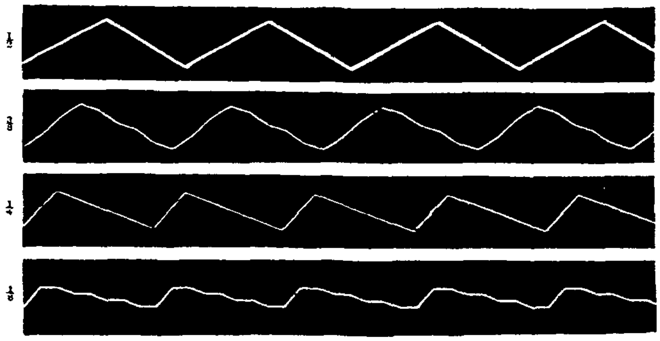
The observations summarized in this table were made with a Gaertner comparator whose stage was driven by a half-millimeter screw with a head reading to thousandths of a millimeter. Each slide was oriented so that the curve to be measured lay along the axis of the micrometer screw and readings were made of the settings necessary to bring



Rubbed at $\frac{1}{2}$.



Rubbed at $\frac{1}{3}$.



Rubbed at $\frac{1}{4}$.

FIGURE 6. Experimental $u\ t$ curves for the simplest aliquot cases. (Cf. figures 3, 4, and 5.)

TABLE III.

Case.	Ratio predicted.	Ratio found.	Per Cent Difference.
$\frac{1}{2}$	.500	.503	+0.6
$\frac{1}{3}$	.333	.332	−0.3
$\frac{1}{4}$	.250	.247	−1.2
$\frac{1}{5}$	.200	.201	+0.5
$\frac{2}{5}$	.200	.202	+1.0
$\frac{1}{6}$	.167	.166	−0.6
$\frac{1}{7}$	.143	.142	−0.7
$\frac{2}{7}$	.143	.141	−1.4
$\frac{3}{7}$	.143	.144	+0.7
$\frac{1}{8}$	.125	.126	+0.8
$\frac{3}{8}$	.125	.123	−1.6
$\frac{1}{9}$	.111	.112	+0.9
$\frac{2}{9}$	.111	.109	−1.8
$\frac{4}{9}$	.111	.112	+0.9
$\frac{1}{10}$	.100	.100	0.0
$\frac{3}{10}$	.100	.100	0.0

Observations to test Krigar-Menzel's law. It is only fair to state that each of the ratios in this table is the best of a number of determinations. The ratio selected in each case does, however, take into account all of the 105 settings which were made on the slide in question.

twenty-one successive vertices under a vertical cross-hair. These gave the lengths of the projections on the t axis of ten consecutive ascents and of the ten descents immediately following them. Since the velocity of the slide when the tracing was made was not in general uniform, the averages of the values thus obtained had to be slightly modified to allow for the distortion of the time scale of the slide. The necessary correction, which was always small, was obtained from the values themselves on the supposition that the acceleration of the slide was uniform during the interval in question — about the fortieth part of a second. Five such measurements were made on different parts of at least three different tracings for each case discussed, so that each of the observed ratios in Table III depends on fifty measurements of each of the quantities involved. This labor seemed to be necessary because, under high magnification, the tracing itself was so wide as to make the setting corresponding to a given vertex somewhat uncertain, and also because it was hoped that this averaging would eliminate the effect of local variations in the time scale of the slide due to any transverse or torsional vibrations that might be present. As a matter of fact such vibrations caused surprisingly little trouble even when the bridge was not used.

Three other sources of error were constantly in evidence. In the first place, it is assumed above that the vertical cross-hair is perpendicular to the axis of the screw, and an error of one degree in this adjustment might, under unfavorable circumstances, cause an error of more than two per cent in the observed ratio.[15] Eighteen sets of observations on photographic reductions of a carefully prepared drawing showed that the angle of error was actually about 40′ and its effect has been allowed for when necessary. A second error is involved in the orientation of the slide itself with respect to the axis of the screw. This was harder to allow for, but the adjustment could be made with considerable accuracy, and furthermore the resulting error would be somewhat diminished when the results for five tracings were averaged. And finally, it was difficult, with the comparatively crude apparatus which was used, to be sure that the slide was moving perpendicularly to the motion of the wire when the tracing was made; but in getting slides for measurement, this adjustment was repeated for each line, so as to take advantage of the averaging process already mentioned. Fortunately the resulting error in all three cases varies directly as the ratio of the amplitude of a tracing to the apparent length of its period, and in all cases this ratio was made very small by increasing the speed of the slide when the tracings were made.

Under the circumstances the agreement of the observed with the predicted values is reasonably satisfactory, the average departure being 0.8%, or, if account be taken of sine, –0.1%, while the average of the probable errors of the individual determinations is 0.5%. It is evident that Kriger-Menzel's law holds for the longitudinal vibrations considered in this paper as well as for the transverse vibrations of a bowed string.

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[15] One advantage of the method of section 2 is that it involves the versed sine of the corresponding angle of error rather than its sine.

§ 5. INTEGRAL SURFACES.

It is usual to solve problems connected with the vibration of bowed strings by Helmholz's method and to express the results as trigonometric series in x and t. A solution for the case 2/5 can easily be obtained in this way, and is

$$u = \frac{5\,V\,T}{\pi^2} \sum_n \frac{1}{n^2} \frac{1}{\cos\frac{n\,\pi}{5}} \sin\frac{n\,\pi\,x}{l} \sin\frac{n\,\pi\,t}{T}, \qquad n \neq 5,\ 10,\ \text{etc.},$$

where V is the velocity of the bow — a form which is entirely satisfactory for the theory of music, where the important thing is to know the intensities of the various harmonics produced, but one which gives very little information of any other kind. In the case of an aliquot point it is possible, with considerable labor, to work out from the trigonometric series a complete description of the configurations and motion of the string itself, because the various series in x or t which present themselves are easily recognizable standard forms;[16] but even in this, the simplest of all the non-aliquot cases, such a fortuitous proceeding would be very difficult, if not impossible,[17] and the only other obvious method of handling this form of the solution, namely by direct computation, is impracticable. The desired information can, however, be obtained by a simple graphical process, which it is the purpose of this section to describe.

An integral surface is a three-dimensional graph which represents the displacement u of any portion of the string at any time, as a function of x and t. Such surfaces can be handled in two dimensions by plotting their contour lines, $u =$ a constant, on the basal $x\,t$ plane, just as would be done on an ordinary map. For a string of length l, one is concerned only with that part of the $x\,t$ plane which lies between the lines $x = 0$ and $x = l$, and the displacement, u, is zero along each of these lines. Also, since the motion of such a string is necessarily periodic with a period $2\,T$, it is necessary to cover only so much of the above mentioned strip as lies between the lines $t = 0$ and $t = 2\,T$. If l and T are taken as the units of length and of time respectively, the differential equation becomes

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[16] See section 3 of this paper.

[17] At least without the assistance of a complete set of experimentally determined $y\,t$ curves.

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2},$$

and the fundamental rectangle is two adjacent unit squares.

Such surfaces have been plotted from the data of Table II, for the aliquot cases, from $\frac{1}{2}$ to $\frac{1}{7}$, and are shown in the top line of the first of the accompanying plates. They have a number of common characteristics, and, in particular:

1. The displacement, u, is zero not only along the lines $x = 0$ and $x = 1$, but along the lines $t = 0$, $t = 1$, and $t = 2$ as well.

2. Each surface is made up entirely of plane facets, whose projections on the basal $x\,t$ plane are the squares formed by two sets of equidistant lines, parallel to the diagonals of the large squares, and determined as follows: one line of each set passes through A, the point of discontinuity in the known section, $u = \Phi\,(t)$, under the bow (see Figure 7); if the points where these lines intersect the line $t = 0$ be called B and C, B determines a parallel adjacent to AC, and C one adjacent to AB.

Conversely, if these two properties be assumed, it is possible to reconstruct the whole surface from the known section $u = \Phi\,(t)$. For the set of squares can be laid out in accordance with the second of the above rules; then the heights of the various corners on some arbitrary scale can be determined as follows (see Figure 7). Assume the height of A to be 4. Then D must be 2, for the known section drops uniformly from A to E. Since A is 4 above C, F must be 4 above G, or $0 + 4 = 4$.[18] Then since F is 4 above H, I must be 4 above D, or $2 + 4 = 6$. And I with A and C determines J. Furthermore, the heights of the three corners still undetermined must be equal to those of D, I, and J respectively, since E = K. The intermediate heights along the sides of the small squares and the contours within them can now be filled in without difficulty.

Fortunately, this same process, when carried through for any rational case that obeys Krigar-Menzel's law, gives the desired solution graphically. To show how it works in the general case, the construction of the surface for $\frac{2}{5}$ will be described in some detail with the help of

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[18] The drop along one edge of an oblique section of a square prism must be equal to that along the edge opposite, and therefore to that along the opposite side of an adjacent prism, etc.

Figure 8. The parallels are laid out as before by means of the points A, B, and C; the height of A is taken arbitrarily as 4; and Krigar-Menzel's law gives $D = 2$. Then since $G = 0$, $F = 0 + 4 = 4$. Also F and H give $I = J + 4 = 4$, and E and I give $K = D - 4 = -2$ and $M = L + 4 = 4$, which brings us back *via* N to the starting point A. One such round can always be made, and it will always cross the known section A E in such a way as to start a second round. In this case either of the pairs D I or D M enables one to make this second round and to complete the figure. In a more complicated case a third round, a fourth round, etc., would follow; and as new points are filled in, former rounds may be repeated, until the surface is completed. The method will always yield one, and only one, surface.

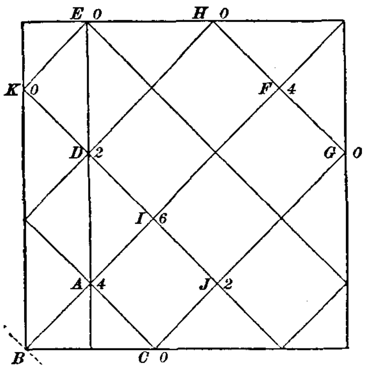

FIGURE 7. Showing the construction of the integral surface for the aliquot case $\frac{1}{5}$.

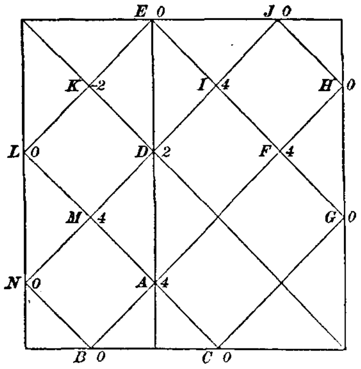

FIGURE 8. Showing the construction of the integral surface for the non-aliquot case $\frac{2}{5}$.

No attempt will be made in this paper to deduce the two rules mathematically from general principles.[19] As is so often necessary in the theory of La Place's equation, they are to be regarded simply as an arbitrary device for producing something that can subsequently be proved to be (*a*) a solution, and (*b*) the only possible solution, satisfying the given conditions. Thus, since such a solution is built up of planes, it is always true that

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[19] It is hoped that the mathematical questions which are suggested by this work can be treated more fully in a later paper. In particular, this graphical method can be so generalized as to give a solution of any kind, whether periodic or not, when its values are known along any three incommeasurably spaced, non-characteristic, parallel lines.

$$\frac{\partial^2 u}{\partial x^2} = 0 = \frac{\partial^2 u}{\partial t^2},$$

and the boundary conditions and Krigar-Menzel's law are obviously satisfied. To show that Young's law is also satisfied, it is necessary to get the Fourier development of the curve $u = \phi(t)$ corresponding to some incommensurable value of x. Krigar-Menzel has proved[20] that any continuous periodic curve, whose period is made up of μ straight lines, can be developed in the form

$$u = \frac{A_0}{2} + \sum_1^{\infty} A_n \cos \frac{n \pi t}{T} + \sum_1^{\infty} B_n \sin \frac{n \pi t}{T},$$

with

$$\left.\begin{matrix} A_n \\ B_n \end{matrix}\right\} = \frac{T}{n^2 \pi^2} \Bigg[(\theta^{\mu}_{\mu-1} - \theta_0^{\,1}) \left\{\begin{matrix} \cos \\ \sin \end{matrix}\right\} \frac{n \pi \tau_0}{T}$$

$$+ (\theta_0^{\,1} - \theta_1^{\,2}) \left\{\begin{matrix} \cos \\ \sin \end{matrix}\right\} \frac{n \pi \tau_1}{T}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$+ (\theta^{\mu-1}_{\mu-2} - \theta^{\mu}_{\mu-1}) \left\{\begin{matrix} \cos \\ \sin \end{matrix}\right\} \frac{n \pi \tau_{\mu-1}}{T} \Bigg],$$

where $2\,T$ is the period, $\tau_0, \tau_1, \ldots \tau_{\mu-1}$ and $\tau_\mu \,(= \tau_0 + 2\,T)$ are the abscissae of the various corners, and $\theta_0^{\,1}, \theta_1^{\,2}, \ldots \theta^{\mu}_{\mu-1}$ are the slopes of the μ lines. Let this formula be applied to a section of any integral surface along a line $x = \epsilon$, where ϵ is a small incommensurable quantity. The left hand edge of an integral surface is always made up of the diagonals of a number of its constituent facets, and each of these planes would be cut by the plane $x = \epsilon$ in a horizontal line. Therefore, the desired section, for a case p/q, is always q long horizontal lines, at various heights, connected by q short oblique lines of various slopes.[21] Beginning with one of the oblique lines, the abscissae of the corners will have one or the other of the following sets of values, according as p is odd or even.

---

[20] Wied. Ann., **49**, 545 (1893).

[21] The stair-like forms already described in section 3 are special cases of this general type of figure.

$$\begin{array}{cccccc}
\tau_0\ , & \tau_1\ , & \tau_2\ , & \tau_3\ , & \ldots & \tau_\mu\,(=\tau_{2q})\ , \\[6pt]
-\epsilon\ , & +\epsilon\ , & \dfrac{2\,T}{q}-\epsilon, & \dfrac{2\,T}{q}+\epsilon, & \ldots & 2\,T-\epsilon\ , \\[10pt]
\dfrac{T}{q}-\epsilon, & \dfrac{T}{q}+\epsilon, & \dfrac{3\,T}{q}-\epsilon, & \dfrac{3\,T}{q}+\epsilon, & \ldots & 2\,T+\dfrac{T}{q}-\epsilon.
\end{array}$$

In either case

$$\theta_1{}^2=\theta_3{}^4=\theta_5{}^6=\ \ldots\ =\theta^{\mu}_{\mu-1}=0\,;$$

and furthermore

$$\theta_0{}^1+\theta_2{}^3+\theta_4{}^5+\ \ldots\ +\theta^{\mu-1}_{\mu-2}=0,$$

because the net gain in the height of the curve through a period is zero. Under these conditions, Krigar-Menzel's formula becomes

$$\begin{aligned}
\left.\begin{matrix}A_n\\B_n\end{matrix}\right\}=\frac{T}{n^2\,\pi^2}\Bigg[&-\theta_0{}^1\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{n\,\pi\,\tau_0}{T}+\theta_0{}^1\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{n\,\pi\,\tau_1}{T}\\
&-\theta_2{}^3\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{n\,\pi\,\tau_2}{T}+\theta_2{}^3\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{n\,\pi\,\tau_3}{T}\\
&\qquad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
&-\theta^{\mu-1}_{\mu-2}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{n\,\pi\,\tau_{\mu-2}}{T}+\theta^{\mu-1}_{\mu-2}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{n\,\pi\,\tau_{\mu-1}}{T}\Bigg]\\
=\mp\frac{2\,T}{n^2\,\pi^2}\Bigg[&\theta_0{}^1\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\left(\frac{n\,\pi}{T}\,\frac{\tau_1+\tau_0}{2}\right)\sin\left(\frac{n\,\pi}{T}\,\frac{\tau_1-\tau_0}{2}\right)\\
&+\theta_2{}^3\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\left(\frac{n\,\pi}{T}\,\frac{\tau_3+\tau_2}{2}\right)\sin\left(\frac{n\,\pi}{T}\,\frac{\tau_3-\tau_2}{2}\right)\\
&\qquad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
&+\theta^{\mu-1}_{\mu-2}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\left(\frac{n\,\pi}{T}\,\frac{\tau_{\mu-1}+\tau_{\mu-2}}{2}\right)\sin\left(\frac{n\,\pi}{T}\,\frac{\tau_{\mu-1}-\tau_{\mu-2}}{2}\right)\Bigg].
\end{aligned}$$

When p is odd, this becomes

$$\begin{aligned}
\left.\begin{matrix}A_n\\B_n\end{matrix}\right\}=\mp\frac{2\,T}{n^2\,\pi^2}\sin\frac{n\,\pi\,\epsilon}{T}\Bigg[&\theta_0{}^1\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\left(\frac{0\,n\,\pi}{q}\right)+\theta_2{}^3\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\frac{2\,n\,\pi}{q}+\ \ldots\\
&+\theta^{\mu-1}_{\mu-2}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\frac{2\,(q-1)\,n\,\pi}{q}\Bigg]\,;
\end{aligned}$$

and when p is even, the form is similar. Whenever n is equal to a multiple of q, both A_n and B_n are zero, the first because each of the sine terms is zero, and the second because each of the cosine terms is one and the sum of the θ's is zero. Young's law is, therefore, always satisfied. And finally, as has been said, it can be proved that Young's and Krigar-Menzel's laws determine the motion of a finite string uniquely.[22] It is, therefore, evident that the method proposed must lead in every case to the desired solution.

A number of solutions, obtained by this method from Krigar-Menzel's law, are shown on the first and second of the accompanying plates. The third plate shows some typical $u\,t$ curves (a) as read off from the various surfaces, and (b) as reproduced experimentally by the methods of section 4.

The surfaces of the first two plates show certain serial relations which are interesting in the light of Young's and Krigar-Menzel's work on the excitation of individual partials. The aliquot cases $(1/k)$ in the first line are obviously related, and are approaching Helmhotz's major solution as a limit as k increases;[23] the amplitude for a given bowing speed is, however, becoming infinite. The mode of vibration which one would actually get in the limiting case, that is, if one were to bow at the end of the string, would be absolute rest, something quite different from the form toward which the surfaces are tending.

A similar serial relation is to be found among the surfaces of the second line. They are the cases which correspond to the various values of the expression $m/(2\,m + 1)$ for integral values of m, just as those of the first line are, essentially, cases which can be grouped under the formula $m/(m+1)$.[24] The series in the second line begins with $\frac{1}{3}$ and approaches $\frac{1}{2}$ as a limit as m increases. The surfaces show a tendency toward a mode of vibration in which the middle of the string would be nearly at rest and each half would be moving in accordance with Helmholtz's major solution; but the amplitude of the motion is, as before, increasing indefinitely. In other words, all the partials which have nodes at the middle of the string are coming out more and more strongly, nearly in the same proportion, as the bowing-point approaches the middle of the string. This is also brought out by the envelopes of these surfaces which are shown in Figure 9. As before, the surface at the limit ($\frac{1}{2}$) is quite different from the limit of the surfaces. It is possible to get a series converging toward $\frac{1}{2}$ from the other side by means of the formula $m/(2\,m - 1)$ as is indicated in Table IV; the approach is through essentially the same set of surfaces, and the same set of partials come out strongly, but in the opposite phase.[25]

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[22] At least within a multiplicative constant which can be determined by means of the velocity law.

[23] The integral surface corresponding to Helmholtz's major solution alone is built up of parts of hyperbolic paraboloids. Its contour lines are parts of rectangular hyperbolae with two adjacent sides of a fundamental square as asymptotes. Its appearance would be much like that of the surface for the case $\frac{1}{7}$.

[24] Any surface $(1 - p/q)$ differs from the related surface (p/q) only in the signs of its displacements. To get the surface $\frac{2}{3}$ from the surface $\frac{1}{3}$ one has only to interpret each hill as a hollow and *vice versa*.

TABLE IV.

	1	2	3	4	5	Limit	5	4	3	2	1	
$\frac{m}{m+1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	1						
$\frac{m}{2m+1}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{4}{9}$		$\frac{1}{2}$	$\frac{5}{9}$	$\frac{4}{7}$	$\frac{3}{5}$	$\frac{2}{3}$	1	$\frac{m}{2m-1}$
$\frac{m}{3m+1}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{3}{10}$			$\frac{1}{3}$		$\frac{4}{11}$	$\frac{3}{8}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{m}{3m-1}$
$\frac{2m}{5m+1}$	$\frac{1}{3}$		$\frac{3}{8}$		$\frac{5}{13}$	$\frac{2}{5}$	$\frac{5}{12}$		$\frac{3}{7}$		$\frac{1}{2}$	$\frac{2m}{5m-1}$

Showing the serial relations among the surfaces of the first and second plates.

Parts of similar series converging toward $\frac{1}{3}$ are shown on the second plate, the formula being $m/(3\,m \pm 1)$. They show very prettily Krigar-Menzel's change of phase in the 3d, 6th, 9th, etc. partials. The envelopes of the first of these series are shown in the upper half of Figure 11, and those of the second series on a reversed scale in the lower half of that figure. It should be compared with Figures 9 and 1, and especially with Figure 10, which is Young's representation of essentially the same fact. Finally, in the lower line of the second plate are two surfaces belonging to the convergences upon $\frac{2}{5}$, the formula being $2\,m/(5\,m \pm 1)$. In them the 5th, 10th, etc., partials are beginning to come out strongly, and are, as usual, in opposite phases.

Notwithstanding this essential difference, the surfaces for $\frac{5}{12}$ and $\frac{5}{13}$ are, in general appearance, very much alike, and a comparison of either of them with the surface for $\frac{2}{5}$ itself seems to show that each of them is

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[25] See the preceding footnote.

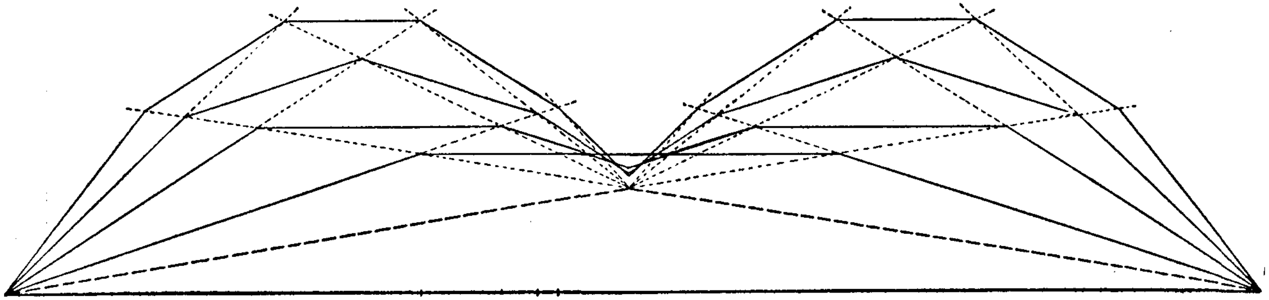

Figure 9. The envelopes, for a single bowing speed, of the cases $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{4}{9}$, showing that as the bowing point approaches $\frac{1}{2}$ the string seems to divide itself more and more definitely into two vibrating segments and tends to vibrate more and more violently. The amplitude of the middle point of the string is decreasing through the values $\frac{4}{3}$, $\frac{6}{5}$, $\frac{8}{7}$, $\frac{10}{9}$, etc., toward 1, which is its actual value in the limiting case. The envelope for that case is drawn with long dashes for comparison with the limit of the envelopes. The horizontal and vertical scales of this figure and of Figure 11 are proportional to those of Figure 1, with which these figures should be compared. They both can be extended indefinitely by simple generalizations of the ray principle already discussed for Figure 1, and each of them furnishes a graphical method for obtaining any envelope belonging to its convergence directly, without the labor of working out the corresponding solution as a whole.

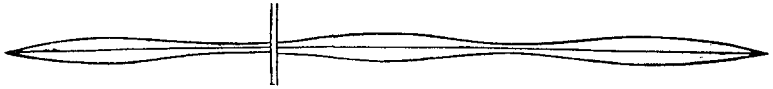

Figure 10. Young's figure of a violin string bowed near a principal node. This should be compared with Figure 11.

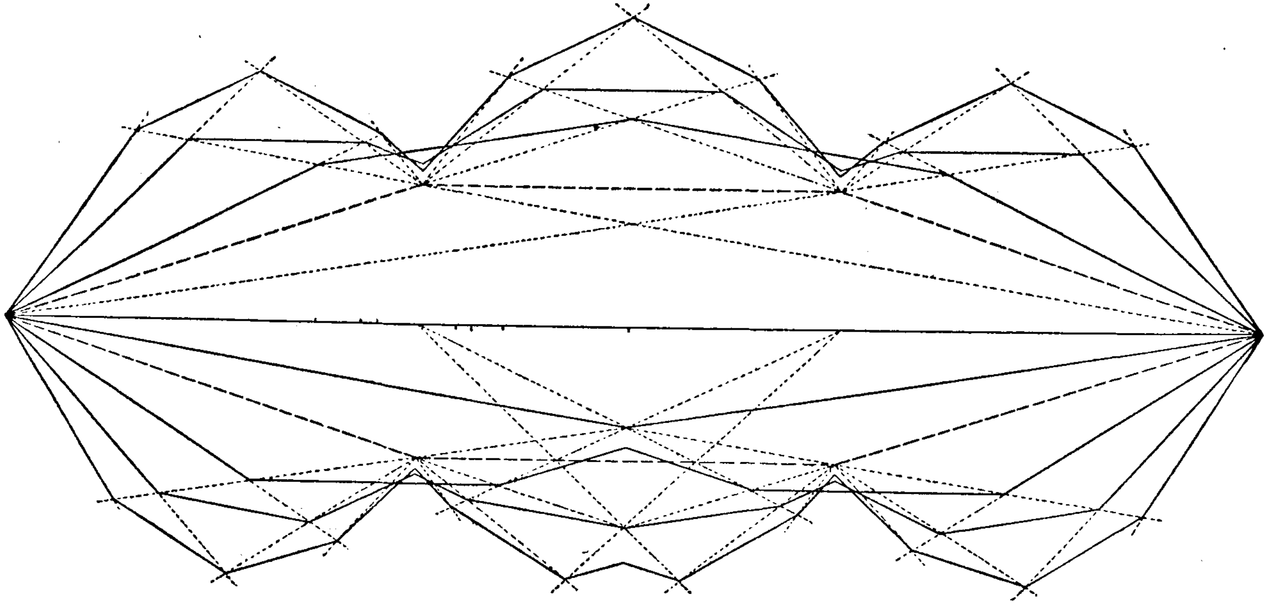

FIGURE 11. In the upper half, the envelopes for the cases $\frac{1}{4}$, $\frac{2}{7}$, and $\frac{3}{10}$, representing one of the convergences upon $\frac{1}{3}$. In the lower half, those for the cases $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{4}{11}$, representing the other convergence. The indentations are at the points $\frac{1}{3}$ and $\frac{2}{3}$ in all cases. The envelope for the limiting case $\frac{1}{3}$ itself is drawn with long dashes. (Cf. Figures 1 and 9).

the result of superposing two relatively independent components, (*a*) the 5th, 10th, etc., partials, always in the Helmholtzian proportion, which dominate the whole solution *near* the point $\frac{2}{5}$, but are completely absent *at* that point, and (*b*) something else which changes more or less continuously near the point $\frac{2}{5}$ and is the whole solution at that point itself. In the surfaces for $\frac{5}{12}$ and $\frac{5}{13}$ this *b* component has not yet been entirely masked by the increasingly important *a* component — hence their similarity. The same thing can be noticed in each of the preceding convergences, and, in general, in any pair of convergences corresponding to a formula of the form $p\,m/(q\,m \pm 1)$; so that, although the complete solution, regarded as a function of the point of excitation, is discontinuous in every interval, nevertheless it is possible to get a general idea of its changes; for as the point of excitation moves along the string, one set of partials after another becomes infinite, jumps through zero to the opposite infinity, and decreases again, dominating the whole solution through a brief interval — which is very brief indeed for all but the lowest partials — and then disappearing into the ever changing but relatively continuous *b* component of subsequent cases.[26] The significance of the points which have been arbitrarily chosen as the successive steps in the convergences of Table IV seems to be that each of them is in its turn the simplest rational point between its predecessor and the goal, and therefore shows the gradual rise of the desired set of partials, while avoiding as much as possible the dominant intervals of extraneous sets.

One other interesting fact is brought out by these graphical solutions. From the method of construction, it is evident that every surface is necessarily symmetrical with respect to the diagonals of its fundamental squares, so that any curve $y = \phi\ (t)$ for a given point x is identical with a curve $y = \psi\ (x)$ for a corresponding time $t = x\,T/l$; that is, any one of the $y\ t$ curves which are reproduced in this and in Krigar-Menzel's paper is a configuration which an "infinite string" would actually take on once in each complete vibration. The variety and irregularity of these configurations show vividly how complicated the motion of a bowed string sometimes is.

SUMMARY.

The results of this investigation may be summarized as follows:

1. The envelope of a string which is rubbed either transversely or

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[26] Of course this has nothing to do with the interesting question as to what happens when the point of excitation actually does move along the string, as, for instance, when a wire is stroked with a bit of rosined chamois.

longitudinally at an aliquot point $(1/k)$ is not Helmholtz's parabola itself, but k chords inscribed in that parabola; and for a given bowing speed, the relative magnitude of the envelopes of the aliquot cases is that indicated in Figure 1. These facts are proved both theoretically and experimentally. The envelopes of the simpler non-aliquot cases have also been worked out theoretically, but it has not seemed worth while to verify them experimentally.

2. Helmholtz's velocity law (see page 695) has been accurately verified for the simpler aliquot cases.

3. Krigar-Menzel's law (see page 696) has been verified for the simpler rational cases, both aliquot and non-aliquot.

4. A graphical method has been developed by means of which the differential equation of the vibrating string can be solved under any set of boundary conditions which the theory of the rubbed string can present. The advantage of this method is that it gives directly the motion of any point of the string, or the configuration of the string as a whole at any time, — information which it is difficult, even in the simplest cases, to obtain by means of trigonometric series.

5. Each of the solutions for the non-aliquot cases $\frac{2}{5}$, $\frac{2}{7}$, and $\frac{3}{7}$ obtained by this method, has been verified at a number of typical points along the string.

6. It has been shown that all rational cases can be grouped into a great number of series or convergences, of which the series of aliquot cases is the simplest example. The solutions belonging to any one of these series are so related to one another that it is possible to predict with considerable accuracy the general characteristics of an unknown solution, and, in particular, to get its envelope, merely by determining to what convergence it belongs. Also these convergences are interesting in connection with the observations of Young and of Krigar-Menzel (see page 694) on the prominence of those partials which have nodes near the point of excitation.

Jefferson Physical Laboratory,
Harvard University.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR.—No. 177.

THE EFFECTS OF INBREEDING, CROSS-BREEDING, AND SELECTION UPON THE FERTILITY AND VARIABILITY OF DROSOPHILA.

BY W. E. CASTLE, F. W. CARPENTER, A. H. CLARK, S. O. MAST, AND W. M. BARROWS.

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By W. E. Castle, F. W. Carpenter, A. H. Clark, S. O. Mast, and W. M. Barrows.

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INTRODUCTION.

PHYSIOLOGICAL characters are inherited no less than morphological ones. In each case there doubtless is in the germ a structural basis on which the development of the peculiarity in question rests. In no instance as yet have we been able to identify beyond question the physical basis of any particular character, but a first step in that direction has been taken in the discovery of specific morphogenic substances in the animal egg and of specific differences among the chromosomes of the germinal nuclei in both sexes. While cytologists attack the problem of heredity from the side of the structure of the germ-cells, it is important that their labors be supplemented by a study of the heritable characters themselves, so that the mutual relations of characters and the modifications which they undergo from generation to generation may be better understood.

Among physiological characters which beyond question are heritable may be mentioned fertility, i. e. the capacity for reproduction. This varies among individuals and among races, as every experienced breeder knows, but the conditions upon which it depends are somewhat uncertain. In some cases external conditions are supposed to induce sterility, as, for example, abundant nutrition and lack of exercise, resulting in excessive vegetative growth without reproductive activity. In other cases inbreeding is assigned as a cause of sterility, or sterility may occur spontaneously without any assignable cause. The relation of inbreeding to sterility has been studied experimentally in mammals by Crampe ('83), Bos ('94), and Guaita ('98); and in birds by Fabre-Domengue ('98). They all find the relation to be a causal one, continuous inbreeding, as of brothers and sisters, resulting in decreased fertility, attended more or less commonly by lack of vigor, diminution in size, partial or complete sterility, and pathological malformations. It was our expectation that similar effects would be observed in the fly, Drosophila, when inbred continuously. This indeed seemed to be the result of our earlier experiments, but more extended observations indicate that close-breeding is not necessarily attended by decreased fertility, in this animal, but that particular degrees of fertility are transmitted in certain families irrespective of the consanguinity of the parents.

MATERIAL AND METHODS.

The organism used in this experiment in close-breeding was *Drosophila ampelophila* Löw, a small dipterous insect known under various popular names such as "the little fruit fly, pomace fly, vinegar fly, wine fly, and pickled fruit fly." These appellations indicate the class of substances in which the insect breeds. During the autumn it will commonly be found in the vicinity of fermenting fruit and decaying vegetable material. It is especially attracted by over-ripe grapes in the process of vinous fermentation. Through breaks in the skin of such grapes it lays its eggs, the resulting larvae feeding on the pulp.

A brief sketch of the life history of Drosophila will show that it is a form well adapted for experiments on inbreeding. The life cycle is rapidly completed, making it easy to obtain a large number of generations in a comparatively short period. The flies can be kept breeding the year round at ordinary room temperature on various fruits, some of which can be readily obtained at any season.

The imagoes become sexually mature within thirty-nine hours after hatching. The female can lay eggs in about forty-eight hours after emergence. The time may be, however, somewhat longer, depending probably on the attractiveness of the material furnishing a place for oviposition. Fermenting and decaying fruit seem to stimulate the laying of the eggs. At the end of three or four days the larvae appear and eat voraciously. The duration of the larval state varies considerably, but under favorable circumstances it may be completed within three days and the pupal state assumed. The larvae commonly seek dry places for pupation, usually leaving the food substance and crawling up the side of the vessel in which they are contained. The duration of the pupal condition, again, varies considerably, but may be completed at the end of about three days.

Allowing thus for the egg, larval, and pupal stages about three days each, and two for the imago to lay eggs, it follows that the whole life cycle may be completed within eleven or twelve days. But the average length of this cycle, when the whole year with its varying conditions is taken into account, would probably lie somewhere between fifteen and twenty days.

The normal sexual proportion in this species appears to be one very nearly approaching equality. Mr. Thomas Ordway states (in a manuscript report) that there seems to be a slight excess of males, but that perhaps the proportion would be somewhat changed if larger numbers than he worked with were taken into account. One can say, at least, that neither sex predominates in any marked degree.

It has been our intention in these experiments to keep external conditions as nearly uniform and as nearly optimal as possible, but to breed only from brothers and sisters through a series of generations. The breeding was at first carried on in small glass battery-jars about 8 cm. deep and 6 cm. wide, each covered with a square of window-glass; later, tumblers of similar dimensions were used, and were found more convenient because of the flaring top. The jars were kept on a table in the laboratory at ordinary room temperature.

A single pair of flies (brother and sister) was put in each breeding-jar, and was kept supplied with an abundance of food. This consisted during generations 1–12 of the *A* series of fermenting grapes, but in subsequent generations of the *A* series, as well as throughout all the other series, it consisted exclusively of banana, in which fermentation was started either by the addition of a little yeast or with a little juice taken from a stock-jar of fermenting banana.

To guard against the possible production of half-brothers and half-sisters through matings of the same female with two or more males, the following method of forming pairs was adopted and continued up to the year 1904–5.

Pupae of the same parentage were put together, a pair in each of a number of jars. When the imagoes emerged, if in any jar they proved to be of different sexes, they were left together; if of the same sex, the flies were rearranged so as to bring those of opposite sex together. Thus each female was ultimately placed in a jar with a single male and allowed to remain there during its entire subsequent life, which usually lasted three or four weeks. Most pairs under these circumstances produced an abundance of eggs within three to five days, and pupae were found about a week later attached to the sides or cover of the breeding-jar. These were removed and counted from day to day before they had an opportunity to emerge as imagoes, and the parents were left in the jar to continue breeding as long as possible.

Pairs which produced no young by the end of two or three weeks were recorded as sterile, and in some cases each parent was then placed with a new mate to ascertain, if possible, in which of the two parents the sterility lay. In some cases one or other of the two was found to be fertile by a different mate, but in no case were both found to be fertile by new mates. This result indicates that the sterility observed is not relative, but absolute; for an animal which is sterile toward one mate is sterile toward *all* mates.

Brief History of the *A* Series.

The experiment which was begun first and has been continued longest constitutes what we shall call the *A* series (see Table I and Figure 1). During the first year of this experiment (1901–2), when it was carried on by F. W. Carpenter, no record was kept of the number of sterile pairs, or the total number of young produced by each pair (except in generation 6, Table I), attention being directed chiefly to the effects of the inbreeding on variability. But beginning with the thirteenth generation, when A. H. Clark took charge of the experiment, records as complete as possible were kept of the number of fertile pairs and the number of young produced by each pair. Whenever a female was accidentally killed or escaped from the breeding-jar (as sometimes happened when pupae were being removed) before she had had an opportunity to lay her full quota of eggs, her brood was not used in making up the average for the generation, though the pair to which she belonged was recorded as fertile.

From generation 13 to generation 24 inclusive, Sept. 1902–June, 1903, the experiment was in charge of A. H. Clark and showed results of a pretty uniform character. Between seventeen and eighteen per cent of the pairs formed were sterile. The average number of young produced by a fertile pair was about 39; the maximum number, 145.

As a control on the results given by the *A* series during this period, pairs were from time to time taken from a stock-jar established in the following way. In October, 1902, flies captured at three different localities several miles apart were placed in a large covered jar within which was a smaller open jar containing fermenting banana. The flies soon increased, forming a vigorous colony, which was given no further attention beyond the occasional addition of fresh food. Three pairs taken from this jar in the latter part of October, 1902, produced remarkably vigorous broods numbering respectively 529, 560, and 260 young, an average of 450 young to a brood. Two pairs taken from the same jar about five weeks later gave a similar result, the broods numbering 623 and 361 respectively.

TABLE I. History of Series *A*.

Inbred Generation.	Average Number Young (Sterile Pairs not included).	Maximum Number of Young in a Brood.	Pairs Fertile.	Pairs Sterile.	When reared.	Observer.	
6	125	151	3	0	March (?) 1902	F. W. Carpenter.	
13	. .	. .	2	1	Sept.–Oct.	A. H. Clark.	
14	. .	55	2	2	October	"	
15	44	115	8	1	November	"	
16	59	75	4	1	Nov.–Dec.	"	
17	28	40	3	2	Dec. 1902–Jan.	"	
18	21	38	13	2	January [1903	"	
19	26	63	11	3	February	"	
20	43	116	12	2	March	"	
21	32	54	9	1	March–Apr.	"	
22	88	145	6	2	Apr.–May	"	
23	33	61	5	0	May	"	
24	72	86	5	1	June	"	
25	. .	. .	8	0 ?	June–July	"	
26	101	121	8	3	July–Aug.	W. E. Castle.	
27	128	138	8	2	August	"	
28	95	119	12	4	Aug.–Sept.	"	
29	153	196	9	4	September	"	
30	101	125	4	3	September	"	
31	4	7	3	4	October	"	
32	62	127	3	2	Oct.–Nov.	"	
33	55	103	7	4	Nov.–Dec.	"	
34	32	125	11	1	Dec. 1903–Jan.	"	
35	29	59	14	1	January [1904	"	
36	45	118	12	0	Jan.–Feb.	"	
37	43	105	6	0	Feb.–March	"	
38	126	257	11	1	March	"	
39	106	225	8	1	March–Apr.	"	
40	123	192	7	0	April	"	
41	174	296	12	1	May	"	
42	158	187	7	3	May	"	
43	63	81	10	2	June	"	
44	. .	. .	6	0	June–July	"	
45	. .	. .	6	0	July	"	
46	. .	. .	4	0	July	"	
47	. .	. .	4	0	August	"	
48	. .	. .	. .	. .	. . .	"	Allowed to interbreed.
49	. .	. .	. .	. .	. . .	"	
50	100	178	6	0	October	W. M. Barrows.	
51	187	210	4	0	November	"	
52	63	93	6	0	December	"	
53	249	372	7	0	January, 1905	"	
54	262	301	5	1 *	Jan.–Feb.	"	
55	308	397	6	0	Feb.–March	"	
56	280	357	6	0	March	"	
57	242	292	6	0	April	"	
58	229	341	6	0	Apr.–May	"	
59	135	229	4	0	May	"	

* ♂ sterile.

A brood reared a month later (in January, 1903) contained 178 young, but was probably incomplete, since it was produced in a very small jar containing an insufficient amount of food, and the food was not renewed, as was the practice in ordinary cultures.

Four control pairs taken from the stock-jar about four months later, in May, 1903, produced broods numbering 174, 142, 79, and 157 respectively. These numbers are very low compared with those obtained from control pairs in the previous fall, but it should be observed that the numbers obtained from the *A* series during this same period (generation 23) are likewise low. Very probably the falling off in both cases was due to low temperatures in the building, at about the time when the steam heat was discontinued, resulting in imperfect fermentation of the food, or directly affecting the egg production of the females. For the broods produced by the *A* series were larger both previous to this period and subsequent to it.

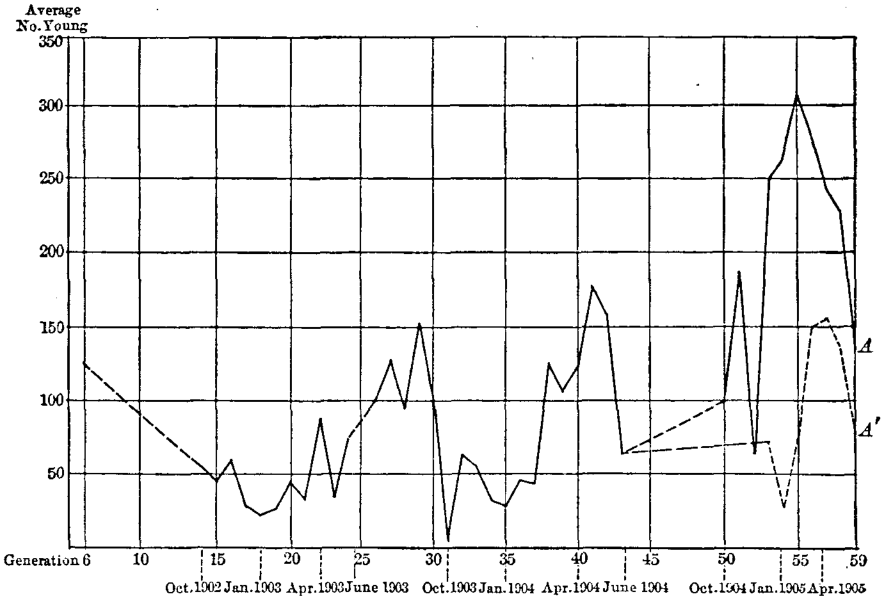


FIGURE 1. Productiveness of *fertile* pairs in the *A* series.

Broods of the control series were throughout this year several times as large as broods of the A *series reared simultaneously under identical conditions.* Further, no sterile pairs were encountered in the control series, though a considerable percentage of sterile pairs was constantly found in the *A* series. The inference naturally drawn from these facts was that inbreeding had decreased the fertility of the *A* series, but subsequent investigations throw doubt on this conclusion.

The *A* series was continued from June, 1903, to October, 1904, by the senior author. The warm weather of the months June–September, 1903, gave the flies optimal conditions for reproduction, and naturally the average number of young in a brood rose somewhat, reaching its highest point in the 29th generation, 153.5 young to a brood. But throughout this period, notwithstanding the favorable conditions, a considerable proportion of the pairs proved sterile, amounting on the average to 28 per cent of the pairs formed. The average brood of a fertile pair was 124 young.

Ten control pairs from outdoor sources, kept under the same conditions and at the same time as generation 27 of the *A* series, were all fertile. In only four out of the ten cases was a full record made of the number of young produced. These four broods numbered 164, 204, 304, and 241 respectively, an average of 228 to a brood, which was just 100 greater than the average-sized brood produced by fertile pairs in generation 27 of the *A* series.

In October, 1903 (31st generation), the productiveness of the *A* series fell off sharply and the series seemed on the verge of extinction. Four out of seven pairs were sterile, and the three fertile pairs produced in all only twelve young. At the same time control pairs from outside were producing vigorous broods containing usually 200 or 300 individuals.

But in the subsequent (32d) generation, the *A* series seemed to recover its normal condition, and so continued for six generations (32–37), fertile pairs producing an average of 41 young. During this period 8 out of 59 pairs, or 13.5 per cent, were sterile.

For the next five generations (38–42) the productiveness of the *A* series was more than doubled. The rise occurred suddenly in generation 38, the parents for this generation having been mated in the last days of February. As this was by no means the most favorable season of the year, and the possibility of an accidental cross seems for a variety of reasons excluded, it is difficult to account for the sudden change, unless it is the expression of a cyclical variation in fertility. In this period 6 out of 51 pairs, or 11.7 per cent, were sterile, while the fertile pairs produced on the average 137 young. Two other series, *M* and *N*, started in the previous October, and kept under similar conditions, were at the same time producing between 200 and 300 young, or even more.

In generation 43 of the *A* series we have indications of the possible return of a period of depression. Ten fertile pairs produced on the average 62.8 young, the maximum number being 81. Two other pairs were sterile. During the next four generations (44–47, reared June–August, 1904), no record was kept of the number of young produced. Twenty pairs in all were used, all of which proved fertile. The young constituting the 47th generation emerged as imagoes about August 15, 1904, and contrary to previous usage were not separated, but were left in the jar where they had emerged, to mate freely *inter se*. About October 10, animals were withdrawn from the jar to start a new series of generations. In the meantime two generations had probably been produced, allowing of possible mating of cousins, rather than of brothers and sisters, as in all previous generations. See page 751 as to the effect of mating cousins.

On October 10, 1904, Mr. W. M. Barrows took charge of certain of the *A* jars, removing ten pairs from jar *A* 47.2 to form what we shall call generation 50. At least eight of the ten pairs were fertile, but they showed wide variation in the size of their broods, ranging from 12 to 239 young. The average size of the six complete broods, that is, broods reared in jars where the mother lived for at least three weeks, and in which the food was properly fermented, was 100. The average was higher still, viz. 187 in generation 51, but it fell to 63 in generation 52, probably as a result of low temperature. It rose again sharply in generation 53, following a transfer to a warm chamber, and continued to rise in the two following generations until an average of 308 was attained in generation 55. Following the attainment of this, for the *A* race, unprecedented maximum, there was a steady fall in the average to 135 in generation 59, where the experiments for the year came to an end.

A second branch of the *A* family (*A*′, Figure 1), taken from the miscellaneously bred descendants of jar *A* 47.3, ran a course nearly parallel with that of the branch already described, but attained a maximum average of only 156. The two were kept under identical conditions.

Low Productiveness Characteristic of the A Stock.

Looking back over the history of the *A* series for fifty-nine generations of almost continuous inbreeding (Figure 1), we see that the average number of young produced by fertile pairs had never been as high as 200 until the fifty-third generation (winter of 1905); usually it was under 100; and about 1 in 5 of the pairs formed had been sterile (at least in the period between the twelfth and forty-fourth generations). Control cultures made under identical conditions, but from stock not inbred, had a productiveness two or three times as great and showed no signs of sterility. The *A* stock, accordingly, is characterized by low productiveness and an inclination to sterility. If these are effects of inbreeding, they began to show themselves early, the small productiveness as early as the sixth generation, the partial sterility as early at least as the thirteenth generation, and probably earlier. But it is very improbable that the low productiveness of race *A* was due to inbreeding, for when the inbreeding experiment was repeated with other stocks of flies no appreciable diminution of fertility occurred.

It cannot have been due to selection, conscious or unconscious, of parents from the least productive broods, for it was the practice of Messrs. Carpenter and Clark "to start a number of cultures and then breed from the one which succeeded best," which would be in general the *most* productive rather than the least productive pair. That selection is a very important factor, will presently appear. It certainly has a much greater influence on fertility than does inbreeding, so that selection from the most productive pairs is able to more than offset the effects of inbreeding. But it was not sufficient in the experiments of Carpenter and Clark to bring the *A* race up to normal productiveness. The low productiveness must have been present in the *A* stock at the beginning of our experiments and have persisted in spite of selection favoring increase of fertility.

Selections for High or Low Productiveness.

The bearing of selection from parental broods more or less productive upon the fertility of the race will be clear from Table II.

In the *A* series, between generation 30 and generation 43, pairs were several times taken simultaneously from parental broods of unequal size. The result is shown in Table II. The larger parental brood produces, as a rule, a larger average filial brood, and with a smaller proportion of sterile pairs. Thus, in the seven generations tabulated (which include all available data), if we weight each generation the same, the following average values are obtained from the lowest and highest parental broods respectively:

	Parental Brood.	Filial Brood.	Per cent Filial Pairs Sterile.
Lowest	96	91	23
Highest	185	104	13

TABLE II.

RESULT OF SELECTING PAIRS SIMULTANEOUSLY FROM BROODS OF DIFFERENT SIZE BUT OF SIMILAR ANCESTRY IN SERIES *A*.

Generation.	Parental Broods.	Average Filial Brood, omitting Sterile Pairs.	Per cent Sterile Pairs.	Number of Filial Broods.
20	134	82	50	4
"	196	119	33	3
33	49	71	50	4
" *	127	53	33	6
35	25	25	14	7
"	50	29	0	3
"	70	20	0	3
"	128	43	0	3
39	145	90	20	5
"	257	127	0	4
40	87	165	0	3
"	225	110	0	3
41	79	161	0	3
"	161	163	17	6
"	192	195	0	4
43	154	46	25	4
"	167	80	25	4

* Two broods in this lot higher than any in foregoing.

This summary shows an unmistakable tendency for the larger parental brood to produce larger filial broods, but *not so much larger* as the difference in the parental broods would lead us to expect. In other words, there occurs *regression toward low productiveness*, which is a fixed racial character of series *A*, or was in the period under discussion. Further evidence as to the effects of selection on fertility, based on series *M* and *N*, will be found on page 769.

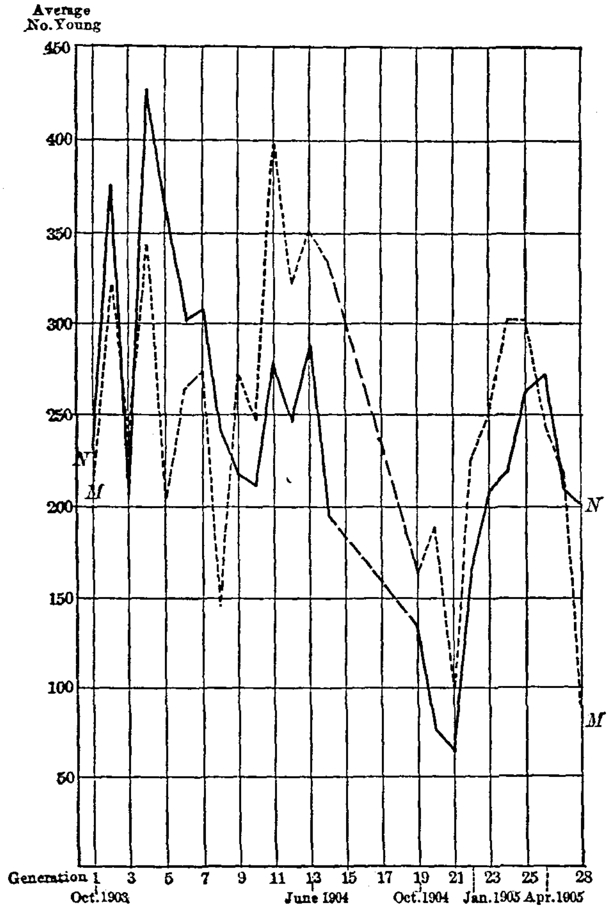


FIGURE 2. Productiveness of *fertile* pairs in the *M* and *N* series.

THE *M* AND *N* SERIES.

Two other inbred series, *M* and *N* (Tables VIII and IX, and Figure 2), started by S. O. Mast, showed during the first fourteen inbred generations productiveness much higher than that of series *A*. The average never fell below 200, except in a single generation of each series, and was usually nearer 300, occasionally rising to 400 or even higher. The *M* series shows a few sporadic cases of sterility; the *N* series, none whatever. The *M* series shows on the whole a rise of the average during the period under discussion; the *N* series, a fall. With the subsequent history of these series we are not at present concerned. Comparison of the three experiments shows, that in Drosophila close inbreeding during fourteen successive generations is not necessarily attended by either decreased productiveness or sterility, but that particular degrees of productiveness are characteristic of particular races. Subsequent experiments indicate that the productiveness of series *M* and that of *N*, during this period, are much nearer the normal than that of series *A*. A more complete account of generations 1–14 of the *M* and *N* series will be found on page 757.

OTHER FAMILIES OF DROSOPHILA.

In the summer of 1903 observations were made on the productiveness of flies obtained from two other sources. A stock, *D*, was reared from eggs found in a banana purchased in a Cambridge fruit store. Another stock, *F*, was formed from flies captured about a garbage can at a distance of half a mile from the laboratory, and at more than a quarter of a mile from the fruit store.

Four pairs from the *D* stock produced broods numbering respectively 164, 204, 304, and 241 young, the average number being 228. Six pairs from the *F* stock produced broods which were only in part recorded. The smallest number credited to a brood was 85; the largest, 220. It is probable, therefore, that the productiveness of this stock was similar to that of stock *D*, though perhaps a little lower. Both stocks, *D* and *F*, resemble *M* and *N*, rather than *A*, in productiveness.

EFFECTS OF CROSS-BREEDING.

The question now arises, if a stock characterized by low productiveness, like *A*, is crossed with a normal stock, what will be the productiveness of the offspring? Further, if Drosophila is indifferent to a considerable amount of inbreeding, will it be indifferent also to cross-breeding? To throw light on these questions, a number of crosses with the *A* stock have been made at various times.

Experiments of A. H. Clark.

In March, 1903, when the *A* series was producing an average of 43⅓ young to a pair, two females from the *A* stock were mated with males from the stock-jar mentioned on page 735. Their broods of offspring numbered 43 and 24 respectively, but the latter may have been incomplete, as the mother died within fourteen days after the culture was started. At any rate, there was no indication of any increased productiveness by *A* females on account of their being mated with males from a different stock. Two pairs, representing a cross reciprocal to the foregoing (*i. e.*, ♀ from stock-jar × ♂ *A*), produced broods numbering 91 and 132 respectively, i. e. two or three times as many young as the first mentioned crosses yielded. As this was about the usual relation of broods of normal series to those of the *A* series, it seems that the productiveness of the female is unaffected by the productiveness of the race to which her mate belongs. Subsequent experiments support this idea. But whether the *offspring* of a female would be affected in productiveness by the cross is an entirely different question. A single experiment was made at that time which bears on this matter. A pair taken from the 132 young produced by the cross, ♀ from stock-jar × ♂ *A*, yielded, in April, 1903, 116 young. The *A* series produced at this time an average of 32.4 young to a brood. The productiveness of the daughter was in this case similar to that of her mother, rather than that of the *A* race from which the father came.

Experiments of W. E. Castle.

More extensive experiments in crossing the *A* stock were made in July and August, 1903, parallel with generation 27 of the *A* series, which, it should be remembered, produced an average of 128 young to a brood, under the favorable summer conditions.

Cross 1. *A* ♀ × *D* ♂.	
Pair 1 . . .	sterile.
Pair 2 . . .	146 young.
Pair 3 . . .	106 "
Pair 4 . . .	150 "
Pair 5 . . .	114 "
Av. for fertile pairs, 129 young.	

Cross 1*a*. *D* ♀ × *A* ♂.	
Pair 1 . . .	sterile.
Pair 2 . . .	337 young.
Pair 3 . . .	315 "
Pair 4 . . .	206 "
Pair 5 . . .	163 "
Av. for fertile pairs, 255 young.	

In Crosses 1 and 1*a*, as in the previously described experiment, females of each race apparently had their productiveness unaffected by the cross-mating; for *A* females produced an average of 129 young by *D* mates, and 128 young by *A* mates. Exactly one pair in five is fruitless in each sort of mating. On the other hand, *D* females produced 255 young by *A* mates, and 228 young by *D* mates (see page 743). The sterility of Pair 1 in Cross 1*a* was probably inherent in the *A* male, as will subsequently appear.

Cross 2. *A* ♀ × *F* ♂.

Pair 1 . . .	142 young.
Pair 2 . . .	162 “
Pair 3 . . .	176 “
Pair 4 . . .	165 “

Average, 161 young.

Cross 2*a*. *F* ♀ × *A* ♂.

Pair 1 . . .	189 young.
Pair 2 . . .	189 “
Pair 3 . . .	186 “
Pair 4 . . .	283 “

Average, 212 young.

The *A* females employed in Cross 2 seem to have been more productive than those employed in Cross 1. Further, the *F* females of Cross 2*a* were less productive than the *D* females employed in Cross 1*a*. Accordingly the averages for the reciprocal crosses, 2 and 2*a*, are not so unlike as are those for Crosses 1 and 1*a*. Nevertheless, in one case as in the other, the *A* females are much the less productive.

Cross 3. *A* ♀ × stock-jar ♂ (p. 735).

Pair 1 . . .	129 young.
Pair 2 . . .	115 “
Pair 3 . . .	150 “
Pair 4 . . .	156 “
Pair 5 . . .	88 young.
Pair 6 . . .	126 “
Pair 7 . . .	113 “

Average, 125 young.

In a third cross (Cross 3), the reciprocal of which was not performed, *A* females were mated with males from the stock-jar (p. 735). They showed about the same productiveness (125) in this case as when mated with their own brothers (128), or with *D* males (129, Cross 1).

A control cross (Cross 4), made at the same time as the foregoing, between *F* females and *D* males, produced an average of 219 young, which agrees closely with the average, 212, produced by *F* females when mated with *A* males (Cross 2*a*).

Cross 4. *F* ♀ × *D* ♂.

Pair 1	199 young.
Pair 2	256 “
Pair 3	263 “
Pair 4	159 “

Average, 219 young.

These several experiments agree in showing that the fecundity of a female is unaffected by the relatedness or unrelatedness of her mate. In other words, the racial fecundity of the male in no case affects the productiveness of his mate, unless he is a wholly sterile animal.

Certain of the young produced by Cross 1, Pair 2, were bred *inter se.* Of ten pairs tested, all were fertile, though the record for one was incomplete on account of an accident. The average number of young produced by the remaining nine pairs was 356. The broods produced by the ten pairs individually were as follows:

Pair.	Young.	Pair.	Young.
G 7.1 . . .	[44] (incomplete)	G 7.6 . . .	393
G 7.2 . . .	338	G 7.7 . . .	412
G 7.3 . . .	408	G 7.8 . . .	388
G 7.4 . . .	339	G 7.9 . . .	269
G 7.5 . . .	359	G 7.10 . . .	297

Average, 356 young.

This is a remarkably high average. The *A* race at this same time (generation 28), and under identical conditions, was producing an average of 95 young to a pair; and pure *D* pairs in the previous generation had produced an average of 228 young, while pure *D* females mated with *A* males had produced 255 young, 100 less than the average brood of their cross-bred offspring.

Young from the reciprocal cross, 1*a*, Pair 5, were likewise mated *inter se.* Eight pairs were tested, all fertile. Their broods were as follows:

Pair.	Young.	Pair.	Young.
G 18.1 . . .	283	G 18.5 . . .	[66] incomplete.
G 18.2 . . .	335	G 18.6 . . .	[178] "
G 18.3 . . .	316	G 18.7 . . .	173
G 18.4 . . .	227	G 18.8 . . .	180

Average, 252 young.

The average number of young, 252, is not much greater than the general average for Cross 1*a*, though the particular brood (Pair 5, Cross 1*a*) from which the parents were derived contained only 163 young, and the average for pure *D* pairs had previously been found to be 228. On the whole, therefore, this cross, like the reciprocal one previously discussed, seems to have increased the productiveness of the cross-bred offspring, making it greater than that of either parent race when purely mated or when cross-mated.

Experiments of S. O. Mast.

The experiments of Mr. Mast, made at various times between November, 1903, and May, 1904, support this conclusion in part. Mr. Mast made crosses (5, 5*a*, 6, and 6*a*) between flies from generations 31–40 of the *A* series (which was then producing about 50 young to a pair), and mates taken from two new series, *M* and *N*, which commonly produced 200–400 offspring to a pair, or even more (see Figure 2, p. 742, and Tables VIII and IX, pp. 758, 759).

Cross 5. *A* ♀ × *M* ♂.

Pair 1 . . .	42 young.
Pair 2 . . .	79 “
Pair 3 . . .	6 “

Average, 42 young.

Cross 5*a*. *M* ♀ × *A* ♂.

Pair 1 . . .	365 young
Pair 2 . . .	379 “
Pair 3 . . .	597 “

Average, 447 young.

Cross 6. *A* ♀ × *N* ♂.

Pair 1 . . .	11 young.
Pair 2 . . .	55 “
Pair 3 . . .	25 “
Pair 4 . . .	22 “
Pair 5 . . .	32 “
Pair 6 . . .	28 “
Pair 7 . . .	84 “
Pair 8 . . .	sterile
Pair 9 . . .	15 “
Pair 10 . . .	36 “
Pair 11 . . .	sterile
Pair 12 . . .	95 “
Pair 13 . . .	90 “

Av. for fertile pairs, 45 young.

Cross 6*a*. *N* ♀ × *A* ♂.

Pair 1 . . .	234 young.
Pair 2 . . .	435 “
Pair 3 . . .	299 “
Pair 4 . . .	279 “
Pair 5 . . .	209 “
Pair 6 . . .	187 “
Pair 7 . . .	[186] “

incomplete.

Av. for pairs 1–6, 274 young.

Mr. Mast also mated females of *A* 36 (average young to a pair, 45) with males taken from a stock-jar started a short time previously with normal individuals similar to the *M* and *N* series. This will be called Cross 7. The reciprocal cross was not made.

Cross 7. *A* ♀ × stock-jar ♂.

Pair 1 . . .	33 young.
Pair 2 . . .	132 “
Pair 3 . . .	22 “

Average, 62 young.

The three crosses, 5, 6, and 7, show a result entirely normal for the *A* series; and the reciprocal crosses, 5*a* and 6*a*, show a result normal for the *M* and *N* series. In each case the productiveness of the mother is unaffected by the character of her mate, precisely as in crosses 1–4, and their reciprocals. Pairs of cross-bred individuals produced by crosses 5–7, when mated brother with sister, showed a marked increase in productiveness as compared with that of their respective inbred mothers.

	Average Mother's Brood.	Average Daughter's Brood.
Cross 5	42	187 (omitting three sterile pairs).
Cross 6	45	229
Cross 7	62	275

In the crosses reciprocal to 5 and 6, viz. 5*a* and 6*a*, the mothers came from races unusually fertile and not extensively inbred; no increase of fertility was observable among their offspring as a result of a cross with the *A* series.

	Average Mother's Brood.	Average Daughter's Brood.
Cross 5 *a*	447	353
Cross 6 *a*	274	274

The cross-breds, like their mothers, showed a high productiveness. This was probably neither increased nor diminished materially by the cross with the unproductive *A* race. The apparent falling off on the part of the young produced by Cross 5*a*, from 447, the average mother's brood, to 353, the average daughter's brood, is due to the unusual size of the maternal average. This in turn was due to the occurrence of a single very large brood, 597, in Cross 5*a*, the other broods numbering 369 and 379 respectively, which is not very different from the average daughter's brood, 353. While the cross-breds from Cross 5*a* were producing broods of this size, the uncrossed *M* race was producing broods averaging 273. While the cross-breds from Cross 6*a* were producing broods averaging 274 in number, the uncrossed *N* race produced a single complete brood of 277 young. We conclude that in the early generations of the *M* series and up to the ninth generation at least of the *N* series, the inbreeding had not materially affected the productiveness of those races, for, among other reasons, a cross with an unrelated and more extensively inbred race (*A*) did not affect the productiveness of the cross-breds, which remained normal, like that of the pure *M* or *N* races respectively.

Mr. Mast's subsequent experiments, however, show that the unproductive *A* race did show its influence in the second cross-bred generation (generation F_2), so that the cross did not eliminate wholly the unproductiveness inherent in the *A* race. For sterility and low productiveness reappeared after skipping a generation, in certain of the grandchildren, as will be shown by an examination of Tables X–XIV. This occurred in two out of five families, following a cross between a normal female and an *A* male. This observation shows that an entirely fertile male may transmit partial or complete sterility of the female sex as a racial character to his grand-daughters, though not apparently to his daughters.

The reciprocal cross, *A* ♀ × normal ♂ (Tables XV–XVIII), resulted in one case in the production of females half of which were sterile (Table XV), the other half being of low productiveness. This case shows that a female of a race inclined to sterility may transmit that character *directly* to her cross-bred offspring (generation F_1). The males of race *A* in Mr. Mast's experiments transmitted sterility to the grand-daughters only, never to the daughters. This difference in heredity through the two sexes would seem to indicate that sterility of the female is dependent upon egg structure rather than sperm structure, the eggs produced by mothers of a fertile race always yielding fertile daughters. But the eggs of cross-bred females, whose father was of an infertile race, produce some of them fertile, some infertile females.

Experiments of W. M. Barrows.

Though the *M* and *N* races, after nine or fewer generations of inbreeding, were not rendered more productive by a cross with race *A*, such was not the case after an additional year of inbreeding. In March, 1905, Mr. Barrows made crosses between the *M* and *N* races (then inbred for twenty-four generations), and between each of these and the *A* race (then inbred for fifty-four generations). See Tables III–V. The cross-mated mothers in each case gave a normal result (F_1), without any increased productiveness on account of the unrelatedness of their mates. But their cross-bred offspring, with equal regularity, showed increased productiveness (F_2), which surpassed that of either parent race under the same conditions.

At the same time Mr. Barrows made crosses between two sub-families of the *A* race (*A* and *A'*), which had arisen from a common ancestry some eleven generations previously. See Figure 1, *A* and *A'*, and Table VI. The two cross-mated mothers had broods considerably smaller than their sub-family (*A'*) was at the time producing, but their cross-bred daughters showed a productiveness just about equal to that of the A′ sub-family under the same conditions. The average brood of the cross-bred mothers was 237, that of the purely mated *A′* mothers was 242.

TABLE III.

EFFECT OF CROSSING RACES *M* AND *N*, EACH INBRED FOR TWENTY-FOUR GENERATIONS.

Generation.	Uncrossed *M* Race, Average Brood.	Uncrossed *N* Race, Average Brood.	$M \times N$ Cross-breds, Average Brood.	Number of Cross-bred Pairs.
24	302	220		
25	302	262	**291** $= F_1$	5
26	244	272	**341** $= F_2$	

TABLE IV.

EFFECT OF CROSSING RACES *A* AND *M*, INBRED FOR FIFTY-FIVE AND TWENTY-FOUR GENERATIONS RESPECTIVELY.

Generations.	Uncrossed *A* Race.	Uncrossed *M* Race.	$A \times M$ Cross-breds.	Number of Cross-bred Pairs.
55, 24	308	302		
56, 25	280	302	**212** $= F_1$	2
57, 26	242	244	**341** $= F_2$	

TABLE V.

EFFECT OF CROSSING RACES *A* AND *N*, INBRED FOR FIFTY-FIVE AND TWENTY-FOUR GENERATIONS RESPECTIVELY.

Generations.	Uncrossed *A* Race.	Uncrossed *N* Race.	$A \times N$ Cross-breds.	Number of Cross-bred Pairs.
55, 24	308	220		
56, 25	280	262	**235** $= F_1$	3
57, 26	242	272	**298** $= F_2$	

TABLE VI.

EFFECT OF A CROSS BETWEEN TWO DIFFERENT LINES (A AND A') OF THE SAME INBRED STOCK.

Generation.	Uncrossed A Race.	Uncrossed A' Race.	$A \times A'$ Cross-breds.	Number of Cross-bred Pairs.
55	308	75		
56	280	150	**117** $= F_1$	2
57	242	156	**237** $= F_2$	

Accordingly it appears that separation for eleven generations had not made the two sub-families of the A race sufficiently unlike to profit by a cross between them. The cross-breds were no more productive than the more productive parental sub-family.

But any two *unrelated* inbred families are clearly mutually benefited by a cross between them. Their offspring are more productive than either parental race (Tables III–V).

Matings of Cousins not more Fruitful than those of Brother and Sister.

That a mating between cousins of an inbred race is not more fruitful than a mating between brother and sister of the same race has been shown by experiments among which only the following need be mentioned. Twelve matings were made between cousins in generation 28 of the A series. Three of these were sterile and two others produced incomplete broods of young. The average brood of the remaining seven pairs was 105. The uncrossed A race, generation 29, reared under identical conditions, consisted of four sterile pairs and nine fertile ones. The latter produced broods averaging 153 in number. Further, the young of the cousins bred *inter se* gave the following result. Four out of fourteen pairs were sterile, the remaining ten produced an average of 112 young. The uncrossed A race, generation 30, reared simultaneously and under identical conditions, gave broods averaging 101, produced by five fertile pairs, three pairs being sterile.

CYCLICAL VARIATIONS IN FERTILITY AND THEIR RELATION TO EXTERNAL CONDITIONS AND TO INBREEDING.

One of the most striking features of the record of the A series is the cyclical rise and fall of the productiveness. (See Figure 1.) This was already low in generation 6, and kept on declining until, in January, 1903 (generation 18), the average had sunk to 21. From this point on a gradual rise followed, which ended abruptly in generation 29 (September, 1903). Then the productiveness fell off until, in January, 1904, the average brood numbered 29 young. Again it began to rise, as it had done the previous year. The rise was more rapid than in 1903, and reached a point somewhat higher. It also culminated (generation 41) sooner (in May rather than in September), and then began to decline again. In the fall of 1904 the productiveness of the *A* series was just about what it had been the previous spring, but showed a tendency to decline. Early in January, about the time recovery had begun in the two previous years, the cultures of the *A* series, together with those of series *M* and *N*, were transferred to a warm chamber. A remarkable increase in productiveness followed, which culminated in March (generation 55). The *A* broods then averaged 308 to a pair, but from that time on they steadily diminished in size, until at the end of May they had fallen to 135.

Each year the *A* series showed a gradual rise of productiveness followed by an abrupt decline. The annual rise follows a depression which occurs under fall conditions, in which low temperature is doubtless an important factor. Out of doors the rise would doubtless come in the spring, but in the warm laboratory it sets in earlier and culminates earlier. The culmination occurs in the three successive years in September, in May, and in March respectively. Each year the *A* series reached a higher maximum, until one branch of it at least seems to have become fully normal in productiveness, though another branch (*A′*, Figure 1) had still about the same productiveness in 1904–5 as in the previous year.

The *M* and *N* series (Figure 2) during the first nine months of inbreeding (generations 1–14) showed no marked change in productiveness. The *M* series showed on the whole a rise, the *N* series a fall, in productiveness. Both were evidently influenced by the same set of external conditions because the *M* and *N* curves generally show corresponding elevations and depressions. The *N* cultures were on the whole more vigorous; they showed no sterility whatever, whereas sterility did occur sporadically in the *M* series. Nevertheless the *N* series was apparently declining in fertility (perhaps because it had been *above* the normal in fecundity), while *M* was rising. In the second year of inbreeding (generations 19–28) both the *M* and the *N* series were less productive than in the previous year. The reverse was true of the *A* series in the same two seasons. Hence it seems not improbable that inbreeding was causing a decline of fertility in series *M* and *N*, while series *A* under continuous inbreeding was gradually recovering its lost fertility.

But it is not certain that the change in productiveness of *M* and of *N* was due to inbreeding; it may have been due wholly to changed conditions. The cultures of the two years were made by different persons, and their methods may have been different enough to account for the difference in results.

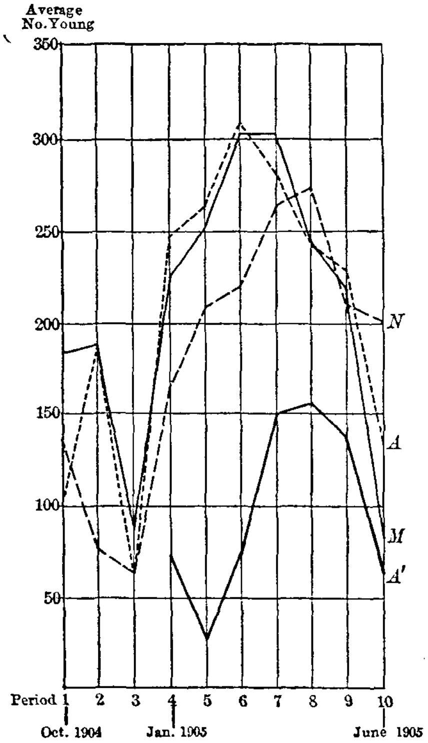


FIGURE 3. Synchronous cultures of 1904–05.

There can be no doubt, however, that a change had occurred in series *A*, which was now twice as productive as it had been the previous year, equalling the *M* and even surpassing the *N* series. This result was so surprising that as soon as it became fully evident, a second line (*A′*) of the *A* series was started, drawn from a different culture jar. This showed a productiveness similar to that of the *A* series the previous year. (See Figure 1, *A* and *A′*.) Accordingly there would seem to have occurred a marked increase of fertility in one jar of the *A* series, but not in another.

When the cultures were transferred to a warm chamber in January, 1905, *M* and *A* responded promptly by a sharp rise in productiveness. Their courses through the next five months were closely parallel (see Figure 3). Series *N* and *A′* responded in a similar way, but without attaining so high an average productiveness, series *N* because of a selection of broods of low productiveness (see page 756), series *A′* because of inherited low productiveness.

In the year 1904–5 two independent lines were maintained by Mr. Barrows in each of the series *A*, *M*, and *N*. The record of each of these lines is shown separately in Figure 4, A and A'', M and M', N and N'. The two lines of each series pass through similar cycles, though not quite synchronously; yet the two lines of each series were reared synchronously. Thus, A reaches a maximum two generations (six or eight weeks) later than A''; M culminates a generation later than M'; and N' culminates four generations later than N. The last case is particularly interesting because in this series alone of the three the attempt was made to select pairs from the most productive and the least productive broods respectively in the two lines. To carry out the selection proved in practice a difficult matter, because one cannot tell in the early stages of the cultures which brood will prove to be largest, so that he must either breed from all the cultures or else use the imagoes which emerge late. To do the former is very laborious, and the latter is undesirable because the late hatched individuals may be under normal size owing to deficient or too sour food.

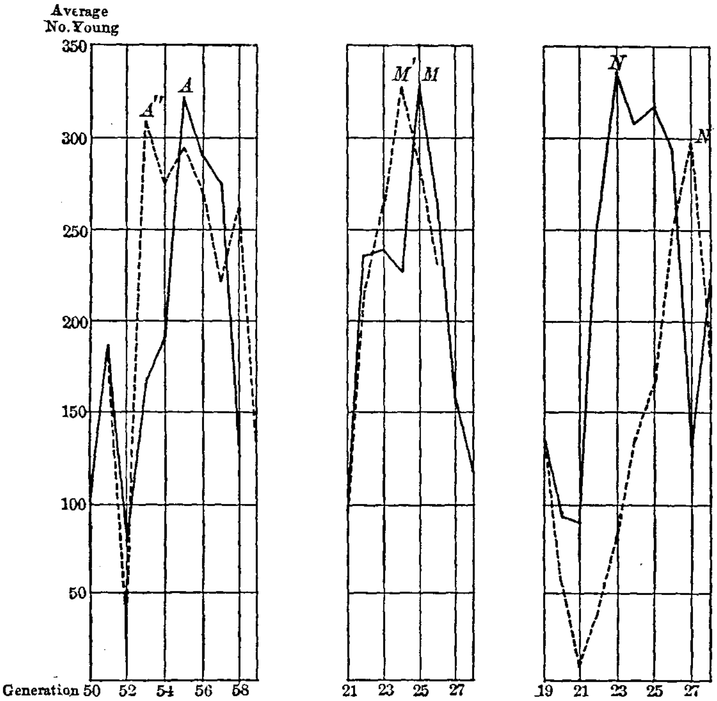
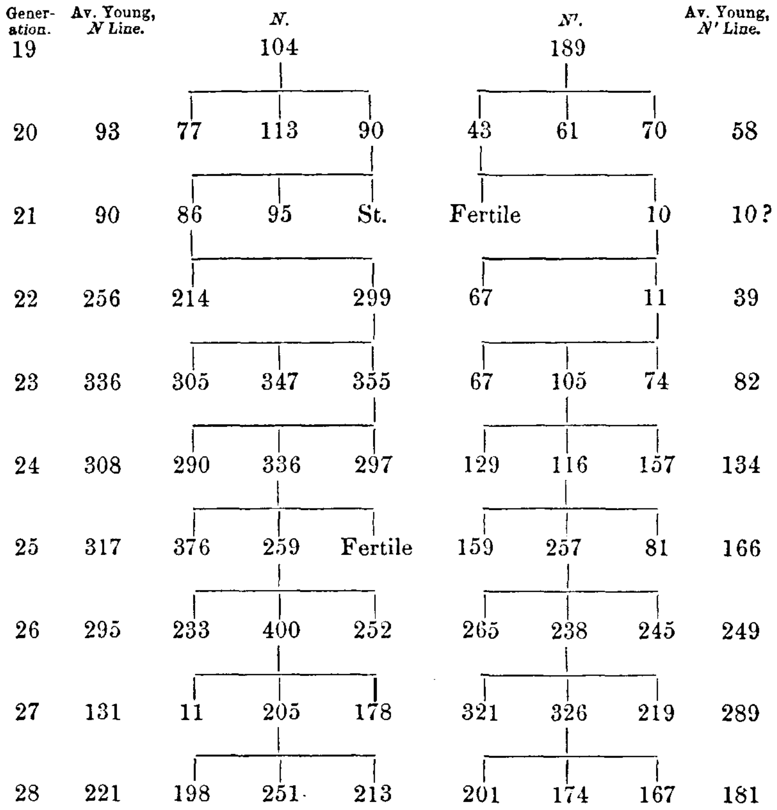


FIGURE 4. Productiveness of the two lines maintained in 1904–1905 in each of the three races, A, M, and N.

The character of the selections made is shown in detail in Table VII, but may be summarized thus:

TABLE VII.

Genealogy of *N* Series, 1904–5 (W. M. Barrows).



Generation.	Av. Young, *N* Line.	*N*.			*N'*.			Av. Young, *N'* Line.
19			104			189		
20	93	77	113	90	43	61	70	58
21	90	86	95	St.	Fertile		10	10?
22	256	214		299	67		11	39
23	336	305	347	355	67	105	74	82
24	308	290	336	297	129	116	157	134
25	317	376	259	Fertile	159	257	81	166
26	295	233	400	252	265	238	245	249
27	131	11	205	178	321	326	219	289
28	221	198	251	213	201	174	167	181

There were usually three broods to choose from in each line, which we may call the largest, the medium-sized, and the smallest, respectively. In each line pairs were as a rule taken from one brood only. The broods selected were as follows:

	Largest.	Medium.	Smallest.
In the *N* line	5	1	3
In the *N'* line	4	2	3

The character of the successive choices in each line was as follows, L standing for largest, M for medium, and S for smallest:

Generation . . .	19	20	21	22	23	24	25	26	27
N line	S	M	S	L	L	L	S	L	L
N′ line	L	S	S?	S	L	S	L	S	L

The so-called smallest broods of the *N* line were larger than the largest broods of the *N′* line in generations 21 and 25, so that the choices of "smallest" broods in the *N* line do not mean much. They were not intended as choices of small broods, and in fact those chosen were not small broods, though they happened to be *less* large than the two other broods of the same line.

The broods selected as smallest in the *N′* line were in most cases really small. The choices of smallest broods coming in succession, in generations 20, 21, and 22 respectively, seem to have given the *N′* line a character of low productiveness, from which, however, it steadily recovered when medium or largest broods were selected. No doubt if the selection of small broods had been consistently followed up in generations 23–27, the *N′* line would have continued lower in productiveness than the *N* line, just as the *A′* line continued lower than the *A* line (Figure 1), though reared under identical conditions and without conscious selection of broods.

These experiments make it clear that Drosophila is in fecundity much influenced by external conditions, such as temperature, upon which is dependent the proper fermentation of its food. This is strikingly shown also in Figure 3. But independently of external conditions some families are characterized by high productiveness, others by low. Improved conditions increase the productiveness of all, by starting them on a cycle of greater productiveness. But the response is more prompt and vigorous on the part of a race normally high in productiveness. Long continued inbreeding may possibly cause a decline in the fertility of Drosophila (for cross-breeding two inbred stocks certainly causes a rise in fertility of the offspring), but the influence of inbreeding on fertility is less than variations in fecundity due to other causes, so that selection of the more fecund broods would certainly maintain a normal fertility even under the closest inbreeding for fifty or more generations.

FURTHER DETAILS CONCERNING EXPERIMENTS OF S. O. MAST.

Crosses were made between a stock of flies (series *A*) inbred for over thirty generations, and two other stocks (series *M* and *N*) inbred for from one to nine generations.

History of the M *and* N *Series.*

The *M* and *N* series were descended from two pairs (*M* and *N*) captured on decaying apples in an orchard three miles north of Cambridge, October 1, 1903. Their progeny were inbred (brother with sister) as a control on the results of the *A* series obtained in previous years, and also to obtain a stock of known productiveness to use in crosses with the *A* series.

In Tables VIII and IX are shown the line of descent and the size of broods of flies, each produced by a pair (brother and sister) descended from one of the original pairs *M* and *N*, which were captured on decaying apples as already stated. The successive generations are designated *M* and *N* respectively, followed by a numeral to indicate the number of generations of inbreeding which had occurred in its production.

No sterile individuals were found in the *N* series, while in the *M* series one sterile female was found in the seventh generation, another in the tenth, and a sterile male in the fourteenth. A second male, probably sterile, was encountered in generation 5. In the three cases of sterility first mentioned, the suspected individuals were proved to be sterile by subsequently mating them with individuals that were known to be fertile. The second matings, like the original ones, proved unfruitful.

In the *M* series, the average brood in the first inbred generation was 213, while the average for the series as a whole is 280, an apparent increase of fertility under close breeding. In the *N* series, generation 1 gave an average of 231 young, while the series as a whole gave 278.5 young to a brood. But the first inbred generation may have been lower than the average because of less perfect methods of treatment. If we divide the series into first and second halves, we find that series *M* is more productive in the second half than in the first, while the reverse is true of *N*. The average brood is in:

	Generations 1–7.	Generations 8–14.
M series . . .	263	296
N series . . .	317	240

TABLE VIII. GENEALOGY OF *M* SERIES (1903–4).

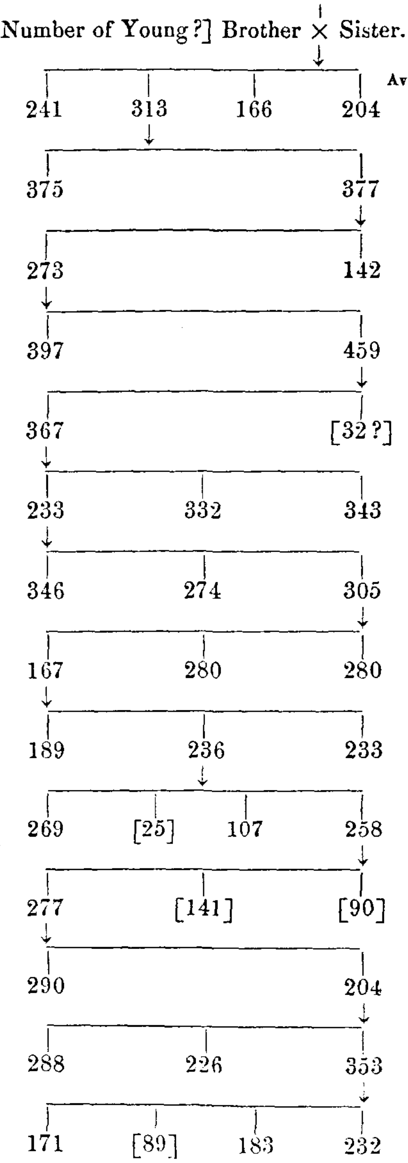
Pair *M*.
[Number of young ?] Brother × Sister
↓

Generation.						Average No. Young.
1	241 ↓	313		166	204	213
2	363 ↓	375		335	210	321
3	109 ↓				338	223
4	396 ↓				290	343
5	? ♂ sterile				205 ↓	205
6	228		237		327 ↓	264
7	♀ sterile		234		313 ↓	273
8	189		182		67 ↓	146
9	247		265 ↓		303	272
10	262 ↓		234		♀ sterile	248
11	351 ↓		385		460	399
12	332 ↓				315	323
13	454	341		281 ↓	335	352
14	335				♂ sterile	335
					Average for series . . .	280

TABLE IX. Genealogy of *N* Series.

Pair *N*.

[Number of Young?] Brother × Sister.

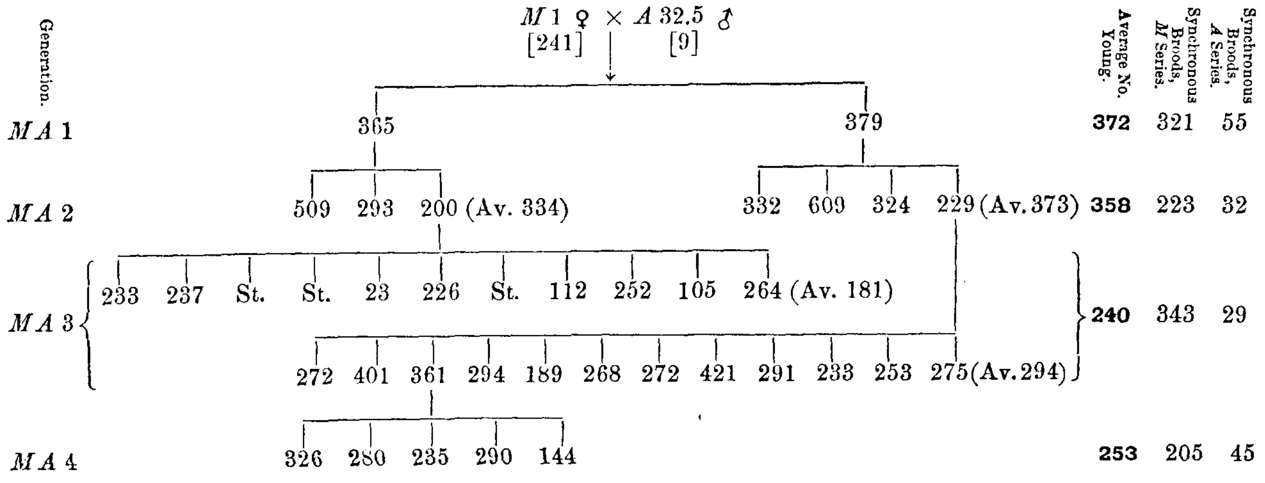


Generation.					Average No. Young.
1	241	313	166	204	231
2	375			377	376
3	273			142	207
4	397			459	428
5	367			[32?]	367
6	233	332		343	302
7	346	274		305	308
8	167	280		280	242
9	189	236		233	219
10	269	[25]	107	258	211
11	277	[141]		[90]	277
12	290			204	247
13	288	226		353	289
14	171	[89]	183	232	195
	Average for series . . .				278.5

TABLE X.

GENEALOGY OF *M A* SERIES.

(Cross 5*a*, page 747; see also Table XI.)


M 1 ♀ × A 32.5 ♂
[241]
[9]
Generation.
Average No. Young.
Synchronous Broods, M Series.
Synchronous Broods, A Series.
M A 1
365
379
372 321 55
M A 2
509 293 200 (Av. 334)
332 609 324 229 (Av. 373)
358 223 32
M A 3
233 237 St. St. 23 226 St. 112 252 105 264 (Av. 181)
272 401 361 294 189 268 272 421 291 233 253 275 (Av. 294)
240 343 29
M A 4
326 280 235 290 144
253 205 45

Dividing the series as nearly as possible into three periods of equal length, we obtain a similar result, as follows:

	Generations 1–5.	Generations 6–9.	Generations 10–14.
M series	261	239	331
N series	322	268	244

Series *M* is more productive in the last five than in the first five generations, while the reverse is true of *N*. Since the changes in the two series are opposite in nature it is improbable that they are due to inbreeding.

Crosses of Series M *and* N *with Series* A.

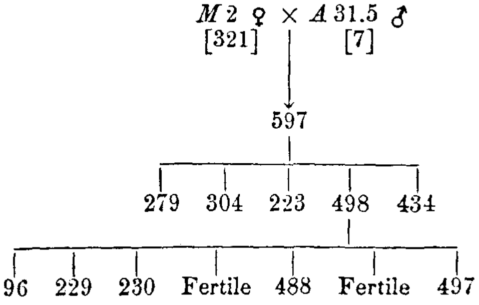
In Tables X–XVII are shown the results of crosses of the *M* and the *N* series with the long inbred *A* series. The cross-bred offspring were usually mated *inter se* (brother with sister) for one or more generations to observe the effects of the cross upon the subsequent history of the race. The generations of cross-breds are designated by the names of the two parental series, the maternal always coming first, followed by a numeral indicating the number of generations which have elapsed since the cross was made. Thus the offspring of the cross, *M* ♀ × *A* ♂, are designated *MA* 1, their children *MA* 2, and their grandchildren *MA* 3. The offspring of the reciprocal cross, *A* ♀ × *M* ♂, are *A M*1, their children and grandchildren *A M*2 and *A M*3 respectively.

The size of the broods, from which came the individuals used in making a cross, is indicated by numerals in brackets placed below the designation of the parental brood.

TABLE XI.

Genealogy of *MA* Series.

(Cross 5*a*, page 745; see also Table X.)

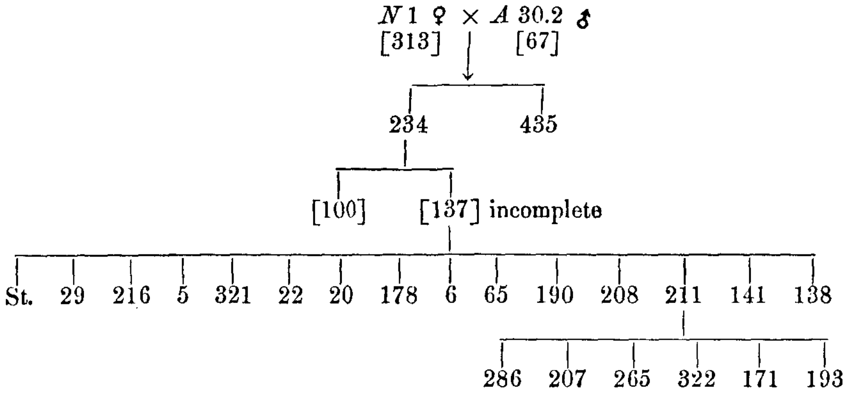


Generation.	Average No. Young.	Synchronous Broods, *M* Series.	Synchronous Broods, *A* Series.
MA 1		223	62
MA 2	**346**	343	55
MA 3	**361**	205	32

TABLE XII.

GENEALOGY OF *N A* SERIES.

(Cross 6*a*, page 747; see also Tables XIII and XIV.)


N 1 ♀ × *A* 30.2 ♂
[313]
[67]
234
435
[100]
[137] incomplete
St.
29
216
5
321
22
20
178
6
65
190
208
211
141
138
286
207
265
322
171
193

Generation.		Average No. Young.	Synchronous Broods, *N* Series.	Synchronous Broods, *A* Series.
N A 1	234, 435	**334**	376	55
N A 2	[100], [137] incomplete	?	207	32
N A 3	St., 29, 216, 5, 321, 22, 20, 178, 6, 65, 190, 208, 211, 141, 138	**125**	428	29
N A 4	286, 207, 265, 322, 171, 193	**241**	367	45

An examination of these tables shows:

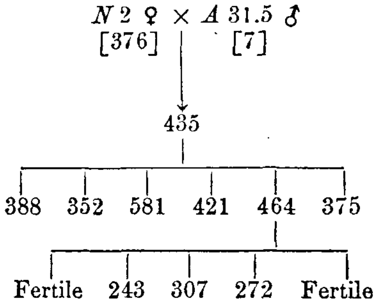
(1) That a female of the *A* series mated with a male of series *M* or *N* does not have her productiveness increased thereby. See Tables XV, XVI, XVII, and XVIII.

(2) That a female of series *M* or *N* mated with a male of the *A* series does not have her productiveness diminished thereby. See Tables X, XI, XII, XIII, and XIV. The reverse seems to be true in certain cases, but this was probably due to the bestowal of special care upon the cross-mated series. In the case of those recorded in Tables XI and XIII this is almost certainly true; the jars of cross-mated flies were in those two instances transferred to a room warmer than that in which the pure *M* and *N* series were reared.

TABLE XIII.

Genealogy of *NA* Series.

(Cross 6*a*, page 747; see also Tables XII and XIV.)



Generation.		Average No. Young.	Synchronous Broods, *N* Series.	Synchronous Broods, *A* Series.
	N 2 ♀ × *A* 31.5 ♂ [376] [7]			
NA 1	435		207	62
NA 2	388 352 581 421 464 375	430	428	55
NA 3	Fertile 243 307 272 Fertile	274	367	32

It follows from (1) and (2) that low productiveness in Drosophila is due to low productiveness of eggs, not of spermatozoa. Complete sterility, however, may be found in either sex, as is shown by the history of both the *A* and the *M* series.

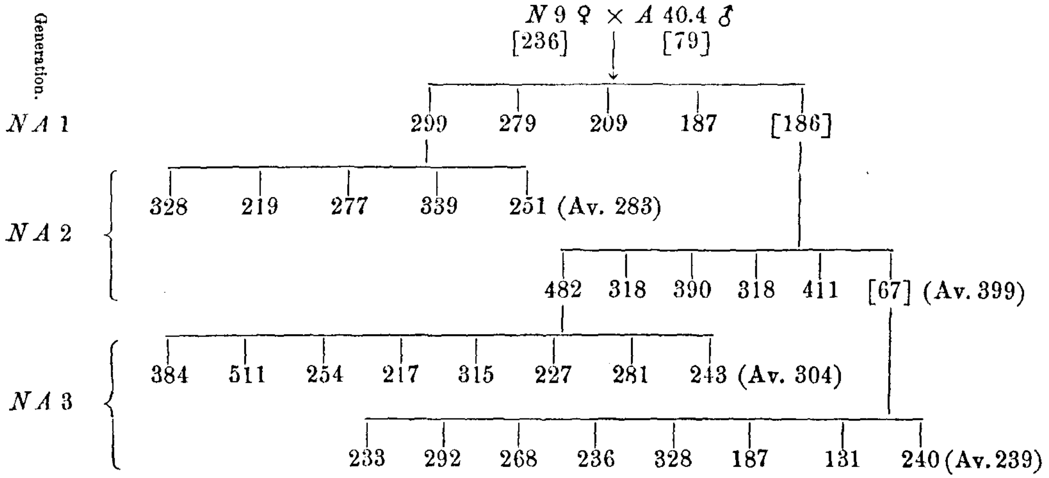
(3) The daughters of an *A* female by an *M* or an *N* male produce more young than their mother did, and more than other *A* mothers do under identical conditions. Their productiveness does not differ materially from that of purely mated *M* and *N* mothers, though *complete* sterility may appear among them.

(4) The daughters of an *M* or an *N* mother by an *A* male are on the whole similar in productiveness to their mother and to purely mated *M* and *N* mothers. None of the daughters has been found sterile.

TABLE XIV.

GENEALOGY OF *NA* SERIES.

(Cross 6*a*, page 747; see also Tables XII and XIII.)


Generation.
N 9 ♀ × *A* 40.4 ♂
[236]
[79]
NA 1
299
279
209
187
[186]
NA 2
328
219
277
339
251 (Av. 283)
482
318
390
318
411
[67] (Av. 399)
NA 3
384
511
254
217
315
227
281
243 (Av. 304)
233
292
268
236
328
187
131
240 (Av. 239)

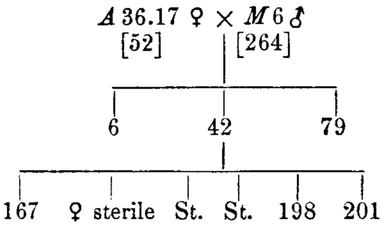
Generation.	Average No. Young.	Synchronous Broods, *N* Series.	Synchronous Broods, *A* Series.
NA 1	**243**	211	174
NA 2	**341**	277	158
NA 3	**272**	247	63

(5) The grandchildren produced by either of the reciprocal crosses between the *A* series and the *M* or *N* series vary much in productiveness. Some are sterile or of low productiveness like mothers of the pure *A* race. These occur oftenest among the young of cross-bred pairs of low productiveness. Others are as productive as the uncrossed *M* or *N* races.

TABLE XV.

Genealogy of *A M* Series.

(Cross 5, page 747.)



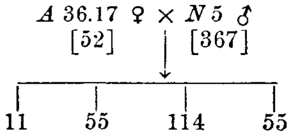
Generation.	Average No. Young.	Synchronous Broods, *A* Series.	Synchronous Broods, *M* Series.
A M 1	**42**	43	273
A M 2	**189**	126	146

(6) It follows from the facts stated in 3–5 that a tendency to low productiveness and sterility on the part of the female may be transmitted through either sex as a racial character. When so transmitted it conforms imperfectly with Mendel's law of alternative inheritance. Fertility is commonly dominant over low productiveness, but not invariably. See Table XV, *A M* 2, in which generation half the pairs are sterile. This case is exceptional. Low productiveness commonly skips a generation following a cross, but may be transmitted *directly* through the *female parent.* Its reappearance after skipping a generation is not certain. If selection is made from the most fertile cross-bred broods, low productive-

TABLE XVI.

A N Series.

(Cross 6, page 747; see also Table XVII.)

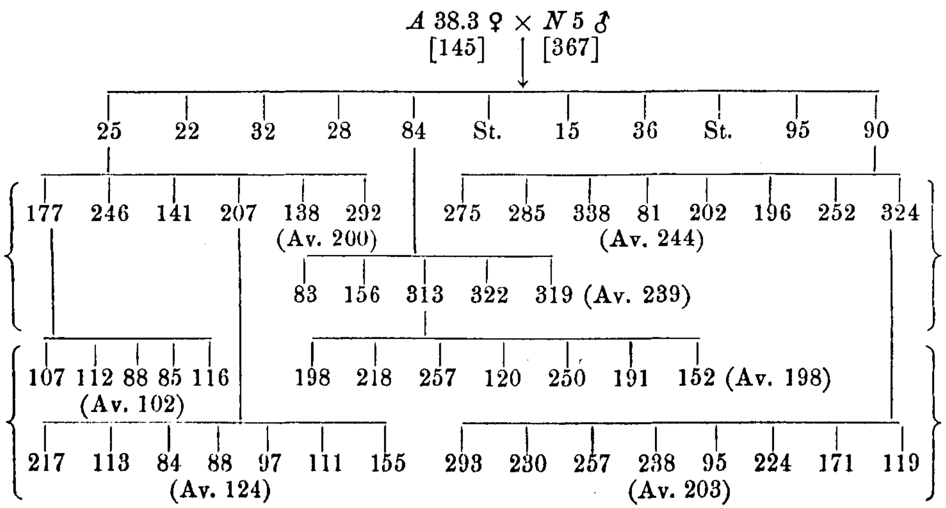


Generation.	Average No. Young.	Synchronous Broods, *A* Series.	Synchronous Broods, *N* Series.
A N 1	**59**	43	302

TABLE XVII.

GENEALOGY OF *A N* SERIES.

(Cross 6, page 747; see also Table XVI.)


A 38.3 ♀ × *N* 5 ♂
[145]
[367]
25
22
32
28
84
St.
15
36
St.
95
90
177
246
141
207
138
292
(Av. 200)
275
285
338
81
202
196
252
324
(Av. 244)
83
156
313
322
319 (Av. 239)
107 112 88 85 116
(Av. 102)
198
218
257
120
250
191
152 (Av. 198)
217
113
84
88
97
111
155
(Av. 124)
293
230
257
238
95
224
171
119
(Av. 203)

Generation.	Average No. Young.	Synchronous Broods, *A* Series.	Synchronous Broods, *N* Series.
A N 1	**47**	106	302
A N 2	**229**	123	308
A N 3	**162**	174	242

ness may not reappear in the following generation. Hence we can only say that low productiveness has a *tendency* to be inherited in alternative fashion, *skipping a generation,* but the alternative character of the inheritance is not sharp as is that of albinism and other typical Mendelian characters.

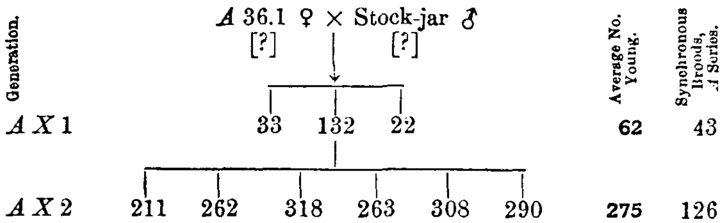
If low productiveness were a typical Mendelian recessive character, we should expect it regularly to reappear after skipping a generation, and to occur on the average in one fourth of the daughters produced by cross-bred pairs. But it does not always skip a generation. See Table XIX. Further, when it does, as expected, skip the first cross-bred generation (F_1) it sometimes fails to reappear in the following generation (F_2), cross-breeding having apparently obliterated the low productiveness.

Nevertheless, considering the fact that it is impossible to draw a sharp line of division between high and low productiveness, the alternative inheritance of the two conditions found respectively in race *A* and in *M* or *N* is remarkably clear.

TABLE XVIII.

A X SERIES.

(Cross 7, page 747.)



If we class as *low* all complete broods of one hundred young or fewer, and as *high* all broods of over one hundred young, then the various pairs are grouped as shown in the left half of Table XIX. But if we class as *low* all broods of less than 150 young, and as *high* all broods of over 150 young, the various pairs are grouped as shown in the right half of Table XIX.

The uncrossed *N* series in all its fourteen generations (Table IX) never produced a complete brood containing as few as 150 young, and the uncrossed *M* series (Table VIII) produced only two such broods, one of 67, the other of 109 young. The *A* series during the period under consideration only occasionally produced broods larger than 150. Accordingly

TABLE XIX.

NUMBERS OF PAIRS LOW OR HIGH, RESPECTIVELY, IN PRODUCTIVENESS, AMONG THE CROSS-BREDS AND AMONG THEIR CHILDREN.

Among Cross-breds from Cross *M* or *N* ♀ × *A* ♂.	Broods under 100.	Broods over 100.	Broods under 150.	Broods over 150.
M A 2, Table X	0	7	0	7
" " XI	0	5	0	5
N A 2, " XIII	0	6	0	6
" " XIV	0	10	0	10
Totals	0	28 All high	0	28 All high
Among Cross-breds from Cross *A* ♀ × *M* or *N* ♂.				
A M 2, Table XV	3	3	3	3
A N 2, " XVII	2	17	4	15
A X 2, " XVIII	0	6	0	6
Totals	5 One in six low	26	7 One in four or five low	24
Among *Children* of Cross-breds from Cross *M* or *N* ♀ × *A* ♂.				
M A 3, Table X	4	7	6	5
" " "	0	12	0	12
" " XI	1	4	1	4
N A 3 " XII	7	8	9	6
" " XIII	0	3	0	3
" " XIV	0	8	0	8
" " "	0	8	1	7
Totals	12 One in five low	50	17 One in three or four low	45
Among *Children* of Cross-breds from Cross *A* ♀ × *M* or *N* ♂.				
A N 3, Table XVII	2	3	5	0
" " "	0	7	1	6
" " "	3	4	5	2
" " "	1	7	2	6
Totals	6 One in four or five low	21	13 One in two low	14

broods of fewer than 100 young are clearly like *A* broods, those of over 150 young are like *M* and *N* broods in size. Those between these limits might fall in either category, but would be more characteristic of the *A* series.

Among the 89 broods reared from the children of cross-breds, 18 (or 20.2 per cent) are under 100 in size, while 30 broods (or 33.7 per cent) are under 150 in size. The expectation based on the hypothesis that low productiveness is a Mendelian recessive character is that 25 per cent of the broods will be low, which value lies between the two extremes named above, 20.2 per cent and 33.7 per cent.

TABLE XX.

Effect of selecting Pairs from Broods more or less Productive but of Similar Ancestry.

Generation in which Selection was made.	Parental Broods.	Average Filial Brood.
M A 1, Table X	365	334
	379	373
M A 2, Table X	200	181
	229	294
A N 1, Table XVII	25	200
	84	239
	90	244
A N 2, Table XVII	177	102
	207	124
	313	198
	324	203

Selection for High or Low Productiveness.

The inheritance of different degrees of fertility is unmistakable throughout these experiments and affords material for selection to act upon. Whenever pairs are taken simultaneously from broods of the same ancestry but of different productiveness, the pairs taken from the more productive brood are invariably more productive, as a glance at Table XX will show. If we weight alike each of the several cases included in Table XX, we obtain the following average values from the *lower* and *higher* broods respectively:

	Parental Brood.	Filial Brood.
Lower	196	197
Higher	232	239

We get here no evidence of regression. Compare page 741.

Is a Single Copulation sufficient to fertilize All the Eggs of a Female?

It is generally assumed that in many insects a single copulation (or several within a brief period) suffices to fertilize all the eggs subsequently produced by the female. That this is not so in Drosophila is indicated by the following observations.

In generation *M* 13, Table II, a female was taken from a jar containing several males and put in a jar with food, where she laid eggs. Pupae were removed from the jar as follows:

July 1	46	
" 5	16	
" 9	13	
" 13	0	
" 16	9	
" 20	32	
" 25	11	(Experiment closed.)

Fresh eggs were found in the jar July 9, and some of these were put into a jar containing some fruit, but none of them hatched. The same day a male was put in with the female, and, as shown above, the eggs laid later were fertile. Likewise a cross-bred female which had been with males for a few days was on June 12 put in a separate jar with food. Pupae were found as follows:

June 26	21	
July 1	21	
" 5	0	
" 9	0	(Male introduced.)
" 13	0	
" 16	29	
" 20	1	

On July 9 a male was introduced, and pupae were again found after none had been found for over a week. These results show conclusively that a single copulation or several within a period of a few days is not necessarily sufficient to fertilize all the eggs in Drosophila.

Abstract of Report of F. W. Carpenter on the Sixth Inbred Generation of the *A* Series.

To test the variability of the inbred flies, as compared with those not inbred, the following variable character was selected. On the proximal segment of the tarsus of the first pair of appendages there is present in the male a secondary sexual organ, comb-like in appearance, containing a variable number of chitinous teeth. This "sexual comb" has probably the function of a clasping organ in copulation. It is the number of teeth in this comb that has been taken as the variable character to be dealt with, the counting being done under the compound microscope.

The male Drosophila can often be distinguished with the unaided eye from the female by its smaller size and the darker color of the posterior segments of the abdomen. However, one can be quite certain of the sex only by examining the fly under the microscope, where the presence or absence of the sexual comb peculiar to the male can be noted.

In counting the teeth in the sexual comb the procedure was as follows: The sex of the preserved fly was determined under a low power of the microscope. If the fly proved to be a male it was removed from the stage of the microscope and the anterior pair of appendages was cut away from the body with a sharp scalpel. In order to do this more easily the head of the fly was first removed. The appendages were then examined under the high power of the microscope and the number of teeth in the sexual comb counted.

The Inbreeding Experiments.

The inbred material was obtained thus. A normal male and female were placed in a jar with food. The pair lived here, and from their offspring several males and several females were selected. Of these, one male and one female were placed in each of several jars with food. Brothers and sisters were thus bred together. The most promising one of these latter cultures was selected to perpetuate the line and the other cultures were rejected. Certain of the offspring of this selected culture were treated in the way just described for their parents, and so the successive generations of inbred flies were obtained.

When the sixth inbred generation was reached, three cultures, designated later as "Families *A*, *B*, and *C*," were carried through, each having as parents a brother and sister belonging to the fifth generation. All of the flies appearing in these cultures were preserved in 70 per cent alcohol, so that the whole number of offspring was obtained. The parents were well supplied with food and kept until they died. The female parent of each culture was transferred at the end of about every ten days to a new jar. Imagoes began to appear in the first jar while the female parent was still alive and laying eggs.

A control stock culture was maintained during the year under a large bell-jar. Here the flies were allowed to breed without interference. They required no care except to be provided from time to time with fresh food.

(*a*) *Effect on fertility and sexual proportion.* In the three cultures of the sixth generation designated "Families *A*, *B*, and *C*," all the offspring were saved. The numbers of these and their sexual proportions were as follows:

	A.	*B.*	*C.*
Males	46 (47.9%)	76 (50.3%)	64 (50.4%)
Females . . .	50	75	63
Totals . . .	96	151	127

No accurate determinations were made of the number of offspring of a normal pair of flies, but from such observations as were made it is probably safe to say that no striking falling off in fertility is to be seen in the sixth inbred generation. No indication of weakness was observed among the flies.

Since the sexual proportion is near equality (possibly with a slight excess of males), the figures given above indicate, on the whole, no marked change in this respect.

(*b*) *Effect on variability.* The number of teeth was counted in the eighty sexual combs of forty males taken at random from the control stock culture, and the results were plotted in the form of a frequency polygon (Figure 5, *N*). In addition the teeth were counted in the same number of combs in the case of each of the inbred cultures, *A*, *B*, and *C*. These results were also plotted in frequency polygons (Figure 5, *A*, *B*, *C*).

It will be seen that the number of teeth in the sexual comb is higher in all the inbred families than in the normal flies (*N*) studied. Both means and modes are higher for the inbred flies. But it is doubtful whether this is the result of inbreeding, for the male employed in starting the inbreeding experiment, the common ancestor of all the inbred families, had twelve teeth in his right comb, ten in the left, numbers higher than the normal and very similar to those prevailing among the inbred descendants of the male in question.

The variability in number of teeth has apparently been neither increased nor diminished by the inbreeding.[1] The inbred flies have a slightly higher standard deviation, as we should expect from the fact that they have higher means, but the coefficient of variability (ratio of standard deviation to mean) is in family *B* a little greater than in the normal flies, while in families *A* and *C* it is a little less.

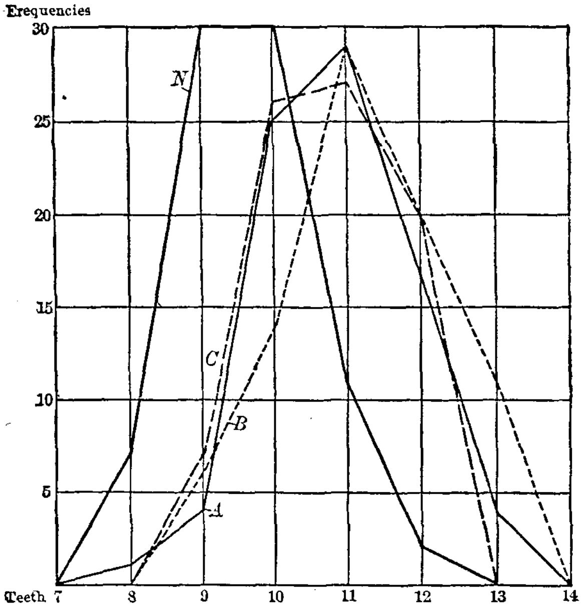


FIGURE 5. Variation in number of teeth on the sex-comb of forty males from each of three different broods (*A*, *B*, and *C*) of a stock inbred for six generations, and of males (*N*) from a stock not inbred.

The number of teeth is about the same, and varies in a similar way on the two sides of the body, as is shown by plotting separately the numbers for each side. The degree of correlation between the numbers on the two sides of the body is very small. This means that the right and left legs are to a great extent independently variable in number of teeth, which would lead one to suppose the variability dependent largely upon local conditions within the pupa.

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[1] An examination was subsequently made by A. H. Clark of the variability of the sexual comb in 45 males of the *fifteenth* inbred generation. This was found to be almost identical in character with what it had been in the sixth generation. The range of variation was still from 8 to 13, with the mode at 11, the mean at 10.79.

Influence of Light of Different Colors.

(*a*) *Influence on fertility.* For about ten weeks flies were allowed to breed in chambers so constructed that the only light admitted passed through colored solutions contained in flattened glass vessels. These colors were red, blue, and green. An orange chamber was also in use, but its culture died out after about six weeks. In addition, flies were bred in white light and in darkness.

Below are given the numbers of imagoes which had appeared in the cultures at the end of the ten weeks:

White.	Dark.	Blue.	Red.	Green.	Orange.
93	67	68	104	73	?

These numbers are of doubtful significance because of several unknown factors entering into the result, such as differences in vigor among the progenitors of the various cultures, possible differences in quality of food, etc. The experiment should be several times repeated before conclusions are drawn.

(*b*) *Influence on phototropic responses of larvae.* The older larvae were observed to be negatively phototropic in their responses to ordinary daylight. In seeking situations for pupation they were found to have distributed themselves as follows in the rectangular glass vessels in which they were contained in the various chambers. The distribution was determined by the position of the pupa cases.

	White.			Blue.			Red.			Green.			Orange.		
	Front.*	Back.*	Sides.	Front.	Back.	Sides.	Front.	Back.	Sides.	Front.	Back.	Sides.	Front.	Back.	Sides.
	9	29	10	15	68	40	21	64	34	18	36	36	8	26	23
Percentage at back	60%			55%			54%			40%			45%		

* "Front" indicates side of vessel toward light, "back," side away from light.

The considerable proportion of individuals which pupated on the front side of the several chambers is doubtless made up largely of individuals which pupated at night, when the directive effect of light would be eliminated, or nearly so. White light evidently has the strongest directive effect, while blue and red are about equally effective, orange and green being less effective still. The observed effects are perhaps due to differences in the intensity rather than in the quality of the illumination in the various chambers.

It is probable that the larvae avoid light of *every* kind at pupation, for in each of the five cultures more pupae are found on the back of the chamber than on its front. No emphasis is laid on the qualitative result obtained because the lights employed were not entirely pure and the intensities were not uniform, so that from this experiment alone it is impossible to say whether or not any one sort of ray is more influential than another in determining the result.

Age at which Sexual Maturity is attained.

Certain experiments were made to determine the earliest time after emergence of the imagoes at which fruitful matings could occur between the sexes. In four cases males and females were left together for twenty-four hours or less, after hatching, and then separated. None of these females laid fertile eggs. Two females which were left with males for thirty-nine hours after hatching and then separated both laid fertile eggs. It follows that the flies become sexually mature at some time between twenty-four and thirty-nine hours after hatching. The time was not more precisely determined.

Is a Male capable of Copulating more than Once?

Yes, as the following experiment shows.

Four unfertilized females were placed in a jar with a single male, which had not yet copulated, and left for twenty-seven hours. Each female was then placed in a separate jar and the male allowed to escape. All the females produced offspring.

As a control on this experiment, two virgin females were placed in a jar with food. After a few days several eggs were observed, but none of these hatched. [This experiment has since been several times repeated, always with the same result. Apparently Drosophila is incapable of parthenogenetic reproduction. W. E. C.]

Studies by W. M. Barrows of Variation in Number of Teeth on Male Sex-Comb, and of Variation in Size.

To determine whether long-continued inbreeding had affected the variability of the character, number of teeth on sex-comb of male, Mr. Barrows has examined individuals from three different inbred series, *A*, *M*, and *N*, and from one stock not inbred, *X*. These experiments show no appreciable effect of inbreeding. In every case the brood reared under the best and the most uniform conditions has the highest average number of teeth, irrespective of whether or not it is inbred. The same may be said of variation in size. Inbreeding has diminished neither the average size nor the variability in size.

TABLE XXI.

Correlation in Number of Spines between Sex-combs of Right and Left Legs, Sixth Inbred Generation, *A* Series.

Left leg (x).

	Spines	9	10	11	12	13	Sums
Right leg (y).	8	. .	1	. .	. .	. .	1
	9	. .	4	1	. .	1	6
	10	3	7	6	10	2	28
	11	6	13	17	11	6	53
	12	2	11	8	6	1	28
	13	. .	1	. .	2	1	4
	Sums	11	37	32	29	11	**120**

Mean number of spines, left leg $= A\,x = 10.93$
" " " right leg $= A\,y = 10.94$
Standard deviation . left leg $= \sigma\,x = 1.453$
" " " . right leg $= \sigma\,y = 1.230$
Coefficient of correlation . . . . $r = .022 \pm .061$

The series and generations studied were *A* 61, *M* 31, *N* 30, and *X* 1. The origin and history of the first three has been described in detail in the earlier parts of this paper. Series *X* originated from two pairs of flies caught in Cambridge in October, 1905. Series *N* died out for lack of food in the thirtieth generation, and the measurements were made from dry dead flies found in the bottom of the jar. This culture shows a unique variability. Series *A*, *M*, and *X* were breeding under the same good conditions and side by side when the specimens were taken for special study.

Out of the preserved material of each of the four series one hundred males were selected at random, the measurements being made as follows:

TABLE XXII.

Correlation in Number of Spines between Sex-combs of Right and Left Legs, Series *A*, Generation 61.

Left leg (x).

Right leg (y).

Spines	8	9	10	11	12	13	14	Sums
7	..	1	..	..	..	..	..	1
8	1	1	1	..	..	..	..	3
9	..	2	9	2	1	..	..	14
10	..	6	10	8	3	..	..	27
11	1	4	10	18	6	1	1	41
12	..	..	1	4	6	..	..	11
13	..	..	..	1	1	..	..	2
14	..	..	..	..	1	..	..	1
Sums	2	14	31	33	18	1	1	**100**

$A\,x = 10.58$ $\sigma\,x = 1.087$

$A\,y = 10.50$ $\sigma\,y = 1.136$

$r = 0.469 \pm .05261$

The fly was held on its back, under a dissecting microscope, by needle points, and the right and left fore legs of the fly were then pulled off and laid on a glass slide. Next, the right hind leg was pulled off, and also laid on the slide. These parts were then covered with a cover glass, and examined under a compound microscope. The number of spines in each sex-comb was counted and recorded. The length of the tibia of the hind leg was ascertained by means of an ocular micrometer, the record being made in divisions of its scale, one division of which was equal to 0.017 mm.

From the data thus gathered were constructed Figures 6 and 7 and Tables XXII–XXVII. Table XXI is based on observations made by F. W. Carpenter (see page 773). He counted the spines on the sex-combs of both fore legs of forty males from each of three jars, *A*, *B*, and *C* of the sixth inbred generation of the *A* series, and also of forty males from a stock culture considered "normal." The variation in number of spines in each of these four lots is shown in Figure 5, and for the three inbred lots combined in Table XXI. That table shows also the degree of correlation in number of spines which exists between the two sides of the body in the sixth inbred generation of the *A* series. The same thing is shown for the sixty-first generation of the *A* series in Table XXII, and for the first inbred generation of the *X* series in Table XXIII. A comparison of these three tables indicates that the correlation between right and left sides is probably not increased by inbreeding, for though

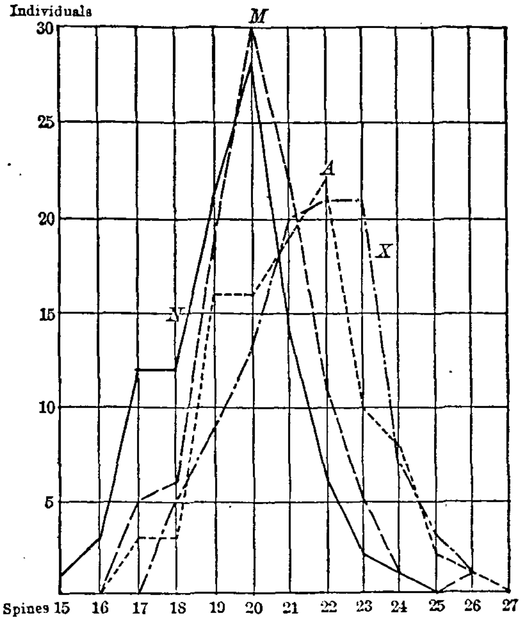

FIGURE 6. Variation in number of spines on the two sex-combs of one hundred males from each of four different series; *A*, from generation *A* 61; *M*, from *M* 31; *N*, from *N* 30; and *X*, from *X* 1.

the correlation is greater in *A* 61 than in *A* 6, it is not materially greater in the former than in the not-inbred *X* series. The close resemblance between *A* 61 and *X* 1 is probably due to the fact that they were reared under similar conditions.

In Table XXVIII are brought together for comparison constants derived from the several correlation tables, XXI–XXVII. From a study of this we learn the close dependence of the variability of Drosophila upon external conditions. Compared with these factors inbreeding is of little consequence.

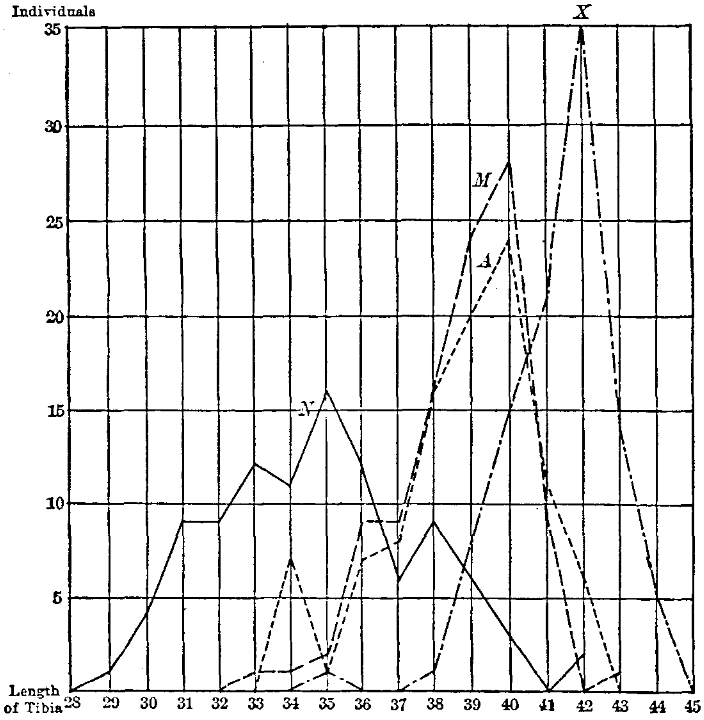

FIGURE 7. Variation in length of tibia of one hundred males from each of four different series; *A*, from generation *A* 61; *M*, from *M* 31; *N*, from *N* 30; and *X*, from *X* 1.

The well-nourished cultures (*A* 61, *M* 31, and *X* 1, Figures 6 and 7) show a higher mean number of spines and a greater tibial length than cultures reared under poor or variable conditions of nutrition, such as

N 30 (Figures 6 and 7) and Carpenter's "normal" individuals (Figure 5, N). This is true irrespective of the inbredness of the different cultures. This conclusion is supported by a comparison of the *A* series in the sixth inbred generation (*A* 6, *B* 6, and *C* 6) and in the sixty-first generation (*A* 61). Conditions were optimum in both cases; the average number of spines and the variability are similar. Contrary to the common opinion, the variability would seem to have been increased rather than diminished by the inbreeding, since the coefficient of variation is greater for the sixty-first than for the sixth inbred generation; but this difference may be due to the small numbers of individuals examined, particularly in the cultures of the sixth generation, each of which furnished only forty males.

TABLE XXIII.

Correlation in Number of Spines between Sex-combs of Right and Left Legs, Series *X*, Generation 1.

Left leg (x).

	Spines	8	9	10	11	12	13	14	Sums
Right leg (y).	8	..	..	1	1	1	..	..	3
	9	..	3	5	..	1	..	..	9
	10	1	3	9	8	4	..	..	25
	11	..	3	11	14	6	1	..	35
	12	..	..	3	14	6	1	..	24
	13	..	..	1	..	2	..	..	3
	14	..	..	..	..	1	..	..	1
	Sums	1	9	30	37	21	2		**100**

$A\,x = 10.74$ $\sigma\,x = 1.094$

$A\,y = 10.81$ $\sigma\,y = 1.186$

$r = 0.301 \pm .06133$

The measurement, tibial length, is probably a good index of relative size of the individuals in the cultures measured. If so, we may infer that the individuals of *X* 1 are larger than those of the other three cultures measured, which fact would indicate that inbreeding tends to diminish size. Yet the effect of this factor is small as compared with the effects of nutrition, for *N* 30 and *M* 31 differ in tibial length by four units, though inbred practically the same amount; but *M* was reared under uniformly good conditions, *N* under varying conditions, which finally became intolerable. Again *M* 31 and *A* 61, which differ by thirty generations in inbredness, but were reared under identical conditions, have an almost identical tibial length. This is exceeded, however, by

TABLE XXIV.

Correlation between Number of Spines in Sex-combs and Length of Tibia, Series *N*, Generation 30.

x = spines (sum of both legs).

y = length of tibia.	15	16	17	18	19	20	21	22	23	24	Sums
29	..	..	1		..	..	..	..	..	..	1
30	..	..	2	2	..	..	..	..	..	..	4
31	1	1	3	2	1	1	..	..	..	..	9
32	..	..	3	1	4	1	..	..	..	..	9
33	..	1	2	2	4	3	..	..	..	..	12
34	..	1	..	3	3	4	..	..	..	..	11
35	..	..	..	2	3	10	1	..	..	..	16
36	..	..	1	..	2	5	4	..	..	..	12
37	..	..	..	..	3	..	..	1	2	..	6
38	..	..	..	..	..	3	3	2	..	1	9
39	..	..	..	..	1	..	3	2	..	..	3
40	..	..	..	..	..	1	2	..	..	..	3
41	..	..	..	..	..	..	..	..	..	..	0
42	..	..	..	..	..	..	1	1	..	..	2
Sums	1	3	12	12	21	28	14	6	2	1	100

$A\,x = 19.38$, $\frac{A}{2} = 9.69$ $\sigma\,x = 1.684$

$A\,y = 34.80$ $\sigma\,y = 2.842$

$r = .7075 \pm .0337$

three units in the *X* 1 culture, which was not inbred beyond a single generation. This difference probably represents, in part at least, an effect of inbreeding, which shows itself between the sixth and the thirty-first generations. Compare the variation in spines of *X* 1, *A* 6–*C* 6, *M* 31, and *A* 61, all of which were reared under optimum conditions. Number of spines is correlated with tibial length, as we shall see, and the number of spines in *A* 6–*C* 6 was slightly higher than in *A* 61. It is fair to suppose that the tibial length also was slightly greater, and so the size. These differences may have been due simply to differences in nutrition, but the fact that *X* 1, which was reared under the same conditions as *A* 61, yet exceeds it in number of spines and tibial length, suggests that *A* 6 may exceed *A* 61 for the same reason, that is, because less inbred.

TABLE XXV.

Correlation between Number of Spines in Sex-combs and Length of Tibia, Series *A*, Generation 61.

x = spines (sum of both legs).

y = length of tibia.	17	18	19	20	21	22	23	24	25	26	Sums
34	3	. .	2	. .	. .	2	. .	. .	. .	. .	7
35	. .	. .	. .	. .	. .	1	. .	. .	. .	. .	1
36	. .	1	1	2	2	1	. .	. .	. .	. .	7
37	. .	1	2	3	1	1	. .	. .	. .	. .	8
38	. .	1	5	3	. .	6	. .	1	. .	. .	16
39	. .	. .	5	3	7	1	1	2	. .	1	20
40	. .	. .	1	2	6	4	5	4	2	. .	24
41	. .	. .	. .	2	3	4	1	1	. .	. .	11
42	. .	. .	. .	1	. .	2	3	. .	. .	. .	6
Sums	3	3	16	16	19	22	10	8	2	1	100

$\frac{A\,x}{2} = 10.55$ $\sigma\,x = 1.8574$

$A\,y = 38.72$ $\sigma\,y = 2.031$

$r = .4687 \pm .0526$

The correlation between number of spines and length of tibia is considerable. It increases as the variation in tibial length increases, being over .70 in *N* 30, in which lot the greatest diversity of tibial lengths occurred; in its production occurred the greatest variations in food supply. The variation in number of spines increases in nearly the same order as the tibial lengths, among the several lots, so that it is clear that substantially the same factors govern both, the correlation between the two becoming more pronounced as the variability becomes greater. This variability is dependent largely upon temperature and food supply, inbreeding being a factor entirely negligible.

TABLE XXVI.

CORRELATION BETWEEN NUMBER OF SPINES IN SEX-COMBS AND LENGTH OF TIBIA, SERIES *M*, GENERATION 31.

x = spines (sum of both legs).

y = length of tibia.

	17	18	19	20	21	22	23	24	25	26	Sums
33	. .	1	. .	. .	. .	. .	. .	. .	. .	. .	1
34	. .	. .	1	. .	. .	. .	. .	. .	. .	. .	1
35	. .	1	1	. .	. .	. .	. .	. .	. .	. .	2
36	. .	1	4	3	1	. .	. .	. .	. .	. .	9
37	1	2	2	1	3	. .	. .	. .	. .	. .	9
38	1	. .	3	9	3	. .	. .	. .	. .	. .	16
39	2	1	5	6	7	3	. .	. .	. .	. .	24
40	1	. .	3	6	5	8	4	1	. .	. .	28
41	. .	. .	. .	4	3	. .	1	. .	. .	1	9
42	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	0
43	. .	. .	. .	1	. .	. .	. .	. .	. .	. .	1
Sums	5	6	19	30	22	11	5	1	0	1	100

$\frac{1}{2} A\,x = 10.115$ $\sigma\,x = 1.568$

$A\,y = 38.70$ $\sigma\,y = 1.723$

$r = .4476 \pm .0539$

TABLE XXVII.

Correlation between Number of Spines in Sex-combs and Length of Tibia, Series *X*, Generation 1.

x = spines (sum of both legs).

y = length of tibia.	18	19	20	21	22	23	24	25	26	Sums
35	. .	. .	. .	. .	. .	1	. .	. .	. .	1
36	. .	. .	. .	. .	. .	. .	. .	. .	. .	0
37	. .	. .	. .	. .	. .	. .	. .	. .	. .	0
38	. .	1	. .	. .	. .	. .	. .	. .	. .	1
39	1	1	1	2	2	1	. .	. .	. .	8
40	1	3	2	2	1	4	. .	2	. .	15
41	2	1	2	5	4	4	2	. .	1	21
42	1	3	6	7	8	6	4	. .	. .	35
43	. .	. .	2	3	4	3	1	1	. .	14
44	. .	. .	. .	1	2	2	. .	. .	. .	5
Sums	5	9	13	20	21	21	7	3	1	100

$\frac{1}{2} A\ x = 10.775$ $\sigma\ x = 1.7485$

$A\ y = 41.38$ $\sigma\ y = 1.461$

$r = .1413 \pm .066$

Summary.

1. Inbreeding probably reduces very slightly the productiveness of *Drosophila*, but the productiveness may be fully maintained under constant inbreeding (brother with sister) if selection is made from the more productive families.

2. In crosses of a race of low productiveness and inclined to sterility (race *A*) with a race of high productiveness (*B*) it has been found that a female of race *A* does not have her fecundity increased by mating with a male of race *B*, and conversely, a female of race *B* does not have her fecundity diminished by a mating with a male of race *A*. Hence every male not actually sterile furnishes an abundance of functional spermatozoa.

TABLE XXVIII.

CONSTANTS DERIVED FROM TABLES XXI–XXVII BROUGHT TOGETHER FOR COMPARISON.

Series and Inbred Generation.	Mean of Spines (A).	$\frac{1}{2}A$ Spines.	Mean of Spines, Right.	Mean of Spines, Left.	Standard Deviation, σ Spines,	Coeff't of Variation, Spines $= \sigma/A$.	Mode of Spines.	Mean Length of Tibia.	Standard Deviation of Tibia (σ).	Correlation (r) Spines and Tibia.
"Normal" (Carpenter)	19.28	9.64 ± .145	9.625	9.650	1.364 ± .1029	14.15	9 and 10	. .	. .	. .
A 6	21.71	10.86 ± .1802	10.87	10.85	1.690 ± .1275	15.56	11	. .	. .	. .
B 6	22.40	11.20 ± .1781	11.125	11.275	1.670 ± .1259	14.91	11	. .	. .	. .
C 6	21.50	10.75 ± .1192	10.825	10.675	1.118 ± .0843	10.40	11	. .	. .	. .
A 61	21.10	10.55 ± .1252	10.50	10.58	1.8574 ± .0885	17.60	11	38.72	2.031 ± .0969	.4687 ± .05261
X 1	21.55	10.775 ± .1179	10.81	10.74	1.7485 ± .0833	16.28	11	41.38	1.461 ± .0697	.1413 ± .066
M 31	20.23	10.115 ± .1058	10.11	10.11	1.568 ± .0748	15.51	10	38.70	1.723 ± .0821	.4476 ± .0539
N 30	19.38	9.69 ± .1136	9.75	9.63	1.684 ± .0795	17.38	9 and 10	34.8	2.842 ± .1355	.7075 ± .0337

3. The cross-breds produced by the mating, *B* ♀ × *A* ♂, are all of high productiveness.

4. The cross-breds produced by a mating *A* ♀ × *B* ♂ are usually, but not always, of high productiveness.

5. The children of both sorts of cross-breds (see 3 and 4) are some of high productiveness like race *B*, others of low productiveness, like race *A*.

6. Low productiveness is inherited after the manner of a Mendelian recessive character in certain of the crosses made, skipping a generation and then reappearing. In other cases it has failed to reappear in generation F_2, indicating its complete extinction by the cross. In a few cases it has failed to be dominated by high productiveness in generation F_1. In such cases the female parent has always been of race *A*. Hence low productiveness (or sterility) of the female may be transmitted directly through the egg from mother to daughter, but only indirectly through the sperm, the character skipping a generation.

7. A cross between two races, one inbred for thirty or more generations and of low productiveness, the other inbred for less than ten generations and of high productiveness, produced offspring like the latter in productiveness, but not superior to it.

8. The same two races crossed after an additional year of inbreeding (about twenty generations) produced offspring superior to either pure race in productiveness.

9. Inbreeding does not affect the variability in number of teeth on the sex-comb of the male, nor the variability in size.

10. Size of the male individual is closely correlated with number of teeth in the male sex-comb.

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THE INFLUENCE OF FREQUENCY UPON THE SELF-INDUCTANCE OF COILS.

By J. G. Coffin.

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It is well known, through the work of Lord Rayleigh,[1] and Stefan,[2] that currents of high frequency in straight wires distribute themselves over the cross-section of the wires very differently from steady currents.

The values of the resistance of such wires, as well as their self-inductance, thereby suffer a change. Resistance undergoes a large increase, self-inductance a small decrease. The theory for straight wires has been hitherto directly applied to wires wound on coils, in want of a better theory for such a case.

Recently, Max Wien [3] and A. Sommerfeld [4] have attacked successfully the problem for wire coils.

Max Wien's results, however, are only applicable to small frequencies, while those of Sommerfeld, although valid for all frequencies, gave only the change in *resistance*.

In the following will be found the derivation of a formula by a method based on that of Sommerfeld for the change of self-inductance with the frequency.

By means of this theory we shall derive a method of assigning the correction to be applied to any coil of a single layer. Its extension to coils of more than one layer is reserved for a future communication. The theory [5] shows that with increasing frequency the currents tend to flow more and more on the inside surface of the coil. The maximum change in self-inductance is then the change due to a diminution of the mean radius of the coil of one half the diameter of the wire, with which the coil is wound.

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[1] Phil. Mag., **21**, 381 (1886).

[2] Wied. Ann., **41**, 400 (1890).

[3] Max Wien, Ann. d. Phys., **14**, 1 (1904).

[4] A. Sommerfeld, Ann. d. Phys., **15**, 673 (1904).

[5] Max Wien, loc. cit.

For example, consider a coil whose mean radius is 27 cms. wound with wire of diameter .06 cms. There is a maximum possible change of 3.5 per cent for very high frequency in the self-inductance of a single turn, while in a coil of 716 turns the total possible change is nearly 0.3 per cent. This is not a negligible amount, so that in standard coils and work of precision it is necessary to know what the corrections are for any frequency.

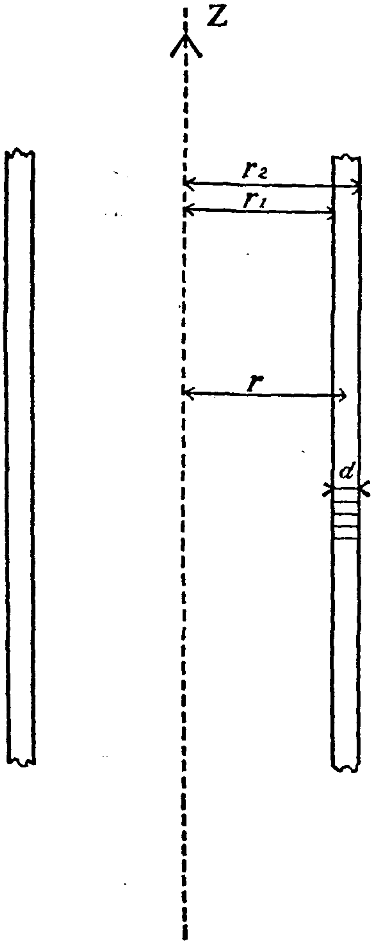

FIGURE 1.

DERIVATION OF THE FORMULA.

Consider an infinite circular current sheet of thickness d, of inside radius r_1 and outside r_2.

Let the symbols $S(\ )$ and $V(\ )$ denote the scalar and vector products respectively. ∇ is the Hamiltonian operator. $V\nabla(\ )$ is the same as curl ( ) or rot ( ), while $S\nabla(\ )$ is the same as div ( ).

By symmetry, the field inside the coil is axial, and a function of r alone, at any given time, and on account of the coil being infinite in length, the field is all inside of it.

In the dielectric, Maxwell's equations are, if we neglect the displacement current,

$$V\nabla H = 0 \text{ and } S\nabla H = 0. \qquad (1)$$

From the second relation we find that H is independent of z, and from the first relation, that it is independent of x and y; hence it is constant inside the coil, and for

$$r \leqq r_1 \text{ assume } H = H_0 \, e^{i\omega t} \qquad (2)$$

where ω is $2\pi \times$ frequency, and H_0 the maximum value of H. Outside the coil, H must be constant, but as it vanishes at infinity it is zero everywhere. Hence, for

$$r \geqq r_2 \qquad H = 0.$$

In the material of the coil, Maxwell's equations are

$$4\pi q = V\nabla H, \quad \frac{dH}{dt} = -V\nabla F, \quad \text{and } q = \sigma F, \tag{3}$$

from which follow $\qquad S\nabla H = 0$ and $S\nabla q = 0 \qquad (4)$

where $\qquad q$ is the current density,

F is the electric force,

σ is the specific conductivity.

Eliminating q and F, by taking the curl of the first equation (3)

$$\begin{aligned} 4\pi V\nabla q = V\nabla V\nabla H &= \nabla S\nabla H - \nabla^2 H \\ &= -\nabla^2 H \qquad \text{by (4)} \end{aligned}$$

and from the second and third of (3)

$$V\nabla q = \sigma V\nabla F = -\sigma\frac{dH}{dt}$$

therefore

$$4\pi\sigma\frac{dH}{dt} = \nabla^2 H. \tag{5}$$

Now assume H in the material of the coil to be given by a function U of r alone (on account of symmetry), multiplied by $H_0 e^{i\omega t}$, or $H = H_0 U e^{i\omega t}$ where

$$\begin{aligned} U &= 1 \text{ when } r = r_1 \\ U &= 0 \quad \text{“} \quad r = r_2. \end{aligned}$$

The differential equation then which U must satisfy by equation (5) becomes in polar co-ordinates

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} + k^2 U = 0 \tag{6}$$

where $k = -i\,4\pi\sigma\omega$.

The solutions of this equation are the Bessel's functions of order zero, J_0 and K_0 and hence

$$H = H_0\,\{A\,J_0\,(kr) + B\,K_0\,(kr)\}\,e^{i\omega t} \tag{7}$$

where A and B are arbitrary constants.

Now, Sommerfeld[6] has shown that the expression

$$H = \sqrt{\frac{r_1}{r}}\,\frac{e^{ik(r_2-r)} - e^{-ik(r_2-r)}}{e^{ik(r_2-r_1)} - e^{-ik(r_2-r_1)}}\,H_0\,e^{i\omega t} \tag{8}$$

is a very good approximation formula for the case in which $k r_1$ and $k r_2$ are large, and he finds also that it is valid even when these quantities are small, this being an unexpected result.

Since

$$4\pi q = V\nabla H = -\frac{\partial H}{\partial r}$$

$$4\pi q = ik\sqrt{\frac{r_1}{r}}\,\frac{e^{ik(r_2-r)} + e^{-ik(r_2-r)}}{e^{ik(r_2-r_1)} - e^{-ik(r_2-r_1)}}\,H_0\,e^{i\omega t} \tag{9}$$

where in differentiating, on account of the largeness of $k r$ only the exponential terms were considered variable.

For the total current I per unit of length we obtain the value

$$I = \frac{1}{N}\int_{r_1}^{r_2} q\,dr = -\frac{1}{4\pi N}\int_{r_1}^{r_2}\frac{dH}{dr}\,dr = \frac{1}{4\pi N}\left\{H_{r_1} - H_{r_2}\right\}$$

$$\therefore\ I = \frac{H_0}{4\pi N}\,e^{i\omega t} \tag{10}$$

where N is the number of turns per unit length of the coil.

The electro-kinetic energy of a system is given by

$$T = \frac{1}{2}\,L\,I^2 \tag{11}$$

where L is its self-inductance, and I is the total current. In the case of alternating currents, the mean square values are meant, or

$$\tilde{T} = \frac{1}{2}\,L\,\tilde{I}^2.$$

Hence we may write

$$L = \frac{2\,\tilde{T}}{\tilde{I}^2} = 2\,\frac{\frac{1}{\tau}\int_0^{\tau} T\,dt}{\frac{1}{\tau}\int_0^{\tau} I^2\,dt} \tag{12}$$

where τ is the period.

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[6] Loc. cit., p. 678.

But the mean square value of an harmonic function is one half the square of its amplitude, and in working with imaginary expressions the square of the amplitude can always be obtained by multiplying any expression by its conjugate. Hence

$$\tilde{I}^2 = \frac{1}{\tau}\int_0^\tau I^2\,dt = \frac{1}{2}\,I\bar{I} \tag{13}$$

where $\bar{I}$ denotes the conjugate to I.

Therefore

$$I^2 = \frac{1}{2}\,I\bar{I} = \frac{1}{2}\left(\frac{H_0}{4\pi N}\right)^2$$

and

$$L = 4\left(\frac{4\pi N}{H_0}\right)^2 \frac{1}{\tau}\int_0^\tau T\,dt. \tag{14}$$

In general,

$$T = \frac{1}{8\pi}\int\int\int_\infty H^2\,dv, \tag{15}$$

if μ, the magnetic permeability, is assumed equal to 1 everywhere, which becomes, in this case,

$$T = \frac{1}{8\pi}\int_0^\infty H^2\,2\pi r\,dr$$

per unit length of coil

$$\begin{aligned} &= \frac{1}{4}\int_0^{r_1} H^2 r\,dr + \frac{1}{4}\int_{r_1}^{r_2} H^2 r\,dr \\ &= T_1 \qquad\qquad\qquad + T_2 \end{aligned} \tag{16}$$

where, in T$_1$, $H = H_0\,e^{i\omega t}$

and in T_2, $H =$ eq. (8).

We may write

$$\frac{1}{\tau}\int_0^\tau T\,dt = \frac{1}{4}\int r\,dr\,\frac{1}{\tau}\int_0^\tau H^2\,dt = \frac{1}{8}\int H\bar{H}\,r\,dr.$$

For T_1 this becomes $\left(\frac{H_0 r_1}{4}\right)^2$. (17)

To obtain the integral T_2 first put in equation (8) $h\,(1 + i)$ for ik.

Then, calling $h\,(r_2 - r) = y$,

and
$$h\,(r_2 - r_1) = y_1,$$

we obtain for H and its conjugate $\bar{H}$, the expressions:

$$H = \sqrt{\frac{r_1}{r}}\,\frac{e^{iy}\,e^{y} - e^{-iy}\,e^{-y}}{e^{iy_1}\,e^{y_1} - e^{-iy_1}\,e^{-y_1}}\,H_0\,e^{i\omega t}$$

and

$$\bar{H} = \sqrt{\frac{r_1}{r}}\,\frac{e^{-iy}\,e^{y} - e^{iy}\,e^{-y}}{e^{-iy_1}\,e^{y_1} - e^{iy_1}\,e^{-y_1}}\,H_0\,e^{-i\omega t}. \tag{18}$$

A conjugate expression is obtained by putting $-i$ for i everywhere in an equation.

Their product becomes

$$H\bar{H} = H_0^2\,\frac{r_1}{r}\,\frac{(e^{2y} + e^{-2y}) - (e^{2iy} + e^{-2iy})}{(e^{2y_1} + e^{-2y_1}) - (e^{2iy_1} + e^{-2iy_1})}$$

or

$$H\bar{H} = H_0^2\,\frac{r_1}{r}\,\frac{\cosh 2\,y - \cos 2\,y}{\cosh 2\,y_1 - \cos 2\,y_1}.$$

Multiplying this expression by $r\,dr$ and integrating from r_1 to r_2 gives

as
$$\int \cosh r\,dr = \sinh r$$

putting
$$x = 2\,h\,(r_2 - r_1) = 2\,h\,d$$

$$T_2 = \frac{H_0^2}{8}\,\frac{r_1\,d}{x}\,\frac{\sinh x - \sin x}{\cosh x - \cos x}.$$

We may now write

$$L_\infty = 4\,\pi^2\,N^2\,r_1^2\left[1 + \frac{2}{3}\,\frac{d}{r_1}\left(\frac{3}{x}\,\frac{\sinh x - \sin x}{\cosh x - \cos x}\right)\right] \tag{19}$$

where $x = 2\,d\sqrt{2\,\pi\,\omega\,\sigma} = (4\,\pi\,d\sqrt{\sigma})\sqrt{n}$ where $\omega = 2\,\pi\,n$

Now, $4\,\pi^2 N^2 r_1^2$ is the self-inductance per unit length of a coil with a mean radius r_1, and is therefore the self-inductance for infinite frequency, according to the remarks of Wien. This is also shown by equation (19), as the second term in parenthesis is zero when $x = \infty$ for $\dfrac{\sinh x}{\cosh x} = 1$, and sinh x and cosh x increase without limit as x increases without limit, as may be seen by their expansions

$$\begin{aligned} \sinh x &= x + \frac{x^3}{3\,!} + \frac{x^5}{5\,!} + \ldots; \quad \sin x = x - \frac{x^3}{3\,!} + \frac{x^5}{5\,!} - \ldots; \\ \cosh x &= 1 + \frac{x^2}{2\,!} + \frac{x^4}{4\,!} + \ldots; \quad \cos x = 1 - \frac{x^2}{2\,!} + \frac{x^4}{4\,!} - \ldots; \end{aligned} \tag{20}$$

Equation (19) may then be written

$$\frac{L_\omega - L_\infty}{L_\infty} = \frac{2}{3}\frac{d}{r_1}\left(\frac{3}{x}\frac{\sinh x - \sin x}{\cosh x - \cos x}\right) = \frac{2}{3}\frac{d}{r_1}\,Q. \tag{21}$$

By this equation we see that the fractional change of self-inductance in any coil may be calculated if the properties of the function

$$Q = \frac{3}{x}\frac{\sinh x - \sin x}{\cosh x - \cos x} \tag{22}$$

be known for all values of x from zero to infinity.

A formula for small frequencies may be derived by expanding the hyperbolic and circular functions and retaining only low powers of the variable x. Q then becomes

$$9 = 3\,\frac{\dfrac{1}{3\,!} + \dfrac{x^4}{7\,!} + \dfrac{x^8}{11\,!} + \ldots}{\dfrac{1}{2\,!} + \dfrac{x^4}{6\,!} + \dfrac{x^8}{10\,!} + \ldots}$$

which, retaining only terms in x^4 is

$$Q = \frac{\dfrac{1}{2} + \dfrac{3\,x^4}{7\,!}}{\dfrac{1}{2} + \dfrac{6\,!}{x^4}} = \frac{1 + \dfrac{6\,x^4}{7\,!}}{1 + \dfrac{2\,x^4}{6\,!}} = 1 + \frac{6\,x^4}{7\,!} - \frac{14\,x^4}{7\,!} = 1 - \frac{8\,x^4}{7\,!} = 1 - \frac{x^4}{630}.$$

This equation is applicable for small frequencies only, and (21) may be written, since $64\,\pi^2 = 630$ very approximately.

$$\frac{L_\omega - L_\infty}{L_\infty} = \frac{2}{3}\frac{d}{r_1}\left(1 - \omega^2 d^4 \sigma^2\right). \tag{23}$$

It agrees as to its form and dimensions with that of Max Wien.[7] When $\omega = 0$, it reduces to $2/3\ d/r_1$. As a verification of the term $2/3\ d/r_1$, the approximation formula for Bessel's formula with *small* argument

$$H = H_0 \sqrt{\frac{r_1}{r}}\,\frac{r_2 - r}{r_2 - r_1}\,e^{i\omega t}\ {}^{8} \tag{24}$$

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[7] Loc. cit., p. 16.

[8] Sommerfeld, loc. cit., p. 679.

was taken and a calculation similar to the one above for the general case made. This completely verified this term, and *proved* that the expression used above equation (21) is actually valid for *small* arguments as well as large. This last expression 2/3 d/r_1 multiplied by $L_\infty = 4\pi^2 N^2 r_1^2$ is

$$L_0 - L_\infty = \frac{8}{3}\pi^2 N^2 r_1 d \tag{25}$$

and is therefore the additional self-inductance due to the field in the wires themselves for steady values of the current, or, in other words, this is the maximum possible change in self-inductance. Calculations by means of (25) show this expression to give, for coils of *finite* length and wires with *round* instead of square cross-sections, the values:

TABLE I.

COMPARISON OF THE THEORY WITH THE TRUE CHANGE IN SELF-INDUCTANCE.

Coil Length.	True Change.	Value of 2/3 d/r_1.	Ratio.
46.0 cms.	.000380 Henry	.000280 Henry	1.35
30.5 "	.000200 "	.000160 "	1.25
15.0 "	.000067 "	.000057 "	1.17
13.0 "	.000051 "	.000044 "	1.16

The column headed "True Change" was computed by taking the difference between the self-inductance of a coil of mean radius a, computed by means of an exact formula, and that of the same coil, using as a mean radius $a - \rho$, where ρ is the radius of the wire. These results, considering the assumptions made in the theory, are in good agreement with it. To make the theory fit to actual facts the second term in the parenthesis in equation (19) should be multiplied by a constant, the average value of which is approximately 1.25, deduced from the above table. We should expect some multiplier to be necessary because a current sheet is the equivalent of square wires without insulation, while, in the coil for which the calculations were made the wires are round and have insulation. We may then write

$$\frac{L_\omega - L_\infty}{L_\infty} = 1.25\,\frac{2}{3}\,\frac{d}{r_1}\left(\frac{3\sinh x - \sin x}{x\cosh x - \cos x}\right). \tag{26}$$

In the proposed use of this theory, however, it is not necessary to employ this constant whose value at best is not satisfactorily determinate.

As cosh x and sinh x increase without limit as x increases, equation (26) may be written, for large values of ω,

$$\frac{L_\omega - L_\infty}{L_\infty} = 1.25\,\frac{2}{3}\,\frac{d}{r_1}\,\frac{3}{x} = \frac{5}{4}\,\frac{d}{r_1}\,\frac{1}{d\sqrt{2\,\pi\,\omega\,\sigma}}. \tag{27}$$

This formula agrees to a constant factor près, with that of Lord Rayleigh,[9] for large values of n for straight wires; i. e.:

$$\frac{L_\omega - L_\infty}{L_\infty} = \sqrt{\frac{R}{2ne}} = 2\,\frac{1}{d\sqrt{2\,\pi\,\omega\,\sigma}}.$$

TABLE II.

TABLE OF VALUES FOR THE CURVES Q, Q_1, AND Q_2, FOR VALUES OF x FROM ZERO TO INFINITY.

x.	Cosh x.	Sinh x.	Cos x.	Sin x.	Q.	Q_1.	Q_2.
0.0	1.0000	0.0000	1.0000	0.0000	1.0000	1.0000	∞
0.5	1.1276	0.5211	0.8776	0.4789	0.9999	0.9999	6.0000
1.0	1.5431	1.1752	0.5402	0.8415	0.9984	0.9984	3.0000
1.5	2.3523	2.1291	0.0706	0.9975	0.9920	0.9918	2.0000
2.0	3.7622	3.6269	–0.4163	0.9092	0.9756	0.973	1.5000
3.0	10.0677	10.0178	–0.9900	0.1409	0.8932	0.871	1.0000
4.0	27.308	27.262	–0.6534	–0.7570	0.7515	0.594	0.7500
5.0	74.205	74.197	+0.2840	–0.9588	0.6100	Negative	0.6000
6.0	202 approx.	202 approx.	0.9603	–0.2790	0.5032	“	0.5000
7.0	573 approx.	573 approx.	0.7537	+0.6574	0.4287	“	0.4289
8.0	Very large and the same as sinh x.	Very large and the same as cosh x.	. . . .	. . . .	0.3750	“	0.3750
9.0			. . . .	. . . .	0.3333	“	0.3333
10.0			. . . .	. . . .	0.3000	“	0.3000
∞			. . . .	. . . .	0.0000	“	0.0000
∞	∞	∞	Indeterm.	Indeterm.	0	–∞	0

Computations were carefully made for the values of the term

$$Q = \frac{3\,\sinh x - \sin x}{x\cosh x - \cos x} \tag{28}$$

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[9] Loc. cit.

for different values of x, as well as of Q_1 and Q_2, where

$$Q_1 = \left(1 - \frac{x^4}{630}\right), \qquad \text{and} \tag{29}$$

$$Q_2 = \frac{3}{x}. \tag{30}$$

Table II contains all of the data.

It is clearly seen that as long as $x < 2$ that formula Q_1 may be used, while if $x > 7$, formula Q_2 is valid. In between these points, the calculated values will serve.

A plot of these three functions on the next page shows still more clearly the relations of the three formulae for different values of x.

MANNER OF USING THE CURVE.

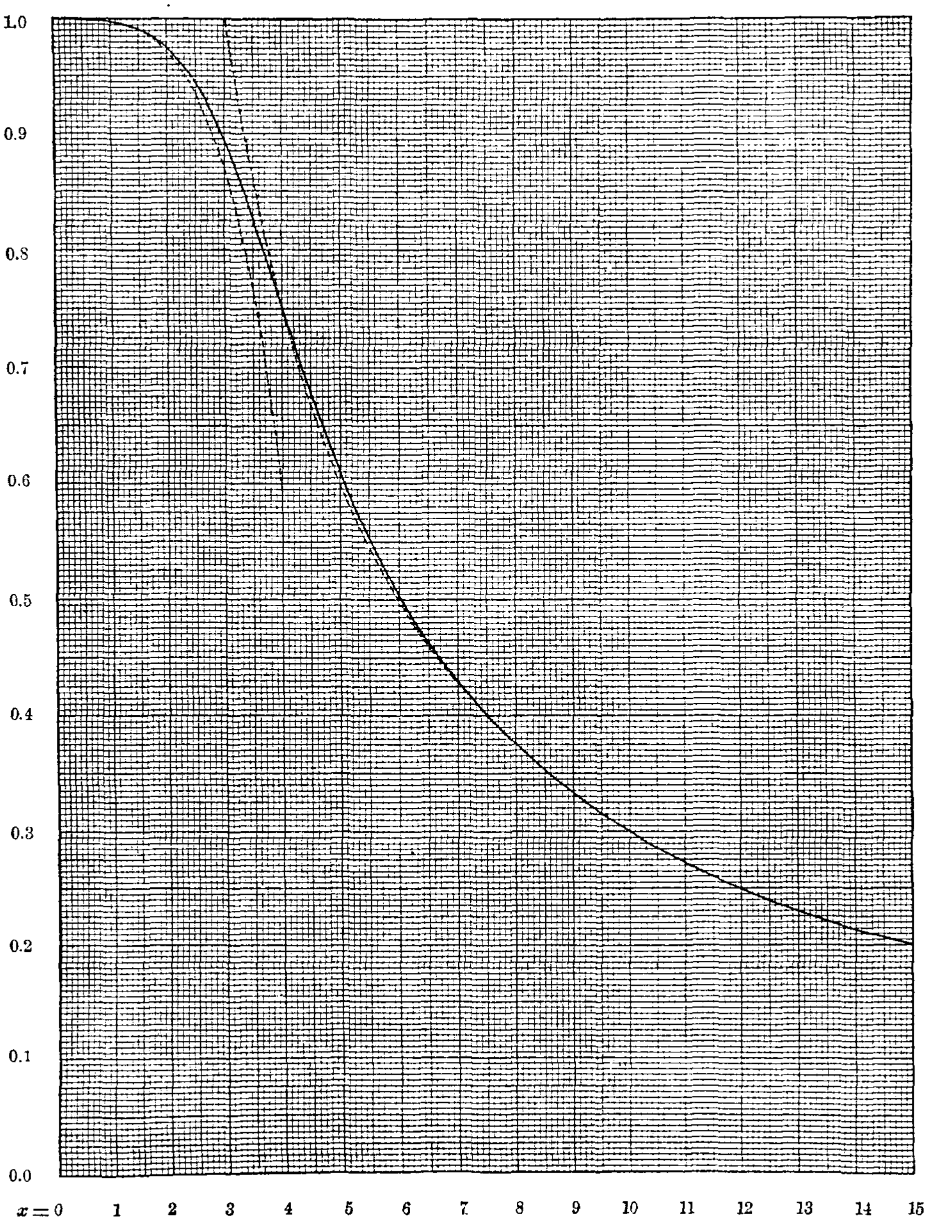
This curve is to be used in the following manner. We assume that in any coil of a single layer, the falling off of the self-inductance takes place according to equation (22), represented by the full-line curve, and the maximum ordinate unity is taken to be the maximum change possible of the self-inductance, which is otherwise calculable. For any given dimensions of the coil, the values of x correspond to definite frequencies, and either a table or a plot of corrections may be derived from the above curve.

For small values of x, and therefore for small values of the frequency, the correction varies proportionately to d^4, while for very large frequencies it is proportionate to $1/d$ by equations (29) and (30).

As the first case is the more usual, it is important, in order to avoid the necessity of any correction, to wind the coil with wires of small diameter. The size necessary to avoid a correction under a given frequency may easily be calculated. As an application of this theory, the following example will be carried out:

Consider a coil of 27 cms. radius $= r_1$ wound with a single layer of wire whose diameter d is .063 cms. Let the conductivity of the wire be taken as 5.9×10^{-4}. Then the relation between x and n is $n = 2630\, x^2$ from (19) where $n =$ frequency. In the following table, column 2, are the values of x corresponding to the frequencies n in column 1. In column 3 are the values of the ordinates corresponding to these values of x taken from the Plot (I).

PLOT I.



Plot of the functions $Q = \frac{3}{x}\frac{\sinh x - \sin x}{\cosh x - \cos x}$ full line.

$Q_1 = (1 - x^4/630)$ dotted line.

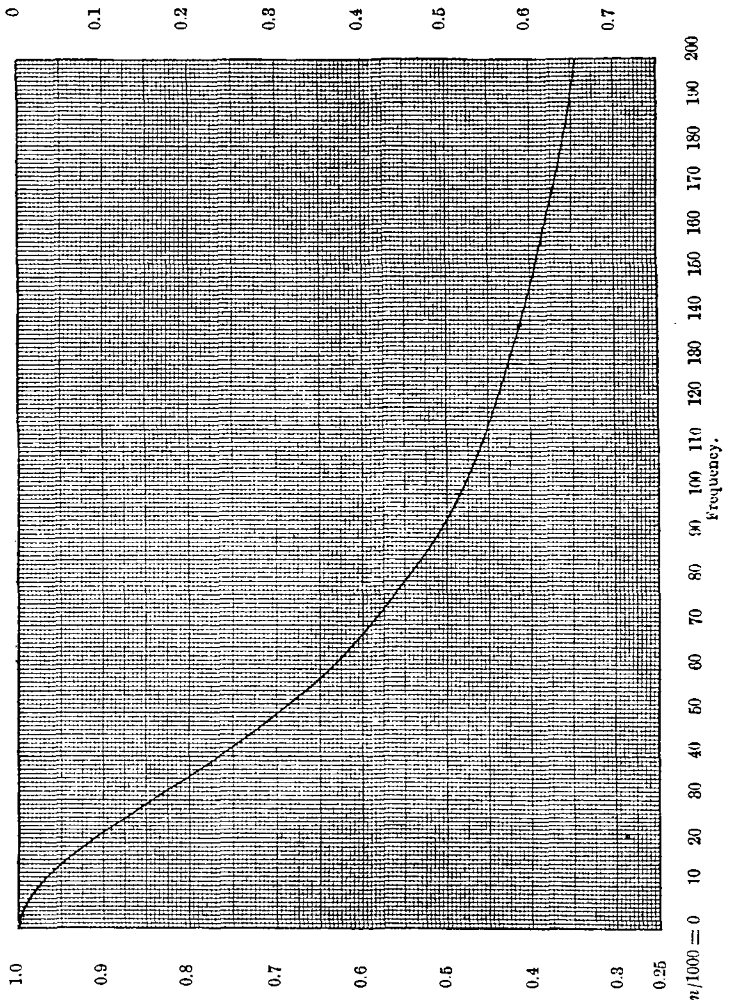
$Q_2 = 3/x$ dotted line.

TABLE III.

n	x	Ordinates.
0	0	1.000
500	0.436	1.000
1000	0.616	0.999
1000	0.755	0.998
3000	1.068	0.996
4000	1.230	0.995
5000	1.380	0.993
6000	1.518	0.990
10000	1.950	0.977
15000	2.380	0.950
20000	2.750	6.916
30000	3.380	0.846
40000	3.900	8.765
50000	4.350	8.700
60000	4.780	0.638
70000	5.150	0.590
80000	5.510	0.550
90000	5.850	0.515
100000	6.170	0.483
120000	6.700	0.463
150000	7.550	0.397
170009	8.050	0.373
200000	8.720	0.344

The plot of the corresponding value of n as abscissae and values of column 3 as ordinates gives a curve of corrections for the above coil. For example, at a frequency of 50,000 there is 0.7 of the total possible change (equal to $L_0\ 2/3\ d/r_1 \times 1.25$ approximately) still to go. Or, the

PLOT II.



Curve showing the fractional part of the maximum decrease for different frequencies in a coil, the wire of which has a diameter of .063 cm.

Ordinates = fraction of total change left unchanged.

$$\text{Abscissae} = \frac{\text{frequencies}}{1000}.$$

For example, at 30,000 ∼ per second the correction is 0.157 × 0.0000618 Henrys in 0.0362 Henrys, or about 1 part in 3600.

correction is 0.3 of the maximum change. The value of the correction may be best obtained by reading *down* from the ordinate unity.

These computations also show that equation (29)

$$q_1 = 1 - \frac{x^4}{630}$$

may be employed up to frequencies of 10,000 for this particular coil, and that equation (30)

$$q_2 = \frac{3}{x}$$

may be used for frequencies beyond 120,000 per second. As the maximum correction is 0.2 per cent of the total, if frequencies under 4000 per second are used, the error will not be over 0.2 per cent × 0.005 = 0.001 per cent, which is practically negligible.

If, however, other things being equal, the diameter of the wire were four times larger than it is, the same corrections would correspond to frequencies 16 times less, or an error of 0.001 per cent is now obtained at a frequency of only 250.

This suggests, if large currents must be carried, that to wind the coil with flat wires would diminish the effect of the frequency upon both the changes in self-inductance and resistance.

The Plots I and II, just derived, show how the self-inductance of a circular *current sheet* falls off with the frequency. In other words, the coil is assumed to be wound with wires of rectangular cross-section. This may cause a doubt as to the applicability of the curve in question.

In the first place, the method of its use does away with the necessity of knowing the constant multiplier of the function Q, as explained above; and in the second place, the following two reasons go to show the entire correctness of the deductions.

Sommerfeld finds that his results agree to a constant près with those of Max Wien, which were deduced for *circular* wires, and both formulae agree remarkably well with the experimental data at hand; this for the increase of resistance with the frequency. We should, therefore, fairly expect the results for the decrease in self-inductance to also be correct to within a constant factor. As there are no data available for the decrease in self-inductance, we have not been able to verify this conclusion. To further assure ourselves, however, we have worked out the problem of the change of self-inductance with the frequency in an infinite straight circular-ring conductor, using the approximation formulae for the Bessel's functions as above given. It was found that the *variable part* of the expression for self-inductance agreed *identically* with that derived for a circular current sheet. This shows that the bending of straight wires into circles, and forming a coil of a single layer therewith, does not affect the *way* in which the self-inductance varies.

We believe, then, that Plot I gives very approximately the law according to which the self-inductance falls off for straight ring-shaped conductors, and probably for any simply shaped square or polygonal hollow straight conductors, as well as for circular coils of a single layer, wound with wires of circular, square, or polygonal cross-sections.

Dept. Physics,
Clark University.

Proceedings of the American Academy of Arts and Sciences.

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RECORDS OF MEETINGS, 1905–1906.

REPORT OF THE COUNCIL: BIOGRAPHICAL NOTICES.

James Munson Barnard. By William R. Ware.
John Bartlett. By M. H. Morgan.

OFFICERS AND COMMITTEES FOR 1906–1907.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

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RECORDS OF MEETINGS.

Nine hundred and fifty-ninth Meeting.

OCTOBER 11, 1905. — STATED MEETING.

The Academy met at the Jefferson Physical Laboratory, Cambridge.

The PRESIDENT in the chair.

There were present thirty-nine Fellows.

The Corresponding Secretary read the following letters: —

From Samuel Williston, accepting Fellowship; from Charles E. Munroe, acknowledging his election as Associate Fellow; from G. Stanley Hall, resigning Fellowship; from the University of Illinois, inviting delegates to the installation of Edmund J. James as President of the University; from Dr. Gustav Fischer, of Jena, soliciting subscriptions for a Memorial to Ernst Abbe; from the Société Entomologique de Belgique, announcing the celebration of its fiftieth anniversary and the publication of a Jubilee volume which will be given to the Academy; from the Directors of the Biblioteca Nazionale Centrale, of Florence, announcing the death of its Librarian, Comm. Desiderio Chilovi; from the Kön. böhmische Gesellschaft der Wissenschaften of Prag, announcing the death of its President, W. W. Ritt. von Tomek; from the Académie royale, d'Archeologie de Belgique Anvers, inviting Members of the Academy to be present at the celebration of the seventy-fifth anniversary of Belgian independence.

The Chair announced the following deaths: —

Edward S. Wood, Resident Fellow in Class I., Section 3; Otto Struve, Foreign Honorary Member in Class I., Section 1.

The following gentlemen were elected members of the Academy: —

Louis Bell, of Boston, to be a Resident Fellow in Class I., Section 2 (Physics).

Arthur Edwin Kennelly, of Cambridge, to be a Resident Fellow in Class I., Section 4 (Technology and Engineering).

Charles Gross, of Cambridge, to be a Resident Fellow in Class III., Section 3 (Political Economy and History).

Kuno Francke, of Cambridge, to be a Resident Fellow in Class III., Section 4 (Literature and the Fine Arts).

Herbert Langford Warren, of Cambridge, to be a Resident Fellow in Class III., Section 4.

Michael Idvorsky Pupin, of New York, to be an Associate Fellow in Class I., Section 2 (Physics).

Wilhelm Ostwald, of Leipzig, to be a Foreign Honorary Member in Class I., Section 3 (Chemistry).

Samuel Henry Butcher, of London, to be a Foreign Honorary Member in Class III., Section 4 (Literature and the Fine Arts), in place of the late Lesley Stephen.

Mr. A. Lawrence Lowell was appointed a delegate to attend the installation of Edmund J. James as President of the University of Illinois, at Urbana, October 17–19, 1905.

The following votes were passed: —

Voted, That the action of the President, William W. Goodwin, in executing in the name and behalf of the Corporation a party wall agreement with the owner of estate numbered 30 on Newbury Street in Boston, dated May 29th, 1905, be and the same hereby is ratified and confirmed as the act of the Corporation.

Voted, That the action of the Treasurer in paying bills for the current expenses of the Academy, in the absence of any appropriation therefor, be hereby approved and confirmed.

Voted, That an appropriation of three thousand dollars ($3,000) be made for the General Expenses of the Academy, as of May, 1905, to be expended during the year 1905-1906.

Voted, That the Treasurer be authorized to sell and transfer any and all Stocks and Bonds belonging to the Academy, with the written approval of one other member of the Finance Committee.

Voted, That the Treasurer be authorized to pay from the income of the General Fund one hundred and sixty dollars ($160) for labels to be used in distributing the publications of the Academy.

Voted, That eight hundred dollars ($800) be appropriated from the income of the Rumford Fund for publication and miscellaneous expenditures.

The Treasurer proposed certain amendments to the Statutes and Standing Votes: —

It was then *Voted*, That the draft of the Statutes and Standing Votes of the Academy, presented by the Treasurer, be referred to a Committee of five, consisting of the President, Recording Secretary, and Treasurer, together with two other members of the Academy, to be appointed by the President.

In accordance with the foregoing vote, the President appointed Professor Morris H. Morgan and Dr. Theodore Lyman members of the Committee to consider the revision of the Statutes and Standing Votes.

Professor John Trowbridge made a Communication on Slow Discharges of Electricity, with Experimental illustrations.

The following papers were presented by title: —

"On the Manner of Growth of a Current in a Nearly-Closed Electromagnet, as Influenced by the Width of the Air-gap." By B. O. Peirce.

"Bibliography of the Hall Effect, with some Titles of Papers on Allied Phenomena." By T. C. McKay. Presented by E. H. Hall.

"The Thermo-luminescence of Fluospar." By H. W. Morse. Presented by John Trowbridge.

"Gaseous and Liquid Inclusions in Fluospar." By H. W. Morse. Presented by John Trowbridge.

"Longitudinal Vibrations of Stretched Strings." By H. N. Davis. Presented by John Trowbridge.

"The Fluorescence of Sodium Vapor and the Resonance Radiation of Electrons." By Robert W. Wood. Presented by Charles R. Cross.

"On a Postulate respecting a Certain Form of Deviation from the Straight Line in a Plane." By Gustavus Hay.

"Slow Moving Electrical Luminous Effects." By John Trowbridge.

"Effect of Magnetic Field on Electric Discharges." By John Trowbridge.

Nine hundred and sixtieth Meeting.

NOVEMBER 8, 1905.

The Academy met at the house of Dr. Alexander Agassiz, Cambridge.

The PRESIDENT in the chair.

There were present eighty Fellows.

The Corresponding Secretary read the following letters: —

From the Sociedad Mexicana de Geografia y Estadistica, Mexico, inviting the Academy to participate in its celebration of the four hundredth anniversary of the birth of Christobal Colon, May 20, 1906; from Charles Gross, Kuno Francke, Herbert Langford Warren, and Albert Edwin Kennelly, accepting Resident Fellowship; from Michael Idvorsky Pupin, accepting Associate Fellowship; from Samuel Henry Butcher, accepting Foreign Honorary Membership; from William Everett, resigning Resident Fellowship; from Baroness von Richthofen, announcing the death of her husband; from the Gesellschaft für Erdkunde, Berlin, announcing a memorial service for Baron von Richthofen to take place October 29, 1905.

The Chair announced the death of Freiherr von Richthofen, Foreign Honorary Member in Class II., Section 1.

The following communications were given: —

"An Account of his Trip to the Eastern Pacific." By Professor Agassiz.

"The International Language, — Esperanto." By Professor Wilhelm Ostwald.

Remarks on Professor Ostwald's paper were made by Professors Morse and Lanman.

The following paper was presented by title: —

"The Electrostatic Field Surrounding Certain Columnar Elements." By P. W. Bridgman. Presented by B. O. Peirce.

Nine hundred and sixty-first Meeting.

DECEMBER 13, 1905.

The Academy met at the house of the President, in Cambridge.

The PRESIDENT in the chair.

There were present thirty-nine Fellows and one Foreign Honorary Member.

In the absence of the Recording Secretary, Professor M. H. Morgan was appointed Recording Secretary, *pro tempore.*

The President announced the following deaths: —

John Bartlett, Resident Fellow in Class III., Section 4; Edward Atkinson, Resident Fellow in Class III., Section 3.

The Corresponding Secretary read the following letters: —

From the Nobel Prize Committee, enclosing a circular relative to the prize in literature; and from Professor A. E. Dolbear, resigning Fellowship.

The following communications were given: —

Professor W. R. Ware — "Memoir of J. M. Barnard."

Professor W. M. Davis — "Notes on South Africa: the Dwyka Glacial Formation and the Karroo Basin Formation."

The following papers were presented by title: —

"The Expansibility and the Compressibility of Ether and of Alcohol in the Neighborhood of their Boiling Points." By Alpheus W. Smith. Presented by Edwin H. Hall.

"On Thermo-electric Heterogeneity in Various Alloys, especially German Silver." By Edwin H. Hall, L. L. Campbell, and S. B. Serviss.

"The Sense of Loudness." By W. C. Sabine.

"The Spectrum of Hydrogen in the Schumann Region of Short Wave Lengths." By Theodore Lyman.

"A Method of Determining Thermal Conductivity." By F. A. Laws, F. L. Bishop, and P. McJunkin.

"The Thermal Conductivity of Lead." By F. L. Bishop. Presented by F. A. Laws.

"The Transition Temperature of Sodic Bromide: a New Fixed Point in Thermometry." By T. W. Richards and R. C. Wells.

"A New Method of Standardizing Thermometers below Zero." By T. W. Richards and F. G. Jackson.

"On the Language of Vitruvius." By M. H. Morgan.

"Friction and Force due to Transpiration as Dependent on Pressure in Gases." By J. L. Hogg. Presented by John Trowbridge.

Nine hundred and sixty-second Meeting.

JANUARY 10, 1906. — STATED MEETING.

The Academy met in its house, 28 Newbury Street.

The PRESIDENT in the chair.

There were present thirty Fellows.

The President made a short address congratulating the Academy on having a home of its own.

The Corresponding Secretary read the following communications: —

From The Franklin Bicentennial Committee, inviting five members of the Academy to the exercises at Symphony Hall on Wednesday, January 17, 1906, in observance of the two hundredth anniversary of the birth of Benjamin Franklin; from G. Lecointe, Scientific Director of the Royal Observatory of Belgium, announcing a project of an International Association for the study of the Polar Regions, being the outcome of the Congress of Mons, held September 24, 1905.

The Committee on the Revision of the Statutes presented the following report: —

The Committee on the Revision of the Statutes of the American Academy respectfully submit a copy of the Revised Statutes and recommend their adoption in this form.

The changes made in the Statutes relate chiefly to the method of making Appropriations, to the approval and payment of Bills, and to the efficient management of the Treasurer's office. The Council is charged with the responsibility of reporting the Appropriations to the Academy, and for this purpose the Chairman of three Committees are made members of the Council in place of the three Vice-Presidents.

The calling of special Meetings is provided for, at which any action heretofore allowed at Stated Meetings may be taken.

Slight changes are made in the Nomination and Election of Fellows and Members, and a few verbal alterations are offered.

There has been a great disparity in the number of Resident Fellows belonging to each of the three Classes, and an Amendment to the Statutes is submitted to remedy this trouble.

The distribution of the publications of the Academy has been omitted from the Statutes and is authorized by the Standing Votes in which few changes have been made.

(Signed) W. W. Goodwin.
M. H. Morgan.
Charles P. Bowditch.
Wm. Watson.
Theodore Lyman.

Boston, January 10, 1906.

The Academy then proceeded to consider in detail the proposed changes in the Statutes.

Chapters I., III., IV., V., VI., VIII., IX., and XI., of the Statutes, and Nos. 2 and 8 of the Standing Votes were unanimously approved, but such marked differences of opinion were expressed in the discussion of Chapters II. and VII. that at the suggestion of the President the discussion of these changes was deferred to the meeting of February 14, and on the motion of the Recording Secretary, it was

Voted, To meet on adjournment February 14.

Professor Trowbridge then made the following communication: "Effect of Magnetic Field on Production of the X Rays."

At the request of the President, Dr. Bell gave a brief description of the electric lights just installed in the building, which were quite novel in character.

On motion of Professor W. M. Davis, a House Committee was established; the President appointed Messrs. Ware, Morgan, and Rotch as this Committee.

On motion of Professor Webster it was

Voted, That the thanks of the Academy be tendered to the Committee for their services in preparing this building for occupancy.

Nine hundred and sixty-third Meeting.

FEBRUARY 14, 1906. — ADJOURNED STATED MEETING.

The PRESIDENT in the chair.

There were present twenty-seven Fellows.

The Corresponding Secretary read the following letters:

From Freifrau von Richthofen, in appreciation of messages of sympathy on the death of her husband; from the Academy of Science of St. Louis, inviting the Academy to be represented at a dinner commemorative of its fiftieth anniversary, to be given March 10, 1906; from the American Philosophical Society, inviting the Academy to participate in the celebration of the two hundredth anniversary of the birth of Benjamin Franklin, to be held in Philadelphia on April 17, 18, 19, and 20, 1906; from the Royal Geographical Society of Australasia, Queensland, inviting the Academy to take part in the celebration of its twenty-first anniversary, to take place in June, 1906; from the Treasurer of the Leidy Memorial Committee, asking contributions for a Memorial to Dr. Joseph Leidy, to be presented to the city of Philadelphia; from the Königlich böhmische Gesellschaft der Wissenschaften, announcing the death of Karl Ritter von Koristka.

On the motion of Professor Webster it was

Voted, That Professor W. Trelease, of St. Louis, be requested to represent the Academy at the fiftieth anniversary of the Academy of Science of St. Louis. It was also

Voted, That the President be empowered to appoint delegates to represent the Academy at the celebration of the birth of Franklin, by the American Philosophical Society in April; and to the celebration of the twenty-first anniversary of the Royal Geographical Society of Australasia.

The President announced the death of A. von Kölliker, Foreign Honorary Member in Class II., Section 3.

The Amendments to the Statutes not acted upon at the last meeting were then considered. The proposed amendments to Chapter II. were rejected. A new paragraph to Chapter X., equalizing the number of Resident Fellows in the three Classes, was also rejected.

It was then

Voted, To amend the Statutes and Standing Votes, as follows: —

Ch. I., Sec. 1. The word "Theology" inserted before the words "Philosophy and Jurisprudence."

Ch. III., Sec. 1. Omit in the first line the clause "from the next retiring Councillors."

Ch. IV., Sec. 1. For the clause "to summon extraordinary meetings upon any urgent occasion" substitute the phrase "to direct the Recording Secretary to call special meetings."

Ch. IV., Sec. 2. Omit.

Ch. IV., Sec. 3. To be numbered "2."

Ch. IV., Sec. 4. To be numbered "3."

Ch. V., Sec. 2. Omit the last sentence.

Ch. V., Sec. 3. Add the following sentence: "All bills incurred on account of the Rumford Fund, within the limits of the appropriation made by the Academy, shall be approved by the Chairman of the Rumford Committee."

Ch. V., Sec. 4. Add the following sentence: "All bills incurred on account of the C. M. Warren Fund, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the C. M. Warren Committee."

Ch. V., Sec. 5. Add the following sentences: "This Committee shall report to the Council in March of each year on the appropriations needed for the coming year. All bills incurred on account of publications, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the Committee on Publication."

Ch. V., Sec. 6. Add the following sentences: "This Committee, through the Librarian, shall report to the Council in March of each year, on the appropriations needed for the Library for the coming year. All bills incurred on account of the Library, within the limits of the appropriations made by the Academy, shall be approved by the Librarian."

Ch. V., Sec. 7. The President and Recording Secretary shall be a Committee on the general expenditures of the Academy. This Committee shall report to the Council in March of each year on the appropriations needed for the general expenditures for the coming year, and either member of the Committee may approve bills incurred on this account within the limits of the appropriations made by the Academy.

Ch. V., Sec. 8. An auditing Committee, to consist of two Fellows, for auditing the accounts of the Treasurer, with power to employ an expert and to approve his bill.

Ch. V., Sec. 9. In the absence of the Chairman of any Committee, bills may be approved by a member of the Committee designated by the Chairman for the purpose.

Ch. VI., Sec. 1. In line 6 omit the words "for nomination."

Ch. VI., Sec. 4. "Every persen taking any books, papers, or documents belonging to the Academy and in the custody of the Recording Secretary, shall give a receipt for the same to the Recording Secretary."

Ch. VII., Sec. 2. Read as follows after the first sentence: —

"He shall pay all bills due by the Academy, when approved by the proper officers (except those of the Treasurer's office, which may be paid without such approval). He shall sign all leases of real estate in the name of the Academy. All transfers of stocks, bonds, and other securities belonging to the Academy shall be made by the Treasurer with the written consent of one member of the Committee of Finance. He shall keep an account of all receipts and expenditures, shall submit his accounts annually to the Auditing Committee, and shall report the same at the expiration of his term of office or whenever called on so to do by the Academy or Council."

Ch. VII., Sec. 4. To be omitted.

Ch. VII., Sec. 5. To be numbered "4."

Chap. VIII., Sec. 2. Add "periodicals, etc." after "books" in line four.

Ch. VIII., Sec. 7. Omit in the second line of the first sentence all after "Academy."

Ch. IX., Sec. 1. After the word "notified" in line five insert the words "or at special meetings called for the purpose," and add the following paragraph: —

"Special meetings shall be called by the Recording Secretary at the request of the President or of a Vice-President or of five Fellows. Notifications of the special meetings shall contain a statement of the purpose for which the meeting is called."

Ch. IX., Sec. 2. Insert the word "Resident" before "Fellows" in line one. Insert the words "or special" after "stated," in line two.

Chap. IX., Sec. 3. Insert the word "Resident" before "Fellow" in line two and omit the words "residing in Boston or the vicinity."

Ch. X.. Sec. 2. Omit the words "for nomination" in line five. Omit the word "written" in line seven. Substitute "any" for "stated" in line nine. In the tenth line substitute for the words "during the interval between two stated meetings" the words "until the next stated meeting."

Ch. X., Sec. 6. Omit.

Chap. X., Sec. 7. To be numbered "6."

Ch. XI., Sec. 1, Line 2. For the word "meeting" substitute "stated meeting, or a special meeting called for the purpose."

Standing Votes.

Vote 2, Line 1. Prefix the words, "Associate Fellows, Foreign Honorary Members and" to the word "Resident."

Omit the second sentence. Add, "Exceptions to this rule may be made in special cases by vote of the Academy."

Vote 8. Add the clause: "and the approval of a bill incurred for such purposes by the Chairman shall be accepted by the Treasurer as proof that such certificate has been given."

On motion of Professor Webster it was

Voted, That the sum of five hundred and fifty dollars ($550) be appropriated from the income of the Rumford Fund for the immediate use of the Rumford Committee.

Professor E. H. Hall proposed certain amendments to the Statutes. In accordance with Chapter XI. of the Statutes, these proposed amendments were referred to the following Committee: —

The President, Corresponding Secretary, Recording Secretary, Treasurer, Professor Morgan, and Dr. Lyman.

The following communications were given: —

"Results of the Franco-American Expedition to explore the Atmosphere in the Tropics." By A. Lawrence Rotch.

"Comparison of Lunar and Hawaiian Craters." By W. H. Pickering.

The following papers were presented by title: —

"The Longitudinal Vibrations of a Rubbed String." By Harvey N. Davis. Presented by W. C. Sabine.

"On the Inconsistency of the Axiom of the Plane with the Supposition that the Sum of the Angles of the Rectilinear Triangle is less than 180°." By G. Hay.

"The Effects of Interbreeding, Crossbreeding, and Selection upon the Fertility and Variability of Drosophila." By W. E. Castle, F. W. Carpenter, A. H. Clark, S. O. Mast, and M. W. Barrows.

"On the Length of the Time of Contact in the Case of a Quick Tap on a Telegraph Key." By B. O. Peirce.

"On the Permeability and the Retentiveness of a Mass of Fine Iron Particles." By B. O. Peirce.

Nine hundred and sixty-fourth Meeting.

March 14, 1906. — STATED MEETING.

The Academy met at its house, 28 Newbury Street.

The PRESIDENT in the chair.

There were present eighteen Fellows.

In the absence of the Recording Secretary, W. M. Davis was appointed Recording Secretary, *pro tempore.*

The President announced the death of Professor Samuel Pierpont Langley, Associate Fellow in Class I., Section 2.

The Corresponding Secretary read a letter from M. P. Porzild, Director of the Danish Artic Station at Disco, Greenland, requesting contributions of publications on Arctic nature and biology.

The Chair announced that in accordance with a vote of the last meeting, the Academy would be represented at the Franklin celebration of the American Philosophical Society by the President and Professor W. M. Davis.

The following gentlemen were elected members of the Academy: —

Comfort Avery Adams, of Cambridge, to be a Resident Fellow in Class I., Section 4 (Technology and Engineering).

Edward Charles Jeffrey, of Cambridge, to be a Resident Fellow in Class II., Section 2 (Botany).

Walter Bradford Cannon, of Cambridge, to be a Resident Fellow in Class II., Section 3 (Zoölogy and Physiology).

Joseph Henry Beale, Jr., of Cambridge, to be a Resident Fellow in Class III., Section 1 (Philosophy and Jurisprudence).

George Foot Moore, of Cambridge, to be a Resident Fellow in Class III., Section 2 (Philology and Archaeology).

Albert Andrew Howard, of Cambridge, to be a Resident Fellow in Class III., Section 2 (Philology and Archaeology).

Thomas Nixon Carver, of Cambridge, to be a Resident Fellow in Class III., Section 3 (Political Economy and History).

Charles Benedict Davenport, of Cold Spring Harbor, N. Y., to be an Associate Fellow in Class II., Section 3 (Zoölogy and Physiology). Transferred from Resident Fellowship.

The Chair appointed the following gentlemen to serve as Nominating Committee: —

E. H. Bradford, J. H. Wright, L. J. Johnson.

In accordance with the report of the Committee appointed February 14, 1906, to whom were referred certain alterations of the Statutes, it was

Voted, To amend the Statutes as follows: —

Chap. I., Sec. 2. After the word "hundred," in the first line, insert the phrase "of whom there shall not be more than eighty in any one of the three classes."

Chap. II., Sec. 2. To read as follows: —

"There shall be nine Councillors, chosen from the Resident Fellows. At each annual meeting, three Councillors shall be chosen, by ballot, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several Sections of the same Class, reporting their action to the Academy."

Chap. II., Sec. 3. To read as follows: —

"The Council shall at its March Meeting receive reports from the Rumford Committee, the C. M. Warren Committee, the Committee on Publication, the Committee on the Library, the President and Recording Secretary, and the Treasurer, proposing the appropriations for their work during the year beginning the following May. The Treasurer at the same meeting shall report on the income which will probably be received on account of the various Funds during the same year.

"At the Annual Meeting, the Council shall submit to the Academy, for its action, a report recommending the appropriations, which in the opinion of the Council should be made for the various purposes of the Academy."

Chap. II., Old Sec. 3. To be numbered 4.

Chap. X., Sec. 3. To read as follows: —

"The nomination and election of Associate Fellows shall take place in the manner prescribed in reference to Resident Fellows."

Chap. X., Sec. 4. To read as follows: —

"The nomination and election of Foreign Honorary Members shall take place in the manner prescribed for Resident Fellows, except that the nomination papers shall be signed by at least seven members of the Council before being presented to the Academy."

It was then

Voted, To meet on adjournment April 11, 1906.

Professor M. L. Fernald gave a communication on "Soil and Climatic Control of Plant Distribution in Eastern America."

Professor Barrett Wendell spoke on the "Universities of France."

The following papers were presented by title: —

"On the Conditions to be satisfied if the Sums of the Corresponding Members of two Pairs of Orthogonal Functions of x and y are to be themselves Orthogonal." By B. O. Peirce.

"On the Correction for the Effect of the Counter Electromotive Force of a Moving-Coil Galvanometer used Ballistically." By B. O. Peirce.

"The Accuracy of Musical Taste with Respect to Architectural Acoustics." By Wallace C. Sabine.

"Variation in Reverberation with Variation in Pitch." By Wallace C. Sabine.

"Athenian Lekythoi with Outline Drawing in Glaze Varnish on a White Ground." By Arthur Fairbanks. Presented by John H. Wright.

"A Theory of Thermo-Electric Action." By Edwin H. Hall.

"Studies in the Eupatorieae. I. Revision of the Genus Piqueria. II. Revision of the Genus Ophryosporus. III. The Genus Helogyne and its Synonyms. IV. Diagnoses and Synonymy of Eupatorieae and of Certain Other Compositae which have been classed with them." By B. L. Robinson.

Nine hundred and sixty-fifth Meeting.

APRIL 11, 1906. — ADJOURNED STATED MEETING.

The Academy met at its house, 28 Newbury Street.

VICE-PRESIDENT TROWBRIDGE in the chair.

There were present twenty-four Fellows.

The Corresponding Secretary read the following letters: —

From Professor Trelease, of St. Louis, giving an account of the discharge of his duties as the representative of the Academy at the celebration of the fiftieth anniversary of the Academy of Science of St. Louis; from Joseph H. Beale, Walter B. Cannon, George F. Moore, and E. C. Jeffrey, accepting Resident Fellowship; from Wilhelm Ostwald, accepting Foreign Honorary Membership; from F. W. Draper, resigning Fellowship; from the Smithsonian Institution, announcing the death of its Secretary, Samuel P. Langley.

The Chair announced the following deaths: —

Professor James M. Peirce, Resident Fellow in Class I., Section 1; Professor Nathaniel S. Shaler, Resident Fellow in Class II., Section 1.

The Corresponding Secretary moved and it was

Voted, That the request made by M. P. Porzild, for publications on Arctic nature and biology, be referred to the Library Committee.

A communication was given by Dr. Louis Bell on "Some Modern Illuminants and the Mechanical Theory of Light."

A communication was also given by Professor W. M. Davis on "The Desirability of a General Annual Meeting of New England Investigators."

At the conclusion of Professor Davis's remarks it was

Voted, that the question of a General Annual Meeting of the Academy and guests be referred to the Council.

The following paper was presented by title: —

"On a simple Device for Measuring the Deflections of a Mirror Galvanometer." By B. O. Peirce.

Nine hundred and sixty-sixth Meeting.

MAY 9, 1906. — ANNUAL MEETING.

The Academy met at its house, Newbury Street.

The PRESIDENT in the chair.

There were present twenty-six Fellows.

The Corresponding Secretary read a communication from the

Société d'Emulation d'Abbeville, asking contributions for a monument in honor of Boucher de Perthes.

The annual report of the Council was read.*

The annual report of the Treasurer was read, of which the following is an abstract: —

At the beginning of the year 1905–1906, a new system of accounts was adopted. The building No. 28 Newbury Street having been bought at a cost in excess of the amount of the Building Fund, this account was transferred to the General Fund. The balance of the General Fund Income was carried to principal, and the General Fund was reduced by setting off to the Publication Fund the sums which had been given for publication purposes.

General Fund.

Receipts.

Investments	$2,553.51	
Assessments	1905.00	
Admission fees	120.00	
Rent of offices	250.00	
Sundries	10.00	$4,838.51
Deficit		173.47
		$5,011.98

Expenditures.

General expenses	$3,424.06	
Library	1,587.92	$5,011.98

Rumford Fund.

Receipts.

Balance April 30, 1905	$191.20
Investments	3,259.42
	$3,450.62

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* See p. 837.

Expenditures.

Research	$1,400.00	
Publication	1,174.67	
Library	157.09	
Publishing Rumford Pamphlets	229.03	
Miscellaneous	38.60	$2,999.39
Balance April 30, 1906		451.23
		$3,450.62

C. M. WARREN FUND.

Receipts.

Balance April 30, 1905	$667.13
Investments	330.93
	$998.06

Expenditures.

Vault rent	$8.00	
Premium on bonds	50.00	$58.00
Balance April 30, 1906		940.06
		$998.06

PUBLICATION FUND.

Receipts.

Investments	$2,677.03	
Sale of Publications	92.58	$2,769.61

Expenditures.

Publication	$1,761.68	
Premium on bonds	21.25	
Vault rent	25.00	$1,807.93
Balance April 30, 1906		961.68
		$2,769.61

The following reports were also presented: —

REPORT OF THE LIBRARIAN.

The most important feature of the year was the removal of the Library from the third story of the Massachusetts Historical Society building to the new home of the Academy, Number 28 Newbury Street. This building (except the stack) was occupied in December, but the books were not moved until January and February of this year. Great care was taken by the movers, — Moore & Co., — and the placing of the books in the fireproof stack in the rear of the building and the current numbers of periodicals in the steel cases in the Meeting-room was successfully accomplished through the careful supervision of Mrs. Holden, the Assistant Librarian. In the same way the large stock of publications of the Academy was transferred during April to the basement of the building, but this must be regarded as a temporary arrangement only. The cost of moving the Library and belongings, which has been charged to the Building Fund, was $525.83.

The accessions during the year have been as follows: —

	Vols.	Parts of Vols.	Pams.	Maps.	Total.
By gift and exchange	237	2280	227	5	2749
By purchase — General Fund . .	16	1228			1244
By purchase — Rumford Fund . .	8	376			384
Total	261	3361	227	5	4377

3 volumes recommended by the Rumford Committee were purchased at an expense of $13.03 from the special appropriation of $50 for books.

84 books have been borrowed from the Library by 27 persons, including 13 Fellows, and one Library (Clark University Library). The Library has been consulted many times when no books were taken.

Of the books borrowed, all have been returned for the annual examination except 10 (taken out by two people). Of the 8 volumes reported missing a year ago, all have been returned.

The expenses charged to the Library are as follows: Miscellaneous, $587.92; Binding, $482.50, General, and $35.20, Rumford Funds; Subscriptions, $416.25, General, and 108.86, Rumford Funds; making a total of $1486.67 for the General and 144.06 for the Rumford Funds.

It is impossible for one person to properly care for a library of this size in addition to the duties of the Assistant Librarian required by the business of the Society, and it is hoped that the type-written catalogue, left unfinished by the death of Dr. Holden, can be completed this year by an experienced cataloguer. It will soon be necessary to re-bind a large number of sets of books, which, owing to the perishable nature of the bindings formerly used, are now crumbling away.

A. Lawrence Rotch, *Librarian.*

May 9, 1906.

Report of the Rumford Committee.

From the appropriation of $1000 in aid of research voted at the last annual meeting, grants have been made to the following persons in aid of the researches specified.

October 11, 1905. Arthur B. Lamb, of Harvard University, Specific Heat of Salt Solutions, $200.

December 13, 1905. John A. Parkhurst, of the Yerkes Observatory, for the purchase of a Hartmann photometer, $225.

Professor Charles B. Thwing, of Syracuse University, Thermo-Electric Power of Metals (second appropriation), $400.

March 14, 1906. Professor Edwin H. Hall, of Harvard University, Thermo-Electric Properties of Metals, $100.

Professor Frederic E. Kester, of the Ohio State University, Joule-Thomson Effect in Gases, $50.

April 11, 1906. Professor Edwin H. Hall, of Harvard University, Thermo-Electric Properties of Metals (additional), $25.

The appropriation of $100, made in 1903 to Professor Sidney D. Townley for the construction of a stellar photometer has been returned to the Treasurer of the Academy, as a change of location precludes carrying out the investigation which he had planned.

The investigation of Professor Dewitt B. Brace on Double Refraction in Gases in an Electrical Field, for which grants had been made amounting to $400, has been brought to a close by his sudden death. The appropriation had been mostly expended in the construction of apparatus, and it is hoped that the research begun by him may be prosecuted by one of his colleagues.

The pamphlet, relative to the scientific results which have been obtained by the application of the bequest of Count Rumford, referred to in the last Annual Report as in press, has been issued, and copies have been sent to a large number of physicists.

The following papers have been published at the expense of the Rumford Fund during the past year, the first mentioned in the Memoirs of the Academy, the others in the Proceedings.

"The Spectrum of Hydrogen in the Region of Extremely Short Wave-Length." By Theodore Lyman.

"Report of an Experiment to detect the FitzGerald-Lorentz Effect." By Edward W. Morley and Dayton C. Miller.

"Surface Tension at the Interface between Certain Liquids and Vapors." By Arthur L. Clark.

"The Transition Temperature of Sodic Bromide: a New Fixed Point in Thermometry." By Theodore W. Richards and Roger C. Wells.

"A New Method of Standardizing Thermometers below 0°C." By Theodore W. Richards and Frederick G. Jackson.

"A Method of determining Thermal Conductivity." By F. A. Laws with F. L. Bishop and P. McJunkin.

"On Thermo-Electric Heterogeneity in Certain Alloys, especially German-Silver." By Edwin H. Hall, L. L. Campbell, and S. B. Serviss.

"Studies on Fluorite."

1. The Fluorescence of Fluorite.
2. The Thermo-Luminescence of Fluorite.
3. Gaseous and Liquid Inclusions in Fluorspar. By Harry W. Morse.

"The Elimination of Thermometric Lag and Accidental Loss of Heat in Calorimetry." By Theodore W. Richards, Lawrence J. Henderson, and George S. Forbes.

1. Two New Methods of avoiding Error in Calorimetry. By Theodore W. Richards.
2. The Constancy of Results obtained by one of the New Methods. By Theodore W. Richards and George S. Forbes.
3. The Comparison of the two Methods, and the Exact Estimation of Adiabatic Rise of Temperature. By Theodore W. Richards and Lawrence J. Henderson.

"Measurement of Various Thermal and Electrical Effects, especially the Thomson Effect in Soft Iron." By Edwin H. Hall, Messrs. Churchill, Campbell, and Serviss.

"Thermal Expansion of Hydrogen and Carbon Dioxide under Constant Pressure." By Theodore William Richards and Kenneth Lamartine Mark.

"The Relative Positions of the Arc and Spark Lines of the Spectra of Titanium, Zinc, and Iron." By Norton A. Kent.

Also reports of progress of researches not yet ready for publication have been received from Messrs. Frederic L. Bishop, John A. Dunne, William J. Humphreys, Ralph S. Minor, Charles B. Thwing, John Trowbridge, Robert W. Wood.

The expenditures of the past year for publication have been much in excess of the ordinary amount. This is due chiefly to the unusually large number of papers which have been presented, mostly based on researches aided by the Rumford Fund. Appropriations, amounting to $1350, have been made by the Academy to meet this expense.

The Committee has given much thought to the question of an award of the Rumford Premium, and has considered the claims of several persons, but has no recommendation to make for the present year.

CHAS. R. CROSS, *Chairman.*

MAY 9, 1906.

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee begs leave to report that the Committee recommended no grants from the Warren Fund during the past year. This was due to the fact that the income received from the fund in 1904 was very small, owing, it is said, to changes and additions that were being made to the Warren Asphalt Plant.

Respectfully submitted,
LEONARD P. KINNICUTT, *Chairman.*

MAY 9, 1906.

REPORT OF THE PUBLICATION COMMITTEE.

During the period beginning May 1, 1905, and ending April 30, 1906, two numbers of the Proceedings, Volume XL. (Nos. 23 and 24), and thirty-two numbers of Volume XLI. have been published, likewise one biographical notice, making in all 820 pages and 14 plates. Thirteen numbers of the Proceedings (Volume XLI., Nos. 1, 2, 5, 6, 10, 12, 16, 20, 21, 22, 25, and 27) were paid for out of the income of the Rumford Fund.

One number of the Memoirs (Volume XIII., No. 3, pp. 121–146, plates III.–VIII.) has been published at the expense of the Rumford Fund.

The unexpended balance of appropriations from the General Fund for publications on May 1, 1905, was $278.55; the appropriation for the year 1905–06, $2,400, and the receipts from sales of publications $92.58

making the whole amount available for publications $2,771.13. The expenditures for publications have been from the General Fund $1,761.68, leaving an unexpended balance from the General Fund of $1,009.45. The expenditures on account of the income from the Rumford Fund have been $1,174.67.

Beginning with Volume XLII., the Proceedings will be printed from newer and better type, and at the top of the cover the volume and number will be indicated in bold Arabic numerals, to aid the Librarian in the distribution of copies.

E. L. MARK, *Chairman.*

MAY 9, 1906.

FINANCIAL REPORT OF THE COUNCIL.

The income, for the year 1906–07, as estimated by the Treasurer, is as follows: —

General Fund	Investments	$1510	
	Assessments	1800	
	Rent of offices	1200	$4510
Publication Fund	Appleton Fund investments	$ 536	
	Centennial Fund investments	2111	$2647
Rumford Fund	Investments		$2511
C. M. Warren Fund	Investments		280

The following appropriations are recommended: —

GENERAL FUND.

House expenses	$1400	
Library expenses	1400	
Books, periodicals, and binding	900	
Expenses of Meetings	250	
Insurance	75	
Treasurer's office	150	$4175

RUMFORD FUND.

Research	$1000	
Periodicals and binding	150	
Books and binding	50	
Publication	700	
To be used at discretion of Committee	250	$2150

C. M. WARREN FUND.

Research	$600

PUBLICATION FUND.

Publication	$2600

In accordance with the recommendations in the foregoing report it was

Voted, To appropriate for the purposes named, the following sums: —

From the income of the General Fund	$4175
From the income of the Rumford Fund	2150
From the income of the C. M. Warren Fund	280
From the accumulated income of the C. M. Warren Fund	320
From the income of the Publication Fund	2600

On the motion of the Treasurer, it was

Voted, That the annual assessment for the ensuing year be ten dollars ($10).

The Corresponding Secretary proposed that the Statutes be so changed as to permit the establishment of a Retired list, which proposition was referred to the Council.

The following report of the Committee on the Library was read: —

The Committee on the Library desire to submit through the Librarian, the following recommendation to be acted upon by the Academy: That the Librarian be authorized, at his discretion, to dispose of, through sale, exchange, or gift, such volumes as are not germane to the purposes of this Library. Exceptions are to be made in the case of copies presented by the authors, other than those received under the Bowdoin bequest, which under the terms of the bequest may be sold.

The Committee desire the opinion of the Academy concerning the disposition of a valuable work, written by John Eliot in the Indian language, of which the value exceeds two hundred dollars.

On motion of the Librarian it was,

Voted, That the Librarian be authorized to dispose of such books as are not germane to the purposes of this Library.

Inasmuch as the Massachusetts Historical Society has refused to charge any rent for the use of the Academy rooms in its building for the past year, it was

Voted, To present to the Massachusetts Historical Society certain volumes which have been selected by the Library Committee, in grateful recognition of the action of that Society.

The annual election resulted in the choice of the following officers and committees: —

WILLIAM W. GOODWIN, *President.*
JOHN TROWBRIDGE, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
JOHN C. GRAY, *Vice-President for Class III.*
EDWIN H. HALL, *Corresponding Secretary.*
WILLIAM WATSON, *Recording Secretary.*
CHARLES P. BOWDITCH, *Treasurer.*
A. LAWRENCE ROTCH, *Librarian.*

Councillors for Three Years.

IRA N. HOLLIS, of Class I.
JAMES C. WHITE, of Class II.
WILLIAM R. WARE, of Class III.

Councillor for One Year.

In place of JOHN H. WRIGHT, resigned,
CHARLES R. LANMAN.

Finance Committee.

WILLIAM W. GOODWIN,
ELIOT C. CLARKE,
FRANCIS BARTLETT.

Rumford Committee.

CHARLES R. CROSS, ERASMUS D. LEAVITT,
EDWARD C. PICKERING, ARTHUR G. WEBSTER,
THEODORE W. RICHARDS, ELIHU THOMSON,
LOUIS BELL.

C. M. Warren Committee.

LEONARD P. KINNICUTT, ROBERT H. RICHARDS,
SAMUEL CABOT, ARTHUR M. COMEY,
ARTHUR MICHAEL, HENRY P. TALBOT,
CHARLES R. SANGER.

The following standing committees were appointed by the Chair: —

Publication Committee.

WALLACE C. SABINE, of Class I., EDWARD L. MARK, of Class II.,
CRAWFORD H. TOY, of Class III.

Library Committee.

HARRY M. GOODWIN, of Class I., SAMUEL HENSHAW, of Class II.,
HENRY W. HAYNES, of Class III.

Auditing Committee.

A. LAWRENCE LOWELL, FREDERICK J. STIMSON.

The following communications were given: —

"Delphi." By Professor W. W. Goodwin.

"Studies in the Text of Plato's Republic." By Professor J. H. Wright.

The following papers were presented by title: —

"On the Convergence and the Differentiation of Certain

Classes of Trigonometric Series." By W. C. Brenke. Presented by Maxime Bôcher.

"On the Ionization of Fused Salts." By H. M. Goodwin and H. A. Wentworth.

"On the Coefficients of Compressibility and Expansion of Ether and Alcohol." By A. W. Smith. Presented by Edwin H. Hall.

"Some Stages in the Spermatogenesis of the Honey Bee." By E. L. Mark and Manton Copeland.

"On the Thomson Effect and the Temperature Coefficient of Thermal Conductivity in Soft Iron between 114° and 204° C." By Edwin H. Hall, L. L. Campbell, and S. B. Serviss.

"Contributions to the Cytology of the Entomophthoraceae." By Lincoln Ware Riddle. Presented by W. G. Farlow.

AMERICAN ACADEMY OF ARTS AND SCIENCES.

REPORT OF THE COUNCIL. — PRESENTED MAY 9, 1906.

BIOGRAPHICAL NOTICES.

JAMES MUNSON BARNARD	WILLIAM R. WARE.
JOHN BARTLETT	M. H. MORGAN.

REPORT OF THE COUNCIL.

The Academy has lost nine members by death since the last report of the Council at the Annual Meeting of May 10, 1905:—

Five Resident Fellows: Edward Atkinson, John Bartlett, James Mills Peirce, Nathaniel S. Shaler, and Edward S. Wood; one Associate Fellow: Samuel P. Langley; three Foreign Honorary members: Otto Struve, Freiherr von Richthofen, A. von Kölliker. Four resident Fellows have resigned; one Resident Fellow has been transferred to Associate Fellowship.

New members have been elected as follows:—

Resident Fellows, 12; Associate Fellows, 1; Foreign Honorary Members, 2. The roll of the Academy now includes 196 Resident Fellows, 98 Associate Fellows, and 72 Foreign Honorary Members.

JAMES MUNSON BARNARD.

MR. BARNARD was born in Boston on the 15th of May, 1819, and died at his home in Milton on the 10th of September, 1904, in the eighty-fourth year of his age. His father was Mr. Charles Barnard, a highly respected commission merchant in the South American trade, in partnership with Mr. Israel Munson. Mr. Barnard was the youngest of his three sons.

As a boy he received his education at the Boston Latin School, but by the time that he had completed his studies there, he had already come under the shadow which was to cloud a great part of his life. He had formed the opinion, which was not without foundation, that he was liable to attacks of hereditary insanity, and this had inspired him with a profound sympathy for the unhappy company among whom he might some day find himself, and with an eager desire to alleviate their burden. Instead, then, of going to college, this boy, not yet out of his teens, determined to go at once to Europe, to study the institutions for the insane, so as to be able to compare them, and the methods adopted in them, with what was doing in this country. In these studies and in travel he spent four or five years, visiting England and the Continent, and finally making a somewhat extended cruise in the Levant, in a boat of his own, flying the Greek and American flags. He thus saw Greece and some parts of Asia Minor and Syria, besides many of the Greek islands.

On his return home he found that his oldest brother, the Rev. Charles Francis Barnard, was engaged in the establishment of the Warren Street Chapel for Boys, as part of the work of the Ministry for the Poor which had been set on foot by Dr. Tuckerman, and was under the care of the Benevolent Fraternity of the Unitarian churches. In the organization and conduct of this well-devised and wisely directed charity Mr. James Barnard at once took an active part, not only doing his share of the work in the Sunday-school, but evening after evening through the week taking charge of a large class of boys, acquainting himself with their personal needs and ambitions, and encouraging them to better their lives. This was when he was between twenty and thirty years of age, — a time of life when personal ambitions and social claims might well have engrossed his time and attention. As long as he lived he was constantly in the receipt of letters coming from men in every part of the country, written in grateful acknowledgment of these fraternal relations. One of them has since his death placed in the Warren Street Chapel a memorial window, believing that he owed everything that had made life most valuable to him to these wise and seasonable counsels.

This work was the beginning of the career of active beneficence and unselfish devotion which Mr. Barnard made his chief occupation, and which was to continue in many and various fields of usefulness for sixty years. He had no liking for business affairs, and cared more about wisely spending the means at his command than about increasing them. What concerned him most was to increase his own knowledge and understanding, and then to give his time and attention to such kinds of work as need to be done, but can be done only by men who have at command both means and leisure, and are minded to devote to it their time and their money.

In pursuance of the first of these ends he presently joined the company of young men who, when Mr. Agassiz came to Cambridge, hastened to place themselves at his feet and to gain inspiration from his example. At a later period he was formally entered as a student in the Lawrence Scientific School, and his name appears in the annual catalogues from 1854, when he was already thirty-five years old, until 1858. He did not graduate, his studies having been somewhat eclectic; but on his finally taking up his connections the corporation bestowed upon him the honorary degree of Master of Arts. These years not only greatly extended the range of his intellectual interests and gave him opportunities which he eagerly embraced of seasonably furthering scientific objects which were in need of a helping hand, but brought into his life relations of personal friendship and good-will, both with his teachers and with his fellow-students, which added greatly to its happiness.

But after all, it was the world of men rather than the world of things that he cared for most, and the establishment of the Social Science Association in 1862 aroused in Mr. Barnard warmer sympathies and a still greater measure of activity. He became one of its most convinced promoters, and for some time, during a vacancy in the secretaryship, maintained the office of the Association in Boston, and made it the headquarters of the department committees upon public health and some special branches of education. The Association, after his death, put on record their appreciation "of his devotion to its social and economical interests, and to the extension of its work in education and art, and of the importance of the results of his zeal."

It was probably this work for the Social Science Association which disposed him first to take an active part in the establishment of the Museum of Fine Arts in 1876, and then to further the policy of furnishing the public schools with casts and photographs of statues and paintings. This idea, which is now generally entertained, and which school committees and public libraries rival the publishers in promoting, Mr. Barnard had already advocated both by precept and by example. As early as 1873 he had obtained in London and placed in the hall of the Girls' High School a complete set of the casts of the Frieze of the Parthenon.

In recognition of these public services Mr. Barnard was in 1869 made a member of this Academy, in Class II, Section 3, on the nomination of Professor Benjamin Peirce, Dr. Jeffries Wyman, Dr. B. A. Gould, and Mr. Alexander Agassiz. This was done, as Mr. Peirce wrote to him in informing him of his election, because of "his patriotic devotion to the best interests of the country, because he had been the strength of the Social Science organization, and because, when anything had to be done for science or literature, he was, from the great respect in which he was held, and the weight of his influence, the first man whose support was sought."

Meanwhile he did not for a moment forget, and he did not neglect, the object of his earliest interest. This he now promoted chiefly by procuring the writing and the publication, at his own expense, of papers upon mental disease, and especially the means of its prevention. These he distributed in large numbers wherever he found that they would be of service. In all this he was assisted by the sympathy and advice of physicians in charge of hospitals for the insane, both public and private. Among these papers the most noticeable, perhaps, are a little book upon "Sanity of Mind," written for him by Dr. David F. Lincoln, and a brief sketch, by Mr. Francis Tiffany, of the "Life of Dr. Philippe Pinel." This was accompanied by a large photograph of the celebrated painting by Tony Robert-Fleury, representing Dr. Pinel striking the fetters from the inmates of the Salpêtrière, which hangs on the walls of that hospital.

At the same time he came to take an increasing interest in the subject of jurisprudence, and especially in international law, and in the promotion of universal peace. This may have sprung from an inherited predisposition, for his father's father, Mr. Samuel Barnard of Deerfield, was, a century ago, one of the eminent legal practitioners of western Massachusetts, known up and down the Connecticut River by the affectionate and by no means disrespectful name of "Lawyer Sam." Here also he had papers of value translated and distributed, especially from the writings of Grotius, and besides these more important undertakings he was constantly in the habit of reprinting and distributing anything that he had found specially interesting or that he thought would be serviceable to others.

The modest scale of living which he had adopted, and which was entirely in accordance with the simplicity of his personal tastes, made it possible for him thus to spend money with considerable freedom. Besides making frequent gifts of books on these subjects to public libraries, both here and abroad, he accumulated several hundred particularly well-chosen volumes upon jurisprudence for his own reading. These, by his will, he bequeathed to the Harvard Law School, together with a sum of money for the increase of their number. This also brought him into personal relations, both abroad and in this country, which, in the somewhat secluded life which he habitually led, were a constant source of satisfaction. These relations were extended and confirmed, from time to time, by brief visits to England and to the Continent. He thus made the acquaintance, among others, of Sir Henry Sumner Maine, of Dr. von Holsendorff in Berlin, and of Mr. John Stuart Mill, with whom he maintained for a number of years a frequent correspondence, and who wrote of him that "he knew of no American of more enlightened opinions or more desirous of using his opportunities of diffusing them."

These activities were interrupted by many years of ill health, but their burden was lightened by the care and sympathy which a perfectly happy marriage had brought; and when, during the last half dozen years of his life, his strength returned, and there was a renewal of energy, his diligence in furthering what had now become his chief interest seemed to increase quite to the end. Happily the anxieties which had so long haunted him, suddenly, after a long period of depression, finally passed away. The clouds that had darkened his sky seemed to vanish. He became in his old age light-hearted and almost gay, more animated and conversable than he had ever been, and even more active than ever in the furthering of all good enterprises, both his own and other people's. His last years seemed plainly the happiest and most serene of them all.

It is sufficiently apparent, even from this record, that here was a man who strove sedulously and successfully to make the most and the best of himself and of his opportunities, and to do the best and most that was in his power, not only for his immediate friends, but for mankind. It is well that the example of so good a man and so good a citizen should not go unrecorded.

JOHN BARTLETT.

John Bartlett was elected a Fellow of the Academy in 1892, when he was seventy-two years of age. His election was the well-deserved recognition by this body of a remarkable career, probably unique in our annals and likely to remain unique. It was the career of one who was not a trained scholar in any of the sections among which our membership is divided, nor a practitioner in any of the learned professions, but a man who in the course of an active and successful mercantile life of fifty-two years found time to devote to literature in every one of them, who was a collector of books with which he enriched a great library, and who produced literary studies that should make scholars and laymen of every English-speaking country grateful when they hear his name.

He was born in Plymouth, June 14, 1820, and came of ancestors of the oldest New England stock on both sides of the house. He was eighth in descent from that Robert Bartlett who, arriving at Plymouth in 1623, married Mary (the daughter of Richard Warren, Mayflower, 1620), whose eldest son, Benjamin, married a granddaughter of Elder

Brewster, and whose grandson Samuel married a granddaughter of John Alden. His maternal ancestor was Antony Thacher, who arrived in Ipswich in 1636, and after whom is named the island off Cape Ann. Fifth in descent from him was James Thacher, Mr. Bartlett's grandfather, "Surgeon in the Revolutionary War, and its historian; he also wrote the History of Plymouth, American Medical Biography, and several professional books. In his treatise on Hydrophobia he anticipated Pasteur by eighty years or more." *

What formal education Mr. Bartlett had, he obtained in the public schools of Plymouth, but he was in the main his own teacher through his reading and his reflections upon his reading. The former was carried on from childhood until past his eightieth year, and the latter ceased only with his death. "My mother told me [he writes] that at the age of three I read to her a verse from the Bible; at the age of nine I had read that book from Alpha to Omega. . . . Before the age of twelve I had read most of the juvenile literature of that period, as well as Pilgrim's Progress, Josephus, Arabian Nights, Opie on Lying, Coelebs in Search of a Wife, Scottish Chiefs, Thaddeus of Warsaw, Cruise of the Midge, Tom Cringle's Log, Paul and Virginia, Telemachus; Cooper's Spy, Pilot, Pioneers, Last of the Mohicans, and Red Rover; Scott's Ivanhoe, Talisman, and Pirate; Gulliver's Travels, and Munchausen."

This list seems worth noting and preserving in this memoir because it shows us the books to which an intelligent American boy was attracted in the years 1820–1832. "Opie on Lying" is probably now the least known of them, but I am told that in its day it made even the "white lie" a perfect terror to children.

In 1836, at the age of sixteen, Mr. Bartlett's business life began. He came to Cambridge and entered the employment of John Owen, who kept the University Bookstore and also did not a little publishing for prominent New England writers of the day. At first engaged in the bookbindery department, he passed to a clerkship in the store, and was so successful that thirteen years later, in 1849, he became the owner of the establishment. In those days the University Bookstore was connected with the University itself; the payments of the bills of students for books were guaranteed by the Corporation, and the relations between the proprietor of the bookstore and the authorities were close. At the back

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* These details about Mr. Bartlett's family and some other quotations which will follow are taken from a MS. book which he gave to Harvard College Library in 1900.

of the shop was a little room in which Mr. Bartlett took much pride, and he has often been heard to speak with regret of its disappearance after he had ceased his connection with the business. In this place were laid out copies of new books and of foreign periodicals and journals, and the professors in the college were in the habit of frequenting it as a sort of reading room. To this custom Mr. Bartlett no doubt owed much of his close friendship with many of the remarkable men of the time. He was fit to be their associate, being already well acquainted with the best literature and, as a bookseller, a *rara avis*, for he had actually read the books which he offered for sale, and could speak with authority on their contents. In 1859, however, he disposed of the store to Sever, Allyn, & Francis, and for the next four years was not engaged in business. During the Civil War he served from November, 1862, to July, 1863, as volunteer paymaster in the South Atlantic Squadron. After this he associated himself with the firm of Little, Brown, & Co. in Boston, becoming a partner in that house in February, 1865. Here his chief work consisted in personally dealing with the authors for whom the firm published, but he was also generally recognized as an expert in all matters connected with the actual manufacture of books. He retired on February 1, 1889, in order to be able to devote his time to finishing his Concordance to Shakespeare.

In these many years of a business career there were three directions in which Mr. Bartlett was particularly active outside of business hours,—in reading, in compiling literary manuals, and in collecting books in two special fields.

He was an omnivorous reader, and remembered and digested what he read. In 1900 he deposited in the University Library a thick pamphlet entitled by him "A Record of Idle Hours." Herein he wrote thus: "In 1837 I was entered as clerk in a bookstore, and found myself in wandering mazes lost. Without a guide, philosopher, or friend I plunged in, driving through the sea of books like a vessel without pilot or master. My clerical duties were unusually onerous, yet I always found time for study and reading; and during my active business life of fifty-two years I devoted much time to these purposes. My library was dukedom large enough, with few exceptions, for all my wants. The following is a list of the books which I have read." Then follow about seventeen hundred titles,* in every department of literature, "from grave to gay, from lively to severe." It extends into the year 1901, when it ceased only on account of his failing eyesight. It is an extraordinary list, including really all that is good (and much that is not) in English literature, prose and poetry, together with translations of works written in foreign tongues. He did not confine himself to one reading of a book that pleased him. For example, he read Lecky's "European Morals" and Darwin's "Descent of Man" three times, the elder D'Israeli's works and the "Annals" and "Agricola" of Tacitus twice, while for Gibbon we find the extraordinary record of four readings. Many of the standard poets and novelists he read over and over again. Nor did he confine himself to the older books. He kept up with the times. Among the titles of the last years of his record we find the first four volumes of Rhodes's "History of the United States," Lord Rosebery's "Napoleon," and, as the next to the last entry in the pamphlet, the "Creevey Papers."

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* In this enumeration, Shakespeare's Works, Scott's Novels, Byron's Poems, and similar titles are counted only as units.

This extraordinary course of reading and the equally extraordinary memory, which he retained unimpaired almost to the very last, were what fitted him to compile "Familiar Quotations," a work too well known to all lovers of literature to need further characterization here. The first edition, a small thin octavo of 295 pages, published in Cambridge in 1855, grew to the tall thick volume of the ninth and last edition of 1891, comprising 1158 pages, with one of the most useful indexes that ever was composed. It includes also nearly five thousand footnotes, consisting of cross references to the earlier sources of the thought expressed in the quotation concerned, to parallel passages, or to obvious imitations. This part of the book was a real work of literary research, and in 1871 Harvard University appreciatively conferred the honorary degree of A.M. upon the author. Of the sale of "Familiar Quotations" the present publishers can give no exact figures, but report that there is no doubt that it has reached nearly a quarter of a million copies in its various editions. A less widely known work was the "Shakespeare Phrase Book," 1881, (pp. 1034), "a concordance of phrases rather than of mere words, taking every sentence which contains an important thought." This is of course superseded for scholars by the later "New and Complete Concordance or Verbal Index to Words, Phrases and Passages in the Dramatic Works of Shakespeare, with a supplementing concordance to his poems," a large quarto of 1910 pages, published by Macmillan & Co. in London, 1894. Upon this great work he and his wife labored lovingly together at intervals during twenty years. She was herself of literary race, being the daughter of Sidney Willard, professor of Hebrew, and granddaughter of President Joseph Willard, both of Harvard University and both Fellows of this Academy. A true helpmate, her sympathies were wholly with him in all his undertakings, and their ideal union seemed only accentuated by the devotion with which he gave himself up to the care of her when, in her last years, her mind was clouded and she knew not what she said or did.

In the course of his business Mr. Bartlett had good opportunities to acquire books for himself, and he employed them well. His own library was in general large and excellent, but in two subjects it was that of a specialist. For many years he was a devoted angler, especially for trout and black bass. Lowell's poem addressed "To Mr. John Bartlett, who had sent me a seven-pound trout" is well known. Naturally, therefore, he began to collect books on his favorite sport, and in 1882 published a catalogue * of his collections. In 1892 he presented this collection, numbering 1014 volumes and 269 pamphlets, to the Harvard College Library, which in turn put forth a complete account of it in the series of Bibliographical Contributions.† His other and smaller collection, also presented to the College, was of Proverbs and Emblems, 140 volumes under the former head, 130 under the latter.

Such were the occupations of this busy man; but he was also a lover of social life and blest with close friendships. His sense of humor and his never-failing cheerfulness made him a delightful companion, and even in his extreme old age, although he abounded in anecdotes of the past, he never lost his interest in the affairs of the day. In his prime he was a member of the Somerset, Union, Temple, and St. Botolph clubs; he also belonged to the Colonial Society of Massachusetts, to the Boston Society of Sons of the Revolution, and was in 1894 elected an honorary member of the Harvard Chapter of Phi Beta Kappa. For many years the famous whist club, composed of himself, James Russell Lowell, John Holmes, and Charles F. Choate, met at his house, and when it was dissolved by the death of two of its members and the removal of a third, some of his younger neighbors formed what they called the "Bartlett Club" to enliven his evenings by coming together there once a week. Thus loving friends surrounded him as he "declined into the vale of

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* Catalogue of Books on Angling, including Ichthyology, Pisciculture, Fisheries, and Fishing Laws. From the Library of a practitioner of more than fifty years' experience in the Art of Angling. Cambridge, 1882, 4°, pp. 77. Supplement, Cambridge, 1886, pp. 26.

† No. 51. The Bartlett Collection. A list of books on Angling, Fishes and Fish Culture, in Harvard College Library, by L. R. Albee. Cambridge, 1896.

years," and the almost blindness of his last lustrum was spared that utter loneliness which is too often the lot of those who have outlived all their contemporaries. He died after a short illness on December 3, 1905, having passed eighty-five years in usefulness and honor.

M. H. MORGAN.

American Academy of Arts and Sciences

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LIST

OF THE

FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to June 1, 1906.)

RESIDENT FELLOWS.—196.

(Number limited to two hundred.)

CLASS I.—*Mathematical and Physical Sciences.*—78.

SECTION I.—17.

Mathematics and Astronomy.

Solon I. Bailey,	Cambridge.
Maxime Bôcher,	Cambridge.
William E. Byerly,	Cambridge.
Seth C. Chandler,	Wellesley Hills.
Gustavus Hay,	Boston.
Percival Lowell,	Boston.
Edward C. Pickering,	Cambridge.
William H. Pickering,	Cambridge.
Henry S. Pritchett,	Boston.
John Ritchie, Jr.,	Roxbury.
Edwin F. Sawyer,	Brighton.
Arthur Searle,	Cambridge.
William E. Story,	Worcester.
Henry Taber,	Worcester.
Harry W. Tyler,	Boston.
O. C. Wendell,	Cambridge.
P. S. Yendell,	Dorchester.

SECTION II.—24.

Physics.

A. Graham Bell,	Washington, D.C.
Louis Bell,	Boston.
Clarence J. Blake,	Boston.
Francis Blake,	Weston.
George A. Campbell,	Boston.
Harry E. Clifford,	Newton.
Charles R. Cross,	Brookline.
A. W. Duff,	Worcester.
H. M. Goodwin,	Roxbury.
Edwin H. Hall,	Cambridge.
Hammond V. Hayes,	Cambridge.
William L. Hooper,	Somerville.
William W. Jacques,	Newton.
Frank A. Laws,	Boston.
Henry Lefavour,	Boston.
Theodore Lyman,	Brookline.
Benjamin O. Peirce,	Cambridge.
A. Lawrence Rotch,	Boston.
Wallace C. Sabine,	Boston.
John S. Stone,	Boston.
Elihu Thomson,	Swampscott.
John Trowbridge,	Cambridge.
A. G. Webster,	Worcester.
Robert W. Willson,	Cambridge.

SECTION III.—19.

Chemistry.

Samuel Cabot,	Boston.
Arthur M. Comey,	Cambridge.
James M. Crafts,	Boston.
Charles W. Eliot,	Cambridge.
Charles L. Jackson,	Cambridge.
Walter L. Jennings,	Worcester.

Leonard P. Kinnicutt,	Worcester.
Charles F. Mabery,	Cleveland, O.
Arthur Michael,	Boston.
George D. Moore,	Worcester.
John U. Nef,	Chicago, Ill.
Arthur A. Noyes,	Boston.
Robert H. Richards,	Jamaica Plain.
Theodore W. Richards,	Cambridge.
Charles R. Sanger,	Cambridge.
Stephen P. Sharples,	Cambridge.
Francis H. Storer,	Boston.
Henry P. Talbot,	Newton.
Charles H. Wing,	Ledger, N. C.

Section IV. — 18.

Technology and Engineering.

Comfort A. Adams,	Cambridge.
Alfred E. Burton,	Boston.
Eliot C. Clarke,	Boston.
Heinrich O. Hofman,	Jamaica Plain.
Ira N. Hollis,	Cambridge.
L. J. Johnson,	Cambridge.
Arthur E. Kennelly,	Cambridge.
Gaetano Lanza,	Boston.
E. D. Leavitt,	Cambridge.
William R. Livermore,	New York.
Hiram F. Mills,	Lowell.
Cecil H. Peabody,	Brookline.
Andrew H. Russell,	Manila.
Albert Sauveur,	Cambridge.
Peter Schwamb,	Arlington.
H. L. Smyth,	Cambridge.
George F. Swain,	Boston.
William Watson,	Boston.

Class II. — *Natural and Physiological Sciences.* — 64.

Section I. — 14.

Geology, Mineralogy, and Physics of the Globe.

H. H. Clayton,	Milton.
Algernon Coolidge,	Boston.
William O. Crosby,	Jamaica Plain.
William M. Davis,	Cambridge.
Benj. K. Emerson,	Amherst.
O. W. Huntington,	Newport, R. I.
Robert T. Jackson,	Cambridge.
T. A. Jaggar, Jr.,	Cambridge.
William H. Niles,	Cambridge.
Charles Palache,	Cambridge.
John E. Pillsbury,	Boston.
Robert DeC. Ward,	Cambridge.
John E. Wolff,	Cambridge.
J. B. Woodworth,	Cambridge.

Section II. — 12.

Botany.

F. S. Collins,	Malden.
Geo. E. Davenport,	Medford.
William G. Farlow,	Cambridge.
Charles E. Faxon,	Jamaica Plain.
Merritt L. Fernald,	Cambridge.
George L. Goodale,	Cambridge.
John G. Jack,	Jamaica Plain.
Edward C. Jeffrey,	Cambridge.
B. L. Robinson,	Cambridge.
Charles S. Sargent,	Brookline.
Arthur B. Seymour,	Cambridge.
Roland Thaxter,	Cambridge.

Section III. — 24.

Zoölogy and Physiology.

Alexander Agassiz,	Cambridge.
Robert Amory,	Boston.

Henry P. Bowditch,	Jamaica Plain.
William Brewster,	Cambridge.
Louis Cabot,	Brookline.
Walter B. Cannon,	Cambridge.
William E. Castle,	Cambridge.
Samuel F. Clarke,	Williamstown.
W. T. Councilman,	Boston.
Harold C. Ernst,	Jamaica Plain.
Edward G. Gardiner,	Boston.
Samuel Henshaw,	Cambridge.
Theodore Hough,	Boston.
John S. Kingsley,	Somerville.
Edward L. Mark,	Cambridge.
Charles S. Minot,	Milton.
Edward S. Morse,	Salem.
George H. Parker,	Cambridge.
William T. Porter,	Boston.
James J. Putnam,	Boston.
Samuel H. Scudder,	Cambridge.
William T. Sedgwick,	Boston.
James C. White,	Boston.
William M. Woodworth,	Cambridge.

SECTION IV. — 14.

Medicine and Surgery.

Edward H. Bradford,	Boston.
Arthur T. Cabot,	Boston.
David W. Cheever,	Boston.
Thomas Dwight,	Nahant.
Reginald H. Fitz,	Boston.
Charles F. Folsom,	Boston.
Frederick I. Knight,	Boston.
Samuel J. Mixter,	Boston.
W. L. Richardson,	Boston.
Theobald Smith,	Jamaica Plain.
O. F. Wadsworth,	Boston.
Henry P. Walcott,	Cambridge.
John C. Warren,	Boston.
Francis H. Williams,	Boston.

CLASS III. — *Moral and Political Sciences.* — 54.

SECTION I. — 10.

Philosophy and Jurisprudence.

James B. Ames,	Cambridge.
Joseph H. Beal, Jr.,	Cambridge.
John C. Gray,	Boston.
Francis C. Lowell,	Boston.
Hugo Münsterberg,	Cambridge.
Josiah Royce,	Cambridge.
Jeremiah Smith,	Cambridge.
Frederic J. Stimson,	Dedham.
Edward H. Strobel,	Cambridge.
Samuel Williston,	Belmont.

SECTION II. — 23.

Philology and Archæology.

Charles P. Bowditch,	Jamaica Plain.
Lucien Carr,	Cambridge.
Franklin Carter,	Williamstown.
Joseph T. Clarke,	Harrow, Eng.
Henry G. Denny,	Roxbury.
J. W. Fewkes,	Washington.
William W. Goodwin,	Cambridge.
Henry W. Haynes,	Boston.
Albert A. Howard,	Cambridge.
Charles R. Lanman,	Cambridge.
David G. Lyon,	Cambridge.
George F. Moore,	Cambridge.
Morris H. Morgan,	Cambridge.
Bennett H. Nash,	Boston.
Frederick W. Putnam,	Cambridge.
Edward Robinson,	Boston.
Edward S. Sheldon,	Cambridge.
Herbert Weir Smyth,	Cambridge.
F. B. Stephenson,	Boston.
Crawford H. Toy,	Cambridge.
John W. White,	Cambridge.
John H. Wright,	Cambridge.
Edward J. Young,	Waltham.

SECTION III. — 11.

Political Economy and History.

Charles F. Adams,	Lincoln.
Thomas N. Carver,	Cambridge.
Andrew McF. Davis,	Cambridge.
Ephraim Emerton,	Cambridge.
A. C. Goodell,	Salem.
Charles Gross,	Cambridge.
Henry C. Lodge,	Nahant.
A. Lawrence Lowell,	Boston.
James F. Rhodes,	Boston.
Charles C. Smith,	Boston.
F. W. Taussig,	Cambridge.

SECTION IV. — 10.

Literature and the Fine Arts.

Francis Bartlett,	Boston.
Arlo Bates,	Boston.
Kuno Francke,	Cambridge.
T. W. Higginson,	Cambridge.
George L. Kittredge,	Cambridge.
Charles Eliot Norton,	Cambridge.
Denman W. Ross,	Cambridge.
William R. Ware,	Milton.
Herbert L. Warren,	Cambridge.
Barrett Wendell,	Boston.

ASSOCIATE FELLOWS. — 98.

(Number limited to one hundred. Elected as vacancies occur.)

Class I. — *Mathematical and Physical Sciences.* — 37.

Section I. — 14.

Mathematics and Astronomy.

Edward E. Barnard,	Williams Bay, Wis.
S. W. Burnham,	Williams Bay, Wis.
George Davidson,	San Francisco.
Fabian Franklin,	Baltimore.
Asaph Hall,	S. Norfolk, Conn.
George W. Hill,	W. Nyack, N.Y.
E. S. Holden,	New York.
Emory McClintock,	Morristown, N.J.
E. H. Moore,	Chicago.
Simon Newcomb,	Washington.
Charles L. Poor,	New York.
George M. Searle,	Washington.
J. N. Stockwell,	Cleveland, O.
Chas. A. Young,	Hanover, N. H.

Section II. — 6.

Physics.

Carl Barus,	Providence, R.I.
G. E. Hale,	Williams Bay, Wis.
T. C. Mendenhall,	Worcester.
A. A. Michelson,	Chicago.
E. L. Nichols,	Ithaca, N. Y.
M. I. Pupin,	New York.

Section III. — 9.

Chemistry.

Wolcott Gibbs,	Newport, R.I.
Frank A. Gooch,	New Haven.
Eugene W. Hilgard,	Berkeley, Cal.
S. W. Johnson,	New Haven.
J. W. Mallet,	Charlottesville, Va.
E. W. Morley,	Cleveland, O.
Charles E. Munroe,	Washington.
J. M. Ordway,	New Orleans.
Ira Remsen,	Baltimore.

Section IV. — 8.

Technology and Engineering.

Henry L. Abbot,	Cambridge.
Cyrus B. Comstock,	New York.
W. P. Craighill,	Charlestown, W. Va.
John Fritz,	Bethlehem, Pa.
James D. Hague,	New York.
F. R. Hutton,	New York.
William Sellers,	Edge Moor, Del.
Robt. S. Woodward,	New York.

Class II. — *Natural and Physiological Sciences.* — 33.

Section I. — 11.

Geology, Mineralogy, and Physics of the Globe.

Cleveland Abbe,	Washington.
George J. Brush,	New Haven.
T. C. Chamberlin,	Chicago.
Edward S. Dana,	New Haven.
Walter G. Davis,	Cordova, Arg.
Samuel F. Emmons,	Washington.
G. K. Gilbert,	Washington.
S. L. Penfield,	New Haven.
R. Pumpelly,	Newport, R.I.
Israel C. Russell,	Ann Arbor.
Charles D. Walcott,	Washington.

SECTION II. — 6.

Botany.

L. H. Bailey,	Ithaca, N. Y.
D. H. Campbell,	Palo Alto, Cal.
J. M. Coulter,	Chicago.
C. G. Pringle,	Charlotte, Vt.
John D. Smith,	Baltimore.
W. Trelease,	St. Louis.

SECTION III. — 9.

Zoölogy and Physiology.

Joel A. Allen,	New York.
W. K. Brooks,	Lake Roland, Md.
C. B. Davenport,	Cold Spring Harbor, N. Y.
F. P. Mall,	Baltimore.
S. Weir Mitchell,	Philadelphia.
H. F. Osborn,	New York.
A. E. Verrill,	New Haven.
C. O. Whitman,	Chicago.
E. B. Wilson,	New York.

SECTION IV. — 8.

Medicine and Surgery.

John S. Billings,	New York.
W. S. Halsted,	Baltimore.
Abraham Jacobi,	New York.
W. W. Keen,	Philadelphia.
William Osler,	Oxford, Eng.
T. Mitchell Prudden,	New York.
Wm. H. Welch,	Baltimore.
H. C. Wood,	Philadelphia.

CLASS III. — *Moral and Political Sciences.* — 27.

SECTION I. — 6.

Philosophy and Jurisprudence.

Joseph H. Choate,	New York.
Melville W. Fuller,	Washington.
William W. Howe,	New Orleans.
Charles S. Peirce,	Milford, Pa.
G. W. Pepper,	Philadelphia.
T. R. Pynchon,	Hartford, Conn.

SECTION II. — 7.

Philology and Archæology.

Timothy Dwight,	New Haven.
B. L. Gildersleeve,	Baltimore.
D. C. Gilman,	Baltimore.
T. R. Lounsbury,	New Haven.
Rufus B. Richardson,	New York.
Thomas D. Seymour,	New Haven.
A. D. White,	Ithaca, N.Y.

SECTION III. — 7.

Political Economy and History.

Henry Adams,	Washington.
G. P. Fisher,	New Haven.
Arthur T. Hadley,	New Haven.
Henry C. Lea,	Philadelphia.
Alfred T. Mahan,	New York.
H. Morse Stephens,	Ithaca.
W. G. Sumner,	New Haven.

SECTION IV. — 7.

Literature and the Fine Arts.

James B. Angell,	Ann Arbor, Mich.
H. H. Furness,	Wallingford, Pa.
R. S. Greenough,	Florence.
Herbert Putnam,	Washington.
Augustus St. Gaudens,	Windsor, Vt.
John S. Sargent,	London.
E. C. Stedman,	Bronxville, N. Y.

FOREIGN HONORARY MEMBERS.—72.

(Number limited to seventy-five. Elected as vacancies occur.)

CLASS I.—*Mathematical and Physical Sciences.*—25.

SECTION I.—7.

Mathematics and Astronomy.

Arthur Auwers,	Berlin.
George H. Darwin,	Cambridge.
Sir William Huggins,	London.
Felix Klein,	Göttingen.
Émile Picard,	Paris.
H. Poincaré,	Paris.
H. C. Vogel,	Potsdam.

SECTION II.—6.

Physics.

Ludwig Boltzmann,	Vienna.
Oliver Heaviside,	Newton Abbot.
F. Kohlrausch,	Marburg.
Joseph Larmor,	Cambridge.
Lord Rayleigh,	Witham.
Joseph J. Thomson,	Cambridge.

SECTION III.—7.

Chemistry.

Adolf Ritter von Baeyer,	Munich.
Marcellin Berthelot,	Paris.
J. H. van't Hoff,	Berlin.
D. Mendeleeff,	St. Petersburg.
Wilhelm Ostwald,	Leipsic.
Sir H. E. Roscoe,	London.
Julius Thomsen,	Copenhagen.

SECTION IV.—5.

Technology and Engineering.

Sir Benjamin Baker,	London.
Lord Kelvin,	Largs.
Maurice Lévy,	Paris.
H. Müller-Breslau,	Berlin.
W. Cawthorne Unwin,	London.

CLASS II.—*Natural and Physiological Sciences.*—23.

SECTION I.—5.

Geology, Mineralogy, and Physics of the Globe.

Sir Archibald Geikie,	London.
Julius Hann,	Vienna.
Albert Heim,	Zurich.
Sir John Murray,	Edinburgh.
Henry C. Sorby,	Sheffield.

SECTION II.—6.

Botany.

E. Bornet,	Paris.
A. Engler,	Berlin.
Sir Joseph D. Hooker,	Sunningdale.
W. Pfeffer,	Leipsic.
H. Graf zu Solms-Laubach,	Strassburg.
Eduard Strasburger,	Bonn.

SECTION III. — 5.

Zoölogy and Physiology.

Sir Michael Foster,	Cambridge.
Ludimar Hermann,	Königsberg.
H. Kronecker,	Bern.
E. Ray Lankester,	London.
Elias Metschnikoff,	Paris.

SECTION IV. — 7.

Medicine and Surgery.

Emil von Behring,	Marburg.
Sir T. L. Brunton,	London.
A. Celli,	Rome.
Sir V. A. H. Horsley,	London.
R. Koch,	Berlin.
Lord Lister,	London.
F. v. Recklinghausen,	Strassburg.

CLASS III. — *Moral and Political Sciences.* — 24.

SECTION I. — 5.

Philosophy and Jurisprudence.

A. J. Balfour,	Prestonkirk.
Heinrich Brunner,	Berlin.
A. V. Dicey,	Oxford.
F. W. Maitland,	Cambridge.
Sir Frederick Pollock, Bart.,	London.

SECTION II. — 7.

Philology and Archæology.

Ingram Bywater,	Oxford.
F. Delitzsch,	Berlin.
W. Dörpfeld,	Athens.
Sir John Evans,	Hemel Hempstead.
H. Jackson,	Cambridge.
J. W. A. Kirchhoff,	Berlin.
G. C. C. Maspero,	Paris.

SECTION III. — 5.

Political Economy and History.

James Bryce,	London.
Adolf Harnack,	Berlin.
Sir G. O. Trevelyan, Bart.,	London.
John Morley,	London.
Pasquale Villari,	Florence.

SECTION IV. — 7.

Literature and the Fine Arts.

E. de Amicis,	Florence.
Gaston Boissier,	Paris.
Georg Brandes,	Copenhagen.
F. Brunetière,	Paris.
S. H. Butcher,	London.
Jean Léon Gérôme,	Paris.
Rudyard Kipling,	Burwash.

STATUTES AND STANDING VOTES.

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11, *and May* 10, 1893, *May* 9, *and October* 10, 1894, *March* 13, *April* 10, *and May* 8, 1895, *May* 8, 1901, *January* 8, 1902, *May* 10, 1905, *February* 14 *and March* 14, 1906.

CHAPTER I.

OF FELLOWS AND FOREIGN HONORARY MEMBERS.

1. The Academy consists of Resident Fellows, Associate Fellows, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy; — Section 2. Physics; — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Theology, Philosophy, and Jurisprudence; — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows shall not exceed two hundred, of whom there shall not be more than eighty in any one of the three classes. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from the Commonwealth. Each Resident Fellow shall

pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

OF OFFICERS.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the annual meeting, on the second Wednesday in May.

2. There shall be nine Councillors, chosen from the Resident Fellows. At each annual meeting, three Councillors shall be chosen, by ballot, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several sections of the same Class, reporting their action to the Academy.

3. The Council shall at its March Meeting receive reports from the Rumford Committee, the C. M. Warren Committee, the Committee on Publication, the Committee on the Library, the President and Record-

ing Secretary, and the Treasurer, proposing the appropriations for their work during the year beginning the following May. The Treasurer at the same meeting shall report on the income which will probably be received on account of the various Funds during the same year.

At the Annual Meeting, the Council shall submit to the Academy, for its action, a report recommending the appropriations which in the opinion of the Council should be made for the various purposes of the Academy.

4. If any office shall become vacant during the year, the vacancy shall be filled by a new election, at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint a Nominating Committee of three Resident Fellows, one for each Class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to direct the Recording Secretary to call special meetings; and to execute or to see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

3. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee on Finance to consist of three Fellows to be chosen by ballot, who shall have, through the Treasurer, full control and management of the funds and trusts of the Academy, with the power of investing and of changing the investment of the same at their discretion.

3. The Rumford Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Academy on all applications and claims for the Rumford premium. They shall also report to the Council in March of each year on all appropriations of the income of the Rumford Fund needed for the coming year, and shall generally see to the due and proper execution of the trust. All bills incurred on account of the Rumford Fund, within the limits of the appropriation made by the Academy, shall be approved by the Chairman of the Rumford Committee.

4. The C. M. Warren Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Council in March of each year on all applications for appropriations from the income of the C. M. Warren Fund for the coming year, and shall generally see to the due

and proper execution of the trust. All bills incurred on account of the C. M. Warren Fund, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the C. M. Warren Committee.

5. The Committee on Publication, to consist of three Fellows, one from each class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and Memoirs shall be entrusted. This Committee shall report to the Council in March of each year on the appropriations needed for the coming year. All bills incurred on account of publications, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the Committee on Publication.

6. The Committee on the Library, to consist of the Librarian *ex officio,* and three other Fellows, one from each class, who shall examine the Library and make an annual report on its condition and management. This Committee, through the Librarian, shall report to the Council in March of each year, on the appropriations needed for the Library for the coming year. All bills incurred on account of the Library, within the limits of the appropriations made by the Academy, shall be approved by the Librarian.

7. The President and Recording Secretary shall be a Committee on the general expenditures of the Academy. This Committee shall report to the Council in March of each year on the appropriations needed for the general expenditures for the coming year, and either member of the Committee may approve bills incurred on this account within the limits of the appropriations made by the Academy.

8. An auditing Committee, to consist of two Fellows, for auditing the accounts of the Treasurer, with power to employ an expert and to approve his bill.

9. In the absence of the Chairman of any Committee, bills may be approved by a member of the Committee designated by the Chairman for the purpose.

CHAPTER VI.

OF THE SECRETARIES.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each

meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

4. Every person taking any books, papers, or documents belonging to the Academy and in the custody of the Recording Secretary, shall give a receipt for the same to the Recording Secretary.

CHAPTER VII.

OF THE TREASURER.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive all moneys due or payable to the Academy and all bequests and donations made to the Academy. He shall pay all bills due by the Academy, when approved by the proper officers (except those of the Treasurer's office, which may be paid without such approval). He shall sign all leases of real estate in the name of the Academy. All transfers of stocks, bonds, and other securities belonging to the Academy shall be made by the Treasurer with the written consent of one member of the Committee of Finance. He shall keep an account of all receipts and expenditures, shall submit his accounts annually to the Auditing

Committee, and shall report the same at the expiration of his term of office or whenever called on so to do by the Academy or Council.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. The Treasurer may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, the Treasurer shall be responsible; or the Treasurer may employ any Trust Company, doing business in Boston, as agent to perform his duties, the compensation of such Assistant Treasurer or agent to be paid from the funds of the Academy.

CHAPTER VIII.

OF THE LIBRARIAN AND LIBRARY.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford, or other special Funds of the Academy, for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

7. The Librarian shall have custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

OF MEETINGS.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings, only, or at meetings adjourned from these and regularly notified, or at special meetings called for the purpose, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

Special meetings shall be called by the Recording Secretary at the request of the President or of a Vice-President or of five Fellows. Notifications of the special meetings shall contain a statement of the purpose for which the meeting is called.

2. Fifteen Resident Fellows shall constitute a quorum for the transaction of business at a stated or special meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Resident Fellow; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

OF THE ELECTION OF FELLOWS AND HONORARY MEMBERS.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them; and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council. No person recommended shall be reported by the Council as a

candidate for election, unless he shall have received the approval of at least five members of the Council present at a meeting. All nominations thus approved shall be read to the Academy at any meeting, and shall then stand on the nomination list until the next stated meeting, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void; and if any Resident Fellow shall neglect to pay his annual assessments for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do.

3. The nomination and election of Associate Fellows shall take place in the manner prescribed in reference to Resident Fellows.

4. The nomination and election of Foreign Honorary Members shall take place in the manner prescribed for Resident Fellows, except that the nomination papers shall be signed by at least seven members of the Council before being presented to the Academy.

5. Three-fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship; and provided that a majority of two-thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

OF AMENDMENTS OF THE STATUTES.

1. All proposed alterations of the Statutes, or additions to them, shall be referred to a committee, and, on their report at a subsequent stated meeting or a special meeting called for the purpose, shall require for

enactment a majority of two-thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded at a stated meeting, or a special meeting called for the purpose by a majority of two-thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

Of Literary Performances.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

STANDING VOTES.

1. Communications of which notice has been given to the Secretary shall take precedence of those not so notified.

2. Associate Fellows, Foreign Honorary Members, and Resident Fellows, who have paid all fees and dues chargeable to them, are entitled to receive one copy of each volume or article printed by the Academy on application to the Librarian personally or by written order within two years of the date of publication. Exceptions to this rule may be made in special cases by vote of the Academy.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium; and the approval of a bill incurred for such purposes by the Chairman shall be accepted by the Treasurer as proof that such certificate has been given.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

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Hay, G., On a Postulate respecting a Certain Form of Deviation from the Straight Line in a Plane, 351–357, 810; On the Inconsistency of the Axiom of the Plane with the Supposition that the Sum of the Angles of the Rectilinear Triangle is Less than 180°, 818.
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Jackson, R. F. *See* Lewis, G. N., and Jackson, R. F.
Jefferson Physical Laboratory, Contributions from, 21, 503, 541, 585, 615, 627, 645, 691.
Jeffrey, Edward Charles, elected Resident Fellow, 819; accepts Fellowship, 822.
Jones, W. W., A Revision of the Genus Zexmenia, 141-167.

Kennelly, Arthur Edward, elected Resident Fellow, 808; accepts Fellowship, 810.
Kent, N. A., The Relative Positions of the Arc and Spark Lines of the Spectra of Titanium, Zinc, and Iron, 279-300.
Kinnicutt, L. P., Report of the C. M. Warren Committee, 828.
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Moore, George Foot, elected Resident Fellow, 819; accepts Fellowship, 822.
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Morse, H. W., Studies on Fluorite: (I) The Fluorescence of Fluorite; (II) The Thermo-Luminescence of Fluorite; (III) Gaseous and Liquid Inclusions in Fluorspar, 585–613, 809.
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Thomson Effect in Soft Iron, Measurements of Various Thermal and Electrical Effects, especially the, 21–55.
Titanium, The Relative Positions of the Arc and Spark Lines of the Spectra of, 279–300.
Tomek, W. W. Ritt. von, Death of, 807.
Transition Temperature of Sodic Bromide, 433–448.
Treasurer, Report of, 823.
Trelease, W., appointed Delegate, 814; Letter from, 822.
Trowbridge, John, Effect of Magnetic Field on Production of the X-Rays, 813; Slow Discharges of Electricity, 809; Electric Discharges. (I) Slow Moving Electrical Luminous Effects; (II) Magnetic Field and Electric Discharges, 627–644, 810.
United States, Descriptions of Spermatophytes from the Southwestern, 233–270.
University of Illinois, Invitation from, 807.

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Vibrations of a Rubbed String, The Longitudinal, 691–727, 809, 818.
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Ware, William R., Notice of J. M. Barnard, 811, 837.
Warren (C. M.) Committee, Report of, 828.
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Warren, Herbert Langford, elected Resident Fellow, 808; accepts Fellowship, 810.
Wave Lengths, Short, The Spectrum of Hydrogen in the Schumann Region of, 811.
Wells, R. C. *See* Richards, T. W., and Wells, R. C.
Wendell, Barrett, The Universities of France, 821.
Wentworth, H. A. *See* Goodwin, H. M., and Wentworth, H. A.
Wheeler, P. *See* Lewis, G. N., and Wheeler, P.
Williston, Samuel, accepts Fellowship, 807.
Wood, Edward S., Death of, 807.
Wood, R. W., The Fluorescence of Sodium Vapor and the Resonance Radiation of Electrons, 809.
Wright, J. H., Studies in the text of Plato's Republic, 832.

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Zinc, The Relative Positions of the Arc and Spark Lines of the Spectra of, 279–300.
Zoölogical Laboratory of the Museum of Comparative Zoölogy at Harvard College, E. L. Mark, Director, Contributions from, 729.

EXPLANATION OF PLATE 2.

FIG. 13. Fluorescence spectrum of fluorite excited by magnesium spark. Magnesium comparison spectra.

FIG. 14. The same crystal excited by iron spark. Iron comparison spectra.

FIG. 15. The same crystal excited by cadmium spark. Cadmium comparison spectra.

EXPLANATION OF PLATE 3.

Fig. 16.

The upper spectrum is the spectrum of the luminescence produced in a piece of fluorspar by heating it to about 200° C. The lower spectrum is that of the magnesium spark.

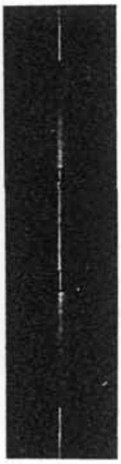


FIG. 1.

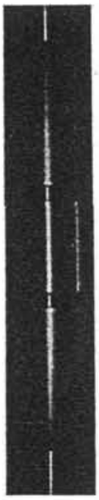


FIG. 2.

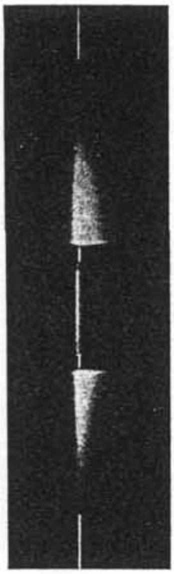
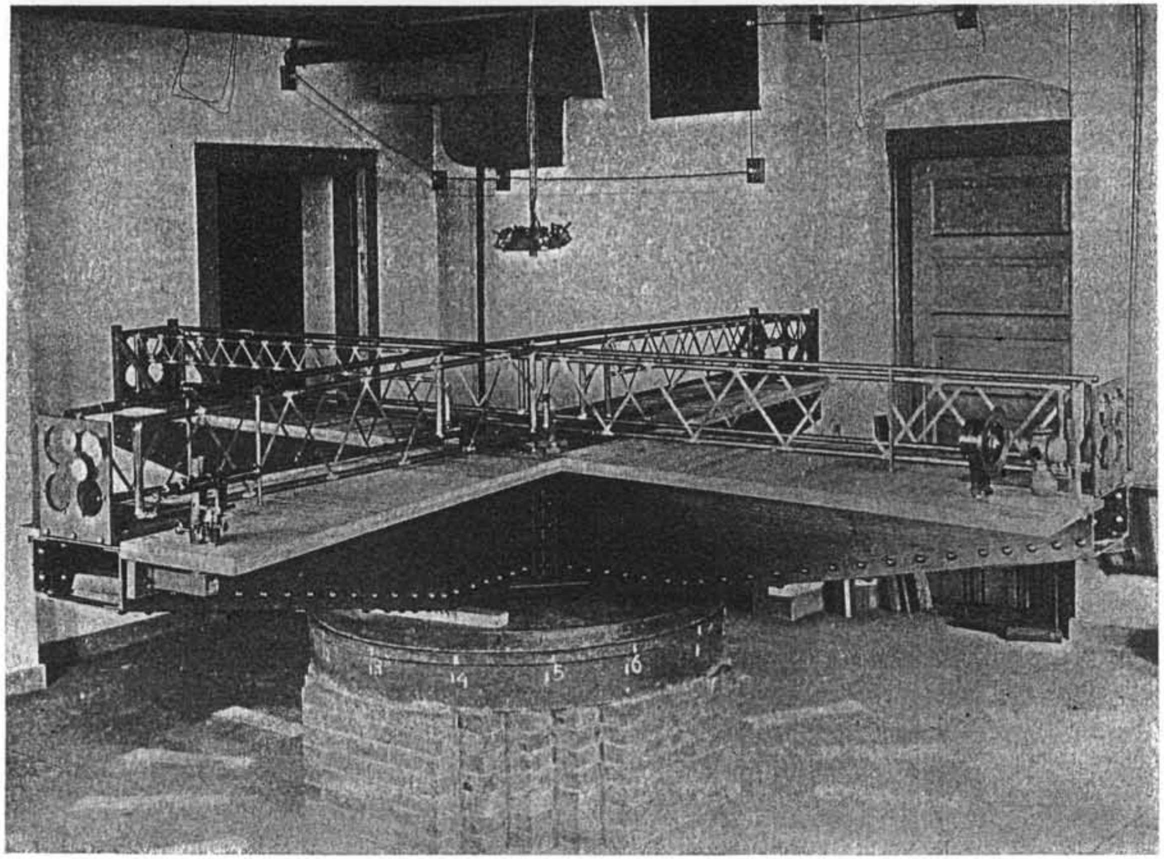


FIG 3.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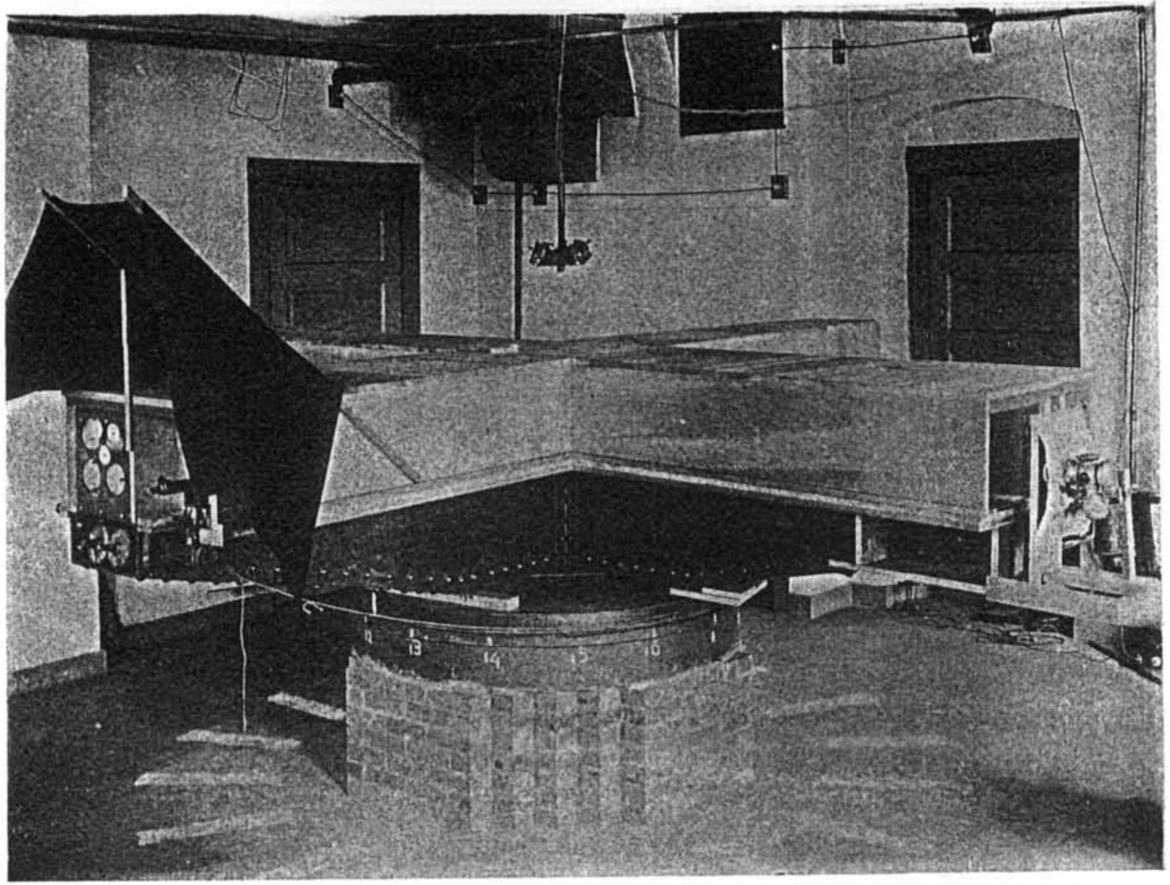
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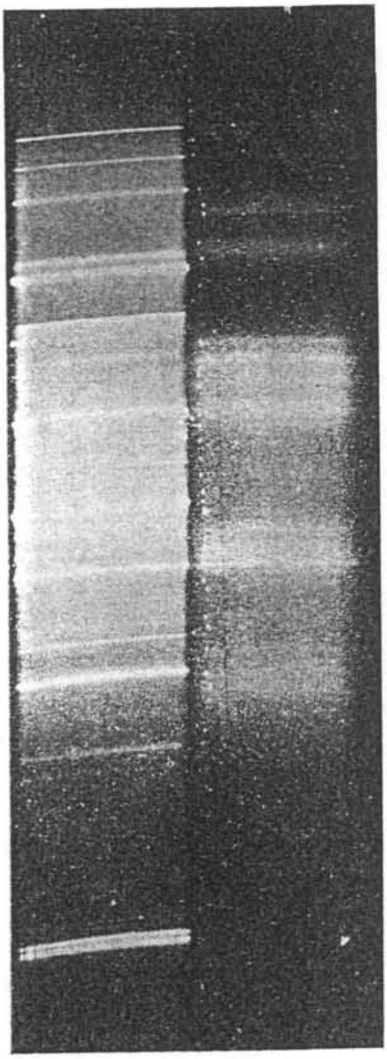


FIG. 16.

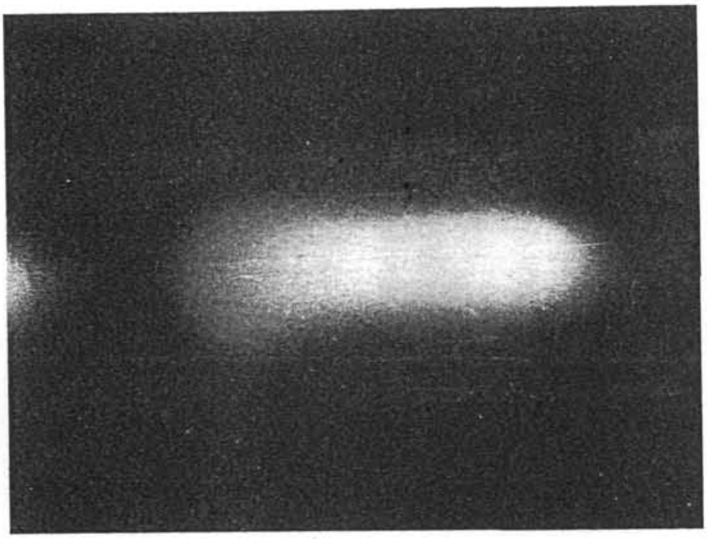
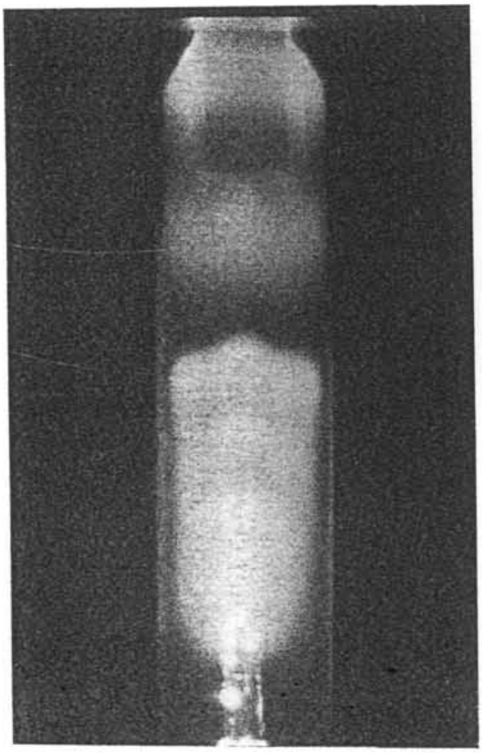


FIG. 1.



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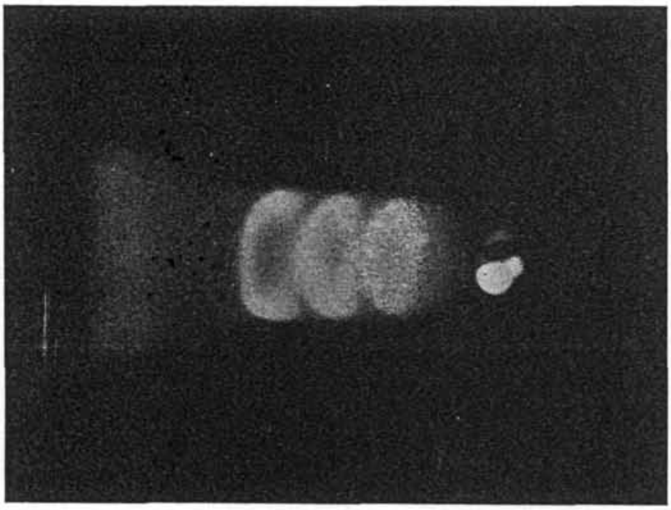


FIG. 3.

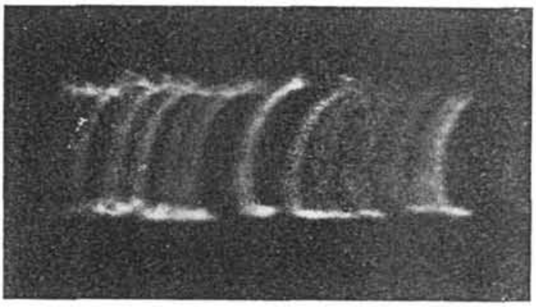


FIG. 4

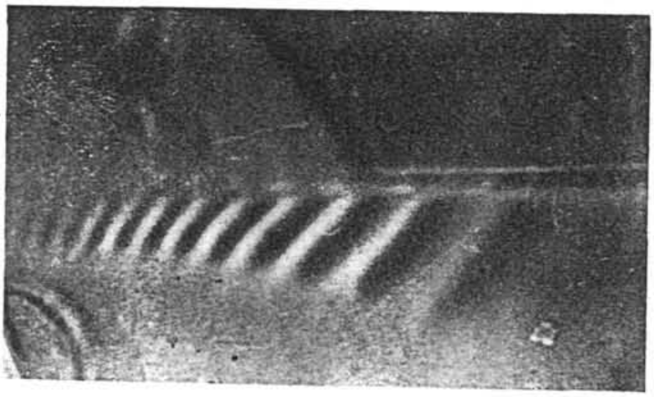


FIG. 5.

FIG. 6.

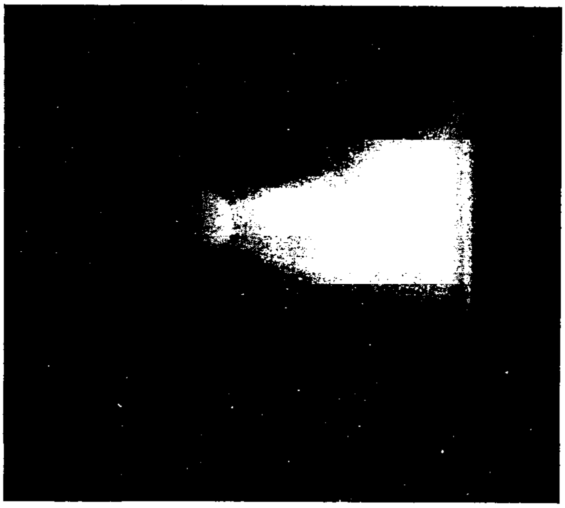


FIG. 7.

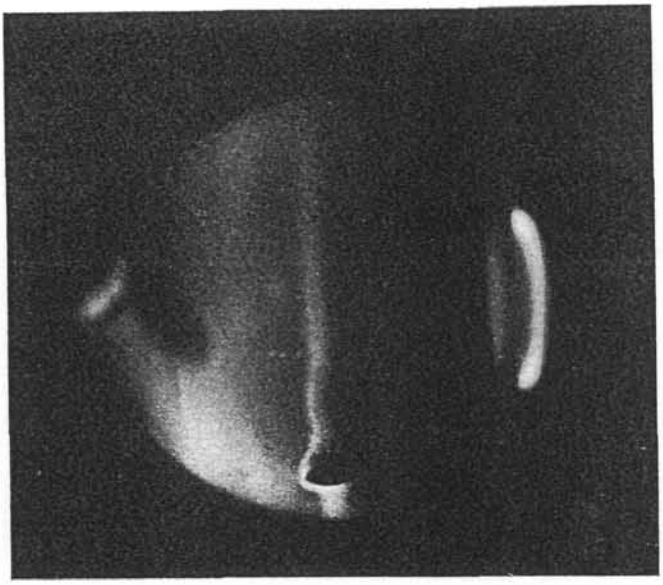


FIG. 8.

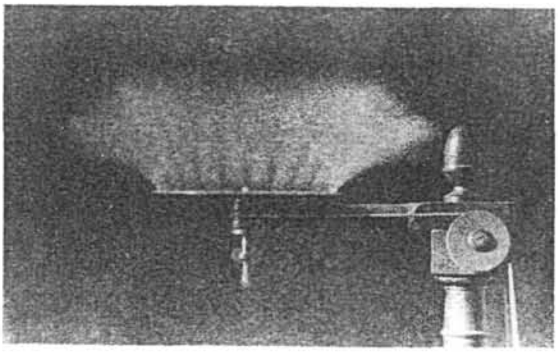
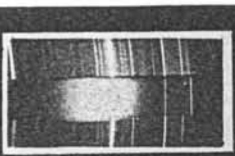


FIG. 9.



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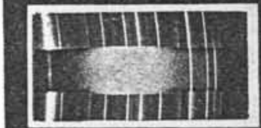

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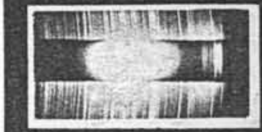

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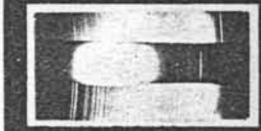

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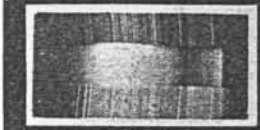
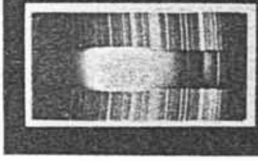


FIG. 5.



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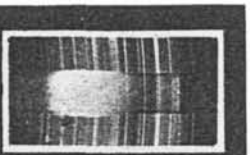

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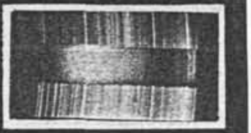

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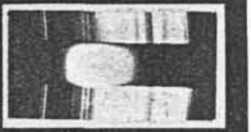

FIG. 9.

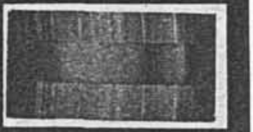

FIG. 10.

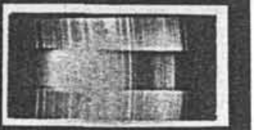

FIG. 11.

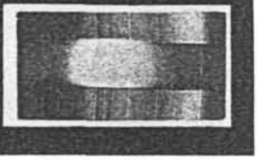

FIG. 12.

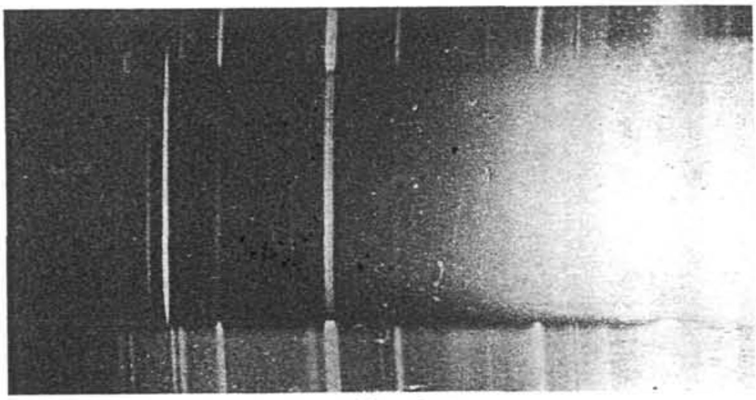

FIG. 13

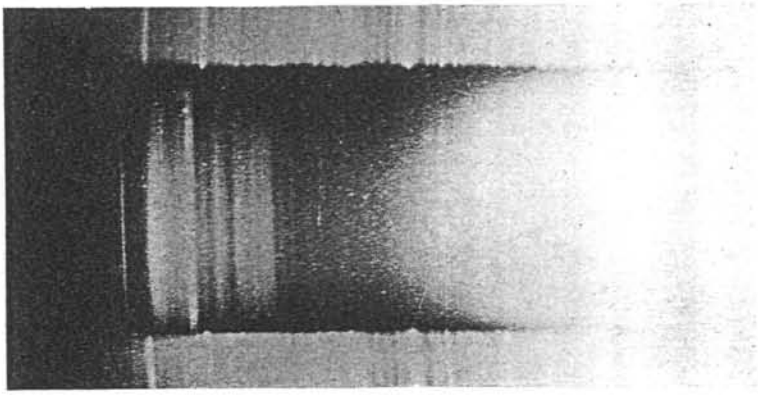

FIG. 14.

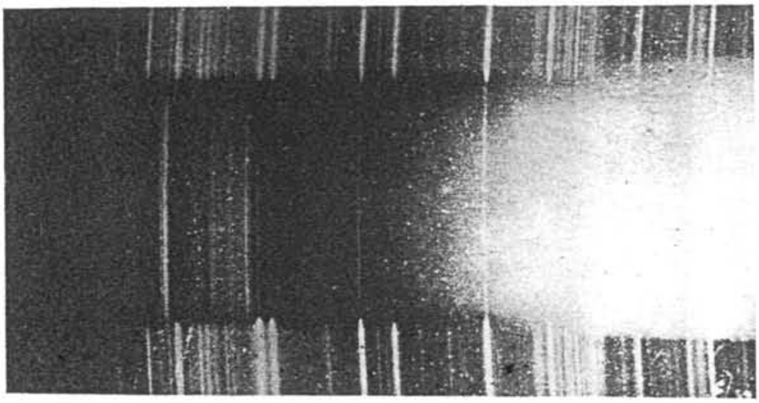

FIG. 15.

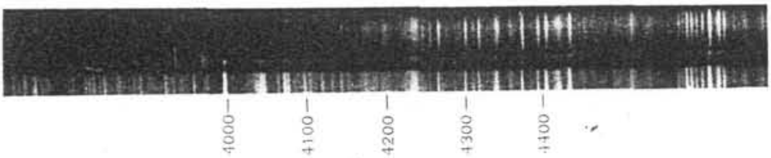
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FIG. A.

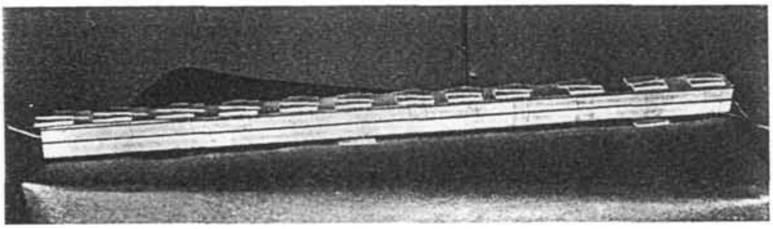

FIG. B.

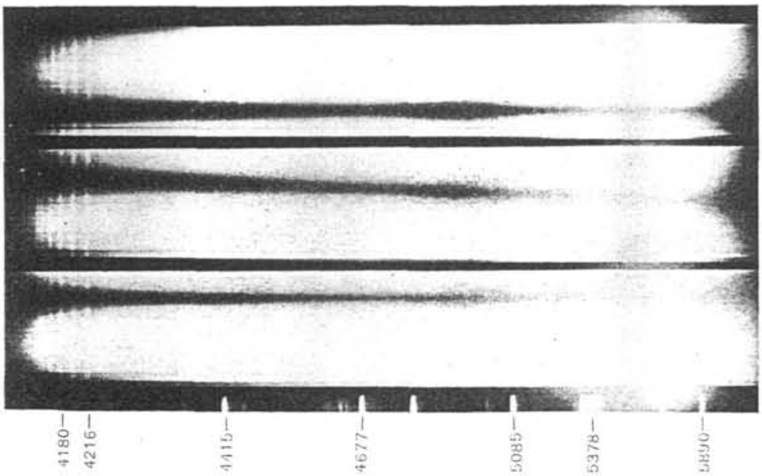
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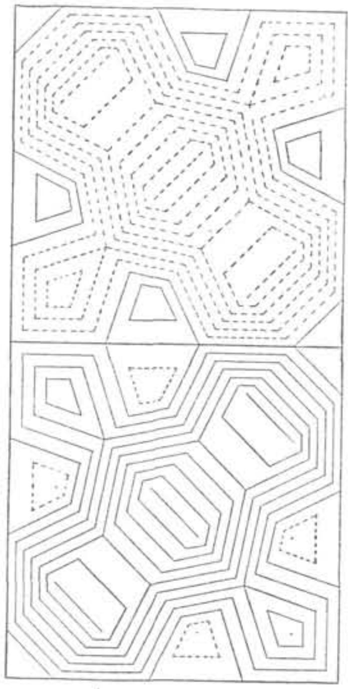
FIG. C.

CASES CONVERGING ON $\frac{1}{3}$.



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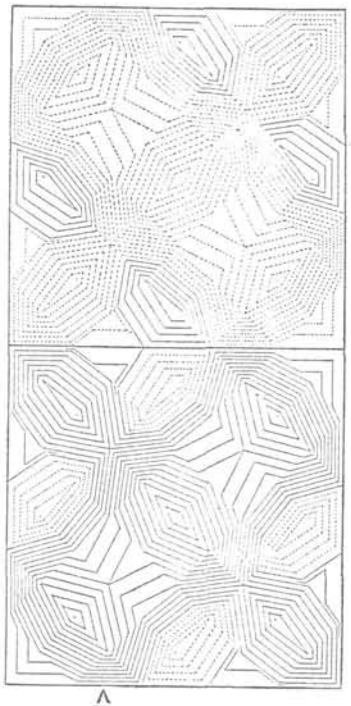
2/7



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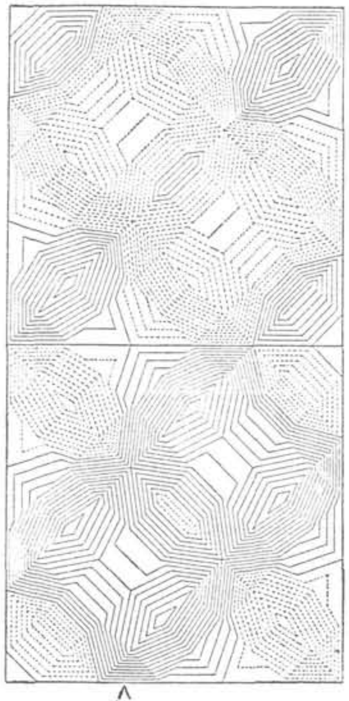
3/10



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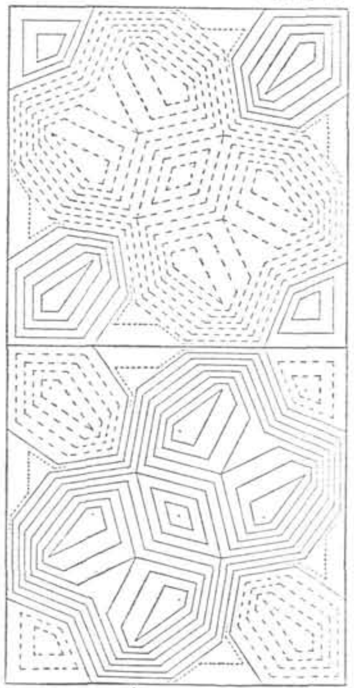
4/11



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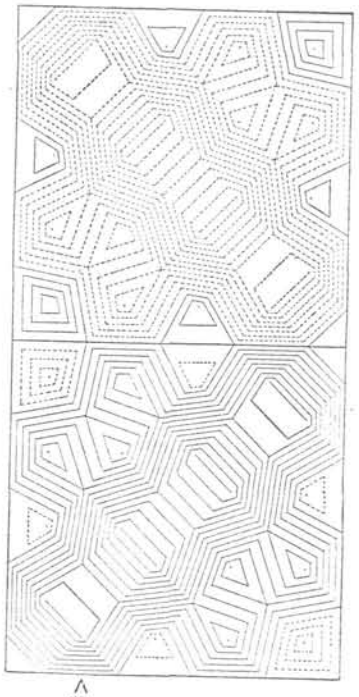
3/8

A CASE IN THE CONVERGENCE ON $\frac{1}{4}$.



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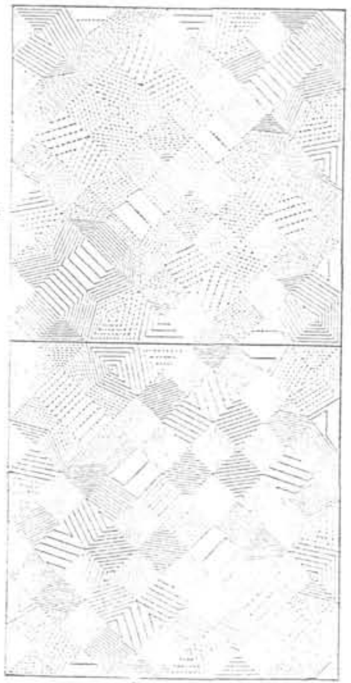
2/9

CASES CONVERGING ON $\frac{2}{5}$.



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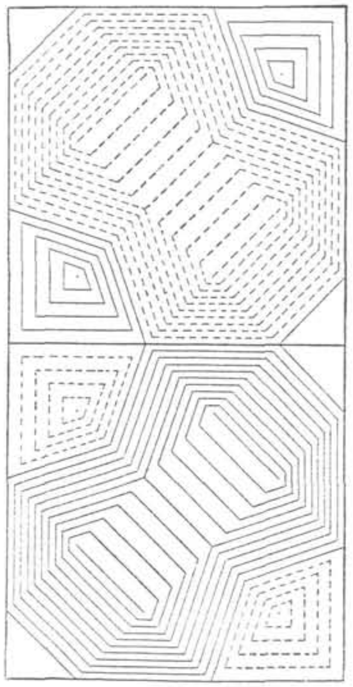
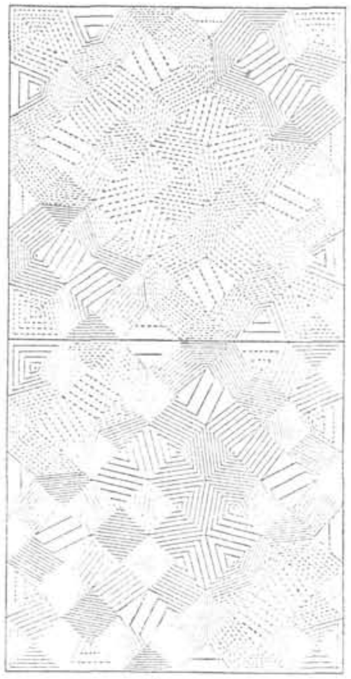
5/13



The limiting case (2/5) itself.



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5/12

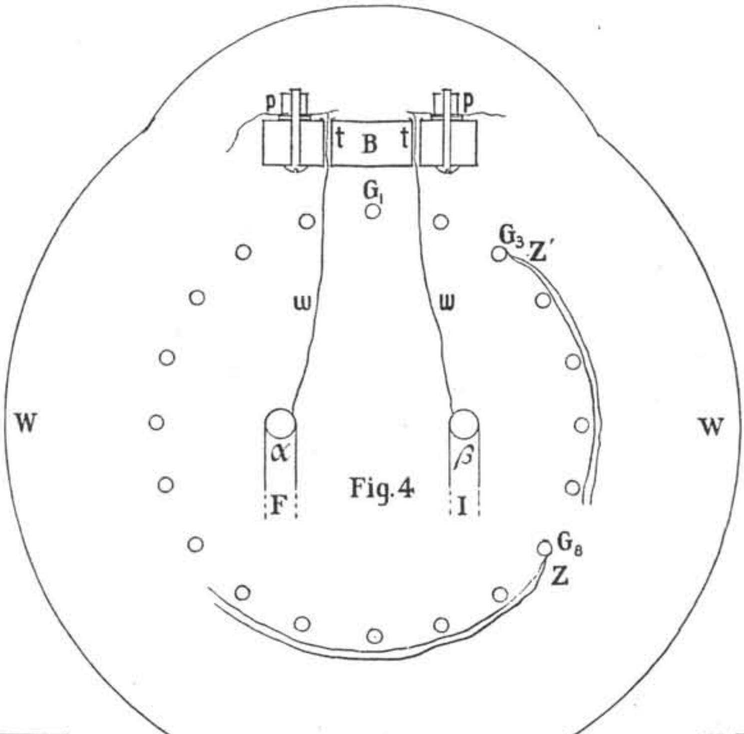

p
p
t
B
t
G_1
G_3
Z′
w
w
W
W
α
β
F
I
G_8
Z

Fig. 4

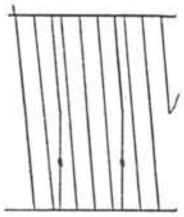


Fig.5

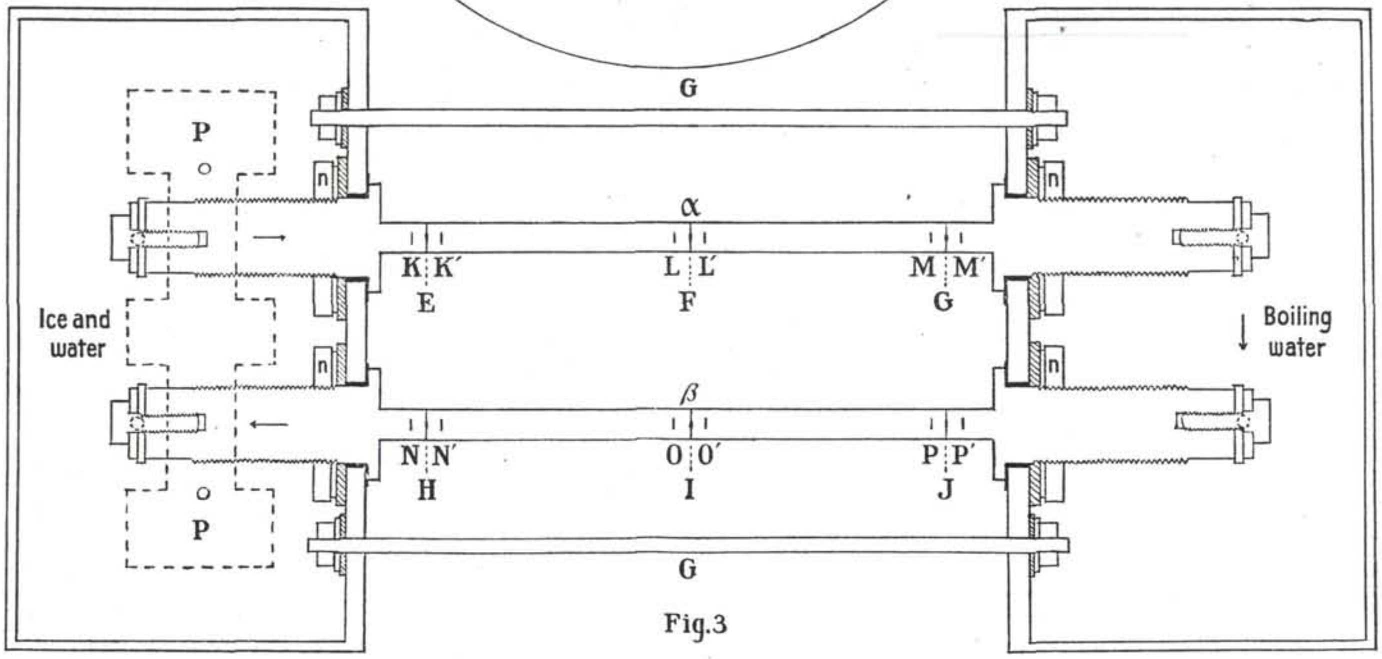
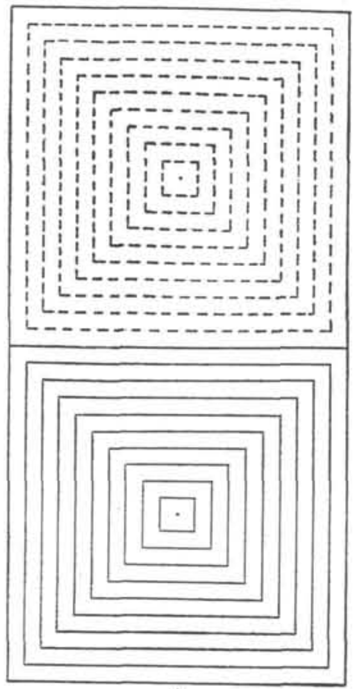
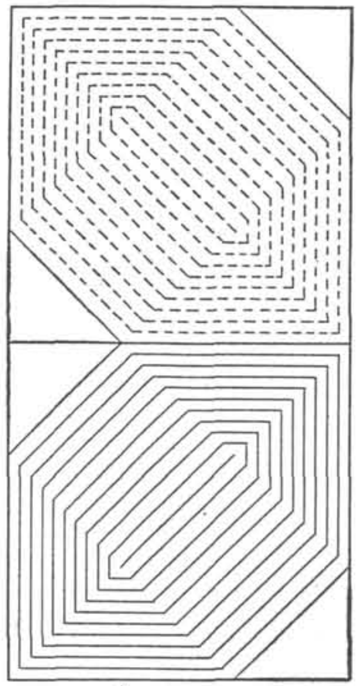

G
P
n
n
α
K K′
L L′
M M′
E
F
G
Ice and water
Boiling water
n
n
β
N N′
O O′
P P′
H
I
J
P
G

Fig.3

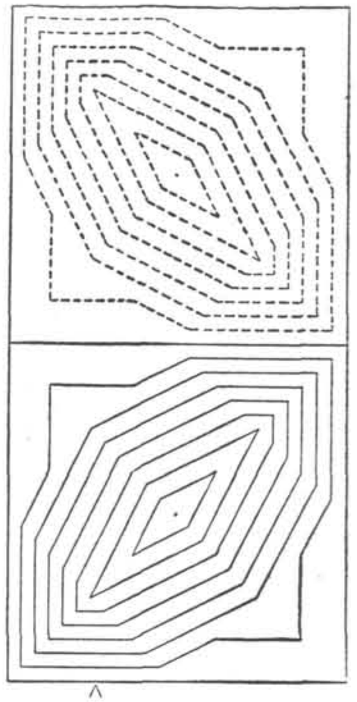
THE ALIQUOT CASES.



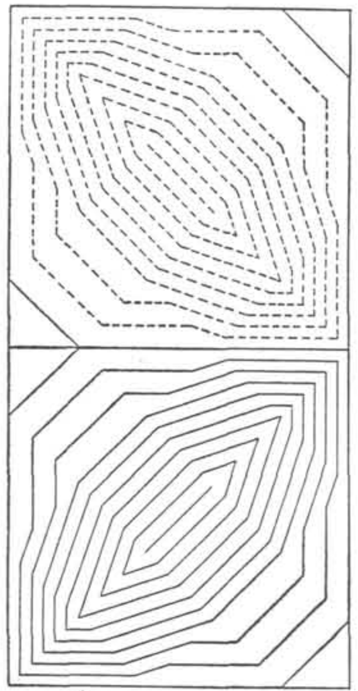
∧ ½



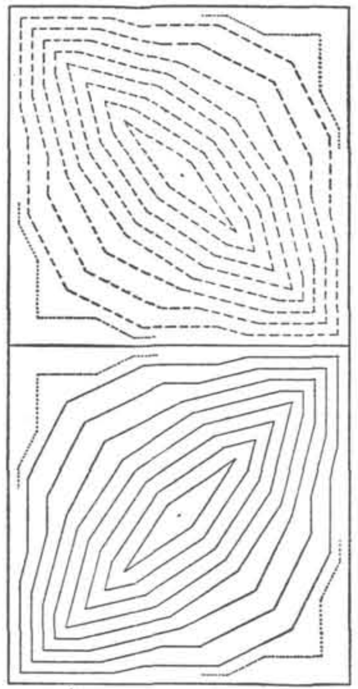
∧ ⅓



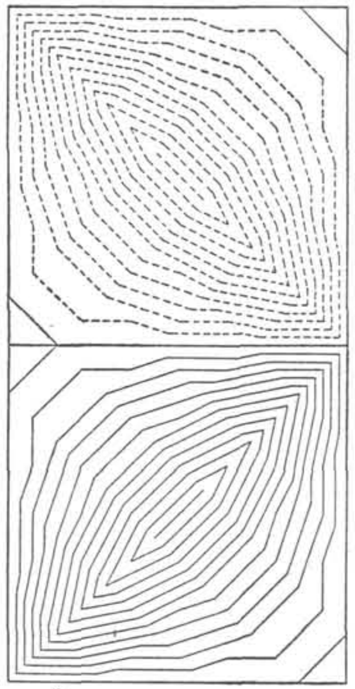
∧ ¼



∧ ⅕

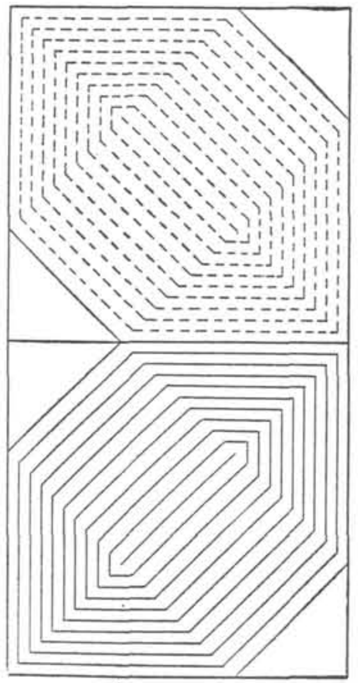


∧ ⅙

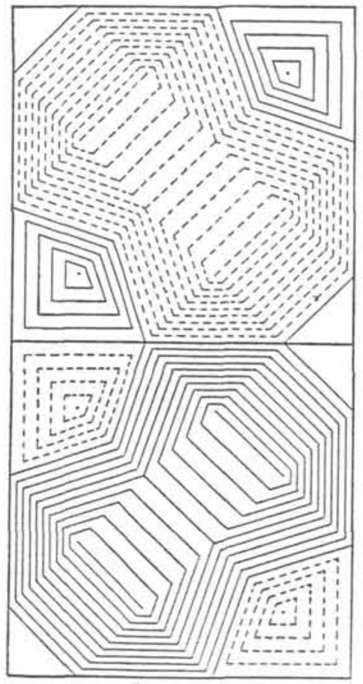


∧ ⅐

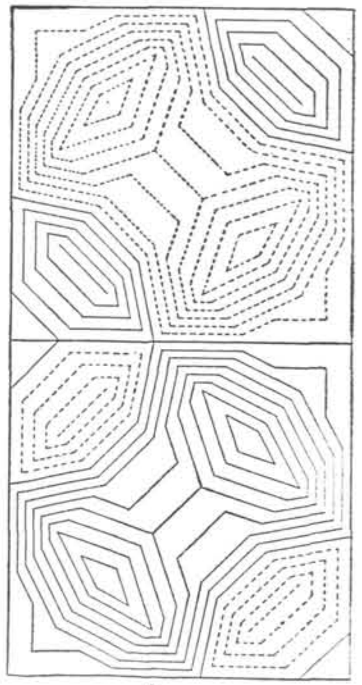
THE RATIONAL CASES CONVERGING ON ½.



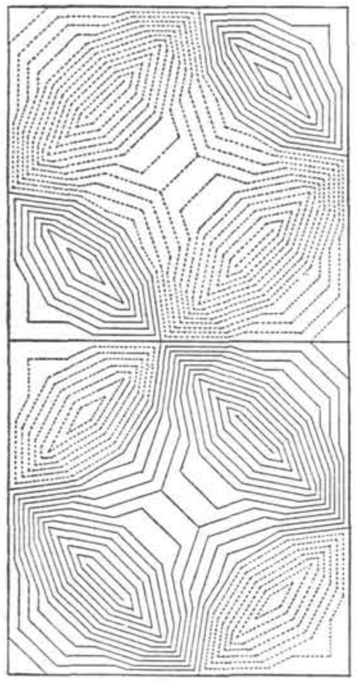
∧ ⅓



∧ ⅖

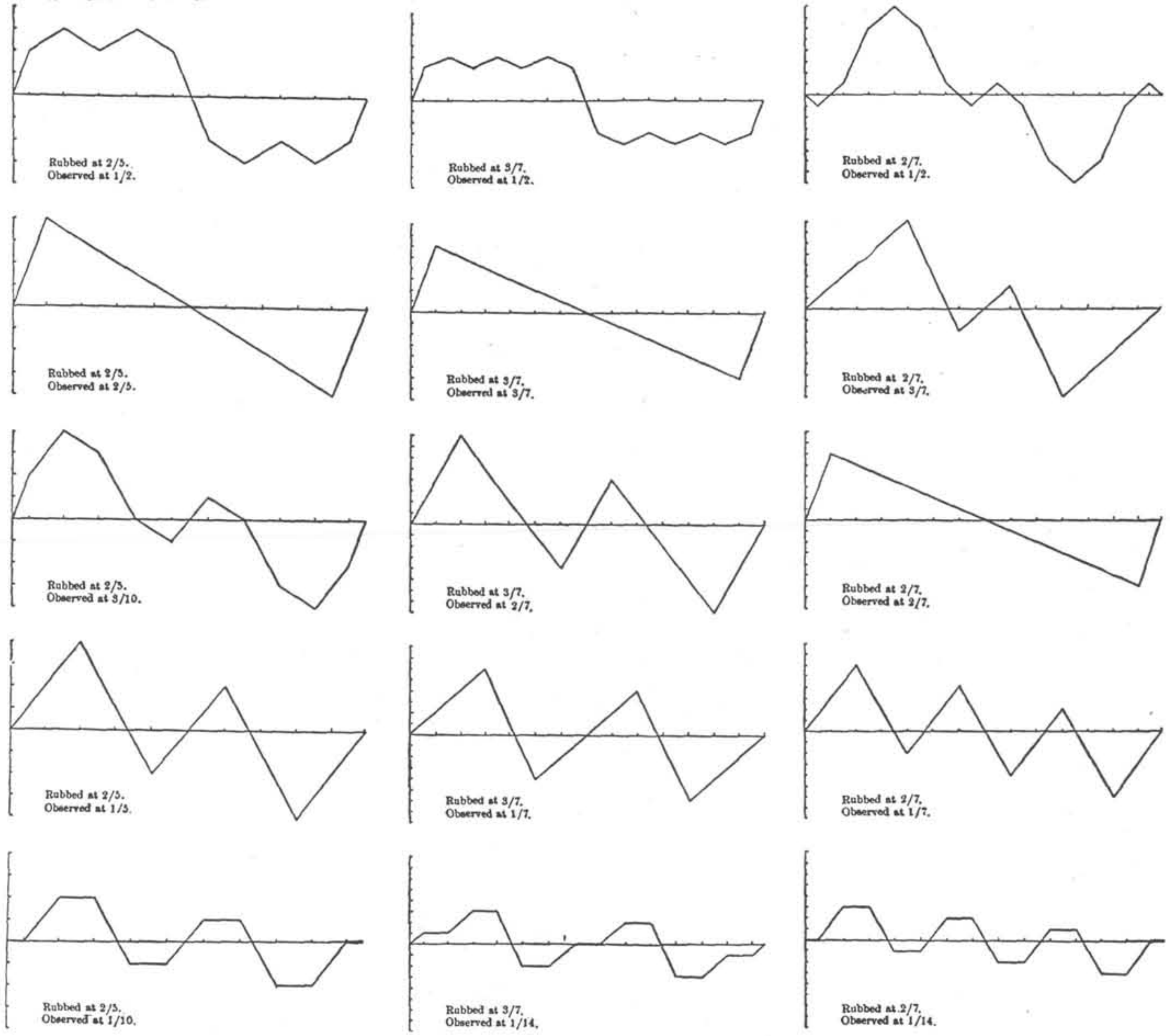


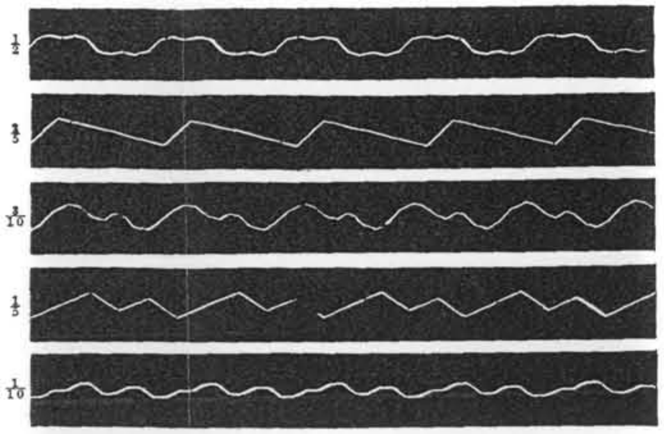
∧ 3/7



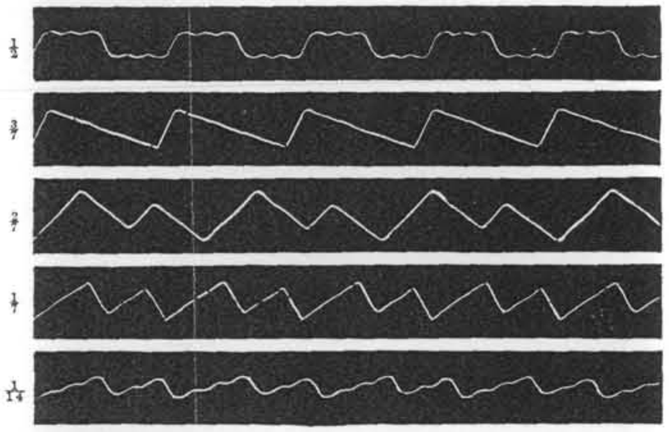
∧ 4/9

The short side of each integral surface is to be taken as the x axis, the long side as the t axis. All contours below sea-level are drawn with dashes. In some cases, parts of the contours $\pm \frac{1}{2}$ are dotted in to show more clearly the nature of the surface. It would not have been easy to draw the surfaces on scales which corresponded to a single bowing speed, or in any other comparable fashion. The number of contours in each surface is, therefore, arbitrary, and their density in different surfaces indicates only roughly the relative slopes or heights of those surfaces under similar conditions. For exact information of this kind the reader is referred to Figures 1, 9, and 11.

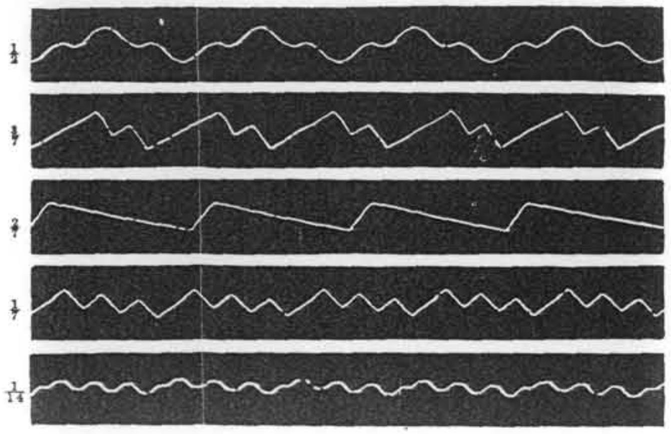




Rubbed at 2/5.



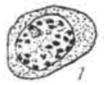
Rubbed at 3/7.



Rubbed at 2/7.

This plate shows, as functions of the time, the displacements of a number of typical points for each of the three simplest non-aliquot cases. The curves in the three columns above were obtained theoretically by means of the corresponding integral surfaces. Those in the column at the right are experimental verifications, obtained by the method of section 4, and much enlarged photographically. The defects are on the original slides.

BLACKMAN:–KARYOSPHERE IN SCOLOPENDRA SUBSPINIPES.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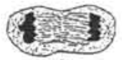
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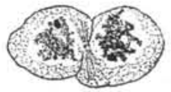
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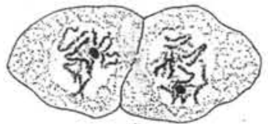
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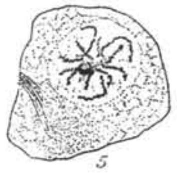
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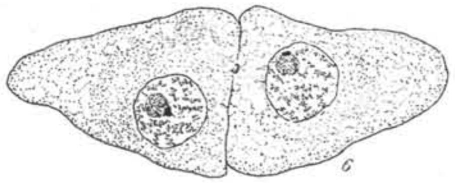
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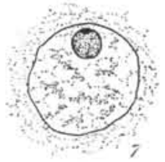
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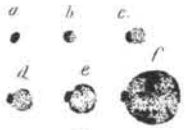

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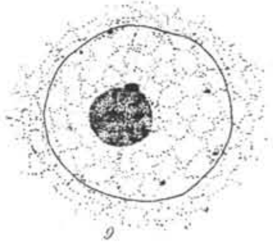
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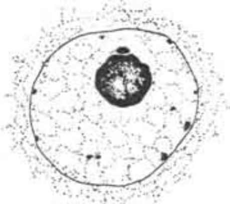
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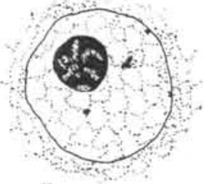
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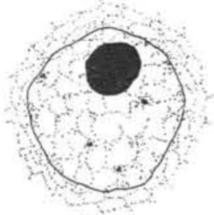
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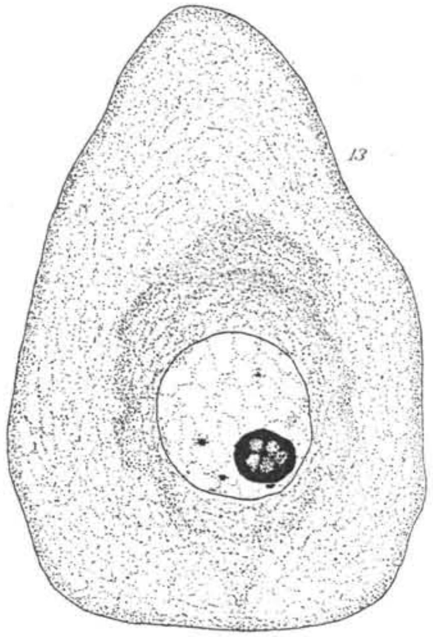
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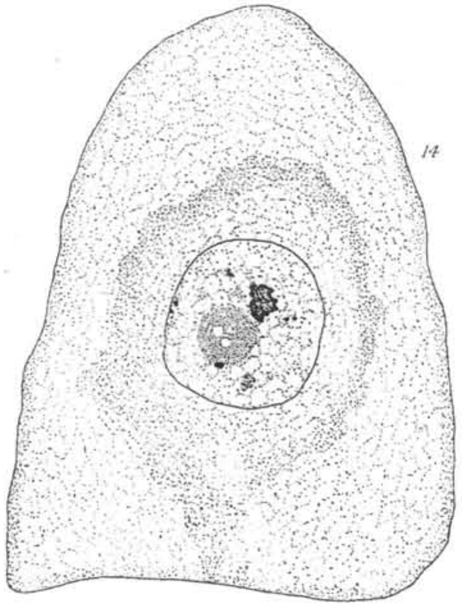
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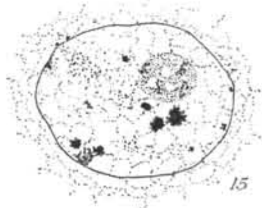
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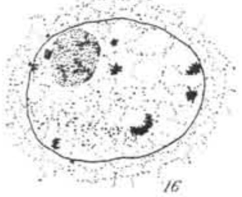
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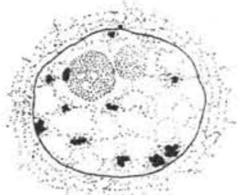
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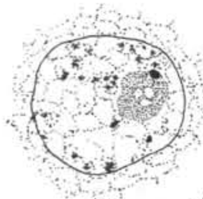
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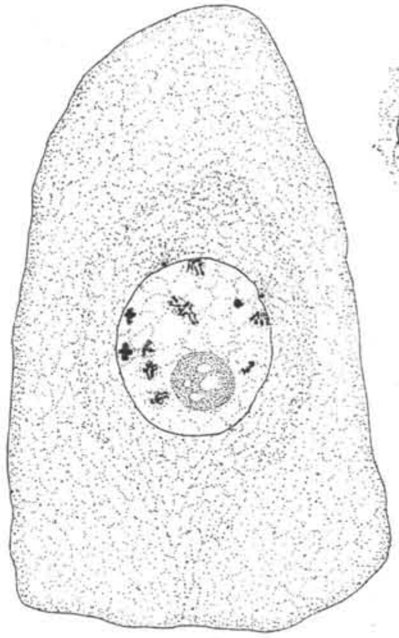
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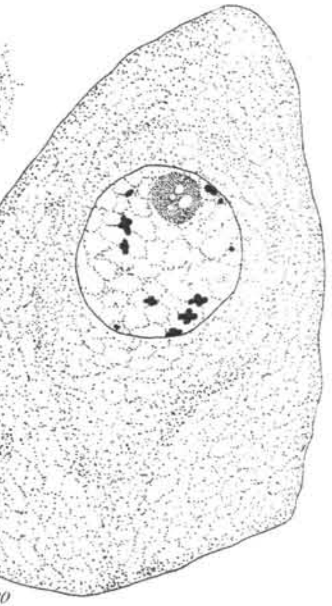
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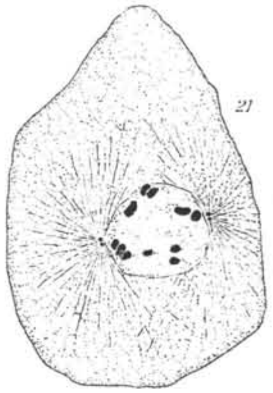
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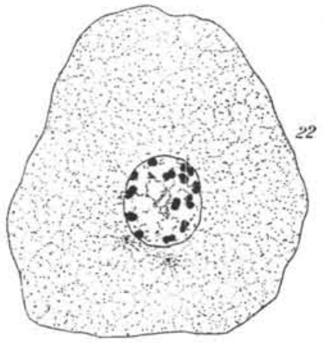
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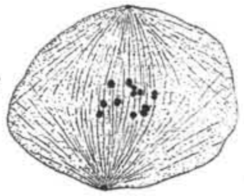
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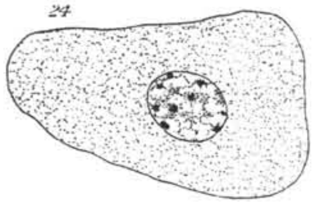
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M.W.B. del.

B. Meisel lith.